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# SPLITTING OF THE PULLBACK OF $T_{\mathbb{P}^3}$ ON AN ELLIPTIC CURVE

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We study the splitting of the pullback of the tangent bundle $T_{\mathbb{P}^3}$ on an elliptic curve under the morphism given by a line bundle of degree less or equal to 5.

**Key words** : Semi-stability; tangent bundle; morphism of projective spaces.

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## 1. INTRODUCTION

Let $X$ be a smooth complex projective variety and $H$ be a very ample divisor on $X$. Let $E$ be a semistable (see section 2 for definition) vector bundle on $X$ with respect to $H$. When $C$ is a general smooth complete intersection curve of sufficiently high degree in $|H|$. Mehta and Ramanathan proved that $E|_C$ is semistable [10]. When $X$ is projective space, it is well known that the tangent bundle on $X$ is semistable with respect to the canonical polarization [9]. If $C$ is a smooth complete intersection curve of two nonlinear hypersurfaces in $\mathbb{P}^3$, it was shown by Biswas, Chaput and Mourougane that $T_{\mathbb{P}^3}|_C$ is semistable [4, Theorem 5]. In the case, when $C$ is a rational curve in $\mathbb{P}^3$ and if $C$ lies on the quadric surface $\mathbb{P}^1 \times \mathbb{P}^1$, the explicit description of the pullback of the tangent bundle $T_{\mathbb{P}^3}$ on $\mathbb{P}^1$ via the normalization map was given by Ascenzi, moreover she also indicated that the nature of singularities of a curve determine the splitting of the restriction of tangent bundle on the curve [7, Theorem 0.2]. Further when $C$ is an elliptic curve embedded into a projective space by a complete linear system, Brenner and Hein showed that the restriction of the tangent bundle of the projective space to the elliptic curve is stable [3, Theorem 1.3]. Motivated by Ascenzi and Brenner-Hein's result, one can ask the following question:

**Question:** What is the explicit description of $\varphi^* T_{\mathbb{P}^3}$, where $C$ is an elliptic curve and $\varphi : C \to \mathbb{P}^3$ is a morphism given by a sublinear system of a complete linear system $|L|$ of a degree $d$ line bundle $L$?

In this paper we give a complete answer of the above question, when $d = 5$. See Theorem 3.1 and Theorem 3.2.

## 2. Preliminaries

Let $X$ be a smooth complex projective variety of dimension $n$. Let $E$ be a vector bundle (= locally free sheaf) over $X$. A subbundle $F$ is a subsheaf of a locally free sheaf $E$ such that $E/F$ is torsion free. A subbundle $F$ of $E$ is proper if $0 < rk(F) < rk(E)$, where $rk(F)$ denotes the rank of $F$. Let $H$ be a very ample line bundle on $X$. The degree a vector bundle $E$ with respect to $H$ is defined as

$$\deg_H(E) := c_1(E).H^{n-1},$$

where $c_1(E)$ is the first Chern class of $E$. The slope of $E$ is defined as

$$\mu_H(E) := \frac{\deg_H(E)}{rk(E)}$$

$E$ is said to be (semi) stable with respect to $H$, if for every proper subbundle $F$ of $E$,

$$\mu_H(F)(\leq) < \mu_H(E).$$

$E$ on $X$ is said to be decomposable, if there is a proper subbundle $F$ of $E$, the following exact sequence

$$0 \to F \to E \to E/F \to 0$$

splits. The vector bundle $E$ on $X$ is indecomposable, if it is not decomposable. Every vector bundle on $\mathbb{P}^1$ of rank $r \geq 2$ is decomposable. Any stable vector bundle on $X$ is indecomposable.

Let $\mathcal{E}(r, d)$ denotes the set of isomorphic classes of indecomposable vector bundles of rank $r$ and degree $d$ on $X$.

