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Research Paper

Finite Element Analysis of Thin Circular Cylindrical Shells

ARUNA RAWAT[1,*], VASANT MATSAGAR[2] and A K NAGPAL[3]

[1]*PhD Research Scholar, Department of Civil Engineering, Indian Institute of Technology (IIT) Delhi, Hauz Khas, New Delhi 110 016, India*

[2]*Associate Professor, Department of Civil Engineering, Indian Institute of Technology (IIT) Delhi, Hauz Khas, New Delhi 110 016, India*

[3]*Emeritus Professor, Department of Civil Engineering, Indian Institute of Technology (IIT) Delhi, Hauz Khas, New Delhi 110 016, India*

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Free vibration finite element (FE) analysis of thin circular cylindrical shells is investigated. The circular cylindrical shell can vibrate in different modes and theoretically infinite modes are possible. The axial mode, m and circumferential mode, n, in any of their combinations define the modes and the corresponding modal frequencies. The shell elements are used to model the thin circular cylindrical shells. The eigenvalues of the shell are extracted using block Lanczos iteration method. Detailed mesh convergence studies are performed for different height or length to radius (H/R) ratios. Importantly, selection of appropriate FE mesh size criteria based on the perimeter and height of the circular cylindrical shell, as well as thickness to radius (h/R) ratio are shown for various boundary conditions. The modal frequencies of the cylindrical shell are investigated for different boundary conditions such as clamped-clamped (C-C), clamped-free (C-F), and simply-supported - simply-supported (S-S). The effects of height to radius (H/R) ratio and thickness to radius (h/R) ratio on the modal frequencies of the cylindrical shells are also studied. For all the considered boundary conditions, the modal frequencies of the cylindrical shells increase with higher circumferential mode number and also with the increase in the h/R ratio. The modal frequencies are observed to be the lowest in the case of the C-F boundary condition of the shell.

Keywords: Boundary Condition; Circular Cylinder; Finite Element; Modal Frequency; Shell

Introduction

Thin shells as structural elements are the most predominantly used in engineering, particularly in civil, mechanical, architectural, aeronautical, and marine engineering. Examples of the shell structures in civil and architectural engineering are large-span roofs, liquid-retaining structures and water tanks, containment shells of nuclear power plants, and concrete arch domes. In mechanical engineering, shell forms are used in piping systems, turbine disks, and pressure vessels technology. Of all existing shell models, the circular cylindrical shell is the most widely used.

Free vibration occurs in the absence of external force, however is initiated by applying initial displacement and/or velocity conditions to the shell. Knowledge of the free vibration characteristics of the thin circular cylindrical shells is important both for understanding the fundamental shell behavior and for designing shell structures for industrial applications. Also, for acoustic and dynamic response calculations, complete information of free vibration characteristics of these structures is needed. The frequencies and mode shapes of the shells are important in the design of such structures.

Many shell theories and methods have been developed over the decades for the analyses of the shells. These shell theories originated from the works reported by Love (1888, 1944) and Flügge (1960) and many more shell theories that neglect some of the terms from the Flügge equations have been proposed

*Author for Correspondence: E-mail: augupta2001@gmail.com. Tel.: +91-875-088-7209

over the years. Among other shell theories are Timoshenko (1921) theory based on one-dimensional (1-D) beam theory, and two-dimensional (2-D) theories include Donnell (1933), Reissner (1941), Sanders (1959) etc. The other theories are based on Rayleigh-Ritz energy methods (Arnold and Warburton, 1949, 1953), closed-form solutions of the governing equations, and iterative solution approaches. Leissa (1973) carried out a comprehensive review and comparison of various shell theories.

Further, the free vibration analysis of the circular cylindrical shell was studied by several researchers using various theories available in the literature. The shells were analyzed for different boundary conditions such as clamped - clamped (C-C), clamped - free (C-F), simply-supported - simply-supported (S-S), simply-supported - clamped (S-C), free - free (F-F), and shear diaphragm-shear diaphragm (SD-SD).

