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ORIGINAL RESEARCH

Dynamical pathology, singular manifold, bilinear forms and solitons on a (3+1)-dimensional Jadaun-Singh equation in aortic dissection

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Abstract Recent soliton advances in *Indian J. Pure Appl. Math.* have been impressive, while as to the dynamical pathology, etc., aortic dissection has been seen as a catastrophic disease influencing the aorta. Hereby, symbolic computation is implemented on a (3+1)-dimensional Jadaun-Singh equation for the dynamical pathology in aortic dissection. Via the singular manifold, etc., auto-Bäcklund transformation, bilinear forms and M-soliton solutions are obtained, for the amplitude of the relevant wave, where M is a positive integer. Our results might assist some studies on the dynamical pathology in aortic dissection and cardiothoracic physicians in pinpointing the latent cases and working on such preventive regimens as the control of hypertension and restriction on physiological activity.

Keywords Singular manifold · Dynamical pathology · Aortic dissection · Symbolic computation · (3+1)-dimensional Jadaun-Singh equation · Bäcklund transformation · Soliton · Bilinear form

Mathematics Subject Classification 37K35 · 37N10 · 76B15 · 35Q35 · 92Bxx

1 Introduction

This paper seconds the recent soliton advances in *Indian J. Pure Appl. Math.*, i.e., Refs. [1–3]. Meanwhile, solitons and related nonlinear evolution equations can be seen, e.g., in Refs. [1–12].

As to the dynamical pathology, etc., aortic dissection has been believed as a serious and life-threatening condition where a tear appears in the wall of the major artery with the blood out of the heart or aorta, while contemporary aortic-wave dynamics has been progressing fast [4,13–19]. Solitons have been thought to result in the unstable wave field and generation of pathological waves, while pathological waves have been able to cause the aneurysmal rupture and progression of aortic dissection [4].

Because the length-to-radius ratio of an aorta is much larger than one, while the axial-circumferential waves formed by the blood flow are comparatively smaller than that radius, Ref. [4] has perused a generalized (3+1)-

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dimensional nonlinear evolution equation to model those waveforms over the tunica intimal surface, i.e., the following (3+1)-dimensional Jadaun-Singh equation for the dynamical pathology in aortic dissection:

$$3v_{rrq} - 2v_{rst} + v_{rrrrs} - 2v_{rs}v_{rr} + 2v_{rs}^2 + 2\,(v_s - v_r)\,v_{rrs} = 0, \tag{1}$$

where the subscripts denote the partial derivatives, v, the amplitude of the relevant wave, is a real analytic function of the spatial coordinates (r, s, q) and temporal coordinate t, v_{rrq} represents the linear term, v_{rrrrs} means the dispersion term, v_{rst} gives the development term, while $v_{rs}v_{rr}$, v_{rs}^2 and $(v_s - v_r)\,v_{rrs}$ are all the nonlinear terms [4]. More explanation on Eq. (1), such as on its relevant parameters, terms and biomedical meaning, can be found in Ref. [4] and references therein.

Ref. [20] has stated that usually in the healthy individuals, diastolic pressure is between 60 and 80 millimetres of mercury (mmHg), while systolic pressure, 90 and 120. Ref. [4] has illustrated that the blood flow around the mean blood pressure, 100 mmHg, is pushed frontward by every left ventricular contraction, and has considered the flows to be Newtonian in the aorta.

On Eq. (1), Lie group analysis with certain similarity reductions and solitons has been done [4].

Relevant contributions [21–34] include, e.g., the aortic dissection as a frontier in the diagnosis and management with its therapeutic management and follow-up [29], impact of certain solitons on the progression of initial lesion in the aortic dissection [30], mathematical modeling and well-posedness of the (3+1)-dimensional shell in disorders of a human vascular system [31] and symmetry analysis and analytic solutions of the Born-Infeld equation arising in nonlinear electrodynamics [32].

However, to our knowledge, there are no Bäcklund transformation and bilinear form for Eq. (1) published as yet. In Section 2 of this paper, with symbolic computation [35–44], we will construct a Bäcklund transformation for Eq. (1). In Section 3, solitonic solutions for Eq. (1) will be presented with symbolic computation, which are different from those in Ref. [4]. In Sections 4 and 5, bilinear forms and M-soliton solutions for Eq. (1) will be symbolically computed out, separately, where M is a positive integer. Section 6 will be our conclusions.