### 2.1 *Vector bundles on Elliptic Curve*

Let $C$ be an elliptic curve over $\mathbb{C}$. For a fixed positive integer $r$, there is a unique indecomposable vector bundle $F_r$ of rank $r$ and degree zero on $C$ with $H^0(F_r) \neq 0$ and for any $E \in \mathcal{E}(r, 0)$, there is a line bundle $L$ of degree zero such that

$$E = F_r \otimes L.$$

Moreover there is a exact sequence

$$0 \to \mathcal{O}_C \to F_r \to F_{r-1} \to 0 \tag{2.1}$$

where $\mathcal{O}_C$ denotes the trivial line bundle on $C$.

Let $r$ and $d$ be positive integers. By fixing a line bundle $A$ on $C$ of degree one, there is a bijection

$$\alpha_{r,d} : \mathcal{E}(h, 0) \to \mathcal{E}(r, d)$$

where $h = (r, d)$ (See [6, Theorem 6]). Let $\alpha_{r,d}(F_h) = E_A(r, d)$. Call it the Atiyah bundle $E_A(r, d)$ of rank $r$ and degree $d$ based at $A$. For $E \in \mathcal{E}(r, d)$, there a line bundle $L$ on $C$ of degree zero such that

$$E = E_A(r, d) \otimes L$$

Following two fundamental results relate between semi-stable and indecomposable vector bundles on $C$.

*Lemma* 2.1 — [5, Lemma 29]. For any positive integers $r$ and $d$, the Atiyah bundle $E_A(r, d)$ on elliptic curve $C$ is semistable.

*Lemma* 2.2 — [5, Lemma 30]. The Atiyah bundle $E_A(r, d)$ is stable if and only if $(r, d) = 1$.

## 3. Splitting Types of The Pullback of Tangent Bundle

A morphism $\varphi : C \to T_{\mathbb{P}^3}$ is said to be non-degenerate if $\varphi(C)$ is not contained in any hyperplanes. We shall assume that the underlying field is the complex numbers. In this section, we obtain the splitting types of the pullback of the tangent bundle $T_{\mathbb{P}^3}$ via the morphisms $\varphi : C \to \mathbb{P}^3$ having a property that $\deg(\varphi^*(\mathcal{O}_{\mathbb{P}^3}(1))) = 5$, when $C$ is an elliptic curve. Further, any rank three and degree five vector bundle $F$ over an elliptic curve $C$ of which the splitting type is given in Theorem 3.1, there is non-degenerate morphism $\varphi : C \to \mathbb{P}^3$ such that $\varphi^* T_{\mathbb{P}^3}(-1) = F$.

*Lemma* 3.1 — Let $C$ be a smooth projective curve and $\varphi : C \to \mathbb{P}^n$ be any morphism of degree $s$. Let $H = \mathcal{O}_{\mathbb{P}^n}(1)$ be the hyperplane bundle on $\mathbb{P}^n$. Then $\deg(\varphi^* \mathcal{O}_{\mathbb{P}^n}(1)) = \deg(\varphi) \deg(\varphi(C))$.

Proof : [2, Lemma 2.1]. □

*Proposition* 3.1 — Let $C$ be a smooth projective curve. Let $\varphi : C \to \mathbb{P}^{n-1}$ be a morphism with a property that $\deg(\varphi^*(\mathcal{O}_{\mathbb{P}^{n-1}}(1))) = d$.

Then the following are equivalent:

(a) $\deg(\varphi(C)) = 1$

(b) $\varphi^* T_{\mathbb{P}^{n-1}} = L_1 \oplus \cdots \oplus L_{n-2} \oplus L_{n-1}$ and $(\deg L_1, ..., \deg L_{n-2}, \deg L_{n-1}) = (d, ..., d, 2d)$.