Sharma and Johns (1972) and Sharma (1974) proposed an approximate method for calculating the natural frequencies of the fixed-free circular cylindrical shells. Soedel (1980) proposed formula for calculating the natural frequencies of the circular cylindrical shells for different boundary conditions. Moussaoui *et al.* (2000) studied the mode shapes and frequencies of thin elastic shells of infinite length. Xuebin (2006) calculated the free vibration frequencies of a thin circular cylindrical shell based on Flügge's shell theory equations for orthotropic materials. Zhang *et al.* (2001) and Xuebin (2008) used wave propagation approach for calculating the free vibration of cylindrical shells for different boundary conditions. Amabili and Païdoussis (2003) presented a comprehensive review of various linear and nonlinear shell models reported in the literature. Amabili (2008) and Kurylov and Amabili (2010) studied the nonlinear vibration of the circular cylindrical shells for different boundary conditions. Farshidianfar and Oliazadeh (2012) used semi-analytical approach for the free vibration analysis of the simply-supported circular cylindrical shells and compared the results with ten different shell theories. Recently, Lee and Kwak (2015) used Rayleigh-Ritz method for free vibration analysis of the circular cylindrical shell and compared the results with different shell theories.

Similarly, finite element method (FEM) was also used for free vibration of the circular cylindrical shell as reported by Zienkiewicz (1969), Ramamurti and Pattabiraman (1976), Kant *et al.* (1994), Zhang *et al.* (2001), Chapelle and Bathe (2011), Oliazadeh *et al.* (2013) and Lee and Kwak (2015).

It is evident from the literature that, as the circular cylindrical shell vibrates in different modes, its analysis becomes intricate, especially analytical/ closed-form solutions may become intractable. Hence, the FEM may be relied on for conducting free vibration analysis of the circular cylindrical shells.

The major objectives of the present study are: (i) to calculate the modal frequencies of the circular cylindrical shells using the FE approach and compare the results with different shell theories published in the literature, (ii) to study the effect of different boundary conditions, such as clamped - clamped (C-C), clamped-free (C-F), and simply-supported - simply-supported (S-S) on the modal frequencies of the shells, (iii) to investigate the effects of different height or length to radius (H/R) ratios, thickness to radius (h/R) ratios, and boundary conditions on the mesh convergence, based on which a ratio of perimeter of circular cylindrical shell to mesh size along the circumference and the ratio of height to mesh size along the height can be estimated, and (iv) to study the effect of H/R and h/R ratios on the modal frequencies of the circular cylindrical shells.

Mathematical Modeling

Figure 1 shows the co-ordinate system used for the cylindrical shell. Figures 2(A) and 2(B) respectively show circumferential and axial vibration modes of the cylindrical shell. The cylindrical shell considered is having height (or length), H; radius, R; constant thickness, h; density, ρ; modulus of elasticity, E; and Poisson's ratio, ν. The reference surface of the shell is taken at its middle surface where an orthogonal

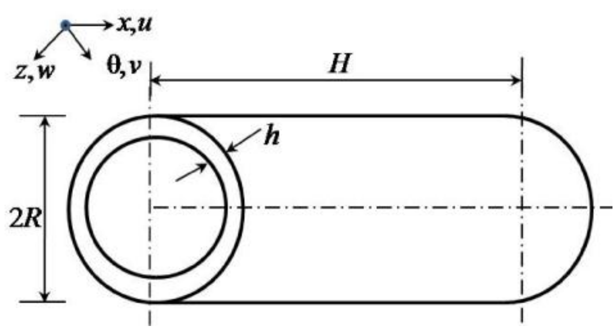


Fig. 1: Co-ordinate system of shell

co-ordinate system (x, θ, z) is defined. The x co-ordinate is taken in the axial direction of the shell, whereas the θ and z co-ordinates are taken respectively in the circumferential and radial directions of the shell. The displacements of the shell are defined by u, v, and w in the x, θ, and z directions, respectively. The circular cylindrical shell vibrates in axial mode, m and the circumferential mode, n, whereas radial mode is ignorable. The circumferential modes are in-and-out deformations in the form of cosine waves, $\cos(n\theta)$, as shown in Fig. 2(A), whereas the axial modes include flexural deformations along the axial direction as shown in Fig. 2(B). Any combination of m and n defines the modes and the corresponding modal frequencies of the circular cylindrical shell.

