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# A CLASSIFICATION OF TETRAVALENT ARC-TRANSITIVE GRAPHS OF ORDER $5p^2$

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Let $s$ be a positive integer. A graph is $s$-transitive if its automorphism group is transitive on $s$-arcs but not on $(s+1)$-arcs. Let $p$ be a prime. In this article a complete classification of tetravalent $s$-transitive graphs of order $5p^2$ is given.

**Key words** : $s$-Transitive graphs; symmetric graphs; Cayley graphs.

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## 1. Introduction

Let $X$ be a finite simple group. We use $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}(X)$ to denote its vertex set, edge set, arc set and its full automorphism group, respectively. For $u, v \in V(X)$, $\{u, v\}$ is the edge incident to $u$ and $v$ in $X$, and $N(u)$ is the neighborhood of $u$ in $X$, that is, the set of vertices adjacent to $u$ in $X$. An $s$-*arc* in a graph is an ordered $(s+1)$-tuple $(v_0, v_1, \cdots, v_{s-1}, v_s)$ of vertices of the graph such that $v_{i-1}$ is adjacent to $v_i$ for $1 \leq i \leq s$, and $v_{i-1} \neq v_{i+1}$ for $1 \leq i \leq s-1$. For a subgroup $G \leq \mathrm{Aut}(X)$, a graph $X$ is said to be $(G, s)$-*arc-transitive* or $(G, s)$-*regular* if $G$ acts transitively or regularly on the set of $s$-arcs of $X$, respectively. A $(G, s)$-arc-transitive graph is said to be $(G, s)$-*transitive* if it is not $(G, s+1)$-arc-transitive. In particular, an $(\mathrm{Aut}(X), s)$-arc-transitive, $(\mathrm{Aut}(X), s)$-regular or $(\mathrm{Aut}(X), s)$-transitive graph is simply called an $s$-*arc-transitive*, $s$-*regular* or $s$-*transitive* graph, respectively. Note that 0-arc-transitive means vertex-transitive, and 1-arc-transitive means *arc-transitive* or *symmetric*.

The first result which gave an upper bound $s$ for $s$-arc transitivity of graphs came from Tutte [31] who proved that there exist no cubic $s$-arc transitive graphs for $s \geq 6$. Tutte' work initiated the study of $s$-arc transitive graphs. Since then, the construction and classification of $s$-arc transitive graphs have received considerable attention (see, for example, [7, 8, 10-16]). After that the most remarkable results in the area is that there exits no $s$-transitive graphs for $s = 6$ and $s \geq 8$, which was obtained by Weiss (see [33]). Further for each value of $s \in \{1, 2, 3, 4, 5, 7\}$ there exists $s$-transitive graphs.

The aim of this paper is to classify all symmetric graphs of order $np$ and valency $k$ for certain values of $n$ and $k$. The classification of $s$-transitive graphs of order $np$ and of valency 3 or $4$ can be obtained from [5, 6, 32], where $1 \leq n \leq 3$. Feng *et al*. [11, 13, 14] classified cubic $s$-transitive graphs of order $np$ with $n = 4, 6, 8$ or 10. Recently Zhou and Feng [37, 38] classified tetravalent $s$-transitive graphs of order $4p$ or $2p^2$. Also Ghasemi and Zhou [17, 19] classified tetravalent $s$-transitive graphs of order $3p^2$ and $4p^2$. The following result is the main result of this paper.

***Theorem* 1.1** — *Let $p > 11$ be a prime and let $X$ be a connected tetravalent arc-transitive graph of order $5p^2$. Then $X$ is $1$-regular, and moreover, $X$ is isomorphic to one of the graphs in Proposition 2.4.*

## 2. Preliminaries

In this section, we introduce some notations and definitions as well as some preliminary results which will be used later in the paper.