PROOF : $(a) \Rightarrow (b)$. Let $T_{\mathbb{P}^{n-1}}$ denotes the tangent bundle on the projective $(n-1)$ space $\mathbb{P}^{n-1}$. We denote $\varphi^*(\mathcal{O}_{\mathbb{P}^{n-1}}(1))$ by $L$. Consider the Euler exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^{n-1}} \to \mathcal{O}_{\mathbb{P}^{n-1}}(1)^n \to T_{\mathbb{P}^{n-1}} \to 0. \tag{3.1}$$

Suppose that $\deg(\varphi(C))$ = 1. Then $\varphi(C) \cong \mathbb{P}^1$. One can easily observe that

$$T_{\mathbb{P}^{n-1}}|_{\varphi(C)} = \mathcal{O}_{\mathbb{P}^1}(1)^{(n-2)} \oplus \mathcal{O}_{\mathbb{P}^1}(2) \tag{3.2}$$

By taking the pullback of the equation (3.2) via $\varphi$, we get

$$\varphi^* T_{\mathbb{P}^{n-1}} = L^{\oplus(n-2)} \oplus L^2.$$

$(b) \Rightarrow (a)$. Suppose that $\varphi^* T_{\mathbb{P}^{n-1}}(-1) = L_1 \oplus \cdots \oplus L_{n-2} \oplus L_{n-1}$ and $(\deg L_1, ..., \deg L_{n-2}, \deg L_{n-1})$ $= (0, ..., 0, d)$. Since $L_i$'s are globally generated (by (3.1)), $H^0(C, L_i) \neq 0$. Since $\deg L_i = 0$ and $H^0(C, L_i) \neq 0$, for all $i = 1, 2, ..., n-2$, $L_i \cong \mathcal{O}_C$, for all $i = 1, 2, ..., n-2$ ([8, Lemma IV.1.2]). Then we have

$$0 \to L^{-1} \xrightarrow{\xi} \mathcal{O}_C^n \xrightarrow{\eta} \mathcal{O}_C^{n-2} \oplus L_{n-1} \to 0. \tag{3.3}$$

By taking the determinant of the above equation, we get

$$L_{n-1} = L.$$

The map $\eta$ induces the map

$$H^0(C, \mathcal{O}_C^n) \to H^0(C, \mathcal{O}_C)^{\oplus(n-2)} \oplus H^0(C, L)$$

defined by

$$\begin{aligned} e_1 &\mapsto (a_{11}, a_{12}, a_{13}, ..., a_{1(n-2)}, s_1) \\ e_2 &\mapsto (a_{21}, a_{22}, a_{23}, ..., a_{2(n-2)}, s_2) \\ &\vdots \\ e_n &\mapsto (a_{n1}, a_{n2}, a_{n3}, ..., a_{n(n-2)}, s_n) \end{aligned}$$

where $\{e_i | i = 1, 2, ..., n\}$ is the standard basis of the $n$ dimensional $\mathbb{C}$-vector space $H^0(C, \mathcal{O}_C^n)$, where $s_1, s_2, s_3, ..., s_n$ are global sections of $L$ on $C$, $a_{ij} \in \mathbb{C}$ but not all zero, since $\eta$ is onto. Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1(n-2)} & s_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2(n-2)} & s_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{(n-1)1} & a_{(n-1)2} & a_{(n-1)3} & \cdots & a_{(n-1)(n-2)} & s_{n-1} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n(n-2)} & s_n \end{bmatrix}.$$

Let $A_i$ be the $(n-1) \times (n-1)$ matrix obtained by omitting the $i^{th}$ row of the matrix $A$. Let $X_i$ denotes $\det(A_i)$.

Let $x \in C$. We denote $(a_{i1}, a_{i2}, a_{i3}, ..., a_{i(n-2)}, s_i)$ by $\omega_i$. Since $\eta$ is onto, the set $\{(\overline{\omega_i})_x | i = 1, 2, ..., n\}$ generates the vector space $\left(\frac{\mathcal{O}_x}{\mathfrak{m}_x}\right)^{n-2} \oplus \frac{L_x}{\mathfrak{m}_x L_x}$. And so the one dimensional vector space $\bigwedge^{n-1}\left(\left(\frac{\mathcal{O}_x}{\mathfrak{m}_x}\right)^{n-2} \oplus \frac{L_x}{\mathfrak{m}_x L_x}\right)$ generated by

$$\{(\overline{\omega_{i_1}})_x \wedge (\overline{\omega_{i_2}})_x \wedge \cdots \wedge (\overline{\omega_{i_{n-1}}})_x | 1 \leq i_1 < \cdots < i_{n-1} \leq n\}.$$

