



Total controllability of nonlocal semilinear functional evolution equations with non-instantaneous impulses

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Abstract In this article, we are discussing a more vital concept of controllability, termed total controllability. We have considered a nonlocal semilinear functional evolution equation with non-instantaneous impulses and finite delay in Hilbert spaces. A set of sufficient conditions of total controllability is obtained for the evolution system under consideration by imposing the theory of C_0 -semigroup and Banach fixed point theorem. We also established the total controllability results for a functional integro-differential equation. Finally, an example demonstrates the feasibility of derived abstract results.

Keywords Total controllability · Non-instantaneous impulse · Functional evolution equations · Nonlocal condition · Banach fixed point theorem

Mathematics Subject Classification 34K06 · 34A12 · 37L05 · 93B05

1 Introduction

Many phenomena and processes that experience abrupt changes in their state at some instants during the evolution process. Such processes are appropriately modelled by impulsive differential equations. Impulsive differential equations have attracted much attention because of their applications in control theory, economics, electrical engineering, biology, (cf. [4, 11, 13]) etc. The classical instantaneous impulses are not capable to explaining the certain dynamics of evolutionary processes in pharmacotherapy. For examples, the introduction of drugs into the bloodstream, in hydrodynamic equilibrium of a person and the consequent immersion for the body are gradual and continuous processes. Such situations can be suitably described by a new type of impulses, known as non-instantaneous impulses, which start at a fixed point and stay active over a finite time interval. In [16], Hernández and O' Regan introduced evolution equations with non-instantaneous impulses. Later Wang et al. in [13], modified the impulsive conditions considered in [16]. Recently, Pierri et al. [35, 36], established the global

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solutions for a class of impulsive abstract differential equations with non-instantaneous impulses. In [41], Zhang et al., considered an evolution system with non-instantaneous impulses and obtained the existence of extremal mild solutions between upper and lower mild solutions by constructing a new monotone iterative method.

On the other hand, there are several real-world phenomena in which the current state of a system depends on the past states and described by the delay differential equations. For example, the logistic reaction-diffusion model with delay, heat conduction in materials with fading memory, neural networks, inferred grinding models and automatic control systems etc, (cf. [14, 28, 32, 40]). For the basic theory of delay differential equations, one can refer to [4, 15].

The notion of controllability plays a significant role in designing and examining the control systems. Controllability refer that the solution of a dynamical control system can steer from an arbitrary initial state to a desired final state by using some suitable control function. The controllability of infinite-dimensional control systems through various fixed point theorems is studied extensively, see for instance, [5, 6, 11, 17, 18, 21–23, 34] etc and the references therein. In [5], Balachandran investigated the controllability condition for a neutral functional integro-differential system. Benchohra et al. in [6], have utilized Schaefer's fixed point theorem to prove the controllability results of the impulsive functional differential inclusions with nonlocal conditions. In [11], Chang derived sufficient conditions for an impulsive functional differential system with infinite delay to be controllable. In [17], authors concerned the controllability of the impulsive functional differential equations with nonlocal conditions, and established sufficient conditions of the controllability by using the measure of noncompactness and Mönch fixed-point theorem. In [33], Balachandran et al., described sufficient conditions of the controllability for the impulsive neutral integro-differential systems having infinite delay. Lijuan et al. in [37], proved the complete controllability for a class of impulsive stochastic integro-differential system using Schaefer's fixed point theorem. Most of the articles in the literature on the controllability, explore the dynamical systems using traditional initial conditions, whereas the nonlocal conditions are more effective to describing countless practical systems. The concept of nonlocal conditions was first introduced by Byszewski in [8], and established the existence and uniqueness of solution to a semilinear evolution equation. Several works have been reported for various systems with nonlocal conditions, see for instance, [2, 7, 8, 27, 31] etc and the references therein.

Most of the paper discussed above have considered instantaneous impulses. However, there are very few works have been reported on the impulsive control system with non-instantaneous impulses. In [19, 29, 30] etc, considered nonlinear control system with non-instantaneous impulses to establish existence and stability of solutions. Recently, Ahmed et al. in [1], considered a non-instantaneous impulsive Hilfer fractional neutral stochastic integro-differential equation with fractional Brownian motion and nonlocal condition. They established the approximate controllability of the considered system by invoking the Sadovskii's fixed point theorem. In [10], sufficient conditions have been established for the total controllability of the second order semilinear differential equation with infinite delay and non-instantaneous impulses. Muslim et al. in [26], established the total controllability and the observability results for a dynamical system with non-instantaneous impulses on time scales. Necessary and sufficient conditions of the total controllability for a class of time-varying switched dynamical systems with impulses have been obtained in [24, 25]. The tools used for that study included the Gramian matrix. Wang et al. in [39], proved the controllability of fractional non-instantaneous nonlinear impulsive differential inclusions by embedding control only in the last time subinterval. Although, in this paper, we have applied the control for each sub interval of time, which gives rise to the essence of total controllability. Moreover, to the best of our knowledge, no work has been reported so far, concerning the total controllability of semilinear functional differential systems with non-instantaneous impulses and nonlocal initial condition. The present study is motivated by this fact. This article is devoted to the total controllability for the nonlocal semilinear functional differential systems with non-instantaneous impulses of the form:

$$\begin{cases} x'(t) = Ax(t) + Bu(t) + \eta(t, x_t), & t \in \bigcup_{j=0}^n (\lambda_j, t_{j+1}] \subset [0, b] = J, \\ x(t) = v_j(t, x(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ x(t) = \phi(t), & t \in [-\beta, 0], \quad \beta > 0, \\ x(0) = \phi(0) + v(x), \end{cases} \quad (1.1)$$

where the state variable $x(\cdot)$ lies in the Hilbert space $\mathbb{X}$ and the linear operator A generates a C_0 -semigroup $T(t)$ on $\mathbb{X}$. The operator $B : \mathbb{U} \rightarrow \mathbb{X}$ is a bounded linear operator and the control function $u(\cdot) \in L^2(J; \mathbb{U})$, where $\mathbb{U}$ is also a Hilbert space. The function ϕ of the initial condition lies in the space D . Also, the nonlinear function



$\eta : J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}] \times D \rightarrow \mathbb{X}$, where

$$D := \{\phi : [-\beta, 0] \rightarrow \mathbb{X} :$$

ϕ is piecewise continuous having jump discontinuity at finite number
of fixed points $\{\beta_0, \beta_1, \dots, \beta_{l+1}\} \subset [-\beta, 0]$, such that

$$-\beta = \beta_0 \leq \beta_1 < \beta_2 < \dots < \beta_l < \beta_{l+1} = 0\},$$

endowed with the norm $\|\phi\|_D = \frac{1}{\beta} \int_{-\beta}^0 \|\phi(\kappa)\|_{\mathbb{X}} d\kappa$. The non-instantaneous impulsive functions $v_j(t, \kappa(t_j^-))$, $t \in (t_j, \lambda_j]$, for $j = 1, \dots, n$, will specify later and the fixed points λ_j and t_j satisfy $0 = \lambda_0 = t_0 < t_1 < \lambda_1 < t_2 < \dots < t_n < \lambda_n < t_{n+1} = b$. The right and left limit of $\kappa(\cdot)$ at the point $t = t_j$ for $j = 1, \dots, n$, exist and denoted by $\kappa(t_j^-)$ and $\kappa(t_j^+)$ respectively, also $\kappa(t_j^-) = \kappa(t_j)$. For every $t \in J = [0, b]$, the function $\kappa_t(\kappa) = \kappa(t + \kappa)$, $-\beta \leq \kappa \leq 0$ such that $\kappa_t \in D$. The function $v : PC(J_\beta = [-\beta, b]; \mathbb{X}) \rightarrow \mathbb{X}$.

The rest of the article is organized as follows: In Sect. 2, we provide the notations, definitions, assumptions and results which are required to prove the desired results. In Sect. 3, we establish sufficient conditions of the total controllability for the system (1.1). The total controllability results are also achieved for a system governed by an integro-differential equation in Sect. 4. An illustration to demonstrate the implementation of the findings is also discussed in the last section.

