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STABILITY ON *S*-SPACE FORM

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In this paper, the stability of the identity map on a compact S-space form is studied. Results related to stability of holomorphic maps on compact domain of S-space form are also discussed.

Key words : Harmonic maps; stability; holomorphic maps; S-space form; K$\ddot{a}$hler manifold.

1. Introduction

Harmonic maps theory takes ideas from both Riemannian geometry and analysis. There are many attractive results about harmonic maps on complex manifolds (see [12, 20]).

In the analogy to the complex case, in the last decade harmonic maps on almost contact metric manifolds were studied [3, 4, 6, 7, 10]. The identity map of a compact Riemannian manifold is a trivial example of a harmonic map but in this case, the theory of the second variation is much complicated and interesting. For example, the stability of the identity map on Einstein manifolds is related to the first eigenvalue of the Laplacian acting on functions [18]. In [19] and [14] we find classifications of compact simply connected irreducible Riemannian symmetric spaces for which the identity map is unstable.

By a well known result, the identity map on the euclidean sphere S^{2n+1} is unstable [18]. More generally, Gherghe, Ianus and Pastore have studied the stability of the identity map on compact Sasakian manifolds with constant $\varphi-$sectional curvature [10].

As a generalization of Sasakian space forms, Alegre, Blair and Carriazo introduced the generalized Sasakian space forms [1]. Stability of the identity map on a compact domain of such a manifold is studied by Rehman [16].

S-manifolds are the generalization of almost complex and contact structure. The paper is organized as follows: After recalling in Section 2 the necessary facts about harmonic maps between general Riemannian manifolds, we give some definitions on S-manifolds and recall in Section 3 how S-space forms are defined. Finally in Section 4, we give some results on the stability of the identity map on S-space form, stability of holomorphic maps on S-space forms are also discussed.

2. Harmonic Maps on Riemannian Manifolds

In this section, we recall some well known facts concerning harmonic maps (see [5] for more details). Let $\phi : (M, g) \longrightarrow (N, h)$ be a smooth map between two Riemannian manifolds of dimensions m and n respectively.

The energy density of ϕ is a smooth function $e(\phi) : M \longrightarrow [0, \infty)$ given by

$$e(\phi)_p = \frac{1}{2}Tr_g(\phi^* h)(p) = \frac{1}{2}\sum_{i=1}^{m} h(\phi_{*p}u_i, \phi_{*p}u_i),$$

for $p \in M$ and any orthonormal basis $\{u_1, \ldots, u_m\}$ of T_pM. If M is a compact Riemannian manifold, the energy $E(\phi)$ of ϕ is the integral of its energy density:

$$E(\phi) = \int_M e(\phi)\upsilon_g,$$

where υ_g is the volume measure associated with the metric g on M. A map $\phi \in C^\infty(M, N)$ is said to be harmonic if it is a critical point of E in the set of all smooth maps between (M, g) and (N, h) i.e., for any smooth variation $\phi_t \in C^\infty(M, N)$ of ϕ $(t \in (-\epsilon, \epsilon))$ with $\phi_0 = \phi$, we have

$$\frac{d}{dt}E(\phi_t)\big|_{t=0} = 0.$$

Now, let (M, g) be a compact Riemannian manifold and $\phi : (M, g) \longrightarrow (N, h)$ be a harmonic map. We take a smooth variation $\phi_{s,t}$ with parameters $s, t \in (-\epsilon, \epsilon)$ such that $\phi_{0,0} = \phi$. The corresponding variation vector fields are denoted by V and W. The Hessian H_ϕ of a harmonic map ϕ is defined by

$$H_\phi(V, W) = \frac{\partial^2}{\partial s \partial t}(E(\phi_{s,t}))\big|_{(s,t)=(0,0)}.$$

The second variation formula of E is [13, 18]:

$$H_\phi(V, W) = \int_M h(J_\phi(V), W)\upsilon_g,$$

where J_ϕ is a second order self-adjoint elliptic operator acting on the space of variation vector fields along ϕ (which can be identified with $\Gamma(\phi^{-1}(TN))$) and is defined by

$$J_\phi(V) = -\sum_{i=1}^{m}(\widetilde{\nabla}_{u_i}\widetilde{\nabla}_{u_i} - \widetilde{\nabla}_{\nabla_{u_i}u_i})V - \sum_{i=1}^{m} R^N(V, d\phi(u_i))d\phi(u_i),$$

for any $V \in \Gamma(\phi^{-1}(TN))$ and any local orthonormal frame $\{u_1, \ldots, u_m\}$ on M. Here R^N is the curvature tensor of (N, h) and $\widetilde{\bigtriangledown}$ is the pull-back connection by ϕ of the Levi-Civita connection of N.

