Indian J Pure Appl Math (2022) 53:1083–1089
https://doi.org/10.1007/s13226-022-00220-1

ORIGINAL RESEARCH

Generalized absolute matrix summability of infinite series and Fourier series

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Received: 30 November 2021 / Accepted: 11 January 2022 / Published online: 25 January 2022

Abstract In this paper, two general theorems on absolute matrix summability of infinite series and Fourier series are obtained. Also, two known theorems dealing with $|\bar{N}, p_n|_k$ summability of infinite series and Fourier series are deduced.

Keywords absolute matrix summability · Fourier series · summability factors · infinite series

Mathematics Subject Classification 26D15 · 40D15 · 40F05 · 40G99 · 42A24

1 Introduction

Let $\sum a_n$ be a given infinite series with partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-k} = p_{-k} = 0, \quad k \geq 1).$$

The sequence-to-sequence transformation

$$\omega_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v$$

defines the sequence (ω_n) of the $(\bar{N}, p_n)$ means of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [8]).

The series $\sum a_n$ is said to be summable $| \bar{N}, p_n |_k$, $k \geq 1$, if (see [1])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{k-1} | \omega_n - \omega_{n-1} |^k < \infty.$$

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ...$$

Communicated by T. S. S. R. K. Rao.

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The series $\sum a_n$ is said to be summable $\mid A, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$ and β is a real number, if (see [24])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid A_n(s) - A_{n-1}(s) \mid^k < \infty.$$

If we take $\beta = 1$, $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then $\mid A, p_n, \beta; \delta \mid_k$ summability reduces to $\mid \bar{N}, p_n \mid_k$ summability.

For any sequence (λ_n), it should be noted that $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$, $\Delta^0\lambda_n = \lambda_n$ and $\Delta^k\lambda_n = \Delta\Delta^{k-1}\lambda_n$ for $k = 1, 2, ...$ (see [8]).

2 Known Result

Bor [2] has proved the following theorem on $\left|\bar{N}, p_n\right|_k$ summability by using a positive non-decreasing sequence.

Theorem 1 *Let (X_n) be a positive non-decreasing sequence and suppose that there exist sequences (γ_n) and (λ_n) such that*

$$\mid \Delta\lambda_n \mid \leq \gamma_n, \tag{1}$$
$$\gamma_n \to 0 \quad as \quad n \to \infty, \tag{2}$$
$$\sum_{n=1}^{\infty} n \mid \Delta\gamma_n \mid X_n < \infty, \tag{3}$$
$$\mid \lambda_n \mid X_n = O(1). \tag{4}$$

If

$$\sum_{v=1}^{n} \frac{\mid t_v \mid^k}{v} = O(X_n) \quad as \quad n \to \infty, \tag{5}$$

where (t_n) is the n-th $(C, 1)$ mean of the sequence (na_n), i.e., $t_n = \frac{1}{n+1}\sum_{v=1}^{n} va_v$, and (p_n) is a sequence such that

$$P_n = O(np_n), \tag{6}$$
$$P_n\Delta p_n = O(p_n p_{n+1}), \tag{7}$$

then the series $\sum_{n=1}^{\infty} a_n \frac{P_n\lambda_n}{np_n}$ is summable $\mid \bar{N}, p_n \mid_k$, $k \geq 1$.

3 Main Result

Some works on absolute summability and absolute matrix summability of infinite series and Fourier series have been done (see [3,5,6,9–12,14–23,25–27]). The aim of this paper is to generalize Theorem 1 by using the $\mid A, p_n, \beta; \delta \mid_k$ summability method.

Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, ... \tag{8}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{9}$$

and

$$A_n(s) = \sum_{v=0}^{n} a_{nv}s_v = \sum_{v=0}^{n} \bar{a}_{nv}a_v \tag{10}$$

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{11}$$

Now, we shall prove the following theorem.

Theorem 2 *Let $A = (a_{nv})$ be a positive normal matrix such that*

$$\overline{a}_{n0} = 1, \ n = 0, 1, \ldots, \tag{12}$$

$$a_{n-1,v} \geq a_{nv}, \ \ for \ n \geq v + 1, \tag{13}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{14}$$

$$| \hat{a}_{n,v+1} | = O(v \mid \Delta_v(\hat{a}_{nv}) \mid). \tag{15}$$

Let (X_n) be a positive non-decreasing sequence. If the conditions (1)–(4), (6), (7) *and*

$$\sum_{v=1}^{n} \left(\frac{P_v}{p_v}\right)^{\beta(\delta k + k - 1) - k + 1} \frac{|t_v|^k}{v X_v^{k-1}} = O(X_n) \quad as \quad n \to \infty, \tag{16}$$