2 Preliminaries

In the present section, we provide some fundamental definitions and required assumptions, which is useful to establish sufficient conditions for the total controllability of the system (1.1). The norms in the state space $\mathbb{X}$ and control space $\mathbb{U}$ are denoted by $\|\cdot\|_{\mathbb{X}}$ and $\|\cdot\|_{\mathbb{U}}$ respectively. The space of all bounded linear operators from $\mathbb{U}$ to $\mathbb{X}$ is denoted by $\mathcal{L}(\mathbb{U}; \mathbb{X})$ equipped with the norm $\|\cdot\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})}$. The notation $\mathcal{L}(\mathbb{X})$, represents the space of all bounded linear operators on $\mathbb{X}$ equipped with the norm $\|\cdot\|_{\mathcal{L}(\mathbb{X})}$.

First, we define the following operators [12]

$$\Gamma_{\lambda_j}^{t_{j+1}} := \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) B B^* T(t_{j+1} - \tau)^* d\tau, \quad j = 0, \dots, n, \quad (2.1)$$

where B^* and $T^*(\cdot)$ denote the adjoint operators of B and $T(\cdot)$ respectively.

Let us define a set

$$\begin{aligned} PC(J; \mathbb{X}) := \{ \kappa : J \rightarrow \mathbb{X} : \kappa(\cdot) \text{ is piecewise continuous with jump discontinuity at} \\ \text{finite number of fixed points } \{t_0, t_1, \dots, t_{j+1}\} \subset J\}, \text{ satisfying} \\ 0 = t_0 \leq t_1, \dots, t_j < t_{j+1} = b, \text{ with } \kappa(t_j^-) = \kappa(t_j) \text{ for } j = 1, \dots, n\}, \end{aligned}$$

and the norm on $PC(J; \mathbb{X})$ is defined by

$$\|\psi\|_{PC} := \sup\{\|\psi(\zeta)\|_{\mathbb{X}}, 0 \leq \zeta \leq b\}.$$

The set $PC(J; \mathbb{X})$ form a Banach space under the norm $\|\cdot\|_{PC}$. Moreover, we also define a set

$$PC(J_\beta; \mathbb{X}) := \{ \kappa : J_\beta \rightarrow \mathbb{X} : \kappa|_{t \in [-\beta, 0]} \in D \text{ and } \kappa|_{t \in J} \in PC(J; \mathbb{X})\},$$

equipped with the norm

$$\|\kappa\|_{PCD} = \frac{1}{\beta} \int_{-\beta}^0 \|\phi(\kappa)\|_{\mathbb{X}} d\kappa + \sup\{\|\psi(\zeta)\|_{\mathbb{X}}, 0 \leq \zeta \leq b\}.$$

It is easily verify that the set $PC(J_\beta; \mathbb{X})$ form a Banach space under the norm $\|\cdot\|_{PCD}$.

In order to examine the main results for the system (1.1), we impose the following assumptions:



Assumption 2.1 (H1) There exist constants $K \geq 1$ and M such that $\|T(t)\|_{\mathcal{L}(\mathbb{X})} \leq K$, and $\|B\|_{\mathcal{L}(\mathbb{U};\mathbb{X})} = M$.
 (H2) Let $\eta : J_1 \times D \rightarrow \mathbb{X}$, $J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}]$ be a continuous function and for some positive constant K_1 , the following condition holds

$$\|\eta(t, x) - \eta(t, y)\|_{\mathbb{X}} \leq K_1 \|x - y\|_D, \quad \forall x, y \in D, \quad t \in J_1.$$

Moreover, there exists a positive constant N such that

$$\|\eta(t, x)\|_{\mathbb{X}} \leq N, \quad \forall t \in J_1, \quad x \in D.$$

(H3) The non-instantaneous impulses $v_j \in C(I_j \times \mathbb{X}; \mathbb{X})$, and there exist positive constants L_{v_j} , for $j = 1, \dots, n$ as

$$\|v_j(t, x) - v_j(t, y)\|_{\mathbb{X}} \leq L_{v_j} \|x - y\|_{\mathbb{X}}, \quad \forall t \in I_j = [t_j, \lambda_j], \quad x, y \in \mathbb{X}.$$

Moreover, there exist positive constants C_{v_j} , for $j = 1, \dots, n$, such that

$$\|v_j(t, x)\|_{\mathbb{X}} \leq C_{v_j}, \quad \forall t \in I_j = [t_j, \lambda_j], \quad x \in \mathbb{X}.$$

(H4) The operators $\Gamma_{\lambda_j}^{t_{j+1}}$, $j = 0, 1, \dots, n$, defined in (2.1), are invertible and there exist positive constants δ^j , such that $\left\|(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}\right\|_{\mathcal{L}(\mathbb{X})} \leq \frac{1}{\delta^j}$, $j = 0, 1, \dots, n$.

(H5) The function $v : PC(J_\beta; \mathbb{X}) \rightarrow \mathbb{X}$ are continuous and there exists a positive constant C_v , such that

$$\begin{aligned} \|v(x) - v(y)\|_{\mathbb{X}} &\leq C_v \|x - y\|_{PCD}, \quad \text{for all } x, y \in PC(J_\beta; \mathbb{X}), \\ \|v(x)\| &\leq \tilde{M}, \quad \text{for all } x \in PC(J_\beta; \mathbb{X}). \end{aligned}$$

In view of the articles [3, 13], we give the following definition:

Definition 2.1 A function $x : [-\beta, b] \rightarrow \mathbb{X}$ is called a mild solution of the system (1.1), if it satisfies the following relations:

$$x(t) = \begin{cases} \phi(t), & t \in [-\beta, 0], \quad \beta > 0, \\ T(t)[\phi(0) + v(x)] + \int_0^t T(t-\tau) [Bu(\tau) + \eta(\tau, x_\tau)] d\tau, & t \in (0, t_1], \\ v_j(t, x(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ T(t - \lambda_j)(v_j(\lambda_j, x(t_j^-))) + \int_{\lambda_j}^t T(t-\tau) [Bu(\tau) + \eta(\tau, x_\tau)] d\tau, & \\ & t \in (\lambda_j, t_{j+1}], \quad j = 1, \dots, n. \end{cases} \quad (2.2)$$

Definition 2.2 The impulsive system (1.1) is said to be exactly controllable on J , if for the initial state $x(0) \in D$ and the arbitrary final state $x^b \in \mathbb{X}$, there exists a control $u \in L^2(J, \mathbb{U})$, such that the mild solution (2.2) satisfies $x(b) = x^b$.

Next, to generalize the above definition, we need the following setting.

Definition 2.3 The impulsive system (1.1) is said to be totally controllable on J , if for the initial state $x(0) \in D$ and the arbitrary final state $\zeta_j \in \mathbb{X}$ of each sub-interval $[\lambda_j, t_{j+1}]$ for $j = 0, \dots, n$, there exists a control $u \in L^2(J, \mathbb{U})$, such that the mild solution (2.2) satisfies $x(t_{j+1}) = \zeta_j$ for $j = 0, \dots, n$.

Remark 2.1 If a system is totally controllable on J , then it is exactly controllable on J . However the converse is not true.



3 Total controllability

The total controllability of the system (1.1), is established in this section. To prove the total controllability of the system, first we obtain the bounds of the feedback control function. In the main theorem 3.1, the existence of mild solution and total controllability of the system is proved by invoking the Banach fixed point theorem.

