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ORIGINAL RESEARCH

Decomposition of hypercubes into sunlet graphs

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Abstract For an integer $k \geq 3$, the sunlet graph of order $2k$, denoted by L_{2k}, is a graph obtained from a cycle of length k by attaching one pendant vertex to each vertex of the cycle. The cycle decompositions of the hypercubes is well studied in the literature. In this paper, we obtain sunlet decompositions of hypercubes using cycle decompositions. We prove that if the hypercube Q_{2n} has a decomposition into cycles of length k, then Q_{mn} has a decomposition into copies of L_{2k} for $m = 4$ or $m \geq 6$. Further, we obtain a necessary and sufficient condition for the existence of a decomposition of Q_n into spanning sunlet graphs. As a consequence, we obtain the decomposition of Q_{mn} into sunlet graphs of various orders using cycle decomposition. In particular, we show that Q_n has an L_q-decomposition for $q = 8$ and $q = 16$ whenever $n = 4$ or $n \geq 6$.

Keywords Decomposition · Hypercube · Sunlet graph

Mathematics Subject Classification 05C51

1 Introduction

The *Cartesian product* of graphs G and H, denoted by $G\Box H$, is a graph with vertex set $V(G\Box H) = V(G) \times V(H)$ and edge set $E(G\Box H) = \{\langle (u, x), (v, y)\rangle : u = v \text{ and } \langle x, y\rangle \in E(H), \text{ or } x = y \text{ and } \langle u, v\rangle \in E(G)\}$. Let n be a positive integer. The *n-dimensional hypercube*, denoted by Q_n, is the Cartesian product of n copies of the complete graph K_2. It is a bipartite, n-regular, n-connected, vertex-transitive and edge-transitive graph with 2^n vertices, and it has a diameter of n. Such beautiful properties make the hypercube one of the popular interconnection networks in parallel and distributing computing [19].

For an integer $k \geq 3$, let C_k denote a cycle on k vertices. The *sunlet graph* of order $2k$, denoted by L_{2k}, is obtained by adding a pendant vertex to each vertex of the cycle C_k. Thus, L_{2k} is a connected unicyclic graph with $2k$ vertices and $2k$ edges in which degree of every vertex is either 1 or 3. It is also known as a corona graph or a k-sun. The graphs L_8 and L_{16} are shown in Fig. 1. Frucht [14] and Grace [17] investigated the properties of gracefulness and harmoniousness in these graphs.

A *decomposition* of a graph G is a collection of subgraphs of G such that the edge sets of these subgraphs partition the edge set of G. For a given graph H, an *H-decomposition* of G is a decomposition into subgraphs,

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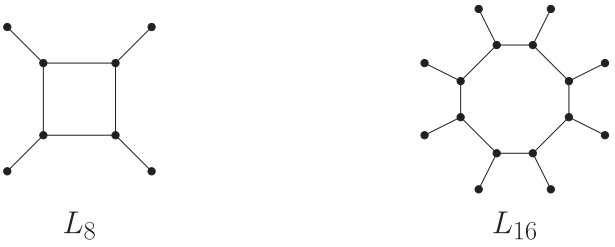


Fig. 1 The sunlet graphs L_8 and L_{16}

each of which is isomorphic to H. It is known that an interconnection network can be viewed as a graph in which processors are vertices and links between processors are edges of that graph. A decomposition of a network into disjoint subnetworks has advantages in network management, performance improvement, security, reducing network congestion, controlling network growth when different system parameters govern each subnetwork; see [1,9,12]. Given the significance of the hypercube as an interconnection network, its decompositions have been extensively studied, including decompositions into cycles [5,10,20,21,26], paths [3,11,20], trees [8,13,18], and regular subgraphs [6,7,22].

The decomposition of graphs into sunlet graphs is mainly studied for the complete graphs, complete bipartite graphs, and various products of these graphs. Fu et al. [16] proved that for $k \in \{6, 10, 14\} \cup \{2^t : t \geq 2\}$, the complete graph K_n has an L_{2k}-decomposition if and only if $n(n-1) \equiv 0 \pmod 4$ and $n \geq 2k$. For $k = 5$, Fu et al. [15] established that K_n has an L_{10}-decomposition if and only if $n \equiv 0, 1, 5, 16 \pmod{20}$. Anitha and Lekshmi [4] studied the sunlet decomposition of K_{2n}, the complete bipartite graph $K_{2n,2n}$ and the Harary graph $H_{4,2n}$. Akwu and Ajayi [2] obtained the necessary and sufficient conditions for decomposing the lexicographic product graphs $K_n \otimes \bar{K}_m$ and $(K_n - I) \otimes \bar{K}_m$ into the sunlet graphs L_{2p}, where p is a prime number. Sowndhariya and Muthusamy [24,25] obtained such conditions for an L_8-decomposition of the tensor product, the wreath product, and the Cartesian product of the complete graphs.