For a regular graph $X$, use $d(X)$ to represent the valency of $X$, and for any subset $B$ of $V(X)$, the subgraph of $X$ induced by $B$ will be denoted by $X[B]$. Let $X$ be a connected vertex-transitive graph, and let $G \leq \mathrm{Aut}(X)$ be vertex-transitive on $X$. For a $G$-invariant partition $\Omega$ of $V(X)$, the *quotient graph* $X_\Omega$ is defined as the graph with vertex set $\Omega$ such that, for any two vertices $B, C \in \Omega$, $B$ is adjacent to $C$ if and only if there exist $u \in B$ and $v \in C$ which are adjacent in $X$. Let $N$ be a normal subgroup of $G$. Then the set $\Omega$ of orbits of $N$ in $V(X)$ is a $G$-invariant partition of $V(X)$. In this case, the symbol $X_\Omega$ will be replaced by $X_N$.

For a positive integer $n$, denote by $\mathbb{Z}_n$ the cyclic group of order $n$ as well as the ring of integers modulo $n$, by $\mathbb{Z}_n^*$ the multiplicative group of $\mathbb{Z}_n$ consisting of numbers coprime to $n$, by $D_{2n}$ the dihedral group of order $2n$, and by $C_n$ and $K_n$ the cycle and the complete graph of order $n$, respectively. We call $C_n$ a $n$-*cycle*.

Let $G$ be a permutation group on a set $\Omega$ and $\alpha \in \Omega$. Denote by $G_\alpha$ the stabilizer of $\alpha$ in $G$, that is, the subgroup of $G$ fixing the point $\alpha$. We say that $G$ is *semiregular* on $\Omega$ if $G_\alpha = 1$ for every

$\alpha \in \Omega$ and *regular* if $G$ is transitive and semiregular. For any $g \in G$, $g$ is said to be *semiregular* if $\langle g \rangle$ is semiregular. The following proposition gives a characterization for Cayley graphs in terms of their automorphism groups.

*Proposition* 2.1 — [2, Lemma 16.3]. A graph $X$ is isomorphic to a Cayley graph on a group $G$ if and only if its automorphism group has a subgroup isomorphic to $G$, acting regularly on the vertex set of $X$.

Let $X$ be a connected symmetric graph and $X_N$ be the quotient graph of $X$. If $X_N$ and $X$ have the same valency, then $X$ is called a *normal cover* of $X_N$. Let $X$ be a connected tetravalent symmetric graph and $N$ an elementary abelian $p$-group. A classification of connected tetravalent symmetric graphs was obtained when $N$ has at most two orbits in [15] and a characterization of such graphs was given when $X_N$ is a cycle in [16].

The following proposition is due to Praeger *et al*., refer to [15, Theorem 1.1] and [27].

*Proposition* 2.2 — Let $X$ be a connected tetravalent $(G, 1)$-arc-transitive graph. For each normal subgroup $N$ of $G$, one of the following holds:

(1) $N$ is transitive on $V(X)$;

(2) $X$ is bipartite and $N$ acts transitively on each part of the bipartition;

(3) $N$ has $r \geq 3$ orbits on $V(X)$, the quotient graph $X_N$ is a cycle of length $r$, and $G$ induces the full automorphism group $D_{2r}$ on $X_N$;

(4) $N$ has $r \geq 5$ orbits on $V(X)$, $N$ acts semiregularly on $V(X)$, the quotient graph $X_N$ is a connected tetravalent $G/N$-symmetric graph, and $X$ is a $G$-normal cover of $X_N$.

Moreover, if $X$ is also $(G, 2)$-arc-transitive, then case $(3)$ can not happen.

The following proposition characterizes the vertex stabilizer of the connected tetravalent $s$-transitive graphs, which can be deduced from [24, Lemma 2.5], or [23, Proposition 2.8], or [22, Theorem 2.2].

*Proposition* 2.3 — Let $X$ be a connected tetravalent $(G, s)$-transitive graph. Let $G_v$ be the stabilizer of a vertex $v \in V(X)$ in $G$. Then $s = 1, 2, 3, 4$ or 7. Furthermore, either $G_v$ is a 2-group for $s = 1$, or $G_v$ is isomorphic to $A_4$ or $S_4$ for $s = 2$; $A_4 \times \mathbb{Z}_3$, $\mathbb{Z}_3 \times S_4$, $S_3 \times S_4$ for $s = 3$; $\mathbb{Z}_3^2 \rtimes \mathrm{GL}(2, 3)$ for $s = 4$; or $[3^5] \rtimes \mathrm{GL}(2, 3)$ for $s = 7$, where $[3^5]$ represents an arbitrary group of order $3^5$.