Theorem 3.1 *Under the Assumptions (H1)–(H5), the control system (1.1) is totally controllable on J , provided that*

$$\max \left\{ \max_{1 \leq j \leq n} \left(\left(1 + \frac{M^2 K^2 b}{\delta^j} \right) \left(K K_1 \gamma b + K L_{v_j} \right) \right), \right. \\ \left. \left(\left(1 + \frac{M^2 K^2 b}{\delta^0} \right) \left(K_1 K \gamma b + K C_v \right) \right), \max_{1 \leq j \leq n} L_{v_j} \right\} < 1, \text{ where } \gamma = \frac{b}{\beta}. \quad (3.3)$$

Proof Let α_1 defined by

$$\alpha_1 = \max \left\{ \left(M K Q b + K N b + K [\|\phi(0)\|_{\mathbb{X}} + \bar{M}] \right), \max_{1 \leq j \leq n} \left(M K Q b + K N b + K C_{v_j} \right), \right. \\ \left. \max_{1 \leq j \leq n} C_{v_j} \right\}.$$

Let $\mathcal{Z} := \{\mathcal{x} \in \text{PC}(J; \mathbb{X}) : \mathcal{x}(0) = \phi(0) + v(\mathcal{x})\}$ be the space endowed with the norm $\|\cdot\|_{\text{PC}}$. We define a set

$$\mathbb{W} := \{\mathcal{x} \in \mathcal{Z} : \|\mathcal{x}\|_{\text{PC}} \leq \alpha_1\},$$

where α_1 is a positive constant. Let us define an operator $\xi : \mathcal{Z} \rightarrow \mathcal{Z}$ as

$$(\xi \mathcal{x})(t) = z(t), z(t) = \begin{cases} T(t)[\phi(0) + v(\tilde{\mathcal{x}})] \\ \quad + \int_0^t T(t-\tau)[Bu(\tau) + \eta(\tau, \tilde{\mathcal{x}}_\tau)]d\tau, & t \in (0, t_1], \\ v_j(t, \tilde{\mathcal{x}}(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ T(t - \lambda_j)v_j(\lambda_j, \tilde{\mathcal{x}}(t_j^-)) \\ \quad + \int_{\lambda_j}^t T(t-\tau)[Bu(\tau) + \eta(\tau, \tilde{\mathcal{x}}_\tau)]d\tau, & t \in (\lambda_j, t_{j+1}], \quad j = 1, \dots, n. \end{cases}$$

Now, we prove the existence of a fixed point of the operator ξ in the following steps.

Step 1: First, let us define the feedback control as

$$u(t) = \sum_{j=0}^n u_j(t) \chi_{(\lambda_j, t_{j+1}]}(t), \quad t \in J, \quad (3.4)$$

where

$$u_j(t) = B^* T(t_{j+1} - t) (\Gamma_{\lambda_j}^{t_{j+1}})^{-1} p_j(\mathcal{x}(\cdot)), \quad t \in (\lambda_j, t_{j+1}], \quad j = 0, \dots, n,$$

with

$$p_0(\mathcal{x}(\cdot)) = \zeta_0 - T(t_1)(\phi(0) + v(\tilde{\mathcal{x}})) - \int_0^{t_1} T(t_1 - \tau) \eta(\tau, \tilde{\mathcal{x}}_\tau) d\tau, \\ p_j(\mathcal{x}(\cdot)) = \zeta_j - T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{\mathcal{x}}(t_j^-))) - \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \eta(\tau, \tilde{\mathcal{x}}_\tau) d\tau \\ \text{for } j = 1, \dots, n,$$



where, $\tilde{x} : J_\beta \rightarrow \mathbb{X}$ such that $\tilde{x}(t) = \phi(t)$, $t \in [-\beta, 0)$ and $\tilde{x}(t) = x(t)$, $t \in J$, and $\zeta_j \in \mathbb{X}$ for $j = 0, \dots, n$ are arbitrary. Using (2.2), we compute

$$\begin{aligned} x(t_1) &= T(t_1)[\phi(0) + v(\tilde{x})] + \int_0^{t_1} T(t_1 - \tau)[Bu_0(\tau) + \eta(\tau, \tilde{x}_\tau)]d\tau \\ &= T(t_1)[\phi(0) + v(\tilde{x})] + \int_0^{t_1} T(t_1 - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \\ &\quad + (\Gamma_0^{t_1})(\Gamma_0^{t_1})^{-1} \left[\zeta_0 - T(t_1)[\phi(0) + v(\tilde{x})] \right. \\ &\quad \left. - \int_0^{t_1} T(t_1 - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \right] \\ &= \zeta_0. \end{aligned}$$

Moreover, we estimate

$$\begin{aligned} x(t_{j+1}) &= T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) + \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau)[Bu(\tau) + \eta(\tau, \tilde{x}_\tau)]d\tau \\ &= T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) + \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \\ &\quad + (\Gamma_{\lambda_j}^{t_{j+1}})(\Gamma_{\lambda_j}^{t_{j+1}})^{-1} \left[\zeta_j - T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) \right. \\ &\quad \left. - \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \right] \\ &= \zeta_j, \text{ for } j = 1, \dots, n. \end{aligned}$$

Hence, the control function (3.4) is suitable for the system (1.1). For $t \in (0, t_1]$, we evaluate

$$\begin{aligned} \|u(t)\|_{\mathbb{U}} &\leq \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_1 - t)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_0^{t_1})^{-1}\|_{\mathcal{L}(\mathbb{X})} \left[\|\zeta_0\|_{\mathbb{X}} + K \|\phi(0) + v(\tilde{x})\|_{\mathbb{X}} \right. \\ &\quad \left. + \left\| \int_0^{t_1} T(t_1 - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \right\|_{\mathbb{X}} \right] \\ &\leq \frac{MK}{\delta_0} [\|\zeta_0\|_{\mathbb{X}} + K [\|\phi(0)\|_{\mathbb{X}} + \bar{M}] + KNb], \\ &\leq Q_0. \end{aligned} \tag{3.5}$$

For $t \in (\lambda_j, t_{j+1}]$, $j = 1, \dots, n$, we estimate

$$\begin{aligned} \|u(t)\|_{\mathbb{U}} &\leq \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_{j+1} - t)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}\|_{\mathcal{L}(\mathbb{X})} \left[\|\zeta_j\|_{\mathbb{X}} + K \|v_j(\lambda_j, \tilde{x}(t_j^-))\|_{\mathbb{X}} \right. \\ &\quad \left. + \left\| \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau)\eta(\tau, \tilde{x}_\tau)d\tau \right\|_{\mathbb{X}} \right] \\ &\leq \frac{MK}{\delta_j} [\|\zeta_j\|_{\mathbb{X}} + KC_{v_j} + KNb], \\ &\leq Q_j. \end{aligned} \tag{3.6}$$

Combining (3.5) and (3.6), we obtain

$$\|u(t)\|_{\mathbb{U}} \leq Q, \text{ for } t \in J, \text{ where } Q = \max_{0 \leq j \leq n} \{Q_j\}.$$

Step 2: Next, we prove that ξ maps bounded set to bounded set. We need to show that for $\alpha_1 > 0$, $\xi(\mathbb{W}) \subseteq \mathbb{W}$. Let us take $t \in (\lambda_j, t_{j+1}]$, $j = 1, \dots, n$ and $y \in \mathbb{W}$, we estimate

$$\begin{aligned} \|(\xi y)(t)\|_{\mathbb{X}} &\leq \|T(t - \lambda_j)v_j(\lambda_j, \tilde{y}(t_j^-))\|_{\mathbb{X}} + \int_{\lambda_j}^t \|T(t - \tau)Bu(\tau)\|_{\mathbb{X}}d\tau + \int_{\lambda_j}^t \|T(t - \tau)\eta(\tau, \tilde{y}_\tau)\|_{\mathbb{X}}d\tau \\ &\leq KC_{v_j} + MKQb + KNb. \end{aligned}$$

Thus, we have

$$\|(\xi y)\|_{\text{PC}} \leq \left(MKQb + KNb + KC_{v_j} \right). \quad (3.7)$$

Now, for $t \in [0, t_1]$ and $y \in \mathbb{W}$, we compute

$$\begin{aligned} \|(\xi y)(t)\|_{\mathbb{X}} &\leq \|T(t)[\phi(0) + v(\tilde{y})]\|_{\mathbb{X}} + \int_0^t \|T(t - \tau)Bu(\tau)\|_{\mathbb{X}}d\tau + \int_0^t \|T(t - \tau)\eta(\tau, \tilde{y}_\tau)\|_{\mathbb{X}}d\tau \\ &\leq K[\|\phi(0)\|_{\mathbb{X}} + \bar{M}] + MKQb + KNb. \end{aligned}$$

Hence, we obtain

$$\|(\xi y)\|_{\text{PC}} \leq \left(MKQb + KNb + K[\|\phi(0)\|_{\mathbb{X}} + \bar{M}] \right). \quad (3.8)$$

