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RECTIFYING AND OSCULATING CURVES ON A SMOOTH SURFACE

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Dedicated to Professor Dr. Bang-Yen Chen on the occasion of his 75^{th} *birthday*

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The main motive of the paper is to look on rectifying and osculating curves on a smooth surface. In this paper we find the normal and geodesic curvature for a rectifying curve on a smooth surface and we also prove that geodesic curvature is invariant under the isometry of surfaces such that rectifying curves remain. We find a sufficient condition for which an osculating curve on a smooth surface remains invariant under isometry of surfaces and also we prove that the component of the position vector of an osculating curve $\alpha(s)$ on a smooth surface along any tangent vector to the surface at $\alpha(s)$ is invariant under such isometry.

Key words : Rectifying curve; osculating curve; isometry of surfaces; first fundamental form; second fundamental form; geodesic curvature; normal curvature.

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1. Introduction

In the Euclidean space $(\mathbb{R}^3)$ the rectifying curves was introduced by Chen [4] and investigated some characterization of such curves. A curve in $\mathbb{R}^3$ whose rectifying plane always contains its position vector is called a rectifying curve. For more features of such curves we instruct the reader to see [3, 5, 6]. In [3], present authors investigated a sufficient condition under which rectifying curves on a smooth surface stay invariant under isometry of surfaces and proved that the component of position vector of such a curve on a smooth surface along the normal to the surface is invariant under the isometry of surfaces which preserves rectifying curve.

The main aim of this paper is to study rectifying and osculating curves on a smooth surface immersed in the Euclidean space. We found normal and geodesic curvature of a rectifying curve and also showed that the geodesic curvature is invariant under the isometry of surfaces which preserves rectifying curve. We find a sufficient condition for the invariancy of an osculating curve on a smooth surface under the isometry of surfaces. We also prove that the component of the position vector of an osculating curve $\alpha(s)$ on a smooth surface along any tangent vector to the surface at $\alpha(s)$ is invariant under the isometry of surfaces which preserves osculating curve.

The paper is structured as follows: Section 2 deals with some of the elementary facts of rectifying curves, osculating curves, geodesic curvature and normal curvature of a curve. Section 3 is devoted to the knowledge of an rectifying curves on a smooth surface and the results about geodesic and normal curvature of such curves (see, Theorem 3.1). The last section is concerned with the study of osculating curves on a smooth surface and deduced the components of position vectors of such a curve along any tangent vector to the surface (see, Theorem 4.1, Theorem 4.2).

2. Preliminaries

In this section, we recall some elementary facts of rectifying curves, osculating curves, isometry of surfaces, geodesic curvature and normal curvature (for details see, [1, 2]) which will be used in the sequel.

Let $\gamma(s)$ be an unit speed parametrized curve in the Euclidean space having at least fourth order continuous derivatives. Let tangent, normal and binormal vectors $\vec{t}$, $\vec{n}$, $\vec{b}$ respectively forms an orthogonal frame at every point on the curve $\gamma(s)$, where $\{\vec{t}, \vec{n}, \vec{b}\}$ is defined as $\vec{t}(s) = \gamma'(s)$, $\vec{n}(s) = \kappa(s)\vec{t'}(s)$ and $\vec{b}(s) = \vec{t}(s) \times \vec{n}(s)$, where $\gamma'(s)$ denote the derivative of $\gamma(s)$ with respect to s. Here $\kappa(s)$ is a positive function. The osculating, normal and rectifying plane at $\gamma(s)$ is generated by $\{\vec{t}, \vec{n}\}$, $\{\vec{n}, \vec{b}\}$ and $\{\vec{t}, \vec{b}\}$ respectively. A curve $\gamma(s)$ in $\mathbb{R}^3$ is called osculating, normal and rectifying if its osculating, normal and rectifying plane always contain position vector of $\gamma(s)$ respectively.

Definition 2.1 — A diffeomorphism f between two smooth surfaces S and $\bar{S}$ in $\mathbb{R}^3$ is an isometry if f takes a curve from S to a curve on $\bar{S}$ of same length.