Note that $\{\omega_{i_1} \wedge \omega_{i_2} \wedge \cdots \wedge \omega_{i_{n-1}} | 1 \leq i_1 < \cdots < i_{n-1} \leq n\} = \{X_1, X_2, ..., X_n\}$. Therefore, $\{X_1, X_2, ..., X_n\}$ is base point free. Thus we get a morphism

$$\pi : C \to \mathbb{P}^{n-1}$$

defined by the equation

$$x \mapsto [X_1(x) : X_2(x) : \cdots : X_n(x)]$$

Without loss of generality, we may assume that $a_{i1} = a_{i2} = \cdots = a_{i(n-2)} = 0$, for $i = n-1, n$ and for all $i \leq n-2$, $a_{ij}$ not all zero, since $\eta$ is onto. Then $X_3 = X_4 = \cdots = X_n = 0$. Therefore $\pi(C) = \mathbb{P}^1$. Since $\varphi$ and $\pi$ are dual to each other, $\varphi(C) = \mathbb{P}^1$. □

*Proposition* 3.2 — Let $C$ be an elliptic curve. Let $\varphi : C \to \mathbb{P}^3$ be a non degenerate morphism having a property that $\deg(\varphi^*(\mathcal{O}_{\mathbb{P}^3}(1))) = 4$. Then $\varphi^* T_{\mathbb{P}^3}$ is stable.

PROOF : [3, Theorem 1.3]. □

*Lemma* 3.2 — Let $E$ be a semistable vector bundle over an elliptic curve $C$ of positive degree $d$. Then $\dim H^0(C, E) = d$.

PROOF : Let $K$ denotes the canonical bundle on $C$. $C$ being an elliptic curve, $K = \mathcal{O}_C$. By Serre's duality we compute $H^1(C, E) = H^0(C, E^* \otimes K) = H^0(C, E^*) = 0$, since a semi-stable vector bundle of negative degree has no non-zero global sections. By applying Riemann-Roch Theorem for vector bundle on a curve, we get

$$\dim H^0(C, E) = d.$$

A vector bundle $E$ on a curve $C$ is said to be globally generated if the natural map

$$H^0(C, E) \otimes \mathcal{O}_C \to E$$

is onto.

*Lemma* 3.3 — A rank $n$ and degree $n$ indecomposable vector bundle $E$ on an elliptic curve $C$ can not be globally generated.

PROOF : Any indecomposable vector bundle on elliptic curve is semistable (see [5, Lemma 29]), hence by Lemma 3.2 we have $\dim H^0(C, E) = n$. Suppose that $E$ is generated by global sections. $i.e.$, there is a natural onto morphism

$$\mathcal{O}_C^n \to E.$$

Therefore, being the morphism onto and both sides of vector bundles have same rank, the vector bundles $\mathcal{O}_C^n$ and $E$ are isomorphic that yields a contraction of the assumption that $E$ is indecomposable. □

***Theorem* 3.1** — *Let $C$ be an elliptic curve. Let $\varphi : C \to \mathbb{P}^3$ be a non-degenerate morphism with a property that $\deg(\varphi^*(\mathcal{O}_{\mathbb{P}^3}(1))) = 5$. Then one of the following holds:*

1. $\varphi^* T_{\mathbb{P}^3}(-1)$ *is stable.*
2. $\varphi^* T_{\mathbb{P}^3}(-1) = M \oplus E$, *where $M$ is a line bundle of degree 2 and $E$ is an indecomposable rank two vector bundle on $C$ of degree 3.*