Similarly for $t \in (t_j, \lambda_j]$, $j = 1, \dots, n$ and $y \in \mathbb{W}$, we obtain

$$\|(\xi y)\|_{\text{PC}} \leq C_{v_j}. \quad (3.9)$$

Summarizing inequalities (3.7), (3.8), (3.9), we have

$$\|(\xi y)\|_{\text{PC}} \leq \alpha_1.$$

Step 3: Next, we will prove that the map ξ is a contraction map. For this, let us take any $x, y \in \mathbb{W}$, $t \in (\lambda_j, t_{j+1}]$, $j = 1, \dots, n$, we compute

$$\begin{aligned} \|(\xi x)(t) - (\xi y)(t)\|_{\mathbb{X}} &\leq \|T(t - \lambda_j)[v_j(\lambda_j, \tilde{x}(t_j^-)) - v_j(\lambda_j, \tilde{y}(t_j^-))]\|_{\mathbb{X}} + \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|\eta(\tau, \tilde{x}_\tau) - \eta(\tau, \tilde{y}_\tau)\|_{\mathbb{X}}d\tau \\ &\quad + \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|B\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})} \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_{j+1} - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}\|_{\mathcal{L}(\mathbb{X})} \|p_j(x) - p_j(y)\|_{\mathbb{X}}d\tau \\ &\leq \|T(t - \lambda_j)\|_{\mathcal{L}(\mathbb{X})} \|v_j(\lambda_j, \tilde{x}(t_j^-)) - v_j(\lambda_j, \tilde{y}(t_j^-))\|_{\mathbb{X}} + \int_{\lambda_j}^t KK_1 \|\tilde{x}_\tau - \tilde{y}_\tau\|_{\mathbb{D}}d\tau \\ &\quad + \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|T(t_{j+1} - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \frac{M^2}{\delta^j} \|p_j(x) - p_j(y)\|_{\mathbb{X}}d\tau. \end{aligned} \quad (3.10)$$

For $\tau \in J$, let us compute

$$\begin{aligned} \|\tilde{x}_\tau - \tilde{y}_\tau\|_{\mathbb{D}} &= \frac{1}{\beta} \int_{-\beta}^0 \|\tilde{x}_\tau(\kappa) - \tilde{y}_\tau(\kappa)\|_{\mathbb{X}}d\kappa = \frac{1}{\beta} \int_{-\beta}^0 \|\tilde{x}(\tau + \kappa) - \tilde{y}(\tau + \kappa)\|_{\mathbb{X}}d\kappa \\ &= \frac{1}{\beta} \int_{\tau-\beta}^{\tau} \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}}du, \text{ where } u = \tau + \kappa. \end{aligned} \quad (3.11)$$



If $\tau < \beta$, then the expression (3.11) is express as

$$\begin{aligned}
 \|\tilde{x}_\tau - \tilde{y}_\tau\|_D &= \frac{1}{\beta} \int_{\tau-\beta}^0 \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du + \frac{1}{\beta} \int_0^\tau \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du \\
 &\leq \frac{1}{\beta} \int_{-\beta}^0 \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du + \frac{1}{\beta} \int_0^b \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du \\
 &\leq \frac{1}{\beta} \int_0^b \sup \|\mathcal{x}(u) - y(u)\|_{\mathbb{X}} du \\
 &\leq \|\mathcal{x} - y\|_{PC} \frac{1}{\beta} \int_0^b du \\
 &\leq \frac{b}{\beta} \|\mathcal{x} - y\|_{PC} \leq \gamma \|\mathcal{x} - y\|_{PC}, \text{ where } \gamma = \frac{b}{\beta}.
 \end{aligned}$$

If $\tau \geq \beta$, then the expression (3.11) is express as

$$\begin{aligned}
 \|\tilde{x}_\tau - \tilde{y}_\tau\|_D &\leq \frac{1}{\beta} \int_0^\tau \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du \\
 &\leq \frac{1}{\beta} \int_0^b \|\tilde{x}(u) - \tilde{y}(u)\|_{\mathbb{X}} du \\
 &\leq \frac{1}{\beta} \int_0^b \sup \|\mathcal{x}(u) - y(u)\|_{\mathbb{X}} du \\
 &\leq \|\mathcal{x} - y\|_{PC} \frac{1}{\beta} \int_0^b du \\
 &\leq \frac{b}{\beta} \|\mathcal{x} - y\|_{PC} \leq \gamma \|\mathcal{x} - y\|_{PC}.
 \end{aligned}$$

Thus, we have

$$\|\tilde{x}_\tau - \tilde{y}_\tau\|_D \leq \gamma \|\mathcal{x} - y\|_{PC}, \tau \in J. \quad (3.12)$$

Using Assumptions 2.1, we estimate

$$\begin{aligned}
 \|p_j(\mathcal{x}) - p_j(y)\|_{\mathbb{X}} &\leq K L_{v_j} \|\mathcal{x} - y\|_{\mathbb{X}} + K K_1 b \|\tilde{x}_\tau - \tilde{y}_\tau\|_D \\
 &\leq K L_{v_j} \|\mathcal{x} - y\|_{\mathbb{X}} + K K_1 \gamma b \|\mathcal{x} - y\|_{PC}, \text{ for } j = 1, \dots, n.
 \end{aligned} \quad (3.13)$$

Combine (3.10), (3.12) and (3.13), we have

$$\begin{aligned}
 &\|(\xi \mathcal{x})(t) - (\xi y)(t)\|_{\mathbb{X}} \\
 &\leq K L_{v_j} \|\mathcal{x} - y\|_{\mathbb{X}} + K K_1 \gamma b \|\mathcal{x} - y\|_{PC} + \frac{M^2 K^2 b}{\delta^j} \left[K L_{v_j} \|\mathcal{x} - y\|_{\mathbb{X}} \right. \\
 &\quad \left. + K K_1 \gamma b \|\mathcal{x} - y\|_{PC} \right].
 \end{aligned}$$

By using the above expression, we obtain

$$\|(\xi \mathcal{x})(t) - (\xi y)(t)\|_{\mathbb{X}} \leq \left(\left(1 + \frac{M^2 K^2 b}{\delta^j} \right) (K K_1 b \gamma + K L_{v_j}) \right) \|\mathcal{x} - y\|_{PC}. \quad (3.14)$$



For $t \in [0, t_1]$ and $x, y \in \mathbb{W}$, we calculate

$$\begin{aligned}
 & \|(\xi x)(t) - (\xi y)(t)\|_{\mathbb{X}} \\
 & \leq \|T(t)[v(\tilde{x}) - v(\tilde{y})]\|_{\mathbb{X}} + \int_0^t \|T(t-\tau)\|_{\mathcal{L}(\mathbb{X})} \|\eta(\tau, \tilde{x}_\tau) - \eta(\tau, \tilde{y}_\tau)\|_{\mathbb{X}} d\tau \\
 & \quad + \int_0^t \|T(t-\tau)\|_{\mathcal{L}(\mathbb{X})} \|B\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})} \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_1-\tau)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_0^{t_1})^{-1}\|_{\mathcal{L}(\mathbb{X})} \|p_0(x) - p_0(y)\|_{\mathbb{X}} d\tau \\
 & \leq \|T(t)\|_{\mathcal{L}(\mathbb{X})} \|v(\tilde{x}) - v(\tilde{y})\|_{\mathbb{X}} + K_1 \int_0^t \|T(t-\tau)\|_{\mathcal{L}(\mathbb{X})} \|\tilde{x}_\tau - \tilde{y}_\tau\|_{\mathbb{D}} d\tau \\
 & \quad + \frac{M^2}{\delta^0} \int_0^t \|T(t-\tau)\|_{\mathcal{L}(\mathbb{X})} \|T(t_1-\tau)^*\|_{\mathcal{L}(\mathbb{X})} \|p_0(x) - p_0(y)\|_{\mathbb{X}} d\tau.
 \end{aligned} \tag{3.15}$$

Further, using Assumptions 2.1, we evaluate

$$\begin{aligned}
 \|p_0(x) - p_0(y)\|_{\mathbb{X}} & \leq K C_v \|\tilde{x} - \tilde{y}\|_{\text{PCD}} + K K_1 b \|\tilde{x}_\tau - \tilde{y}_\tau\|_{\mathbb{D}} \\
 & \leq K C_v \|x - y\|_{\text{PC}} + K K_1 \gamma b \|x - y\|_{\text{PC}}.
 \end{aligned} \tag{3.16}$$