The decomposition into sunlet graph of order eight has been investigated for hypercubes in [23], where it was shown that Q_n has an L_8-decomposition if $n = 4$ or $n \geq 6$. In this paper, we obtain the decomposition of Q_n into sunlet graphs of various orders. The sunlet decomposition of a hypercube is closely related to the cycle decomposition. The cycle decomposition of hypercubes is well-studied in the literature. It is known that the hypercube Q_n has a C_k-decomposition whenever $k \in \{n, 2n, 4n\} \cup \{2^t : 2 \leq t \leq n\} \cup \{n2^{t+1} : 4n \leq n2^{t+1} \leq \frac{2^{2n-1}}{n}\}$; see [5,10,20,21,26].

Our idea is to obtain a sunlet decomposition of a hypercube from its cycle decomposition. Because a cycle decomposition requires all vertices to have an even degree, the hypercube Q_n with odd n cannot have a cycle decomposition. Therefore we focus on even-dimensional hypercubes. We establish that if the hypercube Q_{2n} has a C_k-decomposition, then Q_{mn} has an L_{2k}-decomposition for every integer $m = 4$ or $m \geq 6$. As a consequence, we obtain a decomposition of Q_{mn} into sunlet graphs of various orders. In particular, we show that Q_m has an L_8-decomposition and an L_{16}-decomposition. These results are proved in Section 3. In Section 2, we prove that Q_n has a decomposition into spanning sunlet graphs if and only if n is a multiple of 4. We also obtain a similar result for an L_n-decomposition.

2 Decomposition into spanning sunlet graphs

In this section, we consider a decomposition of a hypercube into spanning sunlet graphs. We prove that the obvious necessary condition for the existence of such a decomposition is sufficient also.

Theorem 2.1 *The hypercube Q_n has a decomposition into copies of a spanning sunlet graph if and only if n is a multiple of* 4.

Proof Suppose Q_n has a decomposition into copies of a spanning sunlet graphs. Any spanning sunlet graph in Q_n has 2^n vertices and so it is isomorphic to L_{2^n}. Let $\mathcal{C}$ be the collection of the copies of L_{2^n} involved in this decomposition. Then $|\mathcal{C}| = \dfrac{|E(Q_n)|}{|E(L_{2^n})|} = \dfrac{n2^{n-1}}{2^n} = \dfrac{n}{2}$. This shows that n is even. We show that $\frac{n}{2}$ is also even. If x is a vertex of Q_n, then it is a vertex of all $\frac{n}{2}$ members of $\mathcal{C}$. Let $d_i(x)$ be the degree of x in ith member of $\mathcal{C}$.

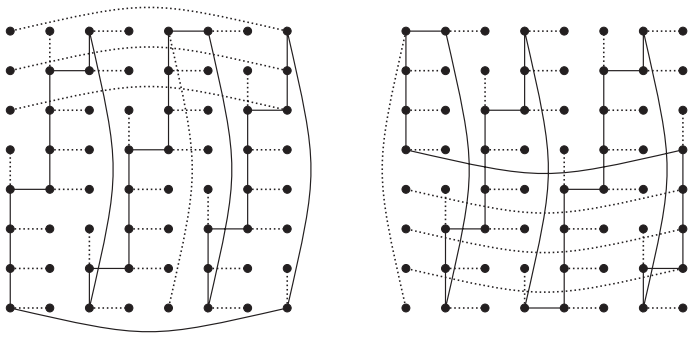


Fig. 2 An L_{64}-decomposition of $C_8 \square C_8$

Then $d_i(x) = 1$ or 3 and further, $n = d_1(x) + d_2(x) + \cdots + d_{\frac{n}{2}}(x)$. Since n is even, the sum on the right side is also even. This implies that $\frac{n}{2}$ is even. Thus n is a multiple of 4.