2 Auto-Bäcklund transformation for Eq. (1)

In the sense of a generalized Laurent series, similar to those in Refs. [39–41], into Eq. (1) we insert a Painlevé expansion,

$$v(r, s, q, t) = \phi^{-J}(r, s, q, t) \sum_{\lambda=0}^{\infty} v_\lambda(r, s, q, t)\,\phi^\lambda(r, s, q, t), \tag{2}$$

around a noncharacteristic movable singular manifold given by an analytic function $\phi = 0$, where J is a positive integer, v_λ's are all the analytic functions with $v_0 \neq 0$, $\phi_r \neq 0$ and $\phi_s \neq 0$. In Eq. (1), we try to balance the powers of ϕ at the lowest orders, obtaining $J = 1$. We then truncate Expansion (2) at the constant-level terms as

$$v(r, s, q, t) = \frac{v_0(r, s, q, t)}{\phi(r, s, q, t)} + v_1(r, s, q, t), \tag{3}$$

substitute Expansion (3) into Eq. (1), make the coefficients of like powers of ϕ to vanish, and with symbolic computation, get the following Painlevé-Bäcklund equations:

$$\phi_r v_0 - \phi_s v_0 + 6\phi_r^2 = 0, \tag{4}$$

$$\begin{aligned}
&6\phi_r^4 v_{0,s} + 2\phi_r^3 v_0 v_{0,s} + 24\phi_s\phi_r^3 v_{0,r} - 6\phi_s\phi_r^2 v_0 v_{0,s} + 8\phi_s\phi_r^2 v_0 v_{0,r} - 4\phi_s^2\phi_r v_0 v_{0,r} + 24\phi_{rs}\phi_r^3 v_0 \\
&\quad + 3\phi_{rs}\phi_r^2 v_0^2 + 36\phi_s\phi_{rr}\phi_r^2 v_0 - 4\phi_s\phi_{rs}\phi_r v_0^2 + 2\phi_s\phi_{rr}\phi_r v_0^2 - \phi_s^2\phi_{rr} v_0^2 = 0,
\end{aligned} \tag{5}$$