Combine (3.12), (3.15) and (3.16), we obtain

$$\|(\xi x)(t) - (\xi y)(t)\|_{\mathbb{X}} \leq \left(\left(1 + \frac{M^2 K^2 b}{\delta^0} \right) (K K_1 \gamma b + K C_v) \right) \|x - y\|_{\text{PC}}. \tag{3.17}$$

Similarly for $t \in (t_j, \lambda_j]$, $j = 1, \dots, n$ and $x, y \in \mathbb{W}$, we compute

$$\begin{aligned}
 \|(\xi x)(t) - (\xi y)(t)\|_{\mathbb{X}} & \leq \|v_j(t, \tilde{x}(t_j^-)) - v_j(t, \tilde{y}(t_j^-))\|_{\mathbb{X}} \\
 & \leq L_{v_j} \|\tilde{x}(t_j^-) - \tilde{y}(t_j^-)\|_{\mathbb{X}} \\
 & \leq L_{v_j} \|x - y\|_{\mathbb{X}}.
 \end{aligned}$$

Moreover, by the above estimate, we obtain

$$\|(\xi x)(t) - (\xi y)(t)\|_{\mathbb{X}} \leq L_{v_j} \|x - y\|_{\text{PC}}. \tag{3.18}$$

After summarizing the inequalities (3.14), (3.17) and (3.18), we have

$$\|(\xi x) - (\xi y)\|_{\text{PC}} \leq L_F \|x - y\|_{\text{PC}}, \tag{3.19}$$

where

$$\begin{aligned}
 L_F = \max \Big\{ & \max_{1 \leq j \leq n} \left(\left(1 + \frac{M^2 K^2 b}{\delta^j} \right) (K K_1 \gamma b + K L_{v_j}) \right), \left(\left(1 + \frac{M^2 K^2 b}{\delta^0} \right) \right. \\
 & \left. (K K_1 \gamma b + K C_v) \right), \max_{1 \leq j \leq n} L_{v_j} \Big\}.
 \end{aligned} \tag{3.20}$$

From (3.19) and (3.3), we conclude that ξ is a contraction map. This proves the existence of a unique fixed point of ξ . Which is a mild solution of (1.1). Therefore, the system (1.1) is totally controllable. $\square$

4 Total controllability of integro-differential equation

In [9], Chalishajar et al., studied the trajectory controllability of an abstract nonlinear integro-differential system in the finite and infinite dimensional space. Bahuguna et al. [20], established the sufficient conditions for the existence and uniqueness of piecewise continuous mild solutions to fractional integro-differential equations in a Banach space with non-instantaneous impulses using the fixed point theorem. In [42], Zhu et al., considered the initial boundary value problem for a class of nonlinear fractional partial integro-differential equations of mixed type with non-instantaneous impulses in Banach spaces. Sufficient conditions of existence and uniqueness of mild solutions for the equations are obtained via general Banach contraction mapping principal.



Here, we consider a functional integro-differential control system with non-instantaneous impulses in Hilbert space $\mathbb{X}$.

$$\begin{cases} x'(t) = Ax(t) + Bu(t) + \int_0^t \kappa(t-s)q(s, t_s)ds, & t \in \bigcup_{j=0}^n (\lambda_j, t_{j+1}] \subset [0, b] = J, \\ x(t) = v_j(t, x(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ x(t) = \phi(t), & t \in [-\beta, 0], \quad t > 0, \end{cases} \quad (4.21)$$

where $x(\cdot)$ is the state variable, $u(\cdot) \in L^2(J; \mathbb{U})$ is the control function, $\mathbb{U}$ is a Hilbert space, $B \in \mathcal{L}(\mathbb{U}; \mathbb{X})$. In order to prove the total controllability of (4.21), we need the following additional assumptions:

Assumption 4.1 (A1) $\kappa_b = \int_0^t |\kappa(s)|ds$.

(A2) The continuous function $q : J_1 \times D \rightarrow \mathbb{X}$, $J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}]$, satisfies Lipschitz continuity condition, there exists a positive constant L_q such that

$$\|q(t, x) - q(t, y)\|_{\mathbb{X}} \leq L_q \|x - y\|_D, \quad \forall x, y \in D, \quad t \in J_1.$$

Moreover, there exists $S > 0$ such that

$$\|q(t, x)\|_{\mathbb{X}} \leq S, \quad \forall t \in J_1, \quad \text{for } x \in D.$$

Definition 4.1 A function $x \in PC([-\beta, b]; \mathbb{X})$ is called a mild solution of the system (4.21), if the following relations are satisfied

$$x(t) = \begin{cases} \phi(t), & t \in [-\beta, 0], \quad \beta > 0, \\ v_j(t, x(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ T(t)\phi(0) + \int_0^t T(t-\tau) \left[Bu(\tau) + \int_0^\tau \kappa(\tau-\eta)q(\eta, x_\eta)d\eta \right] d\tau, & t \in (0, t_1], \\ T(t-\lambda_j)(v_j(\lambda_j, x(t_j^-))) + \int_{\lambda_j}^t T(t-\tau) \left[Bu(\tau) \right. \\ \left. + \int_0^\tau \kappa(\tau-\eta)q(\eta, x_\eta)d\eta \right] d\tau, & t \in (\lambda_j, t_{j+1}], \quad j = 1, \dots, n. \end{cases} \quad (4.22)$$

Theorem 4.1 If the assumptions (H1), (H3)-(H4) and (A1)-(A2) are fulfilled, then the control system (4.21) is totally controllable, provided that

$$\max \left\{ \max_{1 \leq j \leq n} \left(\left(KL_{v_j} + KL_q \kappa_b \gamma b \right) \left(1 + \frac{M^2 K^2 b}{\delta^j} \right) \right), \right. \\ \left. \left(1 + \frac{M^2 K^2 b}{\delta^0} \right) KL_q \kappa_b \gamma b, \max_{1 \leq j \leq n} L_{v_j} \right\} < 1, \quad \text{where } \gamma = \frac{b}{\beta}. \quad (4.23)$$

Proof Let α_2 defined by

$$\alpha_2 = \max \left\{ \left(K \|\phi(0)\|_{\mathbb{X}} + MKRb + K Sb \kappa_b \right), \max_{1 \leq j \leq n} \left(KC_{v_j} \right. \right. \\ \left. \left. + MKRb + K Sb \kappa_b \right), \max_{1 \leq j \leq n} C_{v_j} \right\}.$$

Let $\mathcal{A} := \{x \in PC(J; \mathbb{X}) : x(0) = \phi(0)\}$ be the space endowed with the norm $\|\cdot\|_{PC}$. We define a set

$$\mathbb{Y} := \{x \in \mathcal{A} : \|\cdot\|_{PC} \leq \alpha_2\},$$



where α_2 is a positive constant. Let us define an operator $\delta : \mathcal{A} \rightarrow \mathcal{A}$ as

$$(\delta x)(t) = z(t),$$

$$z(t) = \begin{cases} T(t)\phi(0) + \int_0^t T(t-\tau) [Bu(\tau) + \int_0^\tau \kappa(\tau-\eta)q(\eta, \tilde{y}_\eta)d\eta] d\tau, & t \in (0, t_1], \\ v_j(t, \tilde{y}(t_j^-)), & t \in (t_j, \lambda_j], \quad j = 1, \dots, n, \\ T(t - \lambda_j)(v_j(\lambda_j, \tilde{y}(t_j^-))) + \int_{\lambda_j}^t T(t-\tau) [Bu(\tau) + \int_0^\tau \kappa(\tau-\eta)q(\eta, \tilde{y}_\eta)d\eta] d\tau, & t \in (\lambda_j, t_{j+1}], \quad j = 1, \dots, n. \end{cases}$$

Now, we prove the existence of a fixed point of the operator δ in the following steps.

