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ORIGINAL RESEARCH

Remembering K.R. Parthasarathy (KRP)

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1 S.R.S. Varadhan

I met KRP when I joined Indian Statistical Institute (ISI) in August '59. We were given a desk and told to do research. I had no idea what to do, I was wasting my time playing bridge and reading mystery books. I came to know KRP and Ranga Rao well. Varadarajan was on his way to the US and gave a course using Kelley's book *General topology*. We were intrigued by the Baire Category theorem and used it to prove the existence of distributions indecomposable under convolutions (prime). They convinced me that we should work together on probability. We chose some problems and collectively worked on them.

ISI had many ponds and I wanted to learn to swim. KRP volunteered to teach me. But had to be at 6 am. Calcutta in winter time can be freezing at 6 am. But did not stop KRP from taking a dip. He frowned on any activity other than working and going for walks.

Communication with the Soviet Union was hard those days and Kolmogorov's report on my thesis did not arrive for a long time. By this time KRP was in Moscow for a long visit and helped pry a report from Kolmogorov which was in Russian and he probably provided the translation.

KRP had gone to England and I visited him when he was in Sheffield, He then returned to Mumbai where I visited him. I had met him and Shyama many times in Delhi when he returned to ISI Delhi.

But I will always remember the early days in Calcutta when we got together at 7 am to discuss mathematics and work on problems that we found interesting.

2 B.L.S. Prakasa Rao

Kalyanapuram Rangachari Parthasarathy (aka) KRP was my teacher when I was studying for my M.Stat. course at the Indian Statistical Institute in the then Calcutta during the years 1960–62. He gave a course on Linear Algebra to us. He was probably 25 years old at that time and working for his Ph.D. at the Indian statistical Institute. His lectures were lucid, clear and authoritative. He prepared for his classes with lecture notes which could be compiled into a book form at a later stage. Whether it was in a classroom teaching students or in a conference presenting a paper or in a seminar hall expounding his research, he was meticulous in his preparation and most of us including me learned these methods from him as his students and, later too, as his colleague at the Indian Statistical Institute, Delhi Centre.

I have not had the pleasure of collaborating in his research activities but he encouraged me in various ways throughout my academic career. It is probably known to most of the colleagues at the Indian Statistical Institute

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that he was instrumental for several of us receiving Bhatnagar awards and Fellowships of the scientific Academies in the country, by nominating us as he recognized our abilities and encouraged us in our academic pursuits at the right time.

His main area of research interest was probability starting with information theory for his Ph.D. and later in the study of probability measures in metric spaces and finally in quantum stochastic calculus and related areas. Professor C.R. Rao was the Ph.D. adviser for KRP and suggested to him to work on problems of information theory. After my M.Stat. degree at the Indian Statistical Institute, I spent an year at ISI and moved to Michigan state University in 1963. My work involved limit distribution theory under shape constraints and as a technical tool, I had to use the theory of weak convergence for stochastic processes. Professor J. Sethuraman, another product from the Indian Statistical Institute, was at Stanford University and prepared a technical report on the theory of weak convergence for probability measures. At the same time, KRP was visiting Manchester at the invitation of Professor J. Gani and he prepared a monograph on the topic "Probability measures on metric spaces". At my request, KRP sent a copy of his unpublished lecture notes to me which was later published as a book by Academic Press. I was at Stanford University working with Professor Herman Rubin. Professor Herman Chernoff of Stanford University, the then Editor for Book Reviews of the "Journal of the American Statistical Association" noted that my work involved the ideas of weak convergence and invited me to review the book *Probability Measures on Metric Spaces*. I had the good fortune of reviewing this book which was my first publication in the review category. KRP included work by himself and other members of his "Indian school of Probabilists" V.S. Varadarajan, R. Ranga Rao, S.R.S. Varadhan in this book. The coinage of the phrase "Indian school of Probabilists" is quite appropriate as this group has contributed extensively to the theory of probability measures on abstract spaces from India, even though three of them went abroad after their graduation with Ph.D. from India. This book is still recognized as a fundamental contribution for all those who apply it in their research in statistics, econometrics and other fields.

Apart from the fact that KRP was a great mathematician and probabilist in particular, he was a pious man. In his own words, "The first twenty years of my life were spent in the intensely religious and ritualistic atmosphere of a South Indian brahmin family confined to the towns of Thanjavur and Madras and the paddy fields of the village of Thalanayar in the Thanjavur district. Emphasis was laid at home on the philosophy and teachings of the vaishnavite saints Ramanuja and Desika, the Hindu epics Ramayana and Mahabharata, the devotional songs of Tamil saints known as the alwars and finally the Bhagavad Gita. To this day, I believe in their basic idea that personal and global peace are possible only through renunciation and action with detachment" (Gani (1986), *The Craft of Probabilistic Modeling*, in the Applied Probability Book Series). Indian Statistical Institute and India have lost a great mathematician. I pay my homage to late Professor KRP.

3 S. Ramasubramanian

Even before Professor K.R. Parthasarathy had moved to IIT Delhi during June–July 1973, his awesome reputation had been the topic of respectful discussion among the faculty of the Mathematics department, where I had just finished my M.Sc. Some of my teachers had suggested that I should try to work with KRP for my Ph.D. At that time I had no inkling as to what "working with KRP" would mean; it would take quite a few years to know!

A few days prior to the actual selection process, I picked up enough courage to knock at his door. After introducing myself, I expressed the desire to work under his supervision for my Ph.D. His response: "Very good, but I would like to test your suitability." As it was quite unexpected, I asked for "time-out" even before the start. It turned out to be a 5 hour long ordeal, spread over a couple of days. I could not answer quite a few questions even after generous hints were given. The questions concerned basic real analysis and matrices, I think. There were occasional satisfactory nods from him, even though I could not provide a complete answer. Anyway, he agreed to be my supervisor; more out of pity, I think. The ordeal made me realise the magnitude of my ignorance.