$$\sum_{n=v+1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k + k - 1) - k + 1} |\hat{a}_{n,v+1}| = O\left(\left(\frac{P_v}{p_v}\right)^{\beta(\delta k + k - 1) - k + 1}\right), \tag{17}$$

$$\sum_{n=v+1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k + k - 1) - k + 1} |\Delta_v(\hat{a}_{nv})| = O\left(\left(\frac{P_v}{p_v}\right)^{\beta(\delta k + k - 1) - k}\right) \tag{18}$$

are satisfied, then the series $\sum_{n=1}^{\infty} a_n \frac{P_n \lambda_n}{n p_n}$ is summable $\mid A, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$ and $-\beta(\delta k + k - 1) + k > 0$.

We need the following lemmas to prove Theorem 2.

Lemma 1 [13] *If (X_n) is a positive non-decreasing sequence, then under the conditions* (2) *and* (3), *we have*

$$n X_n \gamma_n = O(1) \quad as \quad n \to \infty, \tag{19}$$

$$\sum_{n=1}^{\infty} \gamma_n X_n < \infty. \tag{20}$$

Lemma 2 [2] *If the conditions* (6) *and* (7) *are satisfied, then we have*

$$\Delta\left(\frac{P_n}{n^2 p_n}\right) = O\left(\frac{1}{n^2}\right). \tag{21}$$

4 Proof of Theorem 2

Let (I_n) denotes A-transform of the series $\sum_{n=1}^{\infty} \frac{a_n P_n \lambda_n}{n p_n}$. Then, we have

$$\bar{\Delta} I_n = I_n - I_{n-1} = \sum_{v=1}^{n} \hat{a}_{nv} \frac{a_v P_v \lambda_v}{v p_v} = \sum_{v=1}^{n} \hat{a}_{nv} \frac{v a_v P_v \lambda_v}{v^2 p_v}$$

by (10) and (11), and we get

$$\begin{aligned} \bar{\Delta} I_n &= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv} P_v \lambda_v}{v^2 p_v}\right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn} P_n \lambda_n}{n^2 p_n} \sum_{r=1}^{n} r a_r \\ &= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv}) \frac{(v+1)}{v^2} \frac{P_v \lambda_v}{p_v} t_v + \sum_{v=1}^{n-1} \frac{\hat{a}_{n,v+1} P_v}{p_v} \Delta \lambda_v t_v \frac{(v+1)}{v^2} \end{aligned}$$

$$+\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\lambda_{v+1}\Delta\left(\frac{P_v}{v^2p_v}\right)t_v(v+1)+\frac{a_{nn}P_n\lambda_n}{n^2p_n}(n+1)t_n=I_{n,1}+I_{n,2}+I_{n,3}+I_{n,4}$$

by applying Abel's transformation.
To complete the proof of Theorem 2, we will prove

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\mid I_{n,r}\mid^k<\infty,\quad for\quad r=1,2,3,4.$$

First, applying Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\mid I_{n,1}\mid^k&=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)\left|\Delta_v(\hat{a}_{nv})\right||\lambda_v||t_v|\right)^k\\&=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k\left|\Delta_v(\hat{a}_{nv})\right||\lambda_v|^k\left|t_v\right|^k\right)\\&\quad\times\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}.\end{aligned}$$

By using (8) and (9), we get $\Delta_v(\hat{a}_{nv})=a_{nv}-a_{n-1,v}$.
Also using (8), (12) and (13), we get

$$\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|=\sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv})\leq a_{nn}.$$

Then, by using (14), (18), (6) and (4), we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\mid I_{n,1}\mid^k&=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k\left|\Delta_v(\hat{a}_{nv})\right||\lambda_v|^k\left|t_v\right|^k\right)\\&=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k\left|\Delta_v(\hat{a}_{nv})\right||\lambda_v|^k\left|t_v\right|^k\right)\\&=O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^k\frac{1}{v^k}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\Delta_v(\hat{a}_{nv})|\\&=O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}|\lambda_v|^{k-1}|\lambda_v|\frac{|t_v|^k}{v}\\&=O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}|\lambda_v|\frac{|t_v|^k}{vX_v^{k-1}}.\end{aligned}$$

Now, by applying Abel's transformation and using the conditions (16), (1), (20) and (4), we get

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\mid I_{n,1}\mid^k&=O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\beta(\delta k+k-1)-k+1}\frac{|t_r|^k}{rX_r^{k-1}}\\&\quad+O(1)|\lambda_m|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}\frac{|t_v|^k}{vX_v^{k-1}}\\&=O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|X_v+O(1)|\lambda_m|X_m\end{aligned}$$

$$
\begin{aligned}
&= O(1)\sum_{v=1}^{m-1}\gamma_v X_v + O(1)|\lambda_m|X_m\\
&= O(1) \quad as \quad m\to\infty.
\end{aligned}
$$