Step 1: First, let us define the feedback control as

$$u(t) = \sum_{j=0}^n u_j(t) \chi_{(\lambda_j, t_{j+1}]}(t), \quad t \in J, \quad (4.24)$$

where

$$u_j(t) = B^*T(t_{j+1} - t)^*(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}h_j(x(\cdot)), \quad t \in (\lambda_j, t_{j+1}], \quad j = 0, \dots, n,$$

with

$$h_0(x(\cdot)) = \zeta_0 - T(t_1)\phi(0) - \int_0^{t_1} T(t_1 - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau,$$

$$h_j(x(\cdot)) = \zeta_j - T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) - \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau$$

for $j = 1, \dots, n$,

where, $\tilde{x} : J_\beta \rightarrow \mathbb{X}$ such that $\tilde{x}(t) = \phi(t)$, $t \in [-\beta, 0)$ and $\tilde{x}(t) = x(t)$, $t \in J$, and $\zeta_j \in \mathbb{X}$ for $j = 0, \dots, n$, are arbitrary. Using (4.22), we compute

$$\begin{aligned} x(t_1) &= T(t_1)\phi(0) + \int_0^{t_1} T(t_1 - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \\ &\quad + \int_0^{t_1} T(t_1 - \tau)Bu(\tau)d\tau \\ &= T(t_1)\phi(0) + \int_0^{t_1} T(t_1 - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \\ &\quad + (\Gamma_0^{t_1})(\Gamma_0^{t_1})^{-1} \left[\zeta_0 - T(t_1)\phi(0) - \int_0^{t_1} T(t_1 - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \right] \\ &= \zeta_0. \end{aligned}$$

Moreover, we estimate

$$\begin{aligned} x(t_{j+1}) &= T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) + \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \\ &\quad + \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau)Bu(\tau)d\tau \\ &= T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) + \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \\ &\quad + (\Gamma_{\lambda_j}^{t_{j+1}})(\Gamma_{\lambda_j}^{t_{j+1}})^{-1} \left[\zeta_j - T(t_{j+1} - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-))) \right. \\ &\quad \left. - \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \int_0^\tau \kappa(\tau - \eta)q(\eta, \tilde{x}_\eta)d\eta d\tau \right] \\ &= \zeta_j, \quad \text{for } j = 1, \dots, n. \end{aligned}$$



Hence, the control function (4.24) is suitable for the system (4.21). For $t \in (0, t_1]$, we estimate

$$\begin{aligned} \|u(t)\|_{\mathbb{U}} &\leq \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_1 - t)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_0^{t_1})^{-1}\|_{\mathcal{L}(\mathbb{X})} \left[\|\zeta_0\|_{\mathbb{X}} + K \|\phi(0)\|_{\mathbb{X}} \right. \\ &\quad \left. + \left\| \int_0^{t_1} T(t_1 - \tau) \int_0^\tau \kappa(\tau - \eta) q(\eta, \tilde{x}_\eta) d\eta d\tau \right\|_{\mathbb{X}} \right] \\ &\leq \frac{MK}{\delta^0} [\|\zeta_0\|_{\mathbb{X}} + K \|\phi(0)\|_{\mathbb{X}} + K Sb\kappa_b], \\ &\leq R_0. \end{aligned} \quad (4.25)$$

For $t \in (\lambda_j, t_{j+1}]$ with $j = 1, \dots, n$, we estimate

$$\begin{aligned} \|u(t)\|_{\mathbb{U}} &\leq \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_{j+1} - t)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}\|_{\mathcal{L}(\mathbb{X})} \left[\|\zeta_j\|_{\mathbb{X}} + K \|v_j(\lambda_j, \tilde{x}(t_j^-))\|_{\mathbb{X}} \right. \\ &\quad \left. + \left\| \int_{\lambda_j}^{t_{j+1}} T(t_{j+1} - \tau) \int_0^\tau \kappa(\tau - \eta) q(\eta, \tilde{x}_\eta) d\eta d\tau \right\|_{\mathbb{X}} \right] \\ &\leq \frac{MK}{\delta^j} [\|\zeta_j\|_{\mathbb{X}} + K C_{v_j} + K Sb\kappa_b], \\ &\leq R_j. \end{aligned} \quad (4.26)$$

Combining (4.25) and (4.26), we obtain

$$\|u(t)\|_{\mathbb{U}} \leq R, \text{ for } t \in J, \text{ where } R = \max_{0 \leq j \leq n} \{R_j\}.$$

Step 2: Here, we show that $\delta(\mathbb{Y})$ is bounded. Let us take $t \in (\lambda_j, t_{j+1}]$, $j = 1, \dots, n$ and $y \in \mathbb{Y}$, we estimate

$$\begin{aligned} \|(\delta y)(t)\|_{\mathbb{X}} &\leq \|T(t - \lambda_j)(v_j(\lambda_j, \tilde{y}(t_j^-)))\|_{\mathbb{X}} + \int_{\lambda_j}^t \|T(t - \tau)Bu(\tau)\|_{\mathbb{X}} d\tau \\ &\quad + \int_{\lambda_j}^t \left\| T(t - \tau) \left[\int_0^\tau \kappa(\tau - \eta) q(\eta, \tilde{y}_\eta) d\eta \right] \right\|_{\mathbb{X}} d\tau. \\ &\leq K C_{v_j} + M K R b + K Sb\kappa_b. \end{aligned}$$

Thus, we have

$$\|(\delta y)\|_{\text{PC}} \leq \left(K C_{v_j} + M K R b + K Sb\kappa_b \right). \quad (4.27)$$

Now, for $t \in [0, t_1]$ and $y \in \mathbb{Y}$, we compute

$$\begin{aligned} \|(\delta y)(t)\|_{\mathbb{X}} &\leq \|T(t)\phi(0)\|_{\mathbb{X}} + \int_0^t \|T(t - \tau)Bu(\tau)\|_{\mathbb{X}} d\tau + \int_0^t \left\| T(t - \tau) \left[\int_0^\tau \kappa(\tau - \eta) q(\eta, \tilde{y}_\eta) d\eta \right] \right\|_{\mathbb{X}} d\tau \\ &\leq K \|\phi(0)\|_{\mathbb{X}} + M K R b + K Sb\kappa_b. \end{aligned}$$

Hence, we get

$$\|(\delta y)\|_{\text{PC}} \leq \left(K \|\phi(0)\|_{\mathbb{X}} + M K R b + K Sb\kappa_b \right). \quad (4.28)$$

Similarly for $t \in (t_j, \lambda_j]$, $j = 1, \dots, n$ and $y \in \mathbb{Y}$, we get

$$\|(\delta y)\|_{\text{PC}} \leq C_{v_j}. \quad (4.29)$$



Using the inequalities (4.27), (4.28) and (4.29), we obtain

$$\|(\delta y)\|_{\text{PC}} \leq \alpha_2.$$

Step 3: Next, we will show that the map δ is a contraction map. Let us take any $x, y \in \mathbb{Y}$, $t \in (\lambda_j, t_{j+1}]$, $j = 1, \dots, n$, we compute

$$\begin{aligned} & \|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \\ & \leq \|T(t - \lambda_j)(v_j(\lambda_j, \tilde{x}(t_j^-)) - T(t - \lambda_j)(v_j(\lambda_j, \tilde{y}(t_j^-)))\|_{\mathbb{X}} \\ & \quad + \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|B\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})} \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_{j+1} - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_{\lambda_j}^{t_{j+1}})^{-1}\|_{\mathcal{L}(\mathbb{X})} \|h_j(x) - h_j(y)\|_{\mathbb{X}} d\tau \\ & \quad + \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \left[\int_0^\tau |\kappa(\tau - \eta)| \|q(\eta, \tilde{x}_\eta) - q(\eta, \tilde{y}_\eta)\|_{\mathbb{X}} d\eta \right] d\tau \\ & \leq \|T(t - \lambda_j)\|_{\mathcal{L}(\mathbb{X})} \|v_j(\lambda_j, \tilde{x}(t_j^-)) - v_j(\lambda_j, \tilde{y}(t_j^-))\|_{\mathbb{X}} \\ & \quad + \frac{M^2}{\delta j} \int_{\lambda_j}^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|T(t_{j+1} - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \|h_j(x) - h_j(y)\|_{\mathbb{X}} d\tau \\ & \quad + \int_{\lambda_j}^t K L_q \kappa_b \|\tilde{x}_\eta - \tilde{y}_\eta\|_{\mathbb{D}} d\tau. \end{aligned} \quad (4.30)$$