Inder K. Rana, Rajendra Bhatia and I were the students of KRP at that time. We started attending KRP's lectures on Mathematics of Quantum Mechanics. I was also trying to understand the (rigorous) construction of Lebesgue measure, Kolmogorov consistency theorem, probability measures on complete separable metric spaces, etc.

Going to KRP with doubts on mathematical topics was also a great experience. The doubts would be quickly cleared. Also, a lengthy discourse on a related topic would follow. Invariably it would be a monologue with occasional clarificatory questions. I was very amazed by the depth and breadth of his knowledge. Slowly one was developing a feel for some aspects of "good mathematics"; of course, it was also quite intimidating.

On the more pleasant side, often the lengthy sessions were punctuated with tea breaks. During these breaks, KRP would recall his days at ISI Kolkata. Apart from talking about his illustrious contemporaries (Professors V.S. Varadarajan, R. Ranga Rao and S.R.S. Varadhan), he would also narrate anecdotes from those days: How Professor C.R. Rao caught him when he tried to get away from the lecture hall, by coming through the other door of the hall; imitate Professor Mahalanobis, etc. He also recalled with awe Professor J.B.S. Haldane's approach to Science. He would talk about Professor Ratan Lal Brahmachari's experimental demonstration of a leaf, starting with *Lo and behold*, and ending it with a profound Sanskrit sloka reciting it in the Bengali way! About the time he had spent in the then Soviet Union (soon after the Cuban missile crisis), recalling an oft-repeated speech of Khrushchev, of course in fluent Russian; an English translation was also added for our benefit. (Incidentally, Professor Debasis Sengupta, IISc, who had visited Russia in the late 1980s or early '90s, once told me that some of the senior faculty in Russia were fondly recalling about the "short man from India who lectured in fluent Russian at the Steklov Institute within a few months of coming to Russia"!)

These narrations were so good that the *dramatis personae* used to come alive (including any characteristic tilt of head while talking), and of course, enriched by the "masala" added!

A small incident at IIT Delhi also comes to my mind. One day, during his second winter in Delhi, he had come to office nattily dressed, with a coat and tie as well. By a coincidence or otherwise, an unusually large number of people including students dropped by his office. My friend Bhatia aptly commented: "KRP has become a tourist attraction today!" (It was not clear if KRP was conscious of the extra attention he was getting. For him, it might just have been "a day out for his coat and tie".)

KRP also used to talk about books (of general interest) he had recently read. Two of the books that I read later impressed me very much: (i) J.D. Bernal: *Science in History*; (ii) Jacques Monod: *Chance and necessity*.

Happily, such tea sessions continued also when he moved over to ISI Delhi, along with the three of us as part of his baggage. (Incidentally, his admiring audience was larger at ISI Delhi.)

Around that time KRP's attention turned to stochastic calculus, perhaps, due to possible connections with quantum mechanics. He began a series of lectures on Brownian motion and stochastic calculus. I found them very interesting, thanks to his customary masterly exposition.

After the preliminaries on Brownian motion, including the esoteric sample path properties, KRP gave a detailed exposition of the construction of Itô integrals. Then the celebrated Itô's formula was proved, followed by strong solutions of stochastic differential equations.

The last topic taken up was the celebrated Cameron–Martin formula. It was presented as giving the Radon–Nikodym derivative (RND) of the measure, induced by a deterministic translation on the Wiener space, with respect to the Wiener measure. KRP also mentioned that Girsanov's formula concerns the analogous situation for a random translation. (I do not remember the exact reason for the lecture series to stop, perhaps, due to shifting to ISI Delhi.)

Among the books available then, a very user-friendly book was: A. Friedman: *Stochastic Differential Equations and Applications, Vol.1*.

When I looked at Girsanov's formula in that book, it seemed to be very strangely phrased and puzzling. After a couple of months, assuming Lipschitz continuity and other nice conditions in the set-up, I slowly realised that Girsanov's formula concerns the RND of the measure induced by the solution of the concerned Stochastic Differential equation (SDE) with respect to the Wiener measure; however, the RND has to be evaluated at the "original" trajectory, but not at the trajectory of the solution. In other words, under the conditions assumed, random translation and SDE are inverses of each other. In the case of a deterministic translation, clearly they are the same. Moreover, Girsanov's formula is stated in a seemingly strange manner so that it is not saddled with strong Lipschitz continuity conditions.

While reporting the above to KRP, I was half expecting that a hole would be found. On the contrary, KRP seemed very pleased that a student was able to spot an error, and proceed ahead. He was very encouraging. In the course of the next few weeks, he suggested some extensions. That done, he wanted the stuff to be written up and typed. Then KRP sent the write-up to an expert for comments.

What followed next was a hard lesson for me. One had assumed that no one had considered the problem of random translations earlier. If a fresh upstart like me could think of a natural problem, it was presumptuous to assume that experts had not already done so. It turned out that the problem had been solved about 6 years earlier. Well, "Don't count your chickens before they hatch."

Of course, KRP was very supportive; perhaps, he also felt a bit guilty. On the positive side, in my opinion, I had got used to the idea of associating stochastic processes with appropriate sample paths, or equivalently with probability measures on corresponding function spaces. In contrast to the drifting that was happening earlier, I felt I had got a mooring.

As a consequence, I did not feel comfortable with quantum Brownian motion, etc.; in fact, I did not feel enthused enough to even try. Unfortunately, the net effect was that I was drifting away from KRP.