*Proposition* 2.4 — [18, Theorem 3.3]. Let $p$ be a prime. A tetravalent graph $X$ of order $5p^2$ is 1-regular if and only if one of the following holds:

**(i)** $p \in \{2, 3, 5, 7, 11\}$;

**(ii)** $X$ is a Cayley graph over $\langle x, y | x^p = y^{5p} = [x, y] = 1\rangle$, with connection set $\{y, y^{-1}, xy, x^{-1}y^{-1}\}$ and $\{y, y^{-2}, xy, x^{-2}y^{-2}\}$;

**(iii)** $X$ is connected arc-transitive circulant graph with respect to every connection set $S$;

**(iv)** $X$ is one of the graphs described in [16, Lemma 8.4].

*Proposition* 2.5 — [16, Theorem 1.2]. Let $X$ be a connected tetravalent symmetric graph of order $5p^2$, where $p > 5$ is a prime. Let $A = \mathrm{Aut}(X)$ and let $N = \mathbb{Z}_p^2$ be a minimal normal subgroup of $A$. Let $K$ denote the kernel of $G$ acting on $N$-orbits. If the quotient graph $X_N$ is a 5-cycle, then $K_v \cong \mathbb{Z}_2$, and $X$ is one-regular.

The finite simple group $G$ is called a $K_n$-group if its order has exactly $n$ distinct prime divisors, where $n \in N$. The following result determines all $K_n$-groups, where $n \in \{3, 4\}$.

*Proposition* 2.6 — [4, 20, 30, 39]. Let $G$ be a finite simple $K_n$-group.

(1) If $n = 3$, then $G$ is isomorphic to one of the following groups:

$$A_5,\ A_6,\ \mathrm{L}_2(7),\ \mathrm{L}_2(8),\ \mathrm{L}_2(17),\ \mathrm{L}_3(3),\ \mathrm{U}_3(3),\ \mathrm{U}_4(2).$$

(2) If $n = 4$ then $G$ is isomorphic to one of the following groups

(i) $A_7$, $A_8$, $A_9$, $M_{11}$, $M_{12}$, $J_2$, $L_2(16)$, $L_2(25)$, $L_2(49)$
$L_2(81)$, $L_2(97)$, $L_2(243)$, $L_2(577)$, $L_3(4)$, $L_3(5)$, $L_3(7)$
$L_3(8)$, $L_3(17)$, $L_4(3)$, $S_4(4)$, $S_4(5)$, $S_4(7)$, $S_4(9)$, $S_6(2)$
$O_8^+(2)$, $G_2(3)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, $U_4(3)$
$U_5(2)$, $Sz(8)$, $Sz(32)$, ${}^3D_4(2)$, ${}^2F_4(2)'$;

(ii) $L_2(r)$, where $r$ is a prime, $r^2 - 1 = 2^a.3^b.v$, $v > 3$ is a prime, $a, b \in \mathbb{N}$;

(iii) $L_2(2^m)$, where $m, 2^m - 1$ and $(2^m + 1)/3$ are primes greater than 3;

(iv) $L_2(3^m)$, where $m, (3^m + 1)/4$ and $(3^m - 1)/2$ are odd primes.

## 3. Main Results

In this section, we classify tetravalent $s$-transitive graphs of order $5p^2$ for each prime $p$. To do so, we need the following lemma.

*Lemma* 3.1 — Let $p > 5$ be a prime and let $n > 1$ be an integer. Let $X$ be a connected tetravalent graph of order $5p^n$. If $A = \mathrm{Aut}(X)$ is transitive on the arc set of $X$, then every minimal normal subgroup of $A$ is solvable.