$$\begin{aligned}
&12v_{0,rs}\phi_r^3 - 3v_{0,s}^2\phi_r^2 + 9v_0\phi_q\phi_r^2 + 6v_0\phi_s v_{1,s}\phi_r^2 + 4v_{0,s}v_{0,r}\phi_r^2 - 6v_0\phi_s v_{1,r}\phi_r^2 + 36v_{0,r}\phi_{rs}\phi_r^2 \\
&\quad + 4v_0 v_{0,rs}\phi_r^2 + 18v_{0,s}\phi_{rr}\phi_r^2 + 18\phi_s v_{0,rr}\phi_r^2 + 18v_0\phi_{rrs}\phi_r^2 + 6\phi_s v_{0,r}^2\phi_r - 6v_0\phi_t\phi_s\phi_r \\
&\quad - 6\phi_s v_{0,s}v_{0,r}\phi_r - 6v_0 v_{0,s}\phi_{rs}\phi_r + 8v_0 v_{0,r}\phi_{rs}\phi_r - 6v_0\phi_s v_{0,rs}\phi_r + 2v_0 v_{0,s}\phi_{rr}\phi_r \\
&\quad + 36\phi_s v_{0,r}\phi_{rr}\phi_r + 36v_0\phi_{rs}\phi_{rr}\phi_r + 3v_0\phi_s v_{0,rr}\phi_r + v_0^2\phi_{rrs}\phi_r + 12v_0\phi_s\phi_{rrr}\phi_r - \phi_s^2 v_{0,r}^2 \\
&\quad - v_0^2\phi_{rs}^2 + 9v_0\phi_s\phi_{rr}^2 - 4v_0\phi_s v_{0,r}\phi_{rs} - 3v_0\phi_s v_{0,s}\phi_{rr} + 3v_0\phi_s v_{0,r}\phi_{rr} + v_0^2\phi_{rs}\phi_{rr} \\
&\quad - v_0\phi_s^2 v_{0,rr} - v_0^2\phi_s\phi_{rrs} = 0,
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\phi_{rr}v_{0,s}^2-2v_{1,s}\phi_r^2v_{0,s}-3\phi_{rr}^2v_{0,s}+2\phi_t\phi_rv_{0,s}+2\phi_r^2v_{1,r}v_{0,s}+2v_{0,r}\phi_{rs}v_{0,s}+4\phi_rv_{0,rs}v_{0,s}\\
&\quad-v_{0,r}\phi_{rr}v_{0,s}+\phi_sv_{0,rr}v_{0,s}-\phi_rv_{0,rr}v_{0,s}+v_0\phi_{rrs}v_{0,s}-4\phi_r\phi_{rrr}v_{0,s}-3v_{0,q}\phi_r^2\\
&\quad+2v_{0,t}\phi_s\phi_r+2v_0\phi_{st}\phi_r+2\phi_t\phi_sv_{0,r}-6\phi_q\phi_rv_{0,r}-4\phi_sv_{1,s}\phi_rv_{0,r}+4\phi_s\phi_rv_{0,r}v_{1,r}\\
&\quad+2v_0\phi_s\phi_{rt}-6v_0\phi_r\phi_{rq}-2v_{0,r}^2\phi_{rs}+2v_0\phi_t\phi_{rs}-4v_0v_{1,s}\phi_r\phi_{rs}+4v_0\phi_rv_{1,r}\phi_{rs}\\
&\quad+2\phi_sv_{0,r}v_{0,rs}-4\phi_rv_{0,r}v_{0,rs}+2v_0\phi_{rs}v_{0,rs}+2v_0\phi_r^2v_{1,rs}-4v_0\phi_s\phi_rv_{1,rs}-3v_0\phi_q\phi_{rr}\\
&\quad-2v_0\phi_sv_{1,s}\phi_{rr}+2v_0\phi_sv_{1,r}\phi_{rr}-12v_{0,r}\phi_{rs}\phi_{rr}-v_0v_{0,rs}\phi_{rr}-12\phi_rv_{0,rs}\phi_{rr}\\
&\quad-2\phi_sv_{0,r}v_{0,rr}-v_0\phi_{rs}v_{0,rr}-12\phi_r\phi_{rs}v_{0,rr}-6\phi_s\phi_{rr}v_{0,rr}+2v_0\phi_s\phi_rv_{1,rr}-v_0v_{0,r}\phi_{rrs}\\
&\quad-12\phi_rv_{0,r}\phi_{rrs}-6v_0\phi_{rr}\phi_{rrs}-6\phi_r^2v_{0,rrs}+v_0\phi_sv_{0,rrs}-v_0\phi_rv_{0,rrs}-4\phi_sv_{0,r}\phi_{rrr}\\
&\quad-4v_0\phi_{rs}\phi_{rrr}-4\phi_s\phi_rv_{0,rrr}-4v_0\phi_r\phi_{rrrs}-v_0\phi_s\phi_{rrrr}=0, \qquad (7)\\
&2v_{0,rs}^2+2\phi_tv_{0,rs}-4v_{1,s}\phi_rv_{0,rs}+4\phi_rv_{1,r}v_{0,rs}-2v_{0,rr}v_{0,rs}-4\phi_{rrr}v_{0,rs}+2v_{0,st}\phi_r+2\phi_{st}v_{0,r}\\
&\quad+2v_{0,s}\phi_{rt}+2\phi_sv_{0,rt}-6v_{0,r}\phi_{rq}-6\phi_rv_{0,rq}+2v_{0,t}\phi_{rs}-4v_{1,s}v_{0,r}\phi_{rs}+4v_{0,r}v_{1,r}\phi_{rs}\\
&\quad-4v_{0,s}\phi_rv_{1,rs}-4\phi_sv_{0,r}v_{1,rs}+4\phi_rv_{0,r}v_{1,rs}-4v_0\phi_{rs}v_{1,rs}+2v_0\phi_{rst}-3v_{0,q}\phi_{rr}\\
&\quad-2v_{0,s}v_{1,s}\phi_{rr}+2v_{0,s}v_{1,r}\phi_{rr}+2v_0v_{1,rs}\phi_{rr}-3\phi_qv_{0,rr}-2\phi_sv_{1,s}v_{0,rr}+2\phi_sv_{1,r}v_{0,rr}\\
&\quad+2v_{0,s}\phi_rv_{1,rr}+2\phi_sv_{0,r}v_{1,rr}+2v_0\phi_{rs}v_{1,rr}-3v_0\phi_{rrq}-2v_0v_{1,s}\phi_{rrs}+2v_0v_{1,r}\phi_{rrs}\\
&\quad-6v_{0,rr}\phi_{rrs}+2v_{0,s}v_{0,rrs}-2v_{0,r}v_{0,rrs}-6\phi_{rr}v_{0,rrs}-2v_0\phi_sv_{1,rrs}+2v_0\phi_rv_{1,rrs}\\
&\quad-4\phi_{rs}v_{0,rrr}-4v_{0,r}\phi_{rrrs}-4\phi_rv_{0,rrrs}-v_{0,s}\phi_{rrrr}-\phi_sv_{0,rrrr}-v_0\phi_{rrrrs}=0, \qquad (8)\\
&4v_{0,rs}v_{1,rs}-2v_{0,rr}v_{1,rs}-2v_{0,rs}v_{1,rr}+2v_{0,rrs}v_{1,s}-2v_{0,rrs}v_{1,r}+2v_{0,s}v_{1,rrs}-2v_{0,r}v_{1,rrs}\\
&\quad-2v_{0,rst}+3v_{0,rrq}+v_{0,rrrrs}=0, \qquad (9)\\
&3v_{1,rrq}-2v_{1,rst}+v_{1,rrrrs}-2v_{1,rs}v_{1,rr}+2v_{1,rs}^2+2\left(v_{1,s}-v_{1,r}\right)v_{1,rrs}=0, \qquad (10)
\end{aligned}$$