Fortunately, Professor Rabi N. Bhattacharya (RNB) happened to spend a semester at ISI Delhi around that time. He would soon become another teacher and mentor having a lasting influence on me. He gave a series of lectures on recurrence/transience of multidimensional diffusion processes. I found the lectures very interesting. Moreover, thanks to his almost self-effacing demeanour, I felt quite comfortable with him.

RNB had already considered the problem of recurrence/transience of homogeneous diffusions processes. So very soon, I started working on recurrence/transience of nonhomogeneous diffusion processes with him. It was also heartening that KRP enthusiastically encouraged my working with RNB. Within a few months after RNB left Delhi, a write-up of our joint work got ready. This became the first half of my thesis.

Professor Varadhan happened to spend a few days at ISI Delhi shortly after. KRP advised me to meet him along with the write-up. Varadhan suggested that I look at the problem of recurrence/transience of projections of diffusions. While working on this problem, I used to meet KRP often to apprise him of the progress. He was very generous with his time, helping me out with proofs, etc. This became the second half of my thesis.

When I wrote up the initial draft of the thesis, he went through it in his usual meticulous fashion, and suggested quite a few changes, both stylistic and some mathematical. Writing the final draft of the thesis could be completed only in the third week of June 1980.

In the meantime I had got a job offer from North-Eastern Hill University, Shillong. I was required to join by the end of June 1980, in time for the new academic year. One may recall that competent mathematical typing facilities were available only in a very few institutions then. Aware of all these, KRP very magnanimously offered to oversee the typing, proof-reading, getting it bound, and having it submitted without delay. I was very much touched. Even today, when I think of this incident, I am moved by his kindness.

Afterwards, especially since I joined ISI Bangalore, we had occasions to meet. He was always interested in the welfare of my family, and used to inquire specifically about my aged father who was staying with me.

At times he would jokingly comment about my not showing interest in quantum probability. In later years, over email exchanges, though it was not explicitly mentioned, I got the feeling that he did not quite approve of the kind of stuff I was looking at. Well, not everyone can be a KRP.

4 Ram Ramaswamy

When the School of Physical Sciences (SPS) was started at JNU in 1986, our biggest academic support came from KRP. Along with Baidyanath Misra, who was the first Dean of the School, KRP started a seminar series that would alternate between JNU and ISI and which would largely be on topics in statistical physics, stochastic phenomena, ergodic theory and such. I have forgotten how long it lasted, but the series was essential in getting the academic program of SPS off to a start.

I had met KRP earlier, just when I came back to India in 1980 and he had asked me to give a talk at a conference he had organised for C.R. Rao's retirement. After my post-doctoral years in the US I had returned without fixing up a job anywhere, and although I had applied for positions to the TIFR and to IIT Madras, nothing had materialised. Seeing that I was at a loose end, KRP – who was then the head of the Stat-Math Unit at ISI Delhi – gave me an office, and also some kind of a position. This was never quite formalised since within a week of this, I got a Visiting Fellowship at the TIFR which was more what I wanted, but I have never forgotten this generosity of KRP's and have tried to emulate it whenever I could.

Over the years, there have been numerous occasions when I needed to consult KRP for academic matters. When Mustansir Barma and I were working on random walks, we wanted to bounce some ideas off him – he gave us a patient hearing and made us formulate our questions better, which was half the battle won! Some years later he actually came across to SPS to give a set of lectures on ergodic theory to me and my students since I had asked him about invariant measures in some dynamical systems we had been studying. There were many more such instances, given the proximity of JNU and ISI, and the fact that he was such a good teacher.

He will be remembered for his dignity and grace, and for the huge difference he made to academics in India by setting such a fine example of what could be achieved by consistent hard work.

5 Vivek S. Borkar

My first contact with KRP was by mail. When I was ready to graduate, I consulted my favourite teacher, Professor David Blackwell, about applying for jobs in India. KRP was one of the names he suggested I write to. I did, and KRP promptly responded with a message that I should give a talk at ISI on return. That did not happen because I got an offer from the TIFR Centre in Bengaluru (now TIFR Centre for Applicable Mathematics) and ended up going there. I first met KRP in person some time soon after joining the TIFR Centre. I recall feeling quite intimidated by him and tended to shy away from him. He must have noticed this, because at one event in IISc, as I was trying to blend into the curtains, he hailed me with a "Hey, why are you hiding there?" With that and subsequent encounters, I gradually got quite comfortable with him.

I met him often either during my visits to ISI Delhi Centre or at various probability events and other mathematics or statistics gatherings. I had many occasions to hear his introductory and semi-introductory talks on quantum probability. Although I did not venture into that direction, one novel insight I got from him which stuck in my mind was what he considered to be an "algebraic" view of probability. He underscored the fact that what we observe in probability are random variables and the collection thereof is simply an (commutative) algebra. Our view that they are measurable functions defined on some probability space is merely a convenient "representation". From that, he would proceed by analogy, beginning with the analogy between indicator functions and projection operators on a Hilbert space, to introduce the algebra of observables in quantum mechanics and their representation as self-adjoint operators on a Hilbert space. It was fascinating. To date I point out in my beginning lectures on measure theoretic probability the algebraic view of probability, mentioning how I learned it from a great mathematician called K.R. Parthasarathy. (My students are from engineering departments; for mathematics students the adjective would have been unnecessary.)

Another aspect of his lectures that stuck in one's mind was his old school style of lecturing. While writing down mathematical expressions on the blackboard in an impeccable handwriting and in an extremely well organized manner, his slow diction and intonation would remind me of the school headmasters I had as a school going kid. Perhaps that's why, when he taught you something, you "stayed taught".