PROOF : Let $v \in V(X)$. Since $A$ is arc-transitive on $X$, by Proposition 2.3, $A_v$ either is a 2-group or has order dividing $2^4 \cdot 3^6$. It follows that $|A| \mid 2^4 \cdot 3^6 \cdot 5 \cdot p^n$ or $|A| = 2^m \cdot 5 \cdot p^n$ for some integer $m$. Let $N$ be a minimal normal subgroup of $A$.

Suppose that $N$ is non-solvable. Since $N$ is minimal, it is a product of isomorphic non-abelian simple groups. First suppose that $|A| \mid 2^4 \cdot 3^6 \cdot 5 \cdot p^n$. Thus $X$ is 2-arc-transitive. Let $X(v)$ be the neighborhood of $v$ in $X$. The 2-arc-transitivity of $A$ implies that $N_v$ acts transitively on $X(v)$ because $N_v \trianglelefteq A_v$. By Proposition 2.2, $N$ acts transitively $V(X)$, a contradiction. Now suppose that $|A| = 2^m \cdot 5 \cdot p^n$. Thus $|N| \mid 2^m \cdot 5 \cdot p^n$ and by Proposition 2.6 we have either $N \cong A_5$ or $N \cong A_6$. Clearly $N$ has at least three orbits on $V(X)$ and $N$ is not regular on $V(X)$. We consider the quotient graph $X_N$. By Proposition 2.2 $X_N \cong C_{p^n}$. Therefore $A/K \cong \mathrm{Aut}(C_{p^n}) \cong D_{2p^n}$, where $K$ is the kernel of $A$ on $V(X_N)$. Let $\Delta$ and $\Delta'$ be two adjacent orbits of $N$ in $V(X)$. Then the subgraph $X[\Delta \cup \Delta']$ of $X$ induced by $\Delta \cup \Delta'$ has valency 2. Since $p > 5$, one has $X[\Delta \cup \Delta'] \cong C_{10}$. The subgroup $K^*$ of $K$ fixing $\Delta$ pointwise also fixes $\Delta'$ pointwise. The connectivity of $X$ and the transitivity of $A/K$ on $V(X_N)$ imply that $K^* = 1$, and consequently, $K \leq \mathrm{Aut}(X[\Delta \cup \Delta']) \cong D_{20}$. Since $K$ fixes $\Delta$, one has $|K| \leq 10$. It follows that $|A| = |A/K||K| \leq 20p^n$. Now by considering the order of $A$ we see that either $|A| = 2.5.p^n$ or $|A| = 2^2.5.p^n$. For the former case $A$ is solvable, a contradiction. Also for the latter case since $N \cong A_5$ or $N \cong A_6$, it implies that $p = 3$, a contradiction. □

Let $X$ be a tetravalent one-regular graph of order $5p^2$. If $p \leq 11$, then $|V(X)| = 20, 45$, $125$, $245$, or $605$. Now, a complete census of the tetravalent arc-transitive graphs of order at most $640$ has been recently obtained by Potočnik, Spiga and Verret [25, 26]. Therefore, a quick inspection through this list (with the invaluable help of `magma` (see [3])) gives the number of tetravalent arc-transitive graphs in the case that $p \leq 11$. Thus in the following theorem we may suppose that $p \geq 13$.

*Theorem* **3.2** — *Let $p > 11$ be a prime and let $X$ be a connected tetravalent graph of order $5p^2$. Then $X$ is $s$-transitive for some positive integer $s$ if and only if it is isomorphic to one of the graphs in Proposition 2.4.*

PROOF : Let $X$ be a tetravalent $s$-transitive graph of order $5p^2$ for a positive integer $s$. If $X$ is one-regular then $X$ is one of the graphs in Proposition 2.4 and so $s = 1$. In what follows, we assume that $p > 11$ and that $X$ is not one-regular. Set $A = \mathrm{Aut}(X)$ and let $P$ be a Sylow $p$-subgroup of $A$. First we prove a claim.

*Claim* : $P$ is not normal in $A$.

