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ORIGINAL RESEARCH

Sufficient conditions for fractional $[a, b]$-deleted graphs

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Abstract Let a and b be two positive integers with $a \leq b$, and let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Let $h : E(G) \to [0, 1]$ be a function. If $a \leq \sum_{e \in E_G(v)} h(e) \leq b$ holds for every $v \in V(G)$, then the subgraph of G with vertex set $V(G)$ and edge set F_h, denoted by $G[F_h]$, is called a fractional $[a, b]$-factor of G with indicator function h, where $E_G(v)$ denotes the set of edges incident with v in G and $F_h = \{e \in E(G) : h(e) > 0\}$. A graph G is defined as a fractional $[a, b]$-deleted graph if for any $e \in E(G)$, $G - e$ contains a fractional $[a, b]$-factor. The size, spectral radius and signless Laplacian spectral radius of G are denoted by $e(G)$, $\rho(G)$ and $q(G)$, respectively. In this paper, we establish a lower bound on the size, spectral radius and signless Laplacian spectral radius of a graph G to guarantee that G is a fractional $[a, b]$-deleted graph.

Keywords Graph · Size · Spectral radius · Signless Laplacian spectral radius · Fractional $[a, b]$-deleted graph

Mathematics Subject Classification 05C50 · 05C70 · 05C72

1 Introduction

In this paper, we only deal with finite undirected graphs which have neither loops nor multiple edges. Let G be a graph with vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$ and edge set $E(G)$. The order and size of G are denoted by $|V(G)| = n$ and $|E(G)| = e(G)$, respectively. For a vertex v of G, the degree of v in G, denoted by $d_G(v)$, is the number of vertices in G which are adjacent to v. Let $\delta(G) = \min\{d_G(v) : v \in V(G)\}$ denote the minimum degree of G. Let $N_G(v)$ denote the neighborhood of a vertex v in G. For any $S \subseteq V(G)$, we use $G[S]$ to denote the subgraph of G induced by S, and write $G - S = G[V(G) \setminus S]$. A vertex subset $S \subseteq V(G)$ is called independent if $G[S]$ has no edges. For two vertices subsets $S, T \subseteq V(G)$ with $S \cap T = \emptyset$, let $E_G(S, T)$ denote the set of edges admitting one end-vertex in S and the other in T and set $e_G(S, T) = |E_G(S, T)|$. As usual, the complete graph of order n is denoted by K_n. Let G_1 and G_2 be two vertex disjoint graphs. We use $G_1 \cup G_2$ to denote the disjoint union of G_1 and G_2. The join $G_1 \vee G_2$ is the graph obtained from $G_1 \cup G_2$ by adding all the edges joining a vertex of G_1 to a vertex of G_2.

Recall that $V(G) = \{v_1, v_2, \cdots, v_n\}$. The adjacency matrix $A(G) = (a_{ij})$ of G is the $n \times n$ symmetric matrix, where $a_{ij} = 1$ if v_i and v_j are adjacent in G, zero otherwise. Let $D(G)$ be the diagonal degree matrix of

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G. Then $Q(G) = D(G) + A(G)$ is called the signless Laplacian matrix of G. The largest eigenvalues of $A(G)$ and $Q(G)$, denoted by $\rho(G)$ and $q(G)$, are called the spectral radius and the signless Laplacian spectral radius of G, respectively.

Let g and f be two integer-valued functions defined on $V(G)$ with $0 \leq g(v) \leq f(v)$ for any $v \in V(G)$. A (g, f)-factor of G is a spanning subgraph F of G such that $g(v) \leq d_F(v) \leq f(v)$ for all $v \in V(G)$. Let a and b are two integers with $1 \leq a \leq b$. If $g \equiv a$ and $f \equiv b$, then a (g, f)-factor of G is called an $[a, b]$-factor of G. A $[k, k]$-factor of G is called a k-factor of G. In particular, a 1-factor is also called a perfect matching. Let $h : E(G) \to [0, 1]$ be a function. If $g(v) \leq \sum_{e \in E_G(v)} h(e) \leq f(v)$ holds for every $v \in V(G)$, then the subgraph of G with vertex set $V(G)$ and edge set F_h, denoted by $G[F_h]$, is called a fractional (g, f)-factor of G with indicator function h, where $E_G(v)$ denotes the set of edges incident with v in G and $F_h = \{e \in E(G) : h(e) > 0\}$. If $g \equiv a$ and $f \equiv b$, then a fractional (g, f)-factor of G is called a fractional $[a, b]$-factor of G. In particular, a fractional (f, f)-factor of G is called a fractional f-factor of G.

