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GEOMETRIC GROUP THEORY AND HYPERBOLIC GEOMETRY: RECENT CONTRIBUTIONS FROM INDIAN MATHEMATICIANS

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Geometric group theory emerged as a distinct branch of mathematics through the seminal work of Gromov [25] in 1987 and since then it has been a very active area of research intermingling with many other fields of mathematics. This paper is a survey of contributions made in the past decade in Geometric group theory by the Indian mathematicians.

Key words : Hyperbolic groups; cannon-Thurston maps.

1. Introduction

Geometric group theory (GGT) is the study of large scale properties of metric spaces, most importantly the Cayley graphs of finitely generated groups. This came into existence by the seminal work of Gromov [25] in 1987. GGT interacts closely with Kleinian groups and low dimensional topology among many other branches of mathematics. However, in this short survey we take the following view. We include all those work published during the past decade which either directly fall under the AMS subject classification as GGT (20F65) or where either the techniques or the results are predominantly coarse geometric in nature. However, for the ease and the continuity of the exposition we sometimes make passing remarks about earlier work too.

2. Hyperbolic Groups and Related Topics

Gromov introduced hyperbolic metric spaces as generalizations of hyperbolic spaces $\mathbb{H}^n$ and hyperbolic groups as the analogue of cocompact lattices in $Isom(\mathbb{H}^n)$. Many aspects of hyperbolic groups are widely studied. We recall that with each (proper) Gromov hyperbolic metric space one can attach the Gromov boundary and a quasi-isometric (QI) embedding between such spaces induce a continuous embedding of their boundaries. The following gives the context for the most remarkable Indian contribution in GGT.

2.1 *The map of Cannon and Thurston and its relatives*

In [13] Cannon and Thurston asked the following question about boundary extensions.

Question: [13, 46]. Suppose S is a closed orientable surface. Suppose that $\pi_1(S)$ acts freely and properly discontinuously on $\mathbb{H}^3$ by isometries such that the quotient manifold has no accidental parabolics. Does the inclusion $\tilde{i} : \tilde{S} \to \mathbb{H}^3$ extend continuously to the boundary?

We extensively borrow from Mjs work [46] for explaining the sequence of developments in this connection. Mj generalizes this question to the following form.

Question : (Mahan Mj). Suppose X is a (proper) Gromov hyperbolic metric space and let $Y \subset X$ be a properly embedded Gromov hyperbolic subspace with respect to the induced length metric from X. Does (or when does) the inclusion map $i : Y \to X$ extend continuously to $\hat{i} : \hat{Y} \to \hat{X}$?

A particularly interesting case (see Q 1.19 in [2]) is when $X = G$ is a Gromov hyperbolic group and $Y = H$ is a Gromov hyperbolic subgroup.

Such a continuous extension is popularized as a Cannon-Thurston map or CT map by Mj as this was inspired by the work of Cannon and Thurston.

Cannon-Thurston answered their question positively for fibers of a closed hyperbolic 3-manifold fibering over circle and for simply degenerate groups with asymptotically periodic ends. Their question kept mathematicians busy for the following three decades. Finally, Mj has completely resolved the general case in a brilliant paper [46].

***Theorem* 2.1** — [46]. *Let ρ be a representation of a surface group $\pi_1(S)$ into $PSl(2, \mathbb{C})$ without accidental parabolics. Let M denote the convex core of $\mathbb{H}^3/\rho(\pi_1(S))$. Further suppose that $i : S \to M$ induces a homotopy equivalence taking parabolic to parabolics. Then the inclusion $\widetilde{i} : \widetilde{S} \to \widetilde{M}$ of universal covers extends continuously to a map $\widehat{i} : \widehat{S} \to \widehat{M}$ between the compactifications of universal covers.*

A few remarks are in order.

(1) Although this theorem can be classified as one about Kleinian group or hyperbolic 3-manifolds, the techniques employed by Mj to solve this problem are completely coarse geometric.

(2) Mj solved a number of special cases before answering the general question. See [41, 43, 44, 47]. We refer the reader to the introduction of the paper [46] of Mj for a detailed account of the

history of activity around this problem and also how the developments towards the proof of the Ending Lamination Theorem [11] have influenced Mj in the context of this problem.

Ramifications of the Cannon-Thurston problem

Since the CT question is a very general one for hyperbolic spaces it has drawn a lot of interests for people to prove analogous results in the context of hyperbolic and relatively hyperbolic groups and spaces. It is difficult to do justice to this development in this short survey since the list of work done on CT maps or those using these ideas is very long. Hence we briefly mention a few of the important developments over the last decade.

(1) Generalizing previous work of Mitra [36], Mj and Pal [48] proved the existence of a CT map for the inclusions of the vertex groups of a graph of relatively hyperbolic group under natural conditions.

(2) Generalizing similar work done by Mitra [35], Pal [60] proves the existence of CT map for a normal relatively hyperbolic subgroup of relatively hyperbolic group under natural conditions.