Suppose to contrary that is $P \trianglelefteq A$. If $P$ is a minimal normal subgroup of $A$ then by Proposition 2.5, $X$ is one-regular, a contradiction. Suppose that $P$ contains a non-trivial subgroup, say $N$, which is normal in $A$. Consider the quotient graph $X_N$ of $X$ relative to the orbit set of $N$, and let $K$ be the kernel of $A$ on $V(X_N)$. Since $p > 11$, one has $|X_N| = 5p$, and hence $d(X_N) = 2$ or $4$ by Proposition 2.2.

Let $d(X_N) = 2$. Then $X_N \cong C_{5p}$ and hence $A/K \cong \mathrm{Aut}(C_{5p}) \cong D_{10p}$. Let $\Delta$ and $\Delta'$ be two adjacent orbits of $N$ in $V(X)$. Then the subgraph $X[\Delta \cup \Delta']$ of $X$ induced by $\Delta \cup \Delta'$ has valency 2. Since $p > 11$, one has $X[\Delta \cup \Delta'] \cong C_{2p}$. The subgroup $K^*$ of $K$ fixing $\Delta$ pointwise also fixes $\Delta'$ pointwise. The connectivity of $X$ and the transitivity of $A/K$ on $V(X_N)$ imply that $K^* = 1$, and consequently, $K \leq \mathrm{Aut}(X[\Delta \cup \Delta']) \cong D_{4p}$. Since $K$ fixes $\Delta$, one has $|K| \leq 2p$. It follows that $|A| = |A/K||K| \leq 20p^2$, and hence $X$ is one-regular.

Let $d(X_N) = 4$. Then $N = K$ and $X_N$ is a tetravalent symmetric graph of order $5p$. Therefore either by [28] $X_N \cong G(5p, 2, 2, 2)$ (for definition of this graph we refer the reader to [28, Section 4]) or by [29, Lemma 3.5] $|X_N| = 85$ and so $p = 17$. In the former case by [28, Theorem 3.5] $X_N$ is one-regular and so $X$ is one-regular, a contradiction. Also for the latter case by [34, Section 4] again $X_N$ is one-regular and so $X$ is one-regular, a contradiction.

Let $M$ be the maximal normal 2-subgroup of $A$ and assume $M > 1$. Since $p > 11$, every orbit of $M$ has length 2 or 4, a contradiction. So $A$ has no non-trivial normal 2-subgroup.

Now we are ready to complete the proof. Let $M$ be a minimal normal subgroup of $A$. By Lemma 3.1 $M$ is an elementary abelian group. Clearly, $M$ is 5-group or $p$-group. First suppose that $M$ is a $p$-group. Thus $|M| = p$ or $p^2$. If $|M| = p^2$, then $M = P$ is a Sylow $p$-subgroup of $A$. By claim, $X$ is one-regular, a contradiction. Suppose that $|M| = p$. Consider the quotient graph $X_M$ of $X$ relative to the orbit set of $M$, and let $K$ be the kernel of $A$ acting on $V(X_M)$. By Proposition 2.2 $d(X_M) = 2$ or $4$. First suppose that $d(X_M) = 4$. Then $X_M$ is a tetravalent symmetric graph of order $5p$ and $K = M$. Then again either by [28] $X_M \cong G(5p, 2, 2, 2)$ or by [28, Lemma 3.5] $|X_M| = 85$ and so $p = 17$. In the former case by [34, Theorem 3.5] $X_M$ is one-regular and so $X$ is one-regular, a contradiction. Also for the latter case by [34, Section 4] again $X_M$ is one-regular and so $X$ is one-regular, a contradiction. Now suppose that $d(X_M) = 2$. Thus $X_M \cong C_{5p}$. Thus $A/K \cong \mathrm{Aut}(C_{5p}) \cong D_{10p}$. Let $\Delta$ and $\Delta'$ be two adjacent orbits of $M$ in $V(X)$. Then the subgraph $[\Delta \cup \Delta']$ of $X$ induced by $\Delta \cup \Delta'$ has valency 2. Since $p > 11$, one has $[\Delta \cup \Delta'] \cong C_{2p}$. The subgroup $K^*$ of $K$ fixing $\Delta$ pointwise also fixes $\Delta'$ pointwise. The connectivity of $X$ and the transitivity of

$A/K$ on $V(X_N)$ imply that $K^* = 1$, and consequently, $K \leq \mathrm{Aut}([\Delta \cup \Delta']) \cong D_{4p}$. Since $K$ fixes $\Delta$, one has $|K| \leq 2p$. It follows that $|A| = |A/K||K| \leq 20p^2$, and hence $A$ is regular on the arcs of $X$, a contradiction.