In connection with quantum probability, I would also like to mention a certain peeve that KRP always carried with him. He mentioned how the Italian mathematicians had taken up quantum probability and were going after it like an army, whereas nothing of the kind happens in India. Coincidentally, only recently while discussing the Dutch and French groups in probability, my friend Rajesh Sundaresan echoed this sentiment when he said, "They work together to develop a field, we work for ourselves."

The memory that stands out, however, is from an event on "Probability and Physics" that took place on a private estate of a retired professor from University of Mysore, who had equipped it for small academic events. It was on the outskirts of Mysore in a very rustic setting. It was December, so the weather was quite chilly and washing hands under the tap was enough to send shivers down one's spine. The place was nice but *sans* modern amenities. The professor briefed us on the routine on our arrival from Bengaluru the evening before the start of the event. We were to get morning tea in a common meeting place in the courtyard while hot water was to be delivered in buckets to our individual rooms. Come morning, I step out looking for hot water and there was no sign of it. KRP, who was my neighbour, came out soon with the same problem on mind. The lack of the much needed hot water was reported to the patriarch of the place and soon enough he was seen shouting in Kannada at an elderly gardener, a thin unshaven man with a beaten look. The gardener was soon seen hobbling about with hot water buckets on his head and we had our much needed hot water baths.

Next morning, we gathered as per instructions for morning tea at the prescribed venue, all of us in our sleeping pyjamas except KRP, who had already bathed and was dressed for the day. As the tea session prolonged over general chitchat, Professor N. Mukunda of IISc, pointed out that the time for the first session was drawing near and we better get back to our rooms. Turning to KRP, he added that the rest of us were not as disciplined and punctual as him, who was ready for the day already. KRP said nothing. As the two of us walked back to our rooms, he mumbled under his breath, "I saw the old man carrying hot water buckets on his head and decided to have cold baths."

There is one last point I need to make because it was always on my mind whenever I thought of KRP. We do draw inspiration from persons of Indian origin who have made seminal contributions to their respective fields. But when these contributions are made outside India, it allows us the loophole of blaming our own underachievements on "Indian conditions". We need such towering personalities *in India* and KRP fitted this role to perfection.

6 R.B. Bapat

My association with KRP was for more than 45 years. In 1975 I joined the first batch of M.Stat. students at ISI Delhi Centre. There were four of us in the batch. KRP taught us two courses. In the beginning he was still at IIT Delhi. So we used to walk to IIT everyday for every class, by cutting across Katwaria Sarai, a village that separates ISI and IIT. We had found a shortcut where we used to jump over a wall and then go to the classroom and then returned. However in the second semester KRP had already joined ISI. The first course was Functional Analysis. I vividly remember the first class that he took. He defined Banach spaces. He simply gave fifteen examples one after another, ranging from very trivial to non-trivial ones, each one with its own justification. It was quite a unique class, something I had never experienced before. The explanation was clear and what was clear on the very first day was his writing on the blackboard. He will not write on the blackboard where already something had been written. He will clean the complete board and then start at the top left hand corner, and proceed very methodically over the board. And this habit we also inculcated in our own teaching. This continued till his later years in the seminars also. So that was my first introduction to KRP.

KRP was always curious about what we are going to do after M.Stat. He very much wanted us to stay at ISI and pursue research there. At least two out of us four had decided to go abroad. He was perhaps not happy with that.

After returning from abroad I joined Mumbai university. I had planned to explore ISI as a job option. At that time KRP was the head of ISI Delhi Centre. I visited there without any formal invitation. I just went there to work with somebody and to meet people. He once called me in the Head's office just to enquire about what I was doing. He asked me about what what support I had got. I said I have no support, I just came on my own. So what was surprising, he immediately brought out a piece of paper and wrote out a fellowship for a month. I think it was some eleven hundred rupees at that time. This was done without any formality or anything. He said you take this, go to the accounts and collect your money. That was sufficient to cover my train expenses as well as my living expenses. This is something I found later on in ISI also in that he always had in mind what exactly the students are doing and what do they want, even though they are not working with him. So that was a very pleasant experience. And not only that he also arranged my talk in the small room near the Head's office which was not a seminar room, where all the professors were present, including Kalyan Mukherjea, B.L.S. Prakasa Rao, B. Ramchandran, S.K. Mitra etc. After the seminar, right then and there he asked me whether I wanted to come to ISI. I sort of said yes, but I needed a formal letter. I went back. After one year a letter came. By that time Ramachandran was the Head, but I am sure that KRP pursued the matter.

After joining ISI my association with KRP was more of a colleague. His area was a little different than mine. So I was not attending his lectures. I would attend the first lecture and skip the rest. My interests were going in a different direction but then we always had the morning tea together along with others. He meticulously came to the morning tea at 11 o'clock where we all met. It was an education to listen to him and learn from his experiences. Over all these 30–40 years he never repeated a story. He had lots of stories about ISI, mathematics, mathematicians in general and current politics. It was very interesting when he spoke something and one had to listen carefully and enjoy.

Later on when I got an A-type flat, it was just next door to KRP. He retired at 60. So almost for three years we were next door neighbours. I could see his lifestyle, which was very simple. He would do all the household work himself along with his wife. He would also clean the steps outside his house which actually belonged to public property or joint property. He would clean the steps going from his house to the ground floor. I think he had a soft corner for the Gandhian philosophy although he never made any fuss about it. He resorted to Khadi and it continued throughout. His simple living was really an example.

KRP was a giant in his area, but at the same time he set an example of modest living and very high achievements. I am sure his work will live forever through his books, his papers and his students. I pay my tribute to his memory.