The index of a harmonic map $\phi : (M, g) \longrightarrow (N, h)$ is defined as the dimension of the largest subspace of $\Gamma(\phi^{-1}(TN))$ on which the Hessian H_ϕ is negative definite. A harmonic map ϕ is said to be stable if the index of ϕ is zero and otherwise is said to be unstable.

The operator $\overline{\triangle}_\phi$ defined by

$$\overline{\triangle}_\phi V = -\sum_{i=1}^{m}(\widetilde{\nabla}_{u_i}\widetilde{\nabla}_{u_i} - \widetilde{\nabla}_{\nabla_{u_i}u_i})V, \qquad V \in \Gamma(\phi^{-1}(TN))$$

is called the rough Laplacian.

Due to the Hodge de Rham Kodaira theory, the spectrum of J_ϕ consists of a discrete set of an infinite number of eigenvalues with finite muiltiplicities and without accumulation points.

3. S-Space Form

As a generalization of both almost complex (in even dimension) and almost contact (in odd dimension) structures, Yano introduced in [21] the notion of f-structure on a smooth manifold of dimension $2n + s$, i.e. a tensor field of type (1,1) and rank $2n$ satisfying $f^3 + f = 0$. The existence of such a structure is equivalent to a reduction of the structural group of the tangent bundle to $U(n) \times O(s)$. Let N be a $(2n + s)$-dimensional manifold with an f-structure of rank $2n$. If there exist s global vector fields $\xi_1, \xi_2, \ldots, \xi_s$ on N such that:

$$f\xi_\alpha = 0, \qquad \eta_\alpha \circ f = 0, \qquad f^2 = -I + \sum \xi_\alpha \otimes \eta_\alpha, \tag{1}$$

where η_α are the dual 1-forms of ξ_α, we say that the f-structure has complemented frames. For such a manifold there exists a Riemannian metric g such that

$$g(X, Y) = g(fX, fY) + \sum \eta_\alpha(X)\eta_\alpha(Y)$$

for any vector fields X and Y on N. See [2].

An f-structure f is normal, if it has complemented frames and

$$[f,f]+2\sum \xi_\alpha \otimes d\eta_\alpha = 0,$$

where $[f,f](X,Y)=N^f(X,Y)$, $X,Y\in TN$ is Nijenhuis torsion of f, defined as

$$[f,f](X,Y)=f^2[X,Y]+[fX,fY]-f[fX,Y]-f[X,fY].$$

Let Ω be the fundamental 2-form defined by $\Omega(X,Y)=g(X,fY)$, X, $Y\in T(N)$. A normal f-structure for which the fundamental form Ω is closed, $\eta_1\wedge\cdots\wedge\eta_s\wedge(d\eta_\alpha)^n\neq 0$ for any α, and $d\eta_1=\cdots=d\eta_s=\Omega$ is called to be an S-structure. A smooth manifold endowed with an S-structure will be called an S-manifold. These manifolds were introduced by Blair in [2].

We have to remark that if we take $s=1$, S-manifolds are natural generalizations of Sasakian manifolds. In the case $s\geq 2$ some interesting examples are given in [2].

If *N* is an S-manifold, then the following formulas are true (see [2]):

$$\overline{\nabla}_X\xi_\alpha=-fX, \qquad X\in T(N), \quad \alpha=1,\dots,s, \tag{2}$$

$$(\overline{\nabla}_X f)Y=\sum\{g(fX,fY)\xi_\alpha+\eta_\alpha(Y)f^2X\}, \quad X,Y\in T(N), \tag{3}$$

where $\overline{\nabla}$ is the Riemannian connection of g. Let *L* be the distribution determined by the projection tensor $-f^2$ and let *M* be the complementary distribution which is determined by f^2+I and spanned by $\xi_1,\dots,\xi_s$. It is clear that if $X\in L$ then $\eta_\alpha(X)=0$ for any α, and if $X\in M$, then $fX=0$. A plane section π on *N* is called an invariant f-section if it is determined by a vector $X\in L(x)$, $x\in$ *N*, such that $\{X,fX\}$ is an orthonormal pair spanning the section. The sectional curvature of π is called the f-sectional curvature. If *N* is an S-manifold of constant f-sectional curvature k, then its curvature tensor has the form