Now suppose that $M$ is a 5-group. Thus $|X_M| = p^2$. If $d(X_M) = 4$, then by Proposition 2.2, $K = M$ is semiregular on $V(X_M)$. Therefore $K = M \cong \mathbb{Z}_5$. Since $P > 11$, $PM = P \times M$ is abelian. Clearly $PM$ is transitive on $V(X)$. Thus $PM$ is regular on $V(X)$, because $|PM| = 5p^2$. Thus by Proposition 2.1, $X$ is a Cayley graph on abelian group of order $5p^2$. By [1, Theorem 1.2], $X$ is normal. If $PM$ is cyclic, then by [35] $X$ is one-regular, a contradiction. Thus $PM$ is not cyclic. Now by [36, Proposition 3.3], $X$ is one-regular, a contradiction. If $d(X_M) = 2$, then with the similar arguments in the above paragraph $X$ is one-regular, a contradiction. □

Now the proof of Theorem 1.1 is complete.

## REFERENCES


1. Y. G. Baik, Y.-Q. Feng, H. S. Sim, and M. Y. Xu, On the normality of Cayley graphs of abelian groups, *Algebra Colloq.*, **5** (1998), 297-304.

2. N. Biggs, *Algebraic graph theory*, second ed., Cambridge University Press, Cambridge, 1993.

3. W. Bosma, C. Cannon, and C. Playoust, The MAGMA algebra system I: the user language, *J. Symbolic Comput.*, **24** (1997), 235-265.

4. Y. Bugeaud, Z. Cao, and M. Mignotte, On simple $K_4$-groups, *J. Algebra.*, **241**(2) (2001), 658-668.

5. C. Y. Chao, On the classification of symmetric graphs with a prime number of vertices, *Tran. Amer. Math. Soc.*, **158** (1971), 247-256.

6. Y. Cheng and J. Oxley, On weakly symmetric graphs of order twice a prime, *J. Combin. Theory B*, **42** (1987), 196-211.

7. M. Conder and C. E. Praeger, Remarks on path-transitivity on finite graphs, *European J. Combin.*, **17** (1996), 371-378.

8. D.Ž. Djoković and G. L. Miller, Regular groups of automorphisms of cubic graphs, *J. Combin. Theory B*, **29** (1980), 195-230.

9. X. G. Fang, C. H. Li, and M. Y. Xu, On edge-transitive Cayley graphs of valency four, *European J. Combin.*, **25** (2004), 1107-1116.

10. Y.-Q. Feng and J. H. Kwak, One-regular cubic graphs of order a small number times a prime or a prime square, *J. Austral. Math. Soc.*, **76** (2004), 345-356.

11. Y.-Q. Feng and J. H. Kwak, Classifying cubic symmetric graphs of order $10p$ or $10p^2$, *Science in China*

*A*, **49** (2006), 300-319.

12. Y.-Q. Feng and J. H. Kwak, Cubic symmetric graphs of order twice an odd prime power, *J. Austral. Math. Soc*., **81** (2006), 153-164.

13. Y.-Q. Feng and J. H. Kwak, Cubic symmetric graphs of order a small number times a prime or a prime square, *J. Combin. Theory B*, **97** (2007), 627-646.

14. Y.-Q. Feng, J. H. Kwak, and K. S. Wang, Classifying cubic symmetric graphs of order $8p$ or $8p^2$, *European J. Combin*., **26** (2005), 1033-1052.