7 R.L. Karandikar

I had heard of Professor K.R. Parthasarathy (KRP) soon after I joined the Ph.D. program at ISI in 1978. Earlier, when I was a M.Stat. student, we did not hear about names of faculty members at ISI other than those who were teaching in Kolkata. I first met KRP in 1979, when he had come to Kolkata as Head of Delhi Centre of ISI for a meeting of the ISI Council. He heard about me being a Ph.D. student with focus on probability theory and stochastic processes and he wanted to meet me. We met after his council meeting ended, and KRP suggested that

instead of sitting in a room and talking, we could talk while we walk in the residential campus around a pond. There were (and still are) two ponds in the residential campus at ISI Kolkata. Walking around the pond in the evening was in any case a favourite activity of faculty and students at ISI Kolkata (and called *taking rounds*), including mine and so I welcomed his suggestion.

Those who knew KRP would recall that he had a lot of energy even after a full day administrative meeting, specially if he was talking about mathematics! That day we walked for almost 3 hours, with KRP going back and forth between talking about his time as a student at ISI Kolkata, the mathematical ideas that could be traced to those days and his mathematical interests at that point of time.

After few rounds, KRP said, "Lets go to the sheds where I stayed as a student. Perhaps they are still there." After we went there, I realised that what he called sheds was then called block III in the hostel. I had stayed in that block earlier. He showed me the room he had stayed nearly two decades ago as a student. The room in block III was as it was when KRP stayed there. KRP also showed me the rooms where his friends – Varadarajan, Varadhan and Ranga Rao stayed in the vicinity of his room, at different times. Now of course, block III along with the other blocks where students stayed till 1980 have been demolished.

During our rounds, KRP talked about how he along with Varadarajan, Ranga Rao and Varadhan (when they were in Calcutta) used to spend most of the day together, studying or talking about mathematics, teaching others what one of them had read. They would also discuss their ideas which could include their views about what is important, and conjectures about the themes they were discussing. Between the four of them, some of them would be at ISI, while others were away on an academic visit.

KRP talked about the topics they studied and discussed during those days, how Varadarajan had taught him various mathematical concepts walking around the same pond we were walking around that day. I may not remember everything we talked about, but I remember him mentioning that in the mathematics world, while the focus of folks dealing with measure theory was shifting from finite dimensional Euclidean spaces to locally compact topological spaces, the four of them had realised that to deal with various questions on stochastic processes, the natural space to consider was a complete separable metric space. He talked at length about their discussion about convergence in distribution of stochastic processes, I could understand this part as I had already been taught the same in M.Stat. On that day and in our subsequent meetings, he talked as to how they had to read various topics right from scratch, such as descriptive set theory, making efforts to locate relevant books or journals.

Several topics that he talked about were included by KRP in the book he had written in the mid '60s, which I could access in the library and read. He also talked about the food they ate and their life in the hostel days in general. He did ask me questions on what topics I have been studying, how much time I devote to studying etc.

While continuing to talk during our first interaction, KRP talked about what he had been reading those days and he asked me some questions about a Lie group valued Brownian motion (perhaps just to see what all topics I had been introduced to!). When I told him that I don't know what is a Lie group, he told me, "Consider the set of all invertible matrices of a fixed dimension with the sole operation being multiplication – let it be denoted by $\mathcal{G}$." He told me that "a $\mathcal{G}$ valued continuous process X_t, starting at the identity ($X_0 = I$) and having stationary independent increments (for $0 \leq s < t$, $X_s^{-1}X_t$ is the increment from time s to time t) is called a $\mathcal{G}$ valued Brownian motion." Here we could instead consider $X_t X_s^{-1}$ as the increment from s to t, and this would be a different process.

All this was of course oral, while we were going around the pond in ISI campus. He asked me to read this from the book by H.P. McKean and digest what the book says about Lie group valued Brownian motions, for he would be posing some interesting problems on this theme in his next visit! In his next visit to ISI Kolkata, he asked me if I have read about the Lie group valued Brownian motion. I told him yes, and for invertible matrix valued Lie groups, I have understood it mostly.

His question was as follows: Let (X_t) be a Lie group valued Brownian motion. Let μ denote the distribution of $\{X_t : 0 \leq t \leq 1\}$. KRP wanted me to explore an analogue of the Cameron–Martin formula in this case. Since we only have one operation (for matrix valued groups, multiplication), this is about the distribution of $Y_t = A_t X_t$ where A_t is a non-random $\mathcal{G}$-valued continuous function. Let ν be the distribution of $\{Y_t : 0 \leq t \leq 1\}$. Under what conditions on $\{A_t : 0 \leq t \leq 1\}$ is ν absolutely continuous w.r.t. μ, and in that case how to obtain the Radon Nikodym derivative of ν w.r.t. μ. Indeed, he said that "once you have done this, you can go on to obtain analogue of the Girsanov formula for the Lie group valued Brownian motion, where $\{A_t : 0 \leq t \leq 1\}$ above is random, but adapted to $\{X_t : 0 \leq t \leq 1\}$." KRP said that any answer even in the two-dimensional case would be interesting, since this would be first such result for the non-commutative case! Each time we met, he would ask the same question and urged me to spend time on this question. Since I would also tell him what I had been reading and doing for my Ph.D., he would tell me that I am going in the right direction and that he was sure that I

could obtain a Girsanov type formula in this non-commutative setting, if I give this question enough importance and time. After I had completed my Ph.D., I made two trips (each of about a month) to ISI Delhi to work with KRP, who by then had moved to Quantum Probability. I did begin to work on this theme with him (and we wrote a paper with Hudson about the same). I moved away from Quantum Probability later.

However, I was able to find a solution to the question KRP had asked me for long – namely, a Girsanov formula for Lie group valued Brownian motions, soon after my Ph.D.