Using Assumptions 4.1, we estimate

$$\|h_j(x) - h_j(y)\|_{\mathbb{X}} = K L_{v_j} \|x - y\|_{\mathbb{X}} + K L_q b \kappa_b \|\tilde{x}_\eta - \tilde{y}_\eta\|_{\mathbb{D}}. \quad (4.31)$$

Combine (3.12), (4.30) and (4.31), we obtain

$$\begin{aligned} & \|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \\ & \leq K L_{v_j} \|x - y\|_{\mathbb{X}} + K L_q \kappa_b \gamma b \|x - y\|_{\text{PC}} \\ & \quad + \frac{M^2 K^2 b}{\delta j} \left[K L_{v_j} \|x - y\|_{\mathbb{X}} + K L_q \kappa_b \gamma b \|x - y\|_{\text{PC}} \right]. \end{aligned}$$

By the above expression, we estimate

$$\begin{aligned} & \|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \\ & \leq \left(\left(K L_{v_j} + K L_q \kappa_b \gamma b \right) \left(1 + \frac{M^2 K^2 b}{\delta j} \right) \right) \|x - y\|_{\text{PC}}. \end{aligned} \quad (4.32)$$

For any $t \in [0, t_1]$ and $x, y \in \mathbb{Y}$, we obtain

$$\begin{aligned} & \|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \\ & \leq \int_0^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|B\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})} \|B^*\|_{\mathcal{L}(\mathbb{X}; \mathbb{U})} \|T(t_1 - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \|(\Gamma_0^{t_1})^{-1}\|_{\mathcal{L}(\mathbb{X})} \|h_0(x) - h_0(y)\|_{\mathbb{X}} d\tau \\ & \quad + \int_0^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \left[\int_0^\tau |\kappa(\tau - \eta)| \|q(\eta, \tilde{x}_\eta) - q(\eta, \tilde{y}_\eta)\|_{\mathbb{X}} d\eta \right] d\tau. \\ & \leq \frac{M^2}{\delta^0} \int_0^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|T(t_1 - \tau)^*\|_{\mathcal{L}(\mathbb{X})} \|h_0(x) - h_0(y)\|_{\mathbb{X}} d\tau \\ & \quad + L_q \kappa_b \int_0^t \|T(t - \tau)\|_{\mathcal{L}(\mathbb{X})} \|\tilde{x}_\eta - \tilde{y}_\eta\|_{\mathbb{D}} d\tau. \end{aligned} \quad (4.33)$$

Using Assumptions 4.1, we estimate

$$\|h_0(x) - h_0(y)\|_{\mathbb{X}} = K L_q b \kappa_b \|\tilde{x}_\eta - \tilde{y}_\eta\|_{\mathbb{D}}. \quad (4.34)$$



Combine (3.12), (4.33) and (4.34), we obtain

$$\|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \leq \left(1 + \frac{M^2 K^2 b}{\delta^0}\right) K L_q \kappa_b \gamma b \|x - y\|_{\text{PC}}. \quad (4.35)$$

Similarly for $t \in (t_j, \lambda_j]$, $j = 1, \dots, n$ and $x, y \in \mathbb{Y}$, we have

$$\begin{aligned} \|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} &\leq \|v_j(t, \tilde{x}(t_j^-)) - v_j(t, \tilde{y}(t_j^-))\|_{\mathbb{X}} \\ &\leq L_{v_j} \|\tilde{x}(t_j^-) - \tilde{y}(t_j^-)\|_{\mathbb{X}} \\ &\leq L_{v_j} \|x - y\|_{\mathbb{X}}. \end{aligned}$$

Using the above estimate, we obtain

$$\|(\delta x)(t) - (\delta y)(t)\|_{\mathbb{X}} \leq L_{v_j} \|x - y\|_{\text{PC}}. \quad (4.36)$$

The inequalities (4.32), (4.35) and (4.36) gives

$$\|(\delta x) - (\delta y)\|_{\text{PC}} \leq L'_F \|x - y\|_{\text{PC}}, \quad (4.37)$$

where

$$\begin{aligned} L'_F = \max \left\{ \max_{1 \leq j \leq n} \left(\left(K L_{v_j} + K L_q \kappa_b \gamma b \right) \left(1 + \frac{M^2 K^2 b}{\delta^j} \right) \right), \right. \\ \left. \left(1 + \frac{M^2 K^2 b}{\delta^0} \right) K L_q \kappa_b \gamma b, \max_{1 \leq j \leq n} L_{v_j} \right\}. \end{aligned} \quad (4.38)$$

By the condition (4.23), we have $L'_F < 1$. Hence, by the estimate (4.37), we conclude that δ is a contraction map. Then δ has a unique fixed point. Which is a mild solution of (4.21). Thus, the system (4.21) is totally controllable. $\square$

The major challenges to construct contraction coefficients L_F , L'_F given in (3.20) and (4.38), and construct the suitable control such that the exact controllability condition holds in each sub-interval $[\lambda_j; t_{j+1}]$ for $j = 0, \dots, n$.

The proposed problems concerned with integer order impulsive systems with time delay (which is finite and independent from a spatial variable). However, in the real-world problems, there are more complicated situations in which the delays depend on the unknown functions. Modeled equations of these problems are frequently called equations with state-dependent delay. In addition, fractional differential equations are a generalization of integer-order differential equations. This kind of equations have applications in many fields of research, including population models, heat diffusion in porous media, imaging processing, viscoelasticity, anomalous diffusion, and many others. Therefore, in future, we will try to focus our study to extend the established results for fractional order semilinear impulsive systems with state-dependent delay.

5 Application

Example 5.1 Let us consider the following transport partial differential equation with impulsive effect and nonlocal condition:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} z(t, \hbar) = \frac{\partial}{\partial \hbar} z(t, \hbar) + \mu(t, \hbar) + F(t, z(t - \beta, \hbar)), \quad 0 \leq \hbar < \infty, \\ \quad \quad \quad t \in (\lambda_j, t_{j+1}), \quad j = 0, \dots, n, \quad \beta > 0, \\ z(0, \hbar) = 0, \quad 0 \leq \hbar < \infty, \\ z(t, \hbar) = v_j(t, z(t_j^-, \hbar)), \quad t \in (\lambda_j, t_j], \quad j = 1, \dots, n, \quad 0 \leq \hbar < \infty, \\ z(t, \hbar) = \phi(t, \hbar) + v(z(t, \hbar)), \quad t \in [-\beta, 0], \quad 0 \leq \hbar < \infty, \end{array} \right. \quad (5.39)$$

where $\phi : [-\beta, 0] \times [0, \infty) \rightarrow \mathbb{R}$, is piecewise continuous function.

Take $J = [0, b]$, $\mathbb{X} = \mathbb{U} = L^2([0, \infty); \mathbb{R})$. Let the operator $A : D(A) \subset \mathbb{X} \rightarrow \mathbb{X}$ be defined as

$$Av = \frac{\partial}{\partial \hbar} v, \text{ where } D(A) = H^1(0, \infty).$$

One can easily verify that $D(A)$ is dense in $\mathbb{X}$ and the operator A with this domain is closed.

Also the operator A is the infinitesimal generator of a contraction semigroup $T(t)$ on $\mathbb{X}$. The semigroup $T(t)_{t \geq 0}$ (see, Example 2.3.7 in [38]) is given by

$$(T(t)v)(\hbar) = v(\hbar + t), \hbar \in [0, \infty), t \in J.$$

Notice that the adjoint $T^*(t)$ is given by

$$(T^*(t)v)(\hbar) = \begin{cases} v(\hbar - t), & \hbar \geq t, \\ 0, & 0 \leq \hbar < t. \end{cases}$$

Next, we define the operator $B : L^2([0, \infty); \mathbb{R}) \rightarrow \mathbb{X}$, such that

$$B(u(t))(\hbar) = u(t)(\hbar) = \mu(t, \hbar), t \in J, \hbar \in [0, \infty).$$

Let us estimate

$$\begin{aligned} \|T(t)v\|_{\mathbb{X}}^2 &= \int_0^\infty |(T(t)v)(\hbar)|^2 d\hbar = \int_0^\infty |v(\hbar + t)|^2 d\hbar \\ &= \int_t^\infty |v(\hbar)|^2 d\hbar \leq \int_0^\infty |v(\hbar)|^2 d\hbar = \|v\|_{\mathbb{X}}^2, \end{aligned}$$

so that we get $\|T(t)\|_{\mathcal{L}(\mathbb{X})} \leq 1$. It is clear that $\|B\|_{\mathcal{L}(\mathbb{U}; \mathbb{X})} = 1$ (as $B = I$ (identity operator)). Hence, the assumption (H1) holds.