$$\begin{aligned}\overline{R}(X,Y,Z,W)=\sum_{\alpha,\beta}\{&g(fX,fW)\eta_\alpha(Y)\eta_\beta(Z)-g(fX,fZ)\eta_\alpha(Y)\eta_\beta(W)+\\&+g(fY,fZ)\eta_\alpha(X)\eta_\beta(W)-g(fY,fW)\eta_\alpha(X)\eta_\beta(Z)\}+\\&+\frac{1}{4}(k+3s)\{g(fX,fW)g(fY,fZ)-g(fX,fZ)g(fY,fW)\}+\\+\frac{1}{4}(k-s)\{F(X,W)F(Y,Z)&-F(X,Z)F(Y,W)-2F(X,Y)F(Z,W)\},\end{aligned} \tag{4}$$

$X,Y,Z,W\in T(N)$. Such a manifold *N*(k) will be called an S-space form. The Euclidean space E^{2n+s} and the hyperbolic space H^{2n+s} are examples of S-space forms.

4. Main Results

Let M be a compact S space form $M(f, \xi_\alpha, \eta_\alpha, g)$, $\alpha = 1, \ldots, s$. We consider the identity map on such a manifold $(\phi = 1_M)$. In this case see [20], the second variation formula is

$$H_{1_M}(V,V) = \int_M h(\overline{\triangle} V, V) v_g - \sum_{i=1}^{2n+s} \int_M h(R(V,u_i)u_i, V) v_g,$$

where $V \in \Gamma(TM)$ and $\{u_1, ..., u_{2n+s}\}$ is a local orthonormal frame on TM.

Let $\{e_i, fe_i, \xi_\alpha\}$ be an orthonormal local f-adapted frame. Then using (4) we have

$$\begin{aligned} g(R(e_i,v)e_i,v) &= -g(fe_i fe_i) \sum_{\alpha,\beta=1}^{s} \eta_\alpha(v)\eta_\beta(v) + \\ &+ \frac{1}{4}(k+3s)\{g(fe_i,fv)g(fvfe_i) - g(fe_i,fe_i)g(fv,fv)\} + \\ &+ \frac{1}{4}(k-s)\{-3g(fv,e_i)g(fv,e_i)\} \qquad (5) \\ g(R(fe_i,v)fe_i,v) &= -g(e_i,e_i) \sum_{\alpha,\beta=1}^{s} \eta_\alpha(v)\eta_\beta(v) + \\ &+ \frac{1}{4}(k+3s)\{g(e_i,fv)g(e_i,fv) - g(e_i,e_i)g(fv,fv)\} + \\ &+ \frac{1}{4}(k-s)\{-3g(fe_i,fv)g(fe_i,fv)\} \qquad (6) \\ g(R(\xi_\alpha,v)\xi_\alpha,v) &= -sg(fv,fv) \qquad (7) \end{aligned}$$

From the above three relations, we get

$$\begin{aligned} \sum_{i=1}^{2n+s} g(R(u_i,V)u_i,V) &= \frac{1}{2}(-k+s-nk-3ns)g(v,v) + \\ &+ \frac{1}{2}(-4n+k-s+nk+3ns) \sum_{\alpha,\beta=1}^{s} \eta_\alpha(v)\eta_\beta(v), \end{aligned}$$

and

$$\begin{aligned} \sum_{i=1}^{2n+s} g(R(u_i,V)u_i,V) &= \frac{1}{2}(-k+s-nk-3ns)g(v,v) + \\ &+ \frac{1}{2}(-4n+k-s+nk+3ns) \sum_{\alpha,\beta=1}^{s} g(v,\xi_\alpha)g(v,\xi_\beta). \end{aligned}$$

For $\alpha = \beta$

$$\begin{aligned}\sum_{i=1}^{2n+s} g(R(u_i, V)u_i, V) &= \frac{1}{2}(-k + s - nk - 3ns)g(v, v) + \\ &+ \frac{1}{2}(-4n + k - s + nk + 3ns)\sum_{\alpha=1}^{s} g(v, \xi_\alpha)^2.\end{aligned}$$

***Theorem* 1** — *Let M be a compact S-space form. If $(-k+s-nk-3ns) \geq 0$ and $(-4n+k-s+nk+3ns) \geq 0$, then the identity map 1_M is stable. In other words, the identity map on s-manifold is stable if k does not belong to the interval* $\left[-\frac{(3n-1)s}{n+1}, \quad -\frac{(3n-1)s}{n+1} + \frac{4n}{n+1}\right]$.