KRP made his first visit to the US as an invited speaker in the Bernoulli society conference in 1983 at Cornell University. Given that his close friends and collaborators of his early academic life had moved to the US two decades ago, it was surprising that this was his first visit. I was at UNC Chapel Hill at that time and I also was at the Bernoulli conference at Ithaca and met KRP a few times. He was very busy, as several participants were meeting him first time and having read his book *Probability Measures on Metric Spaces*, they wanted to talk to the author!

This was his first major talk on Quantum Probability. He had earlier given a talk at the ISI Golden Jubilee Conference in Bangalore in 1982, but the framework had changed. While having Tea and at lunch time, KRP made it a point to tell whoever was near him that this is his first US visit and his talk made it clear that he is not sure if he wants to come again to this country!

When I returned to India in 1984, I came to ISI Delhi and from then on, I met KRP regularly. The ISI Campus in Delhi is small and most of the faculty stayed on Campus. We could see that his lifestyle was very simple and mathematics was his sole passion.

But KRP liked to relate stories – about mathematics and mathematicians. At ISI Delhi, we used to have tea in the canteen at 11 am and KRP would be there everyday and would be telling stories. And expressing his views, such as "going to see a film is a waste of time – you should instead prove a Theorem." Here, "going to see a film" could be replaced by "having a meeting of faculty!" This had been his view from his student days.

It was in the early '90s that emails became the norm as compared to airmail letters. KRP did not like it and complained that he does not want to learn new things now and waste time. He would rather use the time to prove a Theorem. While he complained, slowly he did devote time to learn, and started using emails. At that time (early '90s), at ISI Delhi, we had to go to the computer center to access emails (there were 2 or 3 terminals for this). When at ISI Delhi we managed to get money to have desktop for every faculty member and one landed over KRP's desk, he complained and said it will sit here and will never be used. Moreover it is taking precious space on his table. We (the "computer committee" at ISI who was responsible for getting the computers and their distribution, setup etc.) just said to him, you write down a request to remove the computer from your table and we will do so. He never wrote, though for months he would complain about the same at tea time. After a few months he started using it! In recent times when his eyesight became very weak, he relied on the computer completely for his work.

Let me recall an incident from January 1982 in Bangalore. ISI had organised several conferences, as part of the Golden Jubilee celebrations of ISI, and in January 1982 the conference on Probability was held. The invited speakers who were present included Professors K. Itô, E.B. Dynkin, G. Kallianpur, P.A. Meyer, H. Tanaka, S.R.S. Varadhan, T. Hida, S. Watanabe, J. Jacod, B. Grigelionis, R.L. Hudson, H.H. Kuo, among others. One evening, we had a special dinner where Professor C.R. Rao was also present. At one point, C.R. Rao, G. Kallianpur, K.R. Parthasarathy, S.R.S. Varadhan were discussing their experiences as academic administrators, and several other folks including me were listening to them. KRP (who then was Head of ISI Delhi center) said, "It is very easy to be an administrator. All one has to do is be democratic", just as he has been. Varadhan interjected, "Democratic and you! You were not democratic even when we were students! One night during that period, I was playing bridge with some of my friends and you barged into the room. And then ignoring everyone else, you said to me to be ready at 5 am as you will teach me xyz Theorem then."

8 B.V. Rajarama Bhat

In the journey of life, certain individuals leave an indelible mark on our hearts and minds, holding a place of profound significance. In my life, Professor K.R. Parthasarathy (KRP) occupies one such unique and cherished position. Reflecting on our interactions, I am overwhelmed with gratitude for the invaluable guidance he bestowed upon me. Here I reflect on my interaction with this legend by recalling a few anecdotes.

Back in 1986–87, while studying in the first year of M.Stat. in Indian Statistical Institute (ISI) Kolkata, some classmates and I were drawn to the renowned faculty and strong academic environment at ISI Delhi. The prospect of learning from luminaries like Professors K.R. Parthasarathy, K.B. Sinha (KBS), R. Bhatia, R.L. Karandikar,

R. Bapat and others was enticing. Amongst us there were also students who wanted to specialize in Economics and they were also attracted by ISI Delhi for similar reasons. After much discussion a sizable chunk of us decided to request for a transfer to ISI Delhi, which was kindly approved by the Institute.

When we moved, the professors of Delhi were very welcoming and clearly we were fortunate to have them as our teachers. But pretty soon we found out that they had very high expectations from us. KRP taught us "Advanced Probability" and as everybody knows he is a great teacher. He was very systematic and had an unmatched capacity to keep us glued to his lectures. I recall him saying "probability is measure theory with a soul", quoting some probabilist. It was a pleasure to attend his lectures every day and we benefited immensely by his insights.

In ISI, the faculty members have the freedom to go beyond the syllabus. In our case, this meant that what was taught had little to do with the prescribed syllabus but more to do with what the professors personally wanted us to learn. Initially, it was very exciting for us. But I must admit that in due course many of us found it quite hard to keep up. So our scores dipped and we had to really sweat it out. Despite the challenges of adapting to the rigorous curriculum, our decision to transfer to ISI Delhi proved transformative. Later my classmates went to different places, for instance Indro (Indranil Biswas) moved to TIFR. Some like Pinu (Pinaki Chattopadhyay) slowly moved away from academics. Amongst students who shifted from Kolkata with the intention of doing mathematics, only myself and Anilesh Mohari stayed on at ISI Delhi for doctoral study.