Next, we will verify the assumption (H4). In order to do this, we compute

$$\begin{aligned} \int_{\lambda_j}^{t_{j+1}} \|B^*T^*(s)v\|_{\mathbb{U}}^2 ds &= \int_{\lambda_j}^{t_{j+1}} \|T^*(s)v\|_{\mathbb{X}}^2 ds \\ &= \int_{\lambda_j}^{t_{j+1}} \int_0^\infty |(T^*(s)v)(\hbar)|^2 d\hbar ds \\ &= \int_{\lambda_j}^{t_{j+1}} \int_0^s |(T^*(s)v)(\hbar)|^2 d\hbar ds + \int_{\lambda_j}^{t_{j+1}} \int_s^\infty |(T^*(s)v)(\hbar)|^2 d\hbar ds \\ &= \int_{\lambda_j}^{t_{j+1}} \int_s^\infty |v(\hbar - s)|^2 d\hbar ds \\ &= \int_{\lambda_j}^{t_{j+1}} \int_0^\infty |v(\hbar)|^2 d\hbar ds \\ &= (t_{j+1} - \lambda_j) \|v\|_{\mathbb{X}}^2. \end{aligned}$$

By taking $\delta' = \min\{(t_{j+1} - \lambda_j) : j = 0, \dots, n\}$, we have

$$\int_{\lambda_j}^{t_{j+1}} \|B^*T^*(s)v\|_{\mathbb{U}}^2 ds \geq \delta' \|v\|_{\mathbb{X}}^2.$$

Hence, the above fact implies that the corresponding linear system is exactly controllable (see, [19]) on $J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}]$. Hence, the assumption (H4) holds.

Let the functions $\kappa : J \rightarrow \mathbb{X}$ and $\phi : [-\beta, 0] \rightarrow \mathbb{X}$ be given by

$$\begin{aligned} \kappa(t)(\hbar) &= z(t, \hbar), \hbar \in [0, \infty), \\ \phi(t)(\hbar) &= \phi(t, \hbar), \hbar \in [0, \infty). \end{aligned}$$



We now consider the non-instantaneous impulse

$$v_j(t, \kappa(t_j^-))(\tilde{h}) := v_j(t, z(t_j^-, \tilde{h})), \quad j = 1, \dots, n.$$

Let us choose

$$v_j(t, \kappa) = t\kappa, \quad \text{for } t \in (t_j, \lambda_j], \quad j = 1, \dots, n.$$

Now we calculate

$$\begin{aligned} \|v_j(t, \kappa) - v_j(t, y)\|_{\mathbb{X}} &= \|t\kappa - ty\|_{\mathbb{X}} = |t|\|\kappa - y\|_{\mathbb{X}} \\ &\leq b\|\kappa - y\|_{\mathbb{X}}. \end{aligned}$$

Hence, impulse functions satisfy (H3).

Case 1: First, let us consider the following nonlinear function and nonlocal condition.

The nonlinear function $\eta : J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}] \times D \rightarrow \mathbb{X}$ is defined as

$$\begin{aligned} F(t, z(t - \beta, \tilde{h})) &= k_0 \sin(z(t - \beta), \tilde{h}), \\ \eta(t, \kappa_t)(\tilde{h}) &= F(t, z(t - \beta, \tilde{h})) = k_0 \sin(\kappa_t), \quad \tilde{h} \in [0, \infty), \end{aligned}$$

where D is defined in section 2. We now estimate

$$\begin{aligned} \|\eta(t, \kappa_t) - \eta(t, y_t)\|_{\mathbb{X}} &= \|k_0 \sin(\kappa_t) - k_0 \sin(y_t)\|_{\mathbb{X}} \\ &= \left\| k_0 \cdot 2 \cos\left(\frac{\kappa_t + y_t}{2}\right) \sin\left(\frac{\kappa_t - y_t}{2}\right) \right\|_{\mathbb{X}} \\ &\leq k_0 \|\kappa_t - y_t\|_D. \end{aligned}$$

Also, $\|\eta(t, \kappa_t)\| \leq k_0$. Hence, the nonlinear function satisfies the Assumption (H2).

Now we consider $v : PC([0, b]; \mathbb{X}) \rightarrow \mathbb{X}$ such that

$$(v(\kappa))(t)(\tilde{h}) := v(z(t, \tilde{h})), \quad t \in [0, b].$$

We can choose the function v as

$$(v(\kappa))(t)(\tilde{h}) := \sum_{j=1}^n \alpha_j z(t_j, \tilde{h}), \quad t_j \in J, \quad \tilde{h} \in [0, \infty),$$

where α_j 's are small enough, then v satisfy Hypothesis (H5) (see [27]). Substituting all of the above expressions in the system (5.39), it can be expressed as an abstract form given in (1.1) satisfying the Assumptions 2.1. Finally, by Theorem 3.1, the semilinear system (5.39) (equivalent to the system (1.1)) is totally controllable.

Case 2: Secondly, let us define nonlinear function and nonlocal condition as follows:

$$\begin{aligned} F(t, z(t - \beta, \tilde{h})) &= \int_0^t (t - s) \rho(s, z(s - \beta, \tilde{h})) ds, \\ \text{where } \rho(s, z(s - \beta, \tilde{h})) &= \frac{e^{-t}|z(t - \beta, \tilde{h})|}{(a + 2e^t)(1 + 2|z(t - \beta, \tilde{h})|)}, \quad a > -1. \end{aligned} \quad (5.40)$$

Let us define $q : J_1 = \bigcup_{j=0}^n [\lambda_j, t_{j+1}] \times D \rightarrow \mathbb{X}$,

$$\begin{aligned} q(t, \kappa_t)(\tilde{h}) &= \rho(s, z(s - \beta, \tilde{h})) = \frac{e^{-t}|\kappa_t|}{(a + 2e^t)(1 + 2|\kappa_t|)} \\ \|q(t, \kappa_t) - q(t, y_t)\| &= \left\| \frac{e^{-t}|\kappa_t|}{(a + 2e^t)(1 + 2|\kappa_t|)} - \frac{e^{-t}|y_t|}{(a + 2e^t)(1 + 2|y_t|)} \right\| \\ &\leq \left\| \frac{e^{-t}}{(a + 2e^t)} \right\| \left\| \frac{|\kappa_t|}{(1 + 2|\kappa_t|)} - \frac{|y_t|}{(1 + 2|y_t|)} \right\| \\ &\leq L_q \|\kappa_t - y_t\|, \end{aligned}$$



for $\kappa, y \in \mathbb{X}$ and $L_q = \frac{1}{a+2}$ and $\|q(t, \kappa_t)\| \leq 1$. Hence the Assumption (A2) is satisfied. Now, the nonlocal condition is defined as

$$\xi(\kappa)(t)(\hbar) := 0. \quad (5.41)$$

Partial differential system (5.39) can be expressed in abstract form (4.21), by substituting nonlinear function (5.40), nonlocal condition (5.41) and all other substitution are same as in case 1. Finally, by applying the Theorem 4.1, we can conclude that the semilinear system (5.39) (equivalent to (4.21)) is totally controllable.

Conclusion: In this work, we have studied the total controllability of nonlocal semilinear functional evolution equations with non-instantaneous impulses and finite delay in Hilbert spaces. Sufficient conditions have been imposed to get the total controllability by using C_0 -semigroup along with the Banach fixed point theorem. We have also established the total controllability results for a functional integro-differential equation. Finally, we investigate the total controllability of the transport equation by the developed results.

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