Definition 2.2 — Let $\gamma(s)$ be an unit speed parametrized curve on a surface S. Then $\gamma'(s)$ and the normal $\vec{N}$ to the surface are perpendicular to each other and also $\gamma'(s)$ and $\gamma''(s)$ are perpendicular. So, $\gamma''(s)$ can be written as a linear combination of $\vec{N} \times \gamma'(s)$ and $\vec{N}$, i.e.,

$$\gamma''(s) = \kappa_g \vec{N} \times \gamma'(s) + \kappa_n \vec{N}.$$

The number κ_g and κ_n are respectively called geodesic curvature and normal curvature of the curve $\gamma(s)$ on a surface S given as follows:

$$\begin{aligned}\kappa_g &= \gamma'' \cdot (\vec{N} \times \gamma') \\ \kappa_n &= \gamma'' \cdot \vec{N}.\end{aligned}$$

3. Rectifying Curve on a Smooth Surface

Let $\phi : U \to S$ be the coordinate chart for a smooth surface S and $\gamma(s) : (c, d) \to S$, where$(c, d) \subset \mathbb{R}$ is an unit speed parametrized curve contained in the image of a surface patch ϕ in the atlas of S, U being an open subset of $\mathbb{R}^2$. Then $\gamma(s)$ is given by,

$$\begin{aligned}(c, d) \to U, \quad s \rightarrowtail (u(s), v(s)), \\ \gamma(s) = \phi(u(s), v(s)).\end{aligned} \tag{1}$$

If $\kappa(s)$ is the curvature of $\gamma(s)$ and $\vec{N}$ is normal to S then the tangent $\vec{t}(s)$, normal $\vec{n}(s)$ and binormal $\vec{b}(s)$ are respectively given by

$$\begin{aligned}\vec{t}(s) &= u'\phi_u + v'\phi_v, \\ \vec{n}(s) &= \frac{1}{k(s)}\{(u'^2\phi_{uu} + 2u'v'\phi_{uv} + v'^2\phi_{vv}) + (u''\phi_u + v''\phi_v)\}, \\ \vec{b}(s) &= \frac{1}{k(s)}\Big[u'^3\phi_u \times \phi_{uu} + 2u'^2v'\phi_u \times \phi_{uv} + u'v'^2\phi_u \times \phi_{vv} + u'^2v'\phi_v \times \phi_{uu} \\ &\quad +2u'v'^2\phi_v \times \phi_{uv} + v'^3\phi_v \times \phi_{vv} + (u'v'' - u''v')\vec{N}\Big].\end{aligned}$$

So, the curve $\gamma(s)$ in S will be called rectifying if $\gamma(s) = \lambda(s)\vec{t}(s) + \mu(s)\vec{b}(s)$, for the functions $\lambda(s)$ and $\mu(s)$, i.e.,

$$\begin{aligned}\gamma(s) &= \lambda(s)(u'\phi_u + v'\phi_v) + \frac{\mu(s)}{k(s)}\Big[u'^3\phi_u \times \phi_{uu} + 2u'^2v'\phi_u \times \phi_{uv} + u'v'^2\phi_u \times \phi_{vv} \\ &\quad +u'^2v'\phi_v \times \phi_{uu} + 2u'v'^2\phi_v \times \phi_{uv} + v'^3\phi_v \times \phi_{vv} + (u'v'' - u''v')\vec{N}\Big],\end{aligned}$$

for some functions $\lambda(s)$ and $\mu(s)$.

***Theorem* 3.1** — *Let f be an isometry between two smooth surfaces S and $\bar{S}$. If $\gamma(s)$ and $\bar{\gamma}(s) = f \circ \gamma(s)$ are the rectifying curves on S and $\bar{S}$ respectively then*

(i) $\kappa_n = u'^2L + 2u'v'M + v'^2N$,

(ii) κ_g *is invariant under the isometry* f*, i.e.,* $\kappa_g = \bar{\kappa}_g$*,*

where κ_n *and* κ_g *are respectively the normal and geodesic curvature of* $\gamma(s)$.