PROOF : It is not very difficult to prove that

$$\int_M h(\overline{\triangle}V, V)v_g = \int_M h(\widetilde{\nabla}V, \widetilde{\nabla}V)v_g, \qquad V \in \Gamma(TM),$$

Now the second varition formula becomes

$$\begin{aligned}H_{1_M}(V, V) &= \int_M h(\widetilde{\nabla}V, \widetilde{\nabla}V) + \int_M (-k + s - nk - 3ns)g(V, V)v_g + \\ &+ \int_M (-4n + k - s + nk + 3ns)\sum_{\alpha=1}^{s} g(V, \xi_\alpha)^2 v_g,\end{aligned}$$

and thus the identity map 1_M is stable if $(-k+s-nk-3ns) \geq 0$ and $(-4n+k-s+nk+3ns) \geq 0$.□

Remark 1 :

- If s=0, the identity map on Kähler manifold is stable if k does not belong to the interval $[0, \quad \frac{4n}{n+1}]$.
- If s=1, the identity map on Sasakian manifold is stable if k does not belong to the interval $[-\frac{(3n-1)}{n+1}, \quad \frac{4n}{n+1}]$.

Now Let $M(f, \xi_\alpha, \eta_\alpha, g)$ be a compact s-manifold of constant f-section curvature k, $N(f', \xi'_\alpha, \eta'_\alpha, h)$ another s-manifold and $F : M \to N$ a (f, f')-holomorphic map. Let this is harmonic map.

In second variation formula we take $V = dFX$, for $X \in \Gamma(TM)$, then we get

$$H_F(dFX, dFX) = \int_M h(\overline{\Delta}dFX) + \sum_{i=1}^{2n+s} R^N(dFX_i, dFX)dFX_i, dFX)v_g. \tag{8}$$

Then using harmonic (f, f')-holomorpic map, with computations as in [10], we have

$$H_F(dFX, dFX) = \int_M h\left(dF\left(\sum R^M(X_i, X)X_i\right), dFX\right)v_g + \int_M h(dFX, dFX)v_g. \tag{9}$$

But

$$\sum_{i=1}^{2n+s} R^M(X_i, X)X_i = \frac{1}{2}(-k+s-nk-3ns)X + \\ + \frac{1}{2}(-4n+k-s+nk+3ns)\sum_{\alpha=1}^{s} g(X,\xi_\alpha)\xi_\beta. \quad (10)$$

Since F is (f, f')-holomorphic, we have $dF(\xi_\beta) = 0$ and thus

$$H_F(dFX, dFX) = -\frac{k-s+nk+3ns}{2}\int_M h(dFX, dFX)\upsilon_g.$$

Therefore we have following result.

***Theorem* 2** — *Let $M(f, \xi_\alpha, \eta_\alpha, g)$ be a compact S-manifold of constant f-sectional curvature k, $N(f', \xi'_\alpha, \eta'_\alpha, h)$ be another S-manifold and $F : M \to N$ a non constant (f, f')-holomorphic map, if $k \geq -\frac{(3n-1)s}{n+1}$, then F is unstable.*

If in above result let $M^{2n+s}(f, \xi_\alpha, \eta_\alpha, g)$ be compact S-manifold and take $s' = 0$ in $N^{2n'+s'}$ we have a (f, J')-holomorphic map $F : (M, f) \to (N, J')$ from a compact S-manifold to a Kähler manifold which is harmonic [17], then by above Theorem we obtain the following result

Corollary 1 — Let $M^{2n+s}(f, \xi_\alpha, \eta_\alpha, g)$ be a compact S-manifold of constant sectional curvature k, $N(J', h)$ be a Kähler-manifold and $F : M \to N$ a non constant (f, J')-holomorphic map, if $k \geq -\frac{(3n-1)s}{n+1}$, then F is unstable.

If in above result we take s=0 in M^{2n+s} and $s' = 0$ in $N^{2n'+s'}$ we have a (J, J')-holomorphic map $F : (M, J) \to (N, J')$ from a Kähler manifold to a Kähler manifold which is harmonic [12], then by above Theorem we obtain the following result.