For the free vibration of a cylindrical shell, the equations of motion based on Flügge (1960) in matrix form are given as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \{0\} \tag{1}$$

where L_{ij} (i, j = 1, 2, 3) are the differential operators with respect to x, θ, and t. The first attempt at solving the Eq. (1) involves the assumption of a synchronous motion

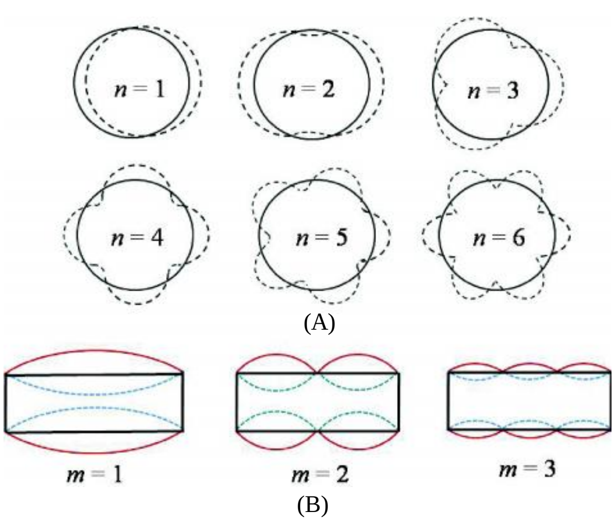


Fig. 2: (A) Circumferential and (B) axial modes of circular cylindrical shell

$$\begin{aligned} u(x,\theta,t) &= U(x,\theta)\, f(t) \\ v(x,\theta,t) &= V(x,\theta)\, f(t) \\ w(x,\theta,t) &= W(x,\theta)\, f(t) \end{aligned} \tag{2}$$

where $f(t)$ is the scalar model coordinate corresponding to the mode shapes $U(x, \theta)$, $V(x, \theta)$, and $W(x, \theta)$. The next step is to use the separation of variables method in order to separate the spatial dependence of the mode shapes between axial and circumferential directions. Hence, the axial, tangential, and radial displacements of the shell wall are varied according to

$$\begin{aligned} u(x,\theta,t) &= Ae^{\lambda_m x} \sin(n\theta)\cos(\omega t) \\ v(x,\theta,t) &= Be^{\lambda_m x} \sin(n\theta)\cos(\omega t) \\ w(x,\theta,t) &= Ce^{\lambda_m x} \sin(n\theta)\cos(\omega t) \end{aligned} \tag{3}$$

where λ_m and n are the axial wave number and the circumferential wave parameter, respectively, A, B, and C are the undetermined constants, and ù is the angular/circular frequency of the natural vibration. Substitution of Eq. (3) into Eq. (1) and use of any of the theories given in the above-mentioned literature, lead to a set of homogeneous equations having the following matrix form

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \{0\} \tag{4}$$

in which C_{ij} (i, j = 1, 2, 3) are coefficients which are the functions of λ_m, n, and a frequency parameter, Ω defined as $\Omega^2 = \left(1-\nu^2\right)\rho\omega^2 R^2 / E$. For the non-trivial solution, the determinant of the coefficient matrix must be zero, such as

$$\det\left(\left[C_{ij}\right]\right) = 0,\ \ i, j = 1,\ 2,\ 3 \tag{5}$$

The characteristics equation is obtained from the expansion of Eq. (4)

$$f(\lambda, \omega) = 0 \tag{6}$$

The boundary conditions for the clamped - clamped (C-C) shell at $x = 0$ and at $x = H$ are $u = v =$

$w = \partial w/\partial x = 0$. And, boundary conditions for the clamped - free (C-F) shell at $x = 0$ are $u = v = w = \partial w/\partial x = 0$, and at $x = H$ are $M_x = N_x = 0$, where M_x is the axial moment and N_x is the axial force in the shell as it deforms. Moreover, for the simply-supported - simply-supported (S-S) shell, the boundary conditions at $x = 0$ and at $x = H$ are $v = w = 0$ and $M_x = N_x = 0$.