PROOF : Let f be an isometry between S and $\bar{S}$. Suppose that $\phi(u, v)$ and $\bar{\phi}(u, v) = f \circ \phi(u, v)$ are surface patches of S and $\bar{S}$ respectively. If $\{E, F, G\}$ and $\{\bar{E}, \bar{F}, \bar{G}\}$ are the coefficients of first fundamental forms of ϕ and $\bar{\phi}$ respectively, then

$$\bar{E} = E, \quad \bar{F} = F \quad \text{and} \quad \bar{G} = G. \tag{2}$$

So

$$\bar{E}_u = (\bar{\phi}_u \cdot \bar{\phi}_u) = (\phi_u \cdot \phi_u)_u = E_u. \tag{3}$$

Similarly

$$\bar{F}_u = F_u, \qquad \bar{G}_u = G_u, \qquad \bar{E}_v = E_v, \qquad \bar{F}_v = F_v, \qquad \text{and} \qquad \bar{G}_v = G_v. \tag{4}$$

Now

$$\begin{aligned} E_u &= (\phi_{uu} \cdot \phi_u)_u = 2\phi_{uu} \cdot \phi_u, \\ \phi_{uu} \cdot \phi_u &= \frac{1}{2} E_u, \end{aligned} \tag{5}$$

and similarly

$$\begin{cases} \phi_{uv} \cdot \phi_v = \frac{1}{2} G_u, \qquad \phi_{uu} \cdot \phi_v = F_u - \frac{1}{2} E_v, \qquad \phi_{uv} \cdot \phi_u = \frac{1}{2} E_v, \\ \phi_{vv} \cdot \phi_v = \frac{1}{2} G_v, \qquad \phi_{vv} \cdot \phi_u = F_v - \frac{1}{2} G_u. \end{cases} \tag{6}$$

If $\gamma(s) = \lambda(s)\vec{t}(s) + \mu(s)\vec{b}(s)$, for some functions $\lambda(s)$ and $\mu(s)$, then

$$\gamma'(s) = \lambda'(s)\vec{t}(s) + \lambda(s)\kappa(s)\vec{n}(s) + \mu'(s)\vec{b}(s) - \mu(s)\tau(s)\vec{n}(s).$$

In [4], Chen obtained a condition for such $\lambda(s)$ and $\mu(s)$ satisfying rectifying equation, which is given by

$$\lambda'(s) = 1, \qquad \lambda(s)\kappa(s) = \mu(s)\tau(s), \qquad \mu'(s) = 0.$$

So

$$\gamma''(s) = \kappa(s)\vec{n}(s).$$

Now

$$\begin{aligned} \kappa_n &= \gamma''(s) \cdot \vec{N} = \kappa(s) n(s) \cdot \vec{N}, \\ &= \{(u'^2\phi_{uu} + 2u'v'\phi_{uv} + v'^2\phi_{vv}) + (u''\phi_u + v''\phi_v)\} \cdot \vec{N}, \\ &= u'^2 L + 2u'v'M + v'^2 N, \end{aligned}$$

where L, M and N are coefficients of the second fundamental form of ϕ.

Again

$$\begin{aligned}
\kappa_g &= \gamma''\cdot(\vec{N}\times\gamma'),\\
&= \gamma''\cdot\{(\phi_u\times\phi_v)\times(u'\phi_u+v'\phi_v)\},\\
&= \{(u'^2\phi_{uu}+2u'v'\phi_{uv}+v'^2\phi_{vv})+(u''\phi_u+v''\phi_v)\}\cdot\{u'(E\phi_v-F\phi_u)+v'(F\phi_v-G\phi_u)\},\\
&= u'u''(EF-FE)+v'u''(F^2-GE)+u'v''(EG-F^2)+v'v''(FG-GF)\\
&\quad +u'^3(E\phi_{uu}\cdot\phi_v-F\phi_{uu}\cdot\phi_u)+u'^2v'(F\phi_{uu}\cdot\phi_v-G\phi_{uu}\cdot\phi_u)+2u'^2v'(E\phi_{uv}\cdot\phi_v\\
&\quad -F\phi_{uv}\cdot\phi_u)+2u'v'^2(F\phi_{uv}\cdot\phi_v-G\phi_{uv}\cdot\phi_u)+u'v'^2(E\phi_{vv}\cdot\phi_v-F\phi_{vv}\cdot\phi_u)\\
&\quad +v'^3(F\phi_{vv}\cdot\phi_v-G\phi_{vv}\cdot\phi_u).
\end{aligned}$$