(3) In [31] Leininger, Mj and Schleimer proved the existence of CT map for the action of a closed hyperbolic surface group on the curve complex of a surface with one puncture and use it to show that the boundary of the curve complex of the punctured surface is connected and locally path connected.

(4) Understanding the point pre-images of a CT map has drawn quite some interests. See for instance [27, 29, 30, 37, 38]. Consider a short exact sequence $1 \to H \to G \to Q \to 1$ of hyperbolic groups with H free or surface group. Using explicit description of CT map, Mj-Rafi in [50] investigated the quasiconvexity of a subgroup K of H in G.

(5) Baker and Riley in a recent work [1] found an example of a hyperbolic group in which the inclusion of a free subgroup does not admit CT map, answering negatively the CT question of Mj for hyperbolic groups in general.

(6) Mj and Series took up the study of the limits of the CT maps for the algebraic and geometric limits of Kleinian groups in a series of papers [53, 54].

2.2 *Rigidity*

Rigidity refers to the phenomena where some weak relation among some mathematical objects implies a strong relation among them. A classical example of this type is provided by the Mostow rigidity theorem or the Margulis super-rigidity theorem. In the context of GGT the relevant notion is that of quasiisometric rigidity or the like. Many important work of this type has been done during the past decade. Some of the notable mentions are [6, 15, 17, 55]. There is a bit of contribution from the Indian mathematicians in this contexts too.

Pattern Rigidity : Mj took up this theme of research in [45] proving the following interesting theorem among other things. (One is referred to [45] for details.)

***Theorem* 2.2** — [45]. *Let X_i be (strongly) hyperbolic relative to the collections of subsets $\mathcal{T}_i$ $(i = 1, 2)$. Let $\phi : \mathcal{T} \to \mathcal{T}$ be a uniformly proper map. Then there exists a quasi-isometry $q : X_1 \to X_2$ which pairs the sets $\mathcal{T}_1$ and $\mathcal{T}_2$ as ϕ does.*

This is a special type of pattern rigidity as introduced by Mosher, Sageev and Whyte [58] (motivated by the work Schwartz [64, 65]) where the patterns are the T_is. Later in [5] Mj and Biswas studied pattern rigidity for uniform lattices in rank-1 symmetric spaces of dimension ≥ 3. Their theorem can be stated in the following form. In what follows PD stands for 'Poincare duality'.

***Theorem* 2.3** — *[5]. Suppose X is a rank-1 symmetric space of dimension ≥ 3 and G_1, G_2 are uniform lattice in $Isom(X)$. Let $H_i \leq G_i$, $i = 1, 2$ be infinite index, quasi-convex subgroups. Let $\mathcal{T}_i$ be the convex hulls of the G_i-translates of the limits sets of H_i's in ∂X. Then any pattern preserving proper bijection from $\mathcal{T}_1$ to $\mathcal{T}_2$ is given by an isometry of X provided one of the following is true:*

- *H_i's are odd dimensional PD-groups.*
- *H_i's are codimension 1 PD-groups.*

Using these techniques Mj and Biswas then proved a QI rigidity result for groups which admit a graph of PD-groups decomposition under some restrictions. See [5] for details. Finally in [45] Mj undertook a detailed study of the groups of pattern preserving maps in the context of hyperbolic PD-groups and proved a number of beautiful structural results for the groups of pattern preserving maps.

Mostow rigidity and related results

This study of pattern rigidity is pursued by Biswas in [9] in the case $X = \mathbb{H}^n$, $n \geq 3$ and by some ingenuous analytic technique he was able to retrieve the Mostow rigidity theorem. Recently, in a similar vein, in [10] Biswas studied when Moebius maps between boundaries of CAT(-1) spaces extend to the interior of the spaces. Among other things he proved the following nice theorem.

***Theorem* 2.4** — *[10]. Let X, Y be proper geodesically complete CAT(-1) spaces such that ∂X*

has at least four points, and let $f : \partial X \to \partial Y$ be a Moebius homeomorphism. Then f extends to a (1, log2)-quasi-isometry $F : X \to Y$, with image log 2-dense in Y.

2.3 *Splitting of groups and combination theorems*

Motivated by 3-manifold topology there has been an extensive study as to when a group admits a splitting into a graph of 'simpler' groups. In [51] Mj, Scott and Swarup used the earlier work of Mj [40] and Scott-Swarup [66] to prove existence of splitting of groups as amalgams over subgroups. Here is a sample theorem from their paper.

***Theorem* 2.5** — *[51]. Let G be a finitely generated, one-ended group and let K be a subgroup which may not be finitely generated. Suppose that e(G, K) $\geq$ 2, and that K is contained in a proper subgroup H of G such that H is almost malnormal in G and e(H) = 1. Then G splits over a subgroup of K.*

Combination theorems

Bestvina and Feighn in a beautiful, short paper [3] gives sufficient conditions for the fundamental group of a graph of hyperbolic groups to be hyperbolic. Inspired by this many authors have proved similar combination theorems and many others have used these results in the past decade. For instance see [8, 12, 14, 23, 32, 33].