By now we had understood that Professor K.R. Parthasarathy is a hard taskmaster and is a strict disciplinarian. After some course work, I was thinking of doing a Ph.D. under him. Some of my friends did try to dissuade me. Their reasons were that it is likely to be very hard, KRP is not practical, he is likely to suggest very difficult problems and that every now and then his interest changes and so on. Amidst whispers of caution from peers, I approached KRP with trepidation, seeking his guidance for my Ph.D. To my great relief he agreed. Little did I know that it would set the stage for a transformative journey of learning and personal growth.

In those days, KRP was developing the theory of quantum stochastic calculus. Professor K.B. Sinha (KBS), who had originally come to ISI Delhi on invitation from KRP had also decided to join hands in this goal. The basic framework for the theory in terms of Boson Fock space operators was already available as a result of KRP's groundbreaking joint work done earlier with Professor Robin Hudson. Gradually most of the foundational results such as the Quantum Itô Formula in full generality and existence and uniqueness results of certain natural quantum stochastic differential equations were in place. KRP thought that it is the right time to write an introductory monograph of this theory, and following his usual practice gave us a lecture series on the topic. Other than myself, the audience included Anilesh, who started working under KBS and some others. Subsequently the book came out, titled *An introduction to quantum stochastic calculus*, and it has become a bible for anyone wanting to learn this subject. Around this time an international conference on Quantum Probability was held at ISI Delhi and we students could gauge the respect the international community had for KRP. Two years after me, Arup Pal joined as a research scholar under KRP's guidance and his interest was the then emerging field of quantum groups. We also had long term visitors such as R. Speicher, J. Martin Lindsay and many others. This way, ISI Delhi became a world renowned center for the subject of Quantum Probability.

KRP's pedagogical prowess was unparalleled, captivating audiences with his lucid presentations and insightful explanations. There was unwavering commitment to present the subject in a very systematic way. He used to point out various directions one should be thinking more and the associated problems we should try to solve. It may sound strange, but it is the reality of the academic world that one of the major problems faced by most Ph.D. students and guides is the difficulty in finding a good problem to work on! Either the problems are too hard, which may take years to solve and these days Ph.D. programs don't have that luxury of indefinite time as it used to be the case in the past, or the problems are too easy and their solutions are not worth publishing. Finding the middle ground is the hard part. KRP never bothered about these issues. Almost everyday he would come up with new research problems. He would attend every colloquium talk of the department in whatever field it is. Invariably he would ask various questions during the lecture and also subsequently, and would share them with everyone around. It was for us to pick up a problem and solve it. He would keep us on our toes. One of his favourite advice to us was that "one should prove at least one lemma a day." That isn't easy!

An important discipline we could see in KRP was that he would meticulously take note of every lecture he attended, using a traditional blue ink pen in his beautiful handwriting. All the notebooks were numbered, dated and then carefully and systematically stored in his office cabinets. Whenever required, he could recall the topic, the speaker and recover the notes! I must admit that I could never reach that level of discipline.

He loathed ostentatious inaugural and valedictory functions of conferences and workshops, which is commonplace in our country. I recall that in one such program, his invited plenary talk was the first talk after the "Inaugural Ceremony". The Vice Chancellor took his own sweet time in arriving and as usual there were several

bouquet presentations and longwinded speeches, and the program got delayed quite a bit. When KRP started his lecture, he began with opining that conferences on Operator Theory, should begin with "Let $\mathcal{H}$ be a Hilbert space," and nothing else is needed!

KRP embodied simplicity and humility in his everyday actions. His Gandhian principles and unassuming demeanour left a lasting impression, reminding us of the virtues of integrity and humility. He also had a great sense of humour. Every working day without fail one could see him from morning 9 am onwards in his office, immersed in his research. He used to put a lot of emphasis on cleanliness. I recall that once, perhaps in the coffee interval of some conference, some tea from a cup he was holding got spilt on his table. He was in the midst of some animated discussion. But he interrupted and said, "Maybe some others won't mind it, but I can't continue the discussion without cleaning it up", and that was that.

Anilesh with KBS extended some theory on solutions of quantum stochastic differential equations from finite to infinite degrees of freedom, and that led to his Ph.D. thesis. I had written some research papers, but they were on different topics and was not cogent enough to make a thesis. I was struggling to finish my Ph.D. KRP was unmindful of that. But KBS and Rajendra Bhatia counselled me not to lose my confidence. It took almost six years to finish my Ph.D. KRP was very excited about the subject of "Weak Markov Flows" we could develop. After submission of the thesis, I thought of going abroad to have some post-doctoral experience. But KRP objected. He felt there is so much work to do in the field of "weak Markov flows" and me going away in the middle would be foolish. He also promised to get some fellowship for me to continue. I was confused. Again Rajendra and KBS came to my rescue. They convinced me that it is better for me to have international experience. I suppose that they also spoke to KRP. Finally, he let me go!

After some years of post-doctoral stints in the University of Pisa (under the mentorship of Professor F. Fagnola), Italy and in the Fields Institute (under the mentorship of Professor G. Elliott), I returned to ISI and became a faculty member in the Bangalore Center. Now I was in a position to invite KRP to Bangalore.

I recall that in one of his visits he complained that the tubelight in his guesthouse room is not working. I reported it to our electrician. Later KRP told me laughingly that the electrician was complaining that he has no assistant, he has to do everything and so on, and finally KRP himself had to hold the ladder and pass the new tubelight to the electrician! So we required one electrician and a mathematician to fix a light bulb.

Except for initial one or two conference visits, he came with Shyama. They were happy to stay in our modest guesthouse. Almost every evening we used to walk several rounds in our verdant campus. Invariably KRP would be telling stories about scientists or about his early days in ISI Kolkata. I got a banyan tree planted by him in the campus and similarly I could get a "Ram phal" planted by him in my farm. I thought it is good to have a record of his account of his life and conducted a formal interview. You may see the video here: https://www.youtube.com/watch?v=C7T9REQAuvc. Thanks to Professor C.S. Aravinda and his team, a written version is available in the science magazine Bhavana: https://bhavana.org.in/a-different-kind-of-mind/.