15. A. Gardiner and C. E. Praeger, On $4$-valent symmetric graphs, *European J. Combin*., **15** (1994), 375-381.

16. A. Gardiner and C. E. Praeger, A characterization of certain families of $4$-valent symmetric graphs, *European. J. Combin*., **15** (1994), 383-397.

17. M. Ghasemi, Tetravalent arc-transitive graphs of order $3p^2$, *Discuss. Math. Graph Theory*, **34** (2014), 567-575.

18. M. Ghasemi and R. Varmazyar, A Family of tetravalent one-regular graphs, *Ars Comb*., **134** (2017), 283-293.

19. M. Ghasemi and J.-X. Zhou, Tetravalent $s$-transitive graphs of order $4p^2$, *Graphs and Combinatorics*, **29**(1) (2013), 87-97.

20. M. Herzog, On finite simple groups of order divisible by three primes only, *J. Algebra*, **120**(10) (1968), 383-388.

21. C. H. Li, *Finite $s$-arc-transitive graphs*, The second internetional workshop on group theory and algebraic combinatorics, Peking University, Beijing, 2008.

22. C. H. Li, The finite vertex-primitive and vertex-biprimitive $s$-transitive graphs for $s \geq 4$, *Tran. Amer. Math. Soc*., **353** (2001), 3511-3529.

23. C. H. Li, Z. P. Lu, and D. Marušič, On primitive permutation groups with small suborbits and their orbital graphs, *J. Algebra*, **279** (2004), 749-770.

24. C. H. Li, Z. P. Lu, and H. Zhang, Tetravalent edge-transitive Cayley graphs with odd number of vertices, *J. Combin. Theory B*, **96** (2006), 164-181.

25. P. Potočnik, P. Spiga, and G. Verret, `http://www.matapp.unimib.it/ spiga/`

26. P. Potočnik, P. Spiga, and G. Verret, *Cubic vertex-transitive graphs on up to 1280 vertices*, arXiv:1201.5317v1 [math.CO].

27. C. E. Praeger, An O'Nan-Scott theorem for finite quasiprimitive permutation groups and an application to 2-arc transitive graphs, *J. London. Math. Soc*., **47** (1992), 227-239.

28. C. E. Praeger, R. J. Wang, and M. Y. Xu, Symmetric graphs of order a product of two distinct primes, *J. Combin. Theory B*, **58** (1993), 299-318.

29. C. E. Praeger and M. Y. Xu, Vertex-primitive graphs of order a product of two distinct primes, *J. Combin. Theory B*, **59** (1993), 245-266.

30. W. J. Shi, On simple $K_4$-groups, *Chines Science Bull.*, **36**(17) (1991), 1281-1283.

31. W. T. Tutte, A family of cubical graphs, *Proc. Camb. Phil. Soc.*, **43** (1947), 621-624.

32. R. J. Wang and M. Y. Xu, A classification of symmetric graphs of order $3p$, *J. Combin. Theory B*, **58** (1993), 197-216.

33. R. Weiss, The nonexistence of $8$-transitive graphs, *Combinatorica*, **1** (1981), 309-311.

34. S. Wilson and P. Potočnik, A Census of edge-transitive tetravalent graphs, http://jan. ucc.nau.edu/swilson/ C4Site/index.html.

35. M. Y. Xu, A note on one-regular graphs, *Chin. Scin. Bull.*, **45** (2000), 2160-2162.

36. J. Xu and M. Y. Xu, Arc-transitive Cayley graphs of valency at most four on abelian groups, *Southeast Asian Bull. Math.*, **25** (2001), 355-363.

37. J.-X. Zhou, Tetravalent $s$-transitive graphs of order $4p$, *Discrete Math.*, **309** (2009), 6081-6086.

38. J.-X. Zhou and Y.-Q. Feng, Tetravalent $s$-transitive graphs of order twice a prime power, *J. Austral. Math. Soc.*, **88** (2010), 277-288.

39. S. Zhang and W. J. Shi, *Revisiting the number of simple $K_4$-groups*, arXiv: 1307.8079v1 [math.NT] (2013).