Now, using (15), (1), (14), (18), (6), (19) and applying Hölder's inequality, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,2}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|\right)^k\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)v\left|\Delta_v(\hat{a}_{nv})\right|\gamma_v|t_v|\right)^k\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k(v\gamma_v)^k\left|\Delta_v(\hat{a}_{nv})\right||t_v|^k\right)\\
&\quad\times\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\right)^{k-1}\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k(v\gamma_v)^k\left|\Delta_v(\hat{a}_{nv})\right||t_v|^k\right)\\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{vp_v}\right)^k(v\gamma_v)^{k-1}(v\gamma_v)|t_v|^k\\
&\quad\times\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\Delta_v(\hat{a}_{nv})|\\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}v\gamma_v\frac{|t_v|^k}{vX_v^{k-1}}.
\end{aligned}
$$

Then, applying Abel's transformation and using the conditions (16), (3), (20) and (19), we get

$$
\begin{aligned}
\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,2}\mid^k &= O(1)\sum_{v=1}^{m-1}\Delta(v\gamma_v)\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\beta(\delta k+k-1)-k+1}\frac{|t_r|^k}{rX_r^{k-1}}\\
&\quad+O(1)m\gamma_m\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}\frac{|t_v|^k}{vX_v^{k-1}}\\
&= O(1)\sum_{v=1}^{m-1}|\Delta(v\gamma_v)|X_v + O(1)m\gamma_m X_m\\
&= O(1)\sum_{v=1}^{m-1}v|\Delta\gamma_v|X_v + O(1)\sum_{v=1}^{m-1}\gamma_v X_v + O(1)m\gamma_m X_m\\
&= O(1) \quad as \quad m\to\infty.
\end{aligned}
$$

Now, using the fact that $\Delta(\frac{P_v}{v^2 p_v}) = O(\frac{1}{v^2})$ by (21) and using (15), (14), (17), (4), we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}|I_{n,3}|^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right)^k\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\times \left(\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})|\right)^{k-1}\\
&= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|^k \frac{|t_v|^k}{v}\right)\\
&= O(1) \sum_{v=1}^{m} |\lambda_{v+1}|^{k-1}|\lambda_{v+1}| \frac{|t_v|^k}{v} \sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\hat{a}_{n,v+1}|\\
&= O(1) \sum_{v=1}^{m} \left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1} |\lambda_{v+1}| \frac{|t_v|^k}{v X_v^{k-1}}.
\end{aligned}
$$

Then, as in $I_{n,1}$, we have

$$
\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} |I_{n,3}|^k = O(1) \quad as \quad m \to \infty.
$$

Finally, as in $I_{n,1}$, we have

$$
\begin{aligned}
\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,4} \mid^k &= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left(\frac{P_n}{n p_n}\right)^k a_{nn}^k |\lambda_n|^k |t_n|^k\\
&= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\lambda_n|^{k-1}|\lambda_n| \frac{|t_n|^k}{n}\\
&= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\lambda_n| \frac{|t_n|^k}{n X_n^{k-1}}\\
&= O(1) \quad as \quad m \to \infty.
\end{aligned}
$$

This completes the proof of Theorem 2.

5 An application

Let f be a periodic function with period 2π and Lebesgue integrable over $(-\pi, \pi)$. The trigonometric Fourier series of f is defined as

$$
f(x) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} C_n(x)
$$

where

$$
a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \quad \text{and} \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx.
$$

Write

$$
\phi(t) = \frac{1}{2}\{f(x+t) + f(x-t)\} \quad \text{and} \quad \phi_1(t) = \frac{1}{t} \int_0^t \phi(u) du.
$$

If $\phi_1(t) \in \mathcal{BV}(0, \pi)$, then $t_n(x) = O(1)$, where $t_n(x)$ is the n-th $(C, 1)$ mean of the sequence $(nC_n(x))$ (see [7]). By using this fact, the following theorem on absolute matrix summability of the trigonometric Fourier series is obtained.

Theorem 3 *If $\phi_1(t) \in \mathcal{BV}(0, \pi)$, and the sequences (p_n), (λ_n), (γ_n) and (X_n) satisfy the conditions of Theorem 2, then the series $\sum_{n=1}^{\infty} \frac{C_n(x) P_n \lambda_n}{n p_n}$ is summable $\mid A, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$ and $-\beta(\delta k + k - 1) + k > 0$.*

6 Conclusion

If we take $\beta = 1$, $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 2 and Theorem 3, then we get two known theorems on absolute Riesz summability of infinite series and Fourier series (see [4]).

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