Conversely, suppose n is a multiple of 4. Then $n = 4q$ for some $q \geq 1$. We write $Q_n = Q_{2q} \square Q_{2q}$. The hypercube Q_{2q} has a decomposition into Hamiltonian cycles, say $Z_1, Z_2, \cdots, Z_q$. Therefore Q_n decomposes into the spanning 4-regular subgraphs $Z_1 \square Z_1, Z_2 \square Z_2, \cdots, Z_q \square Z_q$. Now, the proof follows from the following claim.

Claim: For even $k \geq 4$, the graph $C_k \square C_k$ decomposes into spanning sunlet graphs.

Let $C_k = \langle 1, 2, \cdots, k \rangle$ with vertex i adjacent to $i + 1$ modulo k. Then the vertex set of the graph $C_k \square C_k$ is $\{(i, j) : i, j = 1, 2, \cdots, k\}$. This graph has two vertex-disjoint cycles $\mathcal{C}$ and $\mathcal{D}$ each of length $\frac{k^2}{2}$, where $\mathcal{C} = \langle (1, 1), (1, 2), \cdots, (1, \frac{k}{2}), (2, \frac{k}{2}), (2, \frac{k}{2}+1), \cdots, (2, k-1), (3, k-1), (3, k), (3, 1), \cdots, (3, \frac{k}{2}-2), \cdots, (k, 1) \rangle$ and $\mathcal{D} = \langle (1, \frac{k}{2} + 1), (1, \frac{k}{2} + 2), \cdots, (1, k), (2, k), (2, 1), \cdots, (2, \frac{k}{2} - 1), (3, \frac{k}{2} - 1), (3, \frac{k}{2}), \cdots, (3, k - 1), \cdots, (k, \frac{k}{2} + 1) \rangle$. Now we adjoin a pendant vertex to each vertex of the cycles $\mathcal{C}$ and $\mathcal{D}$ in the lexicographic order as per the availability of the vertex in $C_k \square C_k$ (see Fig. 2 for an illustration). Thus we get two edge-disjoint spanning subgraphs of $C_k \square C_k$ which are isomorphic to the sunlet graph L_{k^2}. These two copies of L_{k^2} together have $2k^2$ edges which is also the number of edges of $C_k \square C_k$ and so, they decompose the graph $C_k \square C_k$. □

Corollary 2.2 *For $n \geq 1$, the hypercube Q_{4n} has an $L_{2^{4n}}$-decomposition.*

When n is a multiple of four, we also obtain an L_n-decomposition of Q_n using a result of Ramras [21], which states that any connected unicyclic subgraph of Q_n with n edges can decompose Q_n.

Lemma 2.3 *For $n \geq 8$, the hypercube Q_n has an L_n-decomposition if and only if n is a multiple of* 4.

Proof Suppose Q_n has an L_n-decomposition. Then L_n is a subgraph of Q_n. Since the number of vertices of any sunlet graph is an even integer, it follows that n must also be even. Further, the graph L_n contains a cycle of length $\frac{n}{2}$. Since Q_n is a bipartite graph, $\frac{n}{2}$ must be an even integer. Thus n is a multiple of 4.

Conversely, suppose n is a multiple of 4. We prove that Q_n has a subgraph isomorphic to L_n. As $\frac{n}{2}$ is even and $4 \leq \frac{n}{2} \leq 2^{n-1}$ and Q_{n-1} is a bipancyclic graph, Q_{n-1} contains a cycle C of length $\frac{n}{2}$. Therefore $C \square K_2$ is a subgraph of $Q_{n-1} \square K_2 = Q_n$. Note that $C \square K_2$ contains two vertex-disjoint copies of C corresponding to the vertices of K_2 and deleting all the edges of one of these two cycles gives the sunlet graph L_n. Thus Q_n contains a subgraph isomorphic to L_n. As L_n is a unicyclcic graph with n edges, by the above result of Ramras [21], Q_n has an L_n-decomposition. □

3 Proof of Main Theorem

In this section, we obtain a sunlet decomposition of a hypercube from a given cycle decomposition. This leads to the following result.

Theorem 3.1 *Let m, n be positive integers such that $m = 4$ or $m \geq 6$. If Q_{2n} has a C_k-decomposition, then Q_{mn} has an L_{2k}-decomposition.*

We divide the proof of this theorem into several lemmas. First, we state the following observation.