Using equations (5) and (6), the last relation can be written as

$$\begin{aligned}
\kappa_g &= v'u''(F^2-GE)+u'v''(EG-F^2)+\frac{1}{2}u'^3(2EF_u-EE_v-FE_u)+\frac{1}{2}u'^2v'(2FF_u-FE_v\\
&\quad -GE_u)+u'^2v'(EG_u-FE_v)+u'v'^2(FG_u-GE_v)+\frac{1}{2}u'v'^2(EG_v-2FF_v-FG_u)\\
&\quad \frac{1}{2}v'^3(FG_v-2GF_v-GG_u).
\end{aligned}$$

If $\bar{\gamma}$ is also a rectifying curve in $\bar{S}$ then $\bar{\kappa}_g$ is given by

$$\begin{aligned}
\bar{\kappa}_g &= v'u''(\bar{F}^2-\bar{G}\bar{E})+u'v''(\bar{E}\bar{G}-\bar{F}^2)+\frac{1}{2}u'^3(2\bar{E}\bar{F}_u-\bar{E}\bar{E}_v-\bar{F}\bar{E}_u)+\frac{1}{2}u'^2v'(2\bar{F}\bar{F}_u-\bar{F}\bar{E}_v\\
&\quad -\bar{G}\bar{E}_u)+u'^2v'(\bar{E}\bar{G}_u-\bar{F}\bar{E}_v)+u'v'^2(\bar{F}\bar{G}_u-\bar{G}\bar{E}_v)+\frac{1}{2}u'v'^2(\bar{E}\bar{G}_v-2\bar{F}\bar{F}_v-\bar{F}\bar{G}_u)\\
&\quad \frac{1}{2}v'^3(\bar{F}\bar{G}_v-2\bar{G}\bar{F}_v-\bar{G}\bar{G}_u).
\end{aligned}$$

By virtue of (2), (3) and (4), the last relation yields

$$\bar{\kappa}_g=\kappa_g.\square$$

4. Osculating Curve on a Smooth Surface

Let $\alpha(s)$ be an unit speed parametrized curve in the atlas ϕ of S. Now, $\alpha(s)$ will be an osculating curve if its position vector always lies in its osculating plane of that curve, i.e., $\alpha(s)=\lambda_1(s)\vec{t}(s)+\lambda_2(s)\vec{n}(s)$, for some function $\lambda_1(s)$ and $\lambda_2(s)$. Therefore,

$$\alpha(s)=\lambda_1(s)(u'\phi_u+v'\phi_v)+\frac{\lambda_2(s)}{\kappa(s)}\{(u'^2\phi_{uu}+2u'v'\phi_{uv}+v'^2\phi_{vv})+(u''\phi_u+v''\phi_v)\},\qquad (7)$$

for some functions $\lambda_1(s)$ and $\lambda_2(s)$.

Differentiating (7) with respect to s, we get

$$\vec{t}(s)=\alpha'(s)=\lambda_1'(s)\vec{t}(s)+\lambda_2'(s)\vec{n}(s)+\lambda_1(s)\kappa(s)\vec{n}(s)+\lambda_2(s)(-\kappa(s)\vec{t}(s)+\tau(s)\vec{b}(s)),$$

and hence

$$\begin{aligned}\lambda_1'(s)-\lambda_2(s)\kappa(s)&=1,\\ \lambda_2'(s)+\lambda_1(s)\kappa(s)&=0,\\ \lambda_2(s)\tau(s)&=0.\end{aligned}$$

The functions λ_1 and λ_2 of an osculating curve (7) can be deduced from the last three relations.

The component of the osculating curve $\alpha(s)$ along the normal $\vec{N}$ to the surface at a point $\alpha(s)$ is given by

$$\begin{aligned}\alpha(s)\cdot\vec{N} &= \Big[\lambda_1(s)(u'\phi_u+v'\phi_v)+\frac{\lambda_2(s)}{\kappa(s)}\{(u'^2\phi_{uu}+2u'v'\phi_{uv}+v'^2\phi_{vv})\\ &\quad +(u''\phi_u+v''\phi_v)\}\Big]\cdot\vec{N},\\ &= \frac{\lambda_2(s)}{\kappa(s)}(u'^2L+2u'v'M+v'^2N),\end{aligned}$$

where L, M and N are coefficients of the second fundamental form of ϕ. So the component of the osculating curve $\alpha(s)$ in S along the normal $\vec{N}$ to the surface S is represented by the coefficient of the second fundamental form of ϕ.