Corollary 2 — Let $M(J, g)$ be a compact Kähler manifold of constant sectional curvature k, $N(J', h)$ be another Kähler-manifold and $F : M \to N$ a non constant (J, J')-holomorphic map, if $k \geq 0$, then F is unstable.

If in above result we take s=1 in M^{2n+s} and $s' = 0$ in $N^{2n'+s'}$ we have a (φ, J')-holomorphic map $F : (M, \varphi) \to (N, J')$ from a Sasakian manifold to a Kähler manifold which is harmonic [8], then by above Theorem we obtain the following result

Corollary 3 — Let $M(\varphi, \xi, \eta, g)$ be a compact Sasakian manifold of constant sectional curvature k, $N(J', h)$ be a Kähler-manifold and $F : M \to N$ a non constant (φ, J')-holomorphic map, if $k \geq -\frac{(3n-1)}{n+1}$, then F is unstable.

We recall now the Weitzenbock formula: Let E be a vector bundle over an m-dimensional Riemannian manifold M. Then for any 1-form $\omega \in A^1(E)$ we have

$$\triangle_1 \omega = \overline{\triangle}\omega - \rho(\omega),$$

where

$$\rho(\omega)(X) = \sum_{i=1}^{m} R(X, u_i)(\omega(u_i)) - \sum_{i=1}^{m} \omega(R(X, u_i)u_i),$$

for any $X \in \Gamma(TM)$. Here $\triangle_1$ is the Laplacian of E-valued 1-forms and $\overline{\triangle}$ is rough Laplacian on $A^1(E)$. Then, using the Weitzenbock formula for $E = M \times R$, we have

$$\triangle_1(V) = \overline{\triangle}V + \sum_{i=1}^{2n+s} R(V, u_i)u_i.$$

Then the second variational formula becomes now

$$H_{1_M}(V, V) = \int_M h(\triangle_1 V, V)\upsilon_g - 2\sum_{i=1}^{2n+s}\int_M h(R(V, u_i)u_i, V)\upsilon_g.$$

***Theorem* 3** — *Let M be a compact S-space form such that $(-4n + k - s + nk + 3ns) \leq 0$. If the first eigenvalue λ_1 of the Laplacian $\triangle_g$ acting on $C^\infty(M)$ satisfies $\lambda_1 < (k - s + nk + 3ns)$, then the identity map 1_M is unstable.*

PROOF : Let λ_1 be the first eigenvalue of the Laplacian $\triangle_g$ acting on $C^\infty(M)$ and f be a non constant eigenfunction of $\triangle_g$ such that $\triangle_g f = \lambda_1 f$. Let $f \in C^\infty(M)$ be taken as $\triangle_g f = \lambda_1(g) f$ and let $V = gradf \neq 0$. Then

$$\begin{aligned}\int_M h(\triangle_1 V, V)\upsilon_g &= \int_M < (d\delta + \delta d)df, df > \upsilon_g = \int_M < d\delta df, df > \upsilon_g \\ &= \lambda_1(g)\int_M < df, df > \upsilon_g \quad (\text{since} \quad \delta df = \triangle_g f) \\ &= \lambda_1(g)\int_M < df, df > \upsilon_g.\end{aligned}$$

Then the second variational formula becomes

$$\begin{aligned}H_{1_M}(V, V) = &\int_M (\lambda_1 - (k - s + nk + 3ns))g(V, V)\upsilon_g + \\ &+ \int_M (-4n + k - s + nk + 3ns)\sum_{\alpha=1}^{s} g(V, \xi_\alpha)^2\upsilon_g.\end{aligned}$$

If the first eigenvalue λ_1 of the Laplacian satisfies $\lambda_1 < (k - s + nk + 3ns)$ and $(-4n + k - s + nk + 3ns) \leq 0$ then 1_M is unstable. $\square$

Using Theorem 2, we have the folowing corollary (see [10]).

Corollary 4 — Let M be a compact Sasakian space form of constant section curvature c such that $c \leq 1$. If the first eigenvalue λ_1 of the Laplacian satisfies $\lambda_1 < c(n+1) + 3n - 1$ then the identity map is unstable.