Lemma 3.2 ([20]) *If the graphs G_1 and G_2 each has an H-decomposition, then the graph $G_1 \square G_2$ has an H-decomposition.*

The *corona product* of two graphs G and H, denoted by $G \circ H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the j^{th} vertex of G to all vertices of the j^{th} copy of H for every j with $1 \leq j \leq |V(G)|$. Hence the sunlet graph L_{2k} is isomorphic to the corona product $C_k \circ K_1$. Note that if a hypercube has a C_k-decomposition, then k must be an even integer with $k \geq 4$. In the following lemma, we construct an L_{2k}-decomposition of the graph $Q_{2n} \circ \bar{K}_n$ from a C_k-decomposition of Q_{2n}, where $\bar{K}_n$ is a graph consisting of n isolated vertices.

Lemma 3.3 *If Q_{2n} has a C_k-decomposition, then the graph $Q_{2n} \circ \bar{K}_n$ has an L_{2k}-decomposition.*

Proof For convenience, denote $Q_{2n} \circ \bar{K}_n$ by G. Suppose the vertices of the subgraph Q_{2n} of G are $v_1, v_2, \cdots, v_r$, where $r = 2^{2n}$. Let T_i be the subgraph of G induced by v_i and the n vertices corresponding to $\bar{K}_n$. Then T_i is isomorphic to the star $K_{1,n}$ with v_i as the vertex of degree n. Note that G decomposes into Q_{2n} and $T_1, T_2, \cdots, T_r$. Suppose Q_{2n} has a C_k-decomposition. Let $\mathcal{C}$ be the collection of the k-cycles of this decomposition. We construct sunlet graphs from the cycles in $\mathcal{C}$ by adjoining edges from $T_1, T_2, \cdots, T_r$. Note that each vertex v_i of Q_{2n} belongs to exactly n cycles from $\mathcal{C}$. Adjoin n edges of the star T_i incident at v_i to these n cycles, with one edge added to each cycle. Repeat this process for all vertices of Q_{2n}, which will use up all edges of $T_1 \cup T_2 \cup \cdots \cup T_r$ and transform each C_k from $\mathcal{C}$ into a sunlet graph L_{2k}. In this way, we obtain an L_{2k}-decomposition of G. □

The following result will be useful in proving to the cases $m = 6$ and $m = 9$ of Theorem 3.1.

Lemma 3.4 *If Q_{2n} has an L_{2k}-decomposition and a C_k-decomposition, then Q_{3n} has an L_{2k}-decomposition.*

Proof Write $Q_{3n} = Q_{2n} \square Q_n$. Since Q_n is an n-regular bipartite graph on 2^n vertices, it has a bipartition (X, Y) with $|X| = |Y| = 2^{n-1}$. Let $X = \{x_i : i = 1, 2, \cdots, 2^{n-1}\}$ and let $Y = \{y_i : i = 1, 2, \cdots, 2^{n-1}\}$. In Q_{3n}, suppose X_i and Y_i are copies of Q_{2n} corresponding to the vertices x_i and y_i of Q_n, respectively. Give directions to the edges of Q_n from X to Y so that the out-degree of each x_i is n and that of each y_i is 0. Let H_i be the subgraph of Q_{3n} obtained by taking union of X_i and all n perfect matchings between X_i and Y_j corresponding to the n outgoing edges from x_i to Y. Then $H_1, H_2, \cdots, H_{2^{n-1}}$ are pairwise edge-disjoint subgraphs of Q_{3n} and they together with $Y_1, Y_2, \cdots, Y_{2^{n-1}}$ decomposes the graph Q_{3n}. Clearly, each H_i is isomorphic to the graph $Q_{2n} \circ \bar{K}_n$. Suppose Q_{2n} has an L_{2k}-decomposition and a C_k-decomposition. Then X_i has a C_k-decomposition and it gives an L_{2k}-decomposition of H_i by Lemma 3.3 for $i = 1, 2, \cdots, 2^{n-1}$. Also, we take an L_{2k}-decomposition of Y_i for $i = 1, 2, \cdots, 2^{n-1}$. This completes an L_{2k}-decomposition of Q_{3n}. □

We prove Theorem 3.1 for the case $m = 4$ in the next lemma.