In mathematical literature, the study on factors and fractional factors of graphs attracted much attention. Some sufficient conditions for graphs to possess [1, 2]-factors were derived by Kelmans [10], Kano, Katona and Király [8], Zhou, Wu and Bian [36], Zhou, Sun and Bian [32], Zhou, Sun and Liu [33], Zhou, Sun and Yang [34], Zhou [25,27], Gao and Wang [4], Zhou, Zhang and Sun [35], Wang and Zhang [18], Wu [23]. Much effort has been devoted to obtain some results on $[a, b]$-factors in graphs by utilizing various graphic parameters such as Fan-type condition [15], toughness [9], binding number and size [21], degree condition [19,30] and others. Gao, Wang and Chen [5], Wang and Zhang [20], Zhou [24,26], Wu [22], Zhou, Pan and Xu [31], Zhou, Liu and Xu [29] presented some sufficient conditions for graphs to possess fractional $[a, b]$-factors.

For any function φ defined on $V(G)$, write $\varphi(S) = \sum_{v \in S} \varphi(v)$, where $S \subseteq V(G)$. In particular, $\varphi(\emptyset) = 0$. In 1970, Lovász [14] characterized a graph with a (g, f)-factor.

Theorem 1.1 ([14]). *Let G be a graph, and let g, f be two nonnegative integer-valued functions defined on $V(G)$ satisfying $g(v) \leq f(v)$ for each $v \in V(G)$. Then G has a (g, f)-factor if and only if for all disjoint subsets S and T of $V(G)$,*

$$f(S) + d_{G-S}(T) - g(T) - q_G(S, T, g, f) \geq 0,$$

where $q_G(S, T, g, f)$ denotes the number of components C of $G - (S \cup T)$ satisfying $g(v) = f(v)$ for all $v \in V(C)$ and $f(V(C)) + e_G(C, T) \equiv 1 \ (mod\ 2)$.

In 1990, Anstee [1] posed a criterion for a graph admitting a fractional (g, f)-factor. Liu and Zhang [13] presented a new proof.

Theorem 1.2 ([1,13]). *Let G be a graph, and let g, f be two functions from $V(G)$ to the nonnegative integers with $g(v) \leq f(v)$ for every $v \in V(G)$. Then G has a fractional (g, f)-factor if and only if*

$$f(S) + d_{G-S}(T) - g(T) \geq 0$$

for any $S \subseteq V(G)$, where $T = \{v : v \in V(G) \setminus S, d_{G-S}(v) < g(v)\}$.

A graph G is defined as a fractional (g, f)-deleted graph if for any $e \in E(G)$, $G - e$ contains a fractional (g, f)-factor. In 2003, Li, Yan and Zhang [12] provided a criterion for a graph to be a fractional (g, f)-deleted graph, which is an extension of Theorem 1.2. For two integers a and b with $1 \leq a \leq b$, if $g \equiv a$ and $f \equiv b$, then a fractional (g, f)-deleted graph is called a fractional $[a, b]$-deleted graph. A fractional $[a, b]$-deleted graph is called a fractional k-deleted graph when $a = b = k$. Kotani [11] obtained a binding number condition for the existence of fractional k-deleted graphs. The following theorem on fractional $[a, b]$-deleted graph is a special case of Li, Yan and Zhang's fractional (g, f)-covered graph theorem.

Theorem 1.3 ([12]). *Let G be a graph and $a \leq b$ be two positive integers. Then G is a fractional $[a, b]$-deleted graph if and only if*

$$b|S| + d_{G-S}(T) - a|T| \geq \varepsilon(S, T)$$

for any $S \subseteq V(G)$, where $T = \{v : v \in V(G) \setminus S, d_{G-S}(v) \leq a\}$ and $\varepsilon(S, T)$ is defined by

$$\varepsilon(S, T) = \begin{cases} 2, \ if\ T\ is\ not\ independent, \\ 1, \ if\ T\ is\ independent\ and\ e_G(T, V(G) \setminus (S \cup T)) \geq 1, \\ 0, \ otherwise. \end{cases}$$

Very recently, O [17] derived a spectral radius condition to guarantee that a graph contains a 1-factor (or a perfect matching). Zhou and Liu [28] presented a spectral radius condition for the existence of an odd $[1, b]$-factor in a graph. Motivated by [12,17,28] directly, it is natural and interesting to show some spectral radius conditions to guarantee the existence of fractional $[a, b]$-deleted graphs. In what follows, we put forward a spectral radius condition and a signless Laplacian spectral radius condition for a graph to be a fractional $[a, b]$-deleted graph, respectively.