PROOF : For $s = 1$, S-space form is Sasakian space form. In this case from Theorem 2, identity map is unstable on Sasakian space form if we have $\lambda_1 < (k - 1 + nk + 3ns)$ and $(-4n + k - 1 + nk + 3n) \leq 0$. That implies if $c \leq 1$ and $\lambda_1 < c(n+1) + 3n - 1$, from Theorem 2 we have that 1_M map is unstable. $\square$

Remark 2 : If M is a Sasakian space form with $c = 1$, then M becomes a space with constant curvature 1, hence isometric to the unit sphere. As we know the first eigenvalue of the Laplacian $\triangle_g$ for the euclidean $(2n+1)$-sphere is $\lambda_1 = (2n+1)$ and therefore, from Corollary 2 $1_{S^{(2n+1)}}$ is an unstable map.

Corollary 5 — Let M be a compact Kähler space form of constant section curvature c such that $c \leq \frac{4n}{n+1}$. If the first eigenvalue λ_1 of the Laplacian satisfies $\lambda_1 < c(n+1)$ then the identity map is unstable.

PROOF : For $s = 0$, S-space form is Kähler space form. In this case from Theorem 2, identity map is unstable on Kähler space form if we have $\lambda_1 < k(n+1)$ and $k \leq \frac{4n}{n+1}$. $\square$

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REFERENCES

1. P. Alegre, D. E. Blair, and Carriazo, Generalized Sasakian-space-forms, *Israel J. Math.*, **141** (2004), 157-183.

2. D. E. Blair, *Riemannian geometry of contact and symplectic manifolds*, Second Edition, Progress in Mathematics, 203. Birkhauser Boston, MA, 2010.

3. E. Boeckx and C. Gherghe, Harmonic maps and cosymplectic manifold, *J. Aust. Math. Soc.*, **76** (2004), 75-92.

4. D. Chinea, Harmonicity on maps between almost contact metric manifolds, *Acta Math. Hungar.*, **126**(4) (2010), 352-362.

5. J. Eells and L. Lemaire, Report on harmonic maps, *Bull. London Math. Soc.*, **20** (1988), 385-524.

6. C. Gherghe and K. Kenmotsu, Energy minimiser maps on C-manifolds, *Differential Geom. Appl.*, **21** (2004), 55-63.

7. C. Gherghe, Harmonicity on cosymplectic manifolds, *Rocky Mountain J. Math.*, **40** (2010).

8. C. Gherghe, Harmonicity on some almost contact metric manifolds, *Rend. Circolo di Palermo, serie II*, **49** (2000), 415-424.

9. C. Gherghe, S. Ianus, and A. M. Pastore, Harmonic maps, harmonic morphism and stability, *Bull. Math. Soc. Sa. Math. Roum.*, (2000), 247-254.

10. C. Gherghe, S. Ianus, and A. M. Pastore, CR manifolds, Harmonic maps and stability, *J. Geometry*, **71** (2001), 42-53.

11. K. Kenmotsu, A class of almost contact Riemannian manifolds, *Tohoku Math. J.*, **24** (1972), 93-103.

12. A. Lichnerowicz, Applications harmoniques et varietes kählleriennes, *Seminario Mat. e. Fis.di Milano*, **39** (1969), 186-195.

13. E. Mazet, La formule de la variation seconde de l' energie au voisinage d' une application harmonique, *J. Diff. Geometry*, **9** (1974), 531-535.

14. Y. Ohnita, Stability of harmonic maps and standard minimal immersion, *Tohoku Math. J.*, **38** (1986), 259-267.

15. P. K. Pandey, R. S. Gupta, and S. Ahmad, B.Y. Chen'S inequalities for bi-slant submanifolds in Kenmotsu space form, *Demonstratio Math.*, **43**(4) (2010), 887-898.

16. N. A. Rehman, Stability on generalized Sasakian space form, *Mathematical Reports*, **17(67)**(1) (2015).

17. N. A. Rehman, Harmonic maps on S-manifolds, *International Journal of Mathematical and its Applications*, **4**(2-B) (2016), 27-34.

18. R. Smith, The second variation formula for harmonic mapping, *proc. Amer. Math Soc.*, **47** (1975), 229-236.

19. H. Urakawa, The first eigenvalue of the Laplacian for a positively curved homogeneous Riemannian manifold, *Compositio Math.*, **59** (1986), 57-71.

20. H. Urakawa, Calculus of variation and harmonic maps, *Amer. Math. Soc. Providence. R.I.*, 1993.

21. K. Yano, *On a structure defined by a tensor field of type (1, 1)satisfying* $f^3 + f = 0$, *Tensor*, **14** (1963), 99-109.