Lemma 3.5 *If Q_{2n} has a C_k-decomposition, then Q_{4n} has an L_{2k}-decomposition.*

Proof We write $Q_{4n} = Q_{2n} \square Q_{2n} = L \square R$, where L and R are isomorphic to Q_{2n}. Let $v_1, v_2, \cdots, v_r$ with $r = 2^{2n}$ be the vertices of R. Then Q_{4n} is obtained by replacing each vertex v_i by a copy L_i of L and each edge $\langle v_i, v_j \rangle$ by the perfect matching M_{ij} between L_i and L_j. Since R is a $2n$-regular and connected graph, it is an Eulerian graph. Hence we can give directions to the edges of R so that there is a directed Eulerian circuit in R. Thus each v_i has in-degree n and out-degree n. Let H_i be the subgraph of Q_{4n} obtained by taking the union of L_i and the n matchings M_{ij} corresponding to n outgoing edges from the vertex v_i in R. It follows that the graphs $H_1, H_2, \cdots, H_r$ decompose the graph Q_{4n}. Observe that each H_i is isomorphic to the graph $L_i \circ \bar{K}_n$ and so is isomorphic to $Q_{2n} \circ \bar{K}_n$. Now, suppose Q_{2n} has a C_k-decomposition. Then, by Lemma 3.3, each H_i has an L_{2k}-decomposition. Thus Q_{4n} has an L_{2k}-decomposition. □

We now complete the proof of Theorem 3.1.

Proof of Theorem 3.1 Suppose Q_{2n} has a C_k-decomposition. Then by Lemma 3.5, Q_{4n} has an L_{2k}-decomposition and so, the result holds for $m = 4$. We prove the result by induction on m. First, we prove it for $m = 6, 7, 8, 9$ and then apply induction for $m \geq 10$.

The result holds for $m = 8$ by Lemma 3.2 as $Q_{8n} = Q_{4n} \square Q_{4n}$. Note that Q_{4n} and Q_{6n} each has a C_k-decomposition by Lemma 3.2 as $Q_{4n} = Q_{2n} \square Q_{2n}$ and $Q_{6n} = Q_{4n} \square Q_{2n}$. Since Q_{4n} has an L_{2k}-decomposition and a C_k-decomposition, we get an L_{2k}-decomposition of Q_{6n} from Lemma 3.4 by replacing n by $2n$. Similarly, the result holds for $m = 9$ by replacing n with $3n$ in Lemma 3.4 as Q_{6n} has a C_k-decomposition.

Now we settle the case $m = 7$. We can write $Q_{7n} = Q_{4n} \square Q_{3n}$. Let DQ_3 be the directed graph obtained from Q_3 as shown in Fig. 3.

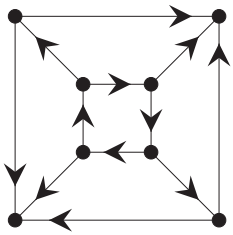


Fig. 3 .

In this directed graph, there are two vertices with 0 out-degree, and each of the remaining six vertices has an out-degree of 2 and an in-degree of 1. We define DQ_{3n} similarly. Note that Q_{3n} is the Cartesian product of n copies of Q_3. Let DQ_{3n} be a directed graph obtained from Q_{3n} by replacing each copy of Q_3 by DQ_3 such that the out-degree of 2^{3n-2} vertices is 0, and the out-degree and the in-degree of the remaining $3 \cdot 2^{3n-2}$ vertices are $2n$ and n, respectively. Let $W_1, W_2, \cdots, W_s$ be the copies of Q_{4n} in Q_{7n} corresponding the vertices of DQ_{3n} with out-degree zero, and let $Y_1, Y_2, \cdots, Y_t$ be the copies of Q_{4n} corresponding the remaining vertices of DQ_{3n}, where $s = 2^{3n-2}$ and $t = 3 \cdot 2^{3n-2}$. For $i = 1, 2, \cdots, t$, let H_i be the subgraph of Q_{7n} obtained by taking union of Y_i and all matchings between Y_i and Y_j if there is a directed edge from the vertex i to j in DQ_{3n}. Then each H_i is isomorphic to the graph $Q_{4n} \circ \bar{K}_{2n}$. Since Q_{4n} has a C_k-decomposition, H_i has an L_{2k}-decomposition by Lemma 3.3. Now, take an L_{2k}-decomposition of each W_j. Since the graph Q_{7n} decomposes into the subgraphs $H_1, H_2, \cdots, H_t$ and $W_1, W_2, \cdots, W_s$, we get an L_{2k}-decomposition of this graph.

Suppose that $m \geq 10$. Assume that the result holds for the hypercube Q_{rn} for any integer r with $6 \leq r \leq m-1$. We write $Q_{mn} = Q_{(m-4)n} \square Q_{4n}$. By induction hypothesis, $Q_{(m-4)n}$ has an L_{2k}-decomposition as $m - 4 \geq 6$. As Q_{4n} has an L_{2k}-decomposition, an L_{2k}-decomposition of Q_{mn} follows from Lemma 3.2. This completes the proof. □

The above theorem also holds for $m = 3, 5$ under an additional condition.