Theorem 1.4 *Let a and b be two positive integers with* $b \geq \max\{a, 3\}$*, and let G be a graph of order n with* $n \geq \max\{a+2, 7\}$*. If*

$$\rho(G) > \rho(K_a \vee (K_{n-a-1} \cup K_1)),$$

then G is a fractional $[a, b]$*-deleted graph.*

Remark 1.5 In what follows, we exhibit that the condition $\rho(G) > \rho(K_a \vee (K_{n-a-1} \cup K_1))$ declared in Theorem 1.4 is sharp, namely, it cannot be replaced by $\rho(G) \geq \rho(K_a \vee (K_{n-a-1} \cup K_1))$.

Let a and b be two positive integers with $a \leq b$ and $G = K_a \vee (K_{n-a-1} \cup K_1)$. Then $\rho(G) = \rho(K_a \vee (K_{n-a-1} \cup K_1))$. Set $S = \emptyset$ and $T = V(K_1)$. It is obvious that T is independent and $e_G(T, V(G) \setminus (S \cup T)) = a \geq 1$. According to the definition of $\varepsilon(S, T)$, we possess $\varepsilon(S, T) = 1$. Thus, we deduce

$$\theta_G(S, T) = b|S| + d_{G-S}(T) - a|T| = 0 + a - a = 0 < 1 = \varepsilon(S, T).$$

By virtue of Theorem 1.3, G is not a fractional $[a, b]$-deleted graph.

Theorem 1.6 *Let a and b be two positive integers with* $b \geq \max\{a, 3\}$*, and let G be a connected graph of order n with* $n \geq \max\{a+2, 7\}$*. If*

$$q(G) > 2n - 4 + \frac{a+1}{n-1}$$

and

$$q(G) > q(K_a \vee (K_{n-a-1} \cup K_1)),$$

then G is a fractional $[a, b]$*-deleted graph.*

Remark 1.7 Next, we claim that the condition $q(G) > q(K_a \vee (K_{n-a-1} \cup K_1))$ declared in Theorem 1.6 cannot be replaced by $q(G) \geq q(K_a \vee (K_{n-a-1} \cup K_1))$. Let a and b be two positive integers with $a \leq b$ and $G = K_a \vee (K_{n-a-1} \cup K_1)$. Then $q(G) = q(K_a \vee (K_{n-a-1} \cup K_1))$. From Remark 1.5, G is not a fractional $[a, b]$-deleted graph. But, I do not know whether the condition $q(G) > 2n - 4 + \frac{a+1}{n-1}$ declared in Theorem 1.6 is sharp or not.

Finally, we pose a sufficient condition to guarantee a graph G to be a fractional $[a, b]$-deleted graph with respect to its size, which plays a key role to verify Theorems 1.4 and 1.6.

Theorem 1.8 *Let a and b be two positive integers with* $b \geq \max\{a, 3\}$*, and let G be a graph of order n with* $n \geq \max\{a+2, 7\}$*. If* $\delta(G) \geq a+1$ *and*

$$e(G) \geq \binom{n-1}{2} + \frac{a+2}{2},$$

then G is a fractional $[a, b]$*-deleted graph.*

The proofs of Theorems 1.4 and 1.6 will be provided in Sections 3 and 4, respectively. The proof of Theorem 1.8 will be provided in Section 2.

2 The proof of Theorem 1.8

In this section, we prove Theorem 1.8, which gives a sufficient condition to ensure that a graph G is a fractional $[a, b]$-deleted graph by utilizing Theorem 1.3.

Proof of Theorem 1.8 Let $\theta_G(S, T) = b|S| + d_{G-S}(T) - a|T|$ for any $S \subseteq V(G)$ and $T = \{v : v \in V(G) \setminus S, d_{G-S}(v) \leq a\}$. Suppose, to the contrary, that G is not a fractional $[a, b]$-deleted graph. According to Theorem 1.3, there exists some subset S of $V(G)$ such that

$$\theta_G(S, T) = b|S| + d_{G-S}(T) - a|T| \leq \varepsilon(S, T) - 1, \tag{2.1}$$

where $T = \{v : v \in V(G) \setminus S, d_{G-S}(v) \leq a\}$. □

Claim 1 $n \geq a + 3$.