PROOF : Let $T_{\mathbb{P}^3}$ denotes the tangent bundle on $\mathbb{P}^3$. Consider the Euler exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^3} \to \mathcal{O}_{\mathbb{P}^3}(1)^4 \to T_{\mathbb{P}^3} \to 0. \quad (3.4)$$

Since $\varphi$ is non-degenerate, $\deg(\varphi) \neq 1$ (Lemma 3.1). By the universal property of the normalization of the curve $\varphi(C)$, $C$ is the de-singularization of $\varphi(C)$.
Taking the pullback of the equation (3.4) via $\varphi$, we have

$$0 \to \mathcal{O}_C \to \varphi^*(\mathcal{O}_{\mathbb{P}^3}(1))^4 \to \varphi^* T_{\mathbb{P}^3} \to 0. \quad (3.5)$$

By twisting the equation (3.5) with $\varphi^*\mathcal{O}_{\mathbb{P}^3}(-1)$,

$$0 \to \varphi^*\mathcal{O}_{\mathbb{P}^3}(-1) \to \mathcal{O}_C^4 \to \varphi^* T_{\mathbb{P}^3}(-1) \to 0. \tag{3.6}$$

Then deg$(\varphi^* T_{\mathbb{P}^3}(-1))$ = 5.

In the case, when $\varphi^* T_{\mathbb{P}^3}(-1)$ is indecomposable vector bundle on the elliptic curve $C$, by Lemma 2.1, $\varphi^* T_{\mathbb{P}^3}(-1)$ is semistable. Moreover it is a stable bundle, since rank and degree of $\varphi^* T_{\mathbb{P}^3}(-1)$ are co-prime (Lemma 2.2).

Let us analyse the case, when $\varphi^* T_{\mathbb{P}^3}(-1)$ is not indecomposable. Then it arises the two cases

(A) $\varphi^* T_{\mathbb{P}^3}(-1) = L_1 \oplus L_2 \oplus L_3$,
where $L_i$'s are line bundles on $C$. By the equation (3.6), the degrees of $L_i$'s are nonnegative.

(B) $\varphi^* T_{\mathbb{P}^3}(-1) = M \oplus E$,
where $M$ is a line bundle on $C$ of non-negative degree and $E$ is an indecomposable rank two vector bundle on $C$ of non-negative degree.

*Case (A)* : Suppose that $\varphi^* T_{\mathbb{P}^3}(-1) = L_1 \oplus L_2 \oplus L_3$. Here we list the all possibilities of the triple:

(i) $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (0, 0, 5)$

(ii) $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (1, 0, 4)$

(iii) $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (0, 2, 3)$

(iv) $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (1, 1, 3)$

(v) $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (1, 2, 2)$

The case $(i)$, $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (0, 0, 5)$ is not possible, since $\varphi$ is non-degenerate (by the Proposition 3.1)

The case (ii), (iv) and (v) are not possible because in this cases, $L_1$ is a globally generated line bundle on the elliptic curve $C$ of degree one (by (3.6)), it yields a contradiction of the fact that a line bundle over an elliptic curve of degree one can not be globally generated.

Now suppose that $(\text{deg}L_1, \text{deg}L_2, \text{deg}L_3) = (0, 2, 3)$. Since $H^0(C, L_1) \neq 0$ and $\deg(L_1) = 0$, $L_1 \cong \mathcal{O}_C$. Therefore the equation (3.6) is

$$0 \to \varphi^*(\mathcal{O}_{\mathbb{P}^3}(-1)) \to \mathcal{O}_C^4 \xrightarrow{\eta} \mathcal{O}_C \oplus L_2 \oplus L_3 \to 0 \tag{3.7}$$