We also want to find the component of $\alpha(s)$ along any tangent vector in the tangent plane to the surface. Since the tangent plane of ϕ at $\alpha(s)$ is generated by ϕ_u and ϕ_v, hence we find the component of $\alpha(s)$ along ϕ_u and ϕ_v respectively. Now

$$\alpha(s)\cdot\phi_u = \Big[\lambda_1(s)(u'\phi_u+v'\phi_v)+\frac{\lambda_2(s)}{\kappa(s)}\{(u'^2\phi_{uu}+2u'v'\phi_{uv}+v'^2\phi_{vv})+(u''\phi_u+v''\phi_v)\}\Big]\cdot\phi_u.$$

Using equation (2), (5) and (6), the last relation can be written as

$$\alpha(s)\cdot\phi_u = \lambda_1(s)(u'E+v'F)+\frac{\lambda_2(s)}{2\kappa(s)}\Big(2u''E+2v''F+u'^2E_u+2u'v'E_v+2v'^2F_v-v'^2G_u\Big),$$

and similarly

$$\alpha(s)\cdot\phi_v=\lambda_1(s)(u'F+v'G)+\frac{\lambda_2(s)}{2\kappa(s)}\Big(2u''F+2v''G+2u'^2F_u-u'^2E_v+2u'v'G_u+v'^2G_v\Big),$$

where E, F and G are coefficients of first fundamental form of ϕ.

***Theorem* 4.1** — *Let S and $\bar{S}$ be two smooth surfaces and $f : S \to \bar{S}$ be an isometry and $\alpha(s)$ be an osculating curve on S. Then $\bar{\alpha}(s) = f \circ \alpha(s)$ is an osculating curve on $\bar{S}$ if*

$$\bar{\alpha}(s) - f_*(\alpha(s)) = \frac{\lambda_2(s)}{\kappa(s)}\Big(u'^2\frac{\partial f_*}{\partial u}\phi_u + 2u'v'\frac{\partial f_*}{\partial u}\phi_v + v'^2\frac{\partial f_*}{\partial v}\phi_v\Big). \tag{8}$$

PROOF : For two smooth surfaces S and $\bar{S}$, consider ϕ and $\bar{\phi}$ respectively as the coordinate charts, where

$$\bar{\phi} = f \circ \phi.$$

Two vectors ϕ_u and ϕ_v generate the tangent plane T_pS at a point p on S. Since $f : S \to \bar{S}$ is an isometry, the differential map f_* of f is an orthogonal matrix of order 3×3. Therefore f_* takes a set of linearly independent vectors $\{\phi_u, \phi_v\}$ of T_pS to $\{\bar{\phi}_u, \bar{\phi}_v\}$ of $T_{f(p)}S$. Then

$$\bar{\phi}_u(u,v) = f_*(\phi(u,v))\phi_u, \tag{9}$$

$$\bar{\phi}_v(u,v) = f_*(\phi(u,v))\phi_v. \tag{10}$$

Again differentiating (7) and (8) partially with respect to both u and v respectively, we get

$$\begin{cases} \bar{\phi}_{uu} &= \frac{\partial f_*}{\partial u}\phi_u + f_*\phi_{uu}, \\ \bar{\phi}_{vv} &= \frac{\partial f_*}{\partial v}\phi_v + f_*\phi_{vv}, \\ \bar{\phi}_{uv} &= \frac{\partial f_*}{\partial v}\phi_u + f_*\phi_{uv} = \frac{\partial f_*}{\partial u}\phi_v + f_*\phi_{uv}. \end{cases} \tag{11}$$