9 Debashish Goswami

I first heard of the legendary mathematician Professor K.R. Parthasarathy (known as KRP in Indian academic circles) when I was a B.Stat. student, perhaps from my teachers or seniors. I was thrilled to know about the only member of the "famous four" (or the "gang of four" in KRP's own witty coinage, the others being V.S. Varadarajan, S.R.S. Varadhan and R. Ranga Rao) of ISI who settled in India. I got my first academic exposure to his work during my M.Stat. first year through the all-time classic *Probability Measures on Metric Spaces* written by him. I saw him from a distance during the month-long stay in the ISI Delhi hostels for the CSO training of our batch. The second year specialisation "Advanced Probability and Related Topics" (A.P.) was offered to our batch both at the Delhi and Bangalore centres of ISI. One of the principal factors for my decision to choose Delhi was the stellar presence of KRP (and several other stalwarts in functional analysis and probability) there. Indeed, it was a decision that shaped and defined my career.

I was lucky to attend the last M.Stat. course taught by KRP before his formal retirement. The name of the course was "harmonic analysis" and it had a unique syllabus, covering both the combinatorial aspects of permutation groups (Young tableaux and so on) on the one hand, and basics of Peter–Weyl theory for compact Lie groups with a detailed analysis of $SU(2)$ and $SO(3)$ on the other. It was such a unique course: combining both discrete and continuous, finite and infinite, with the common theme of symmetry given by group actions which is ubiquitous in mathematics and physics. We were literally spellbound by the beautiful picture of the subject painted by KRP with his masterly brushstrokes, moving smoothly and elegantly from a broad perspective including motivation from physics to rigorous and precise mathematical details. I think I can trace the roots

of my own interest in quantum symmetry, a recurrent theme in my research career, to this course which I was privileged to take from one of the greatest masters of the subject at an early stage of my career. His occasional brief remarks not directly related to the syllabus were keys to hidden treasures, often leading an inquisitive student to discover unexpected links between apparently disparate subjects. I vividly remember how he encouraged us to read Persi Diaconis' invited lecture given at the International Congress of Mathematicians on a connection between the representation theory of permutation groups and the statistical theory of design of experiments. Another such remark was about irreducible representations of the Poincaré group and elementary particles. He was a perfectionist to the highest level. It was amazing to see how meticulously KRP used to prepare his class-notes even though he had perhaps given the same course several times before.

The academic charisma and personality of KRP and KBS (Professor K.B. Sinha) attracted me to join ISI Delhi as a research scholar and work in quantum or noncommutative probability. Although KBS was my official supervisor, I discussed with and learned extensively from both KRP and KBS. In fact, they had a long history of collaboration and were co-creators of a large part of the foundations of the subject. Of course, KRP was one of the founding fathers of quantum probability. It was a great opportunity for a young research scholar like me to learn the subject straight from the horse's mouth. KRP used to drop by the research scholars' office quite often to call me to his office and tell something that he was thinking at the moment, not necessarily having any direct relation to my thesis problems. However, I could understand the deep and ever-lasting impact of such diverse discussions which greatly helped me expand the horizon of knowledge and gave me some sense of unity in diversity in mathematics. He also taught me how to write a paper in a proper way, as the editor of *Sankhya* (an ISI journal) to which I submitted my very first paper (not on my thesis topic but on some problem of classical probability). KRP, with a lot of patience, asked me to make several rounds of revision until he was fully satisfied with the writing. This was a very important lesson for a beginner.

However, academics constitutes only a part of the holistic and enriching personal interaction I had with KRP during my Ph.D. in Delhi. It had an amazing spectrum covering literature, politics, music, culture and what not! I enjoyed the amusing anecdotes about his life in ISI Kolkata during his own Ph.D. and his experience in the USSR during his post-doctoral research with A.N. Kolmogorov (for example, how he could not explain the meaning of vegetarian food to Russians). I listened with awe to his recitation of original Sanskrit verses from Jayadeva's Gita-Govindam and other classics. His profound interest and depth in literature (including Sanskrit and Russian) and culture and his simple lifestyle inspired me in many ways. His personal affection to me went well beyond academic encouragement. After the completion of my Ph.D. I left Delhi and later joined the Kolkata centre of ISI. Since then, I could meet KRP only occasionally. However, I always felt the same aura of affection when we met. He used to call me "Debu", an affectionate nickname abbreviating my first name. When I gave him the news of my wedding by email, telling him the name of my wife (Gopa), he immediately wrote back with his characteristic humour that it is only natural that a "Goswami" (my last name) should get married to "Gopa", referring to the Vaishnavite tradition and literature, of which he had a deep love and knowledge. I was lucky to introduce my family to him on a couple of occasions when he visited Kolkata and his blessings will remain a lifelong asset for them. It is a pity that I could not visit Delhi during the last few years, partly due to the pandemic and also due to my personal preoccupation with my family issues. The last time I met KRP was a conference in Bangalore (KBS Fest) in 2019, to celebrate the 75th birthday of Professor K.B. Sinha. I was getting information about his deteriorating health from colleagues from Delhi. When the pandemic was over, I had a thought to take my family on a trip to Delhi, or at least having Delhi as a place of halt and taking the opportunity to visit the ailing KRP at his residence. Unfortunately, I was too late to carry out such a plan.

The academic legacy of KRP will remain with all those who had the privilege to come in contact with him as students, colleagues and collaborators. For me, he will remain as a towering lighthouse in my life, both academically and personally.

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