with v_1 being a seed solution for Eq. (1) [39–41].

As to $v(r,s,q,t)$, the amplitude of the relevant wave for the dynamical pathology in aortic dissection, Eqs. (3)-(10) form an auto-Bäcklund transformation, because of the mutual consistency of Eqs. (3)-(10), or, the explicit solvability of Eqs. (3)-(10) with respect to ϕ, v_0 and v_1, to be seen below. Other Bäcklund transformations can be seen, e.g., in Refs. [42–44].

3 Three families of the solitonic solutions for Eq. (1)[1]

Next, assume that

$$\phi(r,s,q,t)=e^{\beta_1 r+\beta_2(s,q,t)}+1 \qquad \text{and} \qquad v_1(r,s,q,t)=\beta_3(q,t)r+\beta_4(s,q,t), \qquad (11)$$

where β_1 is a real constant while $\beta_2(s,q,t)$, $\beta_3(q,t)$ and $\beta_4(s,q,t)$ are the real differentiable functions with $\beta_1\neq 0$ and $\beta_{2,s}\neq 0$ since $\phi_r\neq 0$ and $\phi_s\neq 0$. Substituting Expressions (11) into Auto-Bäcklund Transformation (3)-(10), we symbolically compute out three explicitly-solvable families of the solitonic solutions for Eq. (1):

- $v^{(I)}(r,s,q,t)=$

$$\begin{aligned}
&\frac{9\beta_1^2}{2\beta_8q+3\beta_8t-3\beta_1+3\beta_9}\tanh\left[\frac{\beta_1r+\left(\frac{2\beta_8q}{3}+\beta_8t+\beta_9\right)s+\beta_6(q,t)}{2}\right]+\beta_3(q,t)r+\frac{\beta_8s^2}{2\beta_1}\\
&-\frac{3\beta_8s^2}{2(2\beta_8q+3\beta_8t+3\beta_9)}+\frac{\beta_{6,t}(q,t)s}{\beta_1}+\beta_3(q,t)s-\frac{9\beta_{6,q}(q,t)s}{2(2\beta_8q+3\beta_8t+3\beta_9)}-\frac{\beta_1^2s}{2}+\beta_7(q,t)\\
&+\frac{9\beta_1^2}{2\beta_8q+3\beta_8t-3\beta_1+3\beta_9}, \qquad (12)
\end{aligned}$$

  where β_8 and β_9 are the real constants while $\beta_6(q,t)$ and $\beta_7(q,t)$ are the real differentiable functions with $2\beta_8q+3\beta_8t+3\beta_9\neq 0$ and $2\beta_8q+3\beta_8t+3\beta_9\neq 3\beta_1$;

[1] Owing to the page limit, in this section, mathematical derivations are omitted.