In [50] Mj and Reeves proved an analogous result for graph of groups with relatively hyperbolic vertex and edge groups under natural conditions.

In [52] Mj and Sardar introduced metric bundles. Under certain natural conditions, in the footsteps of Bestvina-Feighn, the authors prove a combination theorem for metric bundles providing sufficient conditions for hyperbolicity. Here is a sample application of their result.

***Theorem* 2.6** — *[52]. Suppose we have a short exact sequence of groups*

$$1 \to K \to G \to Q \to 1$$

where K is a non-elementary hyperbolic group and Q is hyperbolic. If the hallway flaring condition is satisfied then G is hyperbolic.

Using the combination theorem they proved a characterization of convex cocompact subgroups of the pure mapping class groups with punctures.

In [51] Mj, Scott and Swarup used the earlier work of Mj [40] and Scott-Swarup [66] to prove splitting of groups as amalgams over subgroups by analyzing the geometry of the cosets of a subgroup.

Pal and Paul [62] generalized Martin's work (see [32]) on combination theorems for acylindrical complex of hyperbolic groups to acylindrical complex of relatively hyperbolic groups.

3. Mapping Class Groups and The Like

Mapping class groups, Teichmüler theory and so on have been a very active area of research during the past decade. Some of the work of notable mention are [4, 6, 7], among many others. There has been some activity by Indian mathematicians in this regard as well (as also noted in the last section).

In [42] Mj introduced a family of interpolating graphs $C(S, \xi)$ of complexity ξ for a surface S and $2 \leq \xi \leq \xi(S)$. For $\xi = 2, 1, \xi(S) - 1$ these are quasi-isometric to the marking graph, the pants graph and the curve graph respectively. He showed that the rank r_ξ of this graph is equal to the largest number of disjoint copies of subsurfaces of complexity greater than ξ which may be embedded in S.

In [20] Gadgil and Pandit worked with connected sums $M = \#_k S^1 \times S^2$ to study splitting of free groups. They defined a notion of geometric intersection number for embedded spheres in M and proved that it agrees with the the algebraic intersection number of Scott and Swarup [67]. The idea was to find an analogue of the curve complex of surfaces in this setting. Similar theme of research is pursued in [19, 21, 22] too.

In [61] Pal generalized a result of Mosher [57] and gave a sufficient condition for a cobounded, Lipschitz path in the Teichmüller space of a punctured surface to be at uniformly bounded distance from a Teichmüller geodesic.

4. Some Other Developments

1. In [16] Das and Mj introduced *controlled Floyd separation*. They constructed an infinitely generated malnormal subgroup K of a free group $\mathbb{F}_2$ on two generators and showed that the double $G = \mathbb{F}_2 *_K \mathbb{F}_2$ has trivial Floyd boundary but it satisfies the controlled separation. This means that given $K \geq 1$ and any two geodesics γ_1, γ_2 in G starting from identity there is $\epsilon > 0$ such that for any unbounded sequences of points $p_n \in \gamma_1, q_n \in \gamma_2$ in the word metric of G, the Floyd length of any K-quasigeodesic joining p_n, q_n is at least ϵ for all $n \in \mathbb{N}$.

2. In [28] Juriaans, Passi, and Prasad studied finite groups G such that the group of units $U(G)$ of the group ring $\mathbb{Z}[G]$ is hyperbolic. They showed that all these unit groups are virtually free. Then Juriaans, Passi, Souza Filho took up the case of unit groups of the group ring over the ring of integers of a number field. However, in both the work the techniques were purely algebraic.

3. In [59] Naolekar and Sankaran showed that for any finitely generated group G the natural homomorphism $\theta_G : Aut(G) \to QI(G)$ is a monomorphism only if the FC-center of G, say $K(G)$, equals the centre $Z(G)$ and the converse holds if $K(G) = Z(G)$ is torsion free. When $K(G)$ is finite they show that $\theta_{\bar{G}}$ is a monomorphism where $\bar{G} = G/K(G)$.

4. In [26] Inamdar and Naolekar showed that if G and H are two finite groups with at least two elements, then there exists a quasi-isometric embedding from Γ_G to Γ_H where Γ_G to Γ_H denote the corresponding Lamplighter groups. Finally they showed that the group of automorphisms of $\Gamma_{\mathbb{Z}_n}$ has infinite index in $\mathcal{Q}I(\Gamma_{\mathbb{Z}_n})$.

5. In [63] Sankaran showed that the group of piecewise-linear homeomorphisms of $\mathbb{R}$ having bounded slopes surjects onto the group $QI(\mathbb{R})$ of all quasi-isometries of $\mathbb{R}$. He moreover showed that $\mathcal{Q}I(\mathbb{R})$ contains the group of compactly supported piecewise-linear homeomorphisms of $\mathbb{R}$, the Richard Thompson group F, and the free group of continuous rank as subgroups.

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