Proof Let $n = a + 2$. In terms of $\delta(G) \geq a + 1$, we infer that G is a complete graph of order $n = a + 2$. Consequently, for any $e \in E(G)$, G has a Hamiltonian cycle C such that $e \in E(C)$. Thus, $G - E(C)$ is a a-factor of G, and also a fractional a-factor of G. Obviously, G is a fractional $[a, b]$-deleted graph, which is contrary to the assumption. Hence, $n \geq a + 3$. This completes the proof of Claim 1. □

Claim 2 $|T| \geq b + 1$.

Proof Assume that $|T| \leq b$. Note that $\varepsilon(S, T) \leq |T|$ by the definition of $\varepsilon(S, T)$. Together with $\delta(G) \geq a + 1$, we deduce

$$\begin{aligned}
\theta_G(S, T) =& b|S| + d_{G-S}(T) - a|T| \\
=& b|S| + d_G(T) - e_G(S, T) - a|T| \\
\geq& b|S| + \delta(G)|T| - |S| \cdot |T| - a|T| \\
\geq& b|S| + (a + 1)|T| - |S| \cdot |T| - a|T| \\
=& (b - |T|)|S| + |T| \\
\geq& |T| \\
\geq& \varepsilon(S, T),
\end{aligned}$$

which contradicts (2.1). Hence, $|T| \geq b + 1$. Claim 2 is proved. □

In view of Claim 2, we derive

$$n \geq |S| + |T| \geq |S| + b + 1. \tag{2.2}$$

According to $e(G) \geq \binom{n-1}{2} + \frac{a+2}{2}$, there exist at most $n - 1 - \frac{a+2}{2}$ edges not in $E_G[V(G) \setminus (S \cup T), T] \cup E(G[T])$. Thus, we obtain

$$d_{G-S}(T) \geq (n - 1 - |S|)|T| - 2\left(n - 1 - \frac{a + 2}{2}\right). \tag{2.3}$$

It follows from (2.1), (2.2), (2.3), $\varepsilon(S, T) \leq 2$, $b \geq \max\{a, 3\}$, $n \geq \max\{a + 2, 7\}$, Claims 1 and 2 that

$$\begin{aligned}
\varepsilon(S, T) - 1 \geq \theta_G(S, T) &= b|S| + d_{G-S}(T) - a|T| \\
&\geq b|S| + (n - 1 - |S|)|T| - 2\left(n - 1 - \frac{a + 2}{2}\right) - a|T| \\
&= b|S| + (n - 1 - |S| - a)|T| - 2\left(n - 1 - \frac{a + 2}{2}\right) \\
&\geq b|S| + (n - 1 - |S| - a)(b + 1) - 2\left(n - 1 - \frac{a + 2}{2}\right) \\
&= (b - 3)n + (n - |S| - b) + n - ab + 3 \\
&\geq (b - 3)(a + 3) + 1 + n - ab + 3 \\
&= n + 3(b - a) - 5 \\
&\geq n - 5 \\
&\geq 2 \\
&\geq \varepsilon(S, T),
\end{aligned}$$

which is a contradiction. This completes the proof of Theorem 1.8. □

3 The proof of Theorem 1.4

In this section, we first introduce some necessary preliminary results, which will be used to verify Theorem 1.4. Hong, Shu and Fang [6], Nikiforov [16] presented an important upper bound on the spectral radius $\rho(G)$.

Lemma 3.1 ([6,16]). *Let G be a graph of order n with minimum degree $\delta(G)$. Then*

$$\rho(G) \le \frac{\delta(G)-1}{2} + \sqrt{2e(G) - n\delta(G) + \frac{(\delta(G)+1)^2}{4}}.$$

The following observation is very useful when we utilize the above upper bound on $\rho(G)$.

Proposition 3.2 ([6,16]). *For a graph G of order n with $e(G) \le \binom{n}{2}$, the function*

$$f(x) = \frac{x-1}{2} + \sqrt{2e(G) - nx + \frac{(x+1)^2}{4}}$$

is decreasing with respect to x for $0 \le x \le n-1$.

Let $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$. Define $A \le B$ if for any $1 \le i, j \le n$, $a_{ij} \le b_{ij}$ and $A < B$ if $A \le B$ and $A \ne B$.