In view of (6) and (11), we get

$$\begin{aligned} \bar{\alpha}(s) &= \lambda_1(s)(u'f_*\phi_u + v'f_*\phi_v') + \frac{\lambda_2(s)}{\kappa(s)}\Big(u''f_*\phi_u + v''f_*\phi_v + u'^2 f_*\phi_{uu} + 2u'v'f_*\phi uv \\ &\quad + v'^2 f_*\phi_{vv} + u'^2\frac{\partial f_*}{\partial u}\phi_u + 2u'v'\frac{\partial f_*}{\partial u}\phi_v + v'^2\frac{\partial f_*}{\partial v}\phi_v\Big), \end{aligned}$$

which can be written as

$$\bar{\alpha}(s) = \lambda_1(s)(u'\bar{\phi}_u + v'\bar{\phi}_v) + \frac{\lambda_2(s)}{\kappa(s)}(u''\bar{\phi}_u + v''\bar{\phi}_v + u'^2\bar{\phi}_{uu} + 2u'v'\bar{\phi}_{uv} + v'^2\bar{\phi}_{vv}),$$

and hence

$$\bar{\alpha}(s) = \bar{\lambda}_1(s)\vec{\bar{t}}(s) + \frac{\bar{\lambda}_2(s)}{\bar{\kappa}(s)}\vec{\bar{n}}(s),$$

for some functions $\bar{\lambda}_1(s)$ and $\bar{\lambda}_2(s)$. Therefore $\bar{\alpha}(s)$ is an osculating curve in $\bar{S}$. □

Note : From the above theorem we admit that the functions $\lambda_1(s)$ and $\bar{\lambda}_1(s)$ for the osculating curves $\alpha(s)$ and $\bar{\alpha}(s)$ on S and $\bar{S}$ respectively does not alter whenever we take an isometry on S to $\bar{S}$. Also $\frac{\bar{\lambda}_2(s)}{\bar{\kappa}(s)} = \frac{\lambda_2(s)}{\kappa(s)}$, i.e., $\lambda_1(s)$ and $\bar{\lambda}_2(s)$ for the osculating curves $\alpha(s)$ and $\bar{\alpha}(s)$ respectively are connected by the curvature functions $\kappa(s)$ and $\bar{\kappa}(s)$.

***Theorem* 4.2** — *Let f be an isometry of two smooth surfaces S and $\bar{S}$. For the osculating curves $\alpha(s)$ and $\bar{\alpha}(s)$ on S and $\bar{S}$ respectively the component of the position vector of the osculating curve along any tangent vector to the surface S at $\alpha(s)$ is invariant under the isometry f, i.e., $\alpha(s)\cdot(a\phi_u + b\phi_v) = \bar{\alpha}(s)\cdot(a\bar{\phi}_u + b\bar{\phi}_v)$, for any two real numbers a and b.*

PROOF : Since $f : S \to \bar{S}$ is an isometry and $\alpha(s)$, $\bar{\alpha}(s)$ are osculating curves on S and $\bar{S}$ respectively, the relations (2), (3) and (4) hold.

Now

$$\begin{aligned}\bar{\alpha}(s)\cdot(a\bar{\phi}_u + b\bar{\phi}_v) \;=\;& a\Big[\bar{\lambda}_1(s)(u'\bar{E} + v'\bar{F}) + \frac{\bar{\lambda}_2(s)}{2\bar{\kappa}(s)}\Big(2u''\bar{E} + 2v''\bar{F} + u'^2\bar{E}_u + 2u'v'\bar{E}_v \\ &+2v'^2\bar{F}_v - v'^2\bar{G}_u\Big)\Big] + b\Big[\bar{\lambda}_1(s)(u'\bar{F} + v'\bar{G}) + \frac{\bar{\lambda}_2(s)}{2\bar{\kappa}(s)}\Big(2u''\bar{F} + 2v''\bar{G} \\ &+2u'^2\bar{F}_u - u'^2\bar{E}_v + 2u'v'\bar{G}_u + v'^2\bar{G}_v\Big)\Big].\end{aligned}$$

By virtue of (2), (3) and (4), the last relation yields

$$\alpha(s)\cdot(a\phi_u + b\phi_v) = \bar{\alpha}(s)\cdot(a\bar{\phi}_u + b\bar{\phi}_v)$$

Therefore the component of an osculating curve $\alpha(s)$ along any tangent vector to the surface S at $\alpha(s)$ is invariant under the osculating curve preserving isometry of surfaces. □

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