Corollary 3.6 *If Q_{2n} has an L_{2k}-decomposition and a C_k-decomposition, then Q_{mn} has an L_{2k}-decomposition for every integer $m \geq 3$.*

Proof By Theorem 3.1, the result holds for $m = 4$ and $m \geq 6$. It also holds for $m = 3$ by Lemma 3.4. The proof of the case $m = 5$ follows by writing $Q_{5n} = Q_{2n} \square Q_{3n}$ and then applying Lemma 3.2 as Q_{2n} and Q_{3n} have an L_{2k}-decomposition. □

We now obtain sunlet decompositions of hypercubes from cycles decompositions and Theorem 3.1. Most of the results about the decomposition of Q_{2n} into cycles available in the literature are summarized in the following theorem.

Theorem 3.7 [5,10,20,21,26] *For a positive integer n, the hypercube Q_{2n} has*

(i). *a C_{2^t}-decomposition for $2 \leq t \leq 2n$.* [10]
(ii). *a C_{2n}-decomposition for $n \geq 2$.* [21]
(iii). *a C_{4n}-decomposition.* [20]
(iv). *a C_{8n}-decomposition for $n \geq 2$.* [26]
(v). *a $C_{n2^{k+1}}$-decomposition for $4n \leq n2^{k+1} \leq \frac{2^{2n-1}}{n}$.* [5]

Corollary 3.8 *For positive integers m and n with $m = 4$ or $m \geq 6$, the hypercube Q_{mn} has*

(i). *an L_{2^t}-decomposition for $3 \leq t \leq 2n+1$ or $t = 4n$.*
(ii). *an L_{4n}-decomposition for $n \geq 2$.*
(iii). *an L_{8n}-decomposition.*
(iv). *an L_{16n}-decomposition for $n \geq 2$.*
(v). *an $L_{n2^{k+2}}$-decomposition for $4n \leq n2^{k+1} \leq \frac{2^{2n-1}}{n}$.*

Proof Cases (ii)-(v) follow directly from Theorem 3.1 and Theorem 3.7(ii)-(v).

We now prove (i). If $3 \leq t \leq 2n+1$, then Q_{mn} has a $C_{2^{t-1}}$-decomposition by Theorem 3.7(i) and so, Q_{2n} has a L_{2^t}-decomposition by Theorem 3.1. Suppose $t = 4n$. Then Q_{4n} has an L_{2^t}-decomposition by Corollary 2.2. By Theorem 3.7(i), Q_{4n} and Q_{6n} each has a $C_{2^{t-1}}$-decomposition. By Lemma 3.4, we get an L_{2^t}-decomposition

of Q_{6n} by replacing n with $2n$ and so, Q_{9n} has an L_{2^t}-decomposition by replacing n with $3n$. By Lemma 3.2, Q_{8n} has an L_{2^t}-decomposition as $Q_{8n} = Q_{4n} \square Q_{4n}$. On the lines of the proof of Theorem 3.1, we get an L_{2^t}-decomposition of Q_{7n}. Thus, by applying induction on m, Q_{mn} has an L_{2^t}-decomposition. This completes the proof. □

Corollary 3.9 *For a positive integer m, Q_m has an L_{16}-decomposition if and only if $m = 4$ or $m \geq 6$.*

Proof If $m = 4$ or $m \geq 6$, then we get an L_{16}-decomposition of Q_m by taking $n = 1$ in Corollary 3.8(i). Conversely, suppose Q_m has an L_{16}-decomposition. Then, obviously $m \geq 4$. We show that $m \neq 5$. If Q_5 has an L_{16}-decomposition, then there will be $\frac{80}{16} = 5$ copies of L_{16} in this decomposition. Since Q_5 is 5-regular, no vertex of it can be a degree 3 vertex of two such copies. This implies that Q_5 has at least $5 \times 8 = 40$ vertices, a contradiction. □

Similarly, we get the following result for L_8-decomposition.

Corollary 3.10 [23] *For a positive integer m, Q_m has an L_8-decomposition if and only if $m = 4$ or $m \geq 6$.*

Also, Corollary 3.8(i) with $n = 2$ gives the following result.

Corollary 3.11 *If m is an even integer with $m = 8$ or $m \geq 12$, then Q_m has an L_{32}-decomposition and an L_{2^8}-decomposition.*

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