- as a solitary-wave special case of $v^{(I)}(r,s,q,t)$,

$$v^{(II)}(r,s,q,t)=\frac{3\beta_1^2}{\beta_9-\beta_1}\tanh\left[\frac{\beta_1 r+\beta_9 s-\frac{\beta_9(\beta_1^3-2\beta_{6b})q}{3\beta_1}+\beta_{6b}t+\beta_{6c}}{2}\right]+\frac{3\beta_1^2}{\beta_9-\beta_1}+\beta_{7a}, \tag{13}$$

  where β_{6b}, β_{6c} and β_{7a} are the real constants with $\beta_9\neq 0$ and $\beta_9\neq\beta_1$;

- $v^{(III)}(r,s,q,t)=$

$$\begin{aligned}&\frac{3\beta_1^2}{\eta_3 e^{\frac{2\eta_2 q}{3}+\eta_2 t}-\beta_1}\tanh\left[\frac{\beta_1 r+\eta_3 e^{\frac{2\eta_2 q}{3}+\eta_2 t}s+\beta_6(q,t)}{2}\right]+\beta_3(q,t)r+\frac{\eta_2\eta_3 e^{\frac{2\eta_2 q}{3}+\eta_2 t}s^2}{2\beta_1}-\frac{\eta_2 s^2}{2}\\&+\frac{\beta_{6,t}(q,t)s}{\beta_1}+\beta_3(q,t)s-\frac{3e^{-\frac{2\eta_2 q}{3}-\eta_2 t}\beta_{6,q}(q,t)s}{2\eta_3}-\frac{\beta_1^2 s}{2}+\beta_7(q,t)+\frac{3\beta_1^2}{\eta_3 e^{\frac{2\eta_2 q}{3}+\eta_2 t}-\beta_1},\end{aligned} \tag{14}$$

  where η_2 and $\eta_3\neq 0$ are the real constants with $\eta_3 e^{\frac{2\eta_2 q}{3}+\eta_2 t}\neq\beta_1$.

As to $v(r,s,q,t)$, the amplitude of the relevant wave for the dynamical pathology in aortic dissection, Solutions (12), (13) and (14) are all different from those in Ref. [4].

4 Bilinear forms for Eq. (1)

On account of the Hirota method [45–49] and symbolic computation, similar to those in Refs. [50,51], we assume that

$$v(r,s,q,t)=\alpha\left[\ln f(r,s,q,t)\right]_r, \tag{15}$$

in which α stands for a real non-zero constant, while $f(r,s,q,t)$ implies a real differentiable function of r, s, q and t.

Using Assumption (15) and integrating Eq. (1) once with respect to r with the integration function equal to zero help us get

$$-2(\ln f)_{rst}+3(\ln f)_{rrq}+2\alpha(\ln f)_{rs}(\ln f)_{rrs}-2\alpha(\ln f)_{rr}(\ln f)_{rrs}+(\ln f)_{rrrs}=0. \tag{16}$$

Based on the formulae [45]

$$(\ln f)_{rst}=\frac{1}{2}\left(\frac{D_s D_t f\cdot f}{f^2}\right)_r, \tag{17a}$$

$$(\ln f)_{rrq}=\frac{1}{2}\left(\frac{D_r D_q f\cdot f}{f^2}\right)_r, \tag{17b}$$

$$(\ln f)_{rs}=\frac{1}{2}\frac{D_r D_s f\cdot f}{f^2}, \tag{17c}$$

$$(\ln f)_{rrs}=\frac{1}{2}\left(\frac{D_r D_s f\cdot f}{f^2}\right)_r, \tag{17d}$$

$$(\ln f)_{rr}=\frac{1}{2}\frac{D_r^2 f\cdot f}{f^2}, \tag{17e}$$

$$(\ln f)_{rrrs}=\frac{1}{2}\left(\frac{D_r^3 D_s f\cdot f}{f^2}-3\frac{D_r^2 f\cdot f}{f^2}\frac{D_r D_s f\cdot f}{f^2}\right)_r, \tag{17f}$$

we find

$$-2\left(\frac{D_s D_t\, f\cdot f}{f^2}\right)_r + 3\left(\frac{D_r D_q\, f\cdot f}{f^2}\right)_r + \left(\frac{D_r^3 D_s\, f\cdot f}{f^2}\right)_r + \frac{D_r D_s\, f\cdot f}{f^2}\left(\alpha\frac{D_r D_s\, f\cdot f}{f^2} - 3\frac{D_r^2\, f\cdot f}{f^2}\right)_r - (\alpha+3)\,\frac{D_r^2\, f\cdot f}{f^2}\left(\frac{D_r D_s\, f\cdot f}{f^2}\right)_r = 0, \tag{18}$$

and then choose

$$\alpha = -3, \qquad \left(D_r D_s + D_r^2\right) f\cdot f = 0, \tag{19}$$

so as to construct out the following bilinear forms for Eq. (1):