Lemma 3.3 ([2,7]). *Let O be an $n \times n$ zero matrix, $A = (a_{ij})$ and $B = (b_{ij})$ be two $n \times n$ matrices with the spectral radius $\rho(A)$ and $\rho(B)$, respectively. If $O \le A \le B$, then $\rho(A) \le \rho(B)$. Furthermore, if B is irreducible and $O \le A < B$, then $\rho(A) < \rho(B)$.*

Proof of Theorem 1.4 We first verify the following claim.

Claim 3 $\delta(G) \ge a+1$.

Proof Assume that $\delta(G) \le a$. Then there exists a vertex $v \in V(G)$ such that $d_G(v) \le a$, which implies that $G \subseteq K_a \vee (K_{n-a-1} \cup K_1)$. According to Lemma 3.3, we infer

$$\rho(G) \le \rho(K_a \vee (K_{n-a-1} \cup K_1)),$$

which contradicts the condition that $\rho(G) > \rho(K_a \vee (K_{n-a-1} \cup K_1))$. Hence, $\delta(G) \ge a+1$. Claim 3 is proved. □

In terms of Claim 3, Lemma 3.1 and Proposition 3.2, we obtain

$$\begin{aligned}\rho(G) \le& \frac{\delta(G)-1}{2} + \sqrt{2e(G) - n\delta(G) + \frac{(\delta(G)+1)^2}{4}} \\ \le& \frac{a}{2} + \sqrt{2e(G) - n(a+1) + \frac{(a+2)^2}{4}}. \end{aligned} \tag{3.1}$$

Note that the graph K_{n-1} is a proper subgraph of the graph $K_a \vee (K_{n-a-1} \cup K_1)$ and the adjacency matrices of connected graphs are irreducible. Then by Lemma 3.3, we have

$$\rho(G) > \rho(K_a \vee (K_{n-a-1} \cup K_1)) > \rho(K_{n-1}) = n-2. \tag{3.2}$$

It follows from (3.1) and (3.2) that

$$e(G) > \frac{(n-1)(n-2)}{2} + \frac{a+1}{2} = \binom{n-1}{2} + \frac{a+1}{2}.$$

By virtue of the integrity of $e(G)$, we get

$$e(G) \ge \binom{n-1}{2} + \frac{a+2}{2}. \tag{3.3}$$

By (3.3), Claim 3 and Theorem 1.8, we see that G is a fractional $[a, b]$-deleted graph. This completes the proof of Theorem 1.4. □

4 The proof of Theorem 1.6

In this section, we first present a necessary preliminary result, which will be used to prove Theorem 1.6. Feng and Yu [3] verified an upper bound on $q(G)$, which has been widely used in the literature.

Lemma 4.1 ([3]). *Let G be a connected graph with n vertices and $e(G)$ edges. Then*

$$q(G) \leq \frac{2e(G)}{n-1} + n - 2.$$

Proof of Theorem 1.6 We first prove the following claim.

Claim 4 $\delta(G) \geq a + 1$.

Proof Suppose, to the contrary, that $\delta(G) \leq a$. Then there exists a vertex $v \in V(G)$ satisfying $d_G(v) \leq a$, which yields that $G \subseteq K_a \vee (K_{n-a-1} \cup K_1)$. In terms of Lemma 3.3, we deduce

$$q(G) \leq q(K_a \vee (K_{n-a-1} \cup K_1)),$$

which contradicts the condition that $q(G) > q(K_a \vee (K_{n-a-1} \cup K_1)$. Consequently, $\delta(G) \geq a + 1$. This completes the proof of Claim 4. □

In view of Lemma 4.1 and the condition that $q(G) > 2n - 4 + \frac{a+1}{n-1}$, we get

$$2n - 4 + \frac{a+1}{n-1} < q(G) \leq \frac{2e(G)}{n-1} + n - 2,$$

which yields that

$$e(G) > \frac{(n-1)(n-2)}{2} + \frac{a+1}{2} = \binom{n-1}{2} + \frac{a+1}{2}.$$

According to the integrity of $e(G)$, we obtain

$$e(G) \geq \binom{n-1}{2} + \frac{a+2}{2}. \tag{4.1}$$

It follows from (4.1), Claim 4 and Theorem 1.8 that G is a fractional $[a, b]$-deleted graph. This completes the proof of Theorem 1.6. □

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Declarations

Competing interests The authors declare that they have no conflicts of interest to this work.

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