The map $\eta$ induces the map

$$H^0(C, \mathcal{O}_C^4) \to H^0(C, \mathcal{O}_C) \oplus H^0(C, L_2) \oplus H^0(C, L_3)$$

defined by

$$\begin{aligned} e_1 &\mapsto (a_1, s_1, t_1) \\ e_2 &\mapsto (a_2, s_2, t_2) \\ e_3 &\mapsto (a_3, s_3, t_3) \\ e_4 &\mapsto (a_4, s_4, t_4) \end{aligned}$$

where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of the four dimensional $\mathbb{C}$-vector space $H^0(C, \mathcal{O}_C^4)$, where $s_1, s_2, s_3, s_4$ are global sections of $L_2$ and $t_1, t_2, t_3, t_4$ are global sections of $L_3$ on $C$. $a_i \in \mathbb{C}$ but not all zero, since $\eta$ is onto. Without loss of generality, we may assume that $a_1 = 1, a_2 = a_3 = a_4 = 0$ , since $\eta$ is onto. Consider the set

$$S = \{Y = s_2t_3 - s_3t_2, Z = s_2t_4 - s_4t_2, W = s_3t_4 - s_4t_3\}$$

By the same argument as in Proposition 3.1, one can check that the set $S$ is base point free. Then, we get the morphism

$$\pi : C \to \mathbb{P}^3$$

defined by

$$x \mapsto [0 : Y(x) : Z(x) : W(x)]$$

Therefore $\pi$ in not non-degenerate. Thus $\varphi$ is not non-degenerate, since $\pi$ and $\varphi$ are dual to each other.

*Case (B)* : When $\varphi^*T_{\mathbb{P}^3} = M \oplus E$, where $M$ is a line bundle on $C$ and $E$ is an indecomposable rank two vector bundle on $C$.

We list all the possibilities of the pair $(M, E)$ in terms of their degrees :

(i) $(\text{deg}M, \text{deg}E) = (0, 5)$

(ii) $(\text{deg}M, \text{deg}E) = (1, 4)$

(iii) $(\text{deg}M, \text{deg}E) = (2, 3)$

(iv) $(\text{deg}M, \text{deg}E) = (3, 2)$

(v) $(\deg M, \deg E) = (4, 1)$

(vi) $(\deg M, \deg E) = (5, 0)$

In the case $(i)$ and $(vi)$, $\varphi$ is not non-degenerate morphism. In the case $(ii)$, by the (3.6) the line bundle $M$ is globally generated line bundle on the elliptic curve $C$ of degree one that yield a contradiction of the fact that says a line bundle of degree one over a elliptic curve can not be globally generated. The case $(iv)$ is also not possible, by Lemma 3.3. In the case $(v)$, we have a globally generated line bundle $\det(E)$ on elliptic curve $C$ of degree one. □

*Recall* : Let $X$ be a variety over a field $\mathbb{F}$. Let $r$ be a fixed positive integer. A vector bundle $E$ over $X$ is said to be generated by sections, if there is an onto morphism of vector bundles on $X$

$$\mathcal{O}_X^r \to E.$$

***Theorem* 3.2** — *Let $C$ be an elliptic curve. Let $F$ be a rank three vector bundle over $C$ of degree $5$ which is one of the form given in Theorem 3.1. Then $F$ is generated by sections. Moreover there is a non-degenerate morphism $\varphi : C \to \mathbb{P}^3$ such that $F = \varphi^* T_{\mathbb{P}^3}(-1)$.*

PROOF : When $F$ is semistable, by Lemma 3.2, $\dim H^0(C, F)$ = 5. When $F = M \oplus E$ where $M$ is a line bundle of degree 2 and $E$ is a rank two indecomposable vector bundle of degree 3. Again by applying Lemma 3.2, we get $\dim H^0(C, E)$ = 3. Therefore $\dim H^0(C, F)$ = 5.

Therefore in both cases $\dim H^0(C, F) = 5$. By suitably choosing four linearly independents global sections, we get an onto morphism

$$\mathcal{O}_C^4 \to F.$$

Equivalently, we get a morphism

$$\varphi : C \to Gr(3, \mathbb{C}^4) = \mathbb{P}^3.$$

$\varphi$ is a required morphism (see [1, Theorem 2.4]).

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