$$\left(-2D_s D_t + 3D_r D_q + D_r^3 D_s\right) f\cdot f = 0, \tag{20a}$$

$$\left(D_r D_s + D_r^2\right) f\cdot f = 0, \tag{20b}$$

under Assumption (15), with D_r, D_s, D_q and D_t defined as [45]

$$D_r^{m_1} D_s^{m_2} D_q^{m_3} D_t^{m_4}\, G(r,s,q,t)\cdot F(r,s,q,t) = \left(\frac{\partial}{\partial r} - \frac{\partial}{\partial r'}\right)^{m_1}\left(\frac{\partial}{\partial s} - \frac{\partial}{\partial s'}\right)^{m_2} \left(\frac{\partial}{\partial q} - \frac{\partial}{\partial q'}\right)^{m_3}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^{m_4} G(r,s,q,t)\, F(r',s',q',t')\Big|_{r'=r,\, s'=s,\, q'=q,\, t'=t}, \tag{21}$$

r', s', q' and t' denoting the formal variables, $G(r,s,q,t)$ indicating a C^∞ function as for r, s, q and t, $F(r',s',q',t')$ implying a C^∞ function as for r', s', q' and t', while m_1, m_2, m_3 and m_4 representing the non-negative integers [45].

As to $v(r,s,q,t)$, the amplitude of the relevant wave for the dynamical pathology in aortic dissection, we will next confirm the mutual consistency of Bilinear Forms (20), by the way of building up certain M-soliton solutions.

5 M-soliton solutions for Eq. (1)

For the purpose of confirming the mutual consistency of Bilinear Forms (20), we are in a position to expand $f(r,s,q,t)$ in Bilinear Forms (20) with respect to a formal expansion parameter ϵ, i.e.,[2]

$$f(r,s,q,t) = 1 + \sum_{\varsigma=1}^{M} \epsilon^{\varsigma} f_{\varsigma}(r,s,q,t), \tag{22}$$

and then to set $\epsilon = 1$, so as to construct the following M-soliton solutions for Eq. (1), concerning $v(r,s,q,t)$, the amplitude of the relevant wave for the dynamical pathology in aortic dissection, i.e.,

Expression (15),

$$f(r,s,q,t) = \sum_{\gamma_i,\gamma_j=0,1} \exp\left\{\sum_{i=1}^{M}\gamma_i\left[\zeta_i r - \zeta_i s + \omega_i q + \frac{1}{2}\left(\zeta_i^3 - 3\omega_i\right)t + \rho_i\right] + \sum_{1\le i<j}^{(M)}\gamma_i\gamma_j\left(\frac{\zeta_j-\zeta_i}{\zeta_j+\zeta_i}\right)^2\right\}, \tag{23}$$

with ς, i and j as three positive integers, $\varsigma \le M$, $i \le M$, $j \le M$, ω_i's and ζ_i's as certain real constants, $\zeta_j \ne \pm\zeta_i$, $f_{\varsigma}(r,s,q,t)$'s as some real differentiable functions with respect to r, s, q and t, the sum $\sum_{\gamma_i,\gamma_j=0,1}$ taken over all the possible combinations of $\gamma_j = 0, 1$, while $\sum_{1\le i<j}^{(M)}$ as the summation over all the possible pairs chosen from the M elements under the condition $i < j$.

[2] similar to those in Refs. [52, 53]

6 Conclusions

Recent soliton advances in *Indian J. Pure Appl. Math.* have been impressive. As to the dynamical pathology, etc., aortic dissection has been believed as a catastrophic disease influencing the aorta, while contemporary aortic wave dynamics has been progressing. Solitons have been thought to result in the unstable wave field and generation of pathological waves, while pathological waves have been able to cause the aneurysmal rupture and progression of aortic dissection.

In this paper, we have implemented some symbolic computation on a (3+1)-dimensional Jadaun-Singh equation for the dynamical pathology in aortic dissection, i.e., Eq. (1). Via the singular manifold, etc., Auto-Bäcklund Transformation (3)-(10) and three families of the solitonic solutions, i.e., Solutions (12), (13) and (14), as well as Bilinear Forms (20) with (15) and M-Soliton Solutions (23) with (15), have been obtained, for $v(r, s, q, t)$, the amplitude of the relevant wave.

We hope that our results, like the ones addressed in Ref. [4], could assist certain studies on the dynamical pathology in aortic dissection and cardiothoracic physicians in pinpointing the latent cases and working on such preventive regimens as the control of hypertension and restriction on physiological activity.

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Declarations

Competing interest The authors declare that they have no conflict of interest.

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