The eigenvalue problem for the modal frequencies of an undamped FE model of the shell is given as

$$\left(-\omega^2 M^{MN} + K^{MN}\right)\Phi^{N} = 0 \qquad (7)$$

where M^{MN} is the mass matrix (which is symmetric and positive definite), K^{MN} is the stiffness matrix (which includes initial stiffness effects if the base state included the effects of nonlinear geometry), Φ^{N} is the eigenvector (the mode of vibration), and M and N are the degrees of freedom.

Finite Element Modeling

In the present study, finite element method (FEM) is used for conducting free vibration analysis of the circular cylindrical shell. Block Lanczos method is used for extracting frequencies of the circular cylindrical shell in the present study. The frequency extraction analysis is performed in the FE software Abaqus® using linear perturbation procedure. The cylindrical shell is modeled using four node quadrilateral shell elements (S4R) with reduced integration and hourglass control. The C-C, C-F, and S-S boundary conditions are applied on both the circular edges of the shell.

The accuracy of the results obtained from the present FE models has been ensured for the circular cylindrical shell by comparing it with the results obtained from the past results reported in the literature. Comparison of the modal frequencies calculated using the present FE approach is made with that published by Lee and Kwak (2015). They presented modal frequencies obtained for the shells by adopting FE approach in ANSYS® and compared with the results obtained from the Flügge (FLH) and the Donnell-Mushtari (DM) theories. An aluminum shell with H = 600 mm, R = 150 mm, h = 1 mm, ρ = 2,770 kg/m^3, E = 71 GPa, and ν = 0.33 is considered. Table 1 shows the modal frequencies (f_{mn}) obtained in the present study for the C-C and C-F boundary conditions of the shell and the results reported by Lee and Kwak (2015). The percentage difference between the modal frequencies obtained by the method adopted here, FLH, and DM theories is shown. It is observed that the modal frequencies obtained from the block Lanczos technique in the present FE approach are duly verified with the FLH theory.

Table 1: Comparison of modal frequencies in Hz between present FEM and results obtained by Lee and Kwak (2015) (*H* = 600 mm, *R* = 150 mm, *h* = 1 mm)

Boundary condition	Mode (m,n)	Present method	ANSYS® (Lee and Kwak, 2015)		FLH (Lee and Kwak, 2015)		DM (Lee and Kwak, 2015)	
		f_{mn}	f_{mn}	Diff. (%)	f_{mn}	Diff. (%)	f_{mn}	Diff. (%)
Clamped-Clamped	(1,5)	371	370	0.3	371	0	379	-2.1
	(1,4)	411	411	0	411	0	415	-1
	(1,6)	429	427	0.5	429	0	439	-2.3
	(1,7)	546	544	0.4	547	-0.2	558	-2.2
	(1,3)	586	586	0	587	-0.2	588	-0.3
	(2,6)	625	624	0.2	633	-1.3	640	-2.3
Clamped-Free	(1,3)	146	146	0	146	0	153	-4.6
	(1,4)	175	174	0.6	175	0	185	-5.4
	(1,2)	242	243	-0.4	242	0	243	-0.4
	(1,5)	263	261	0.8	263	0	274	-4
	(1,6)	382	378	1.1	381	0.3	391	-2.3
	(2,5)	381	380	0.3	381	0	389	-2.1

Numerical Study and Discussion

Free vibration analysis using FEM, carried out in the present study, aims to compare the performance of the thin circular cylindrical shell. The effects of different height or length to radius (H/R) ratios, thickness to radius (h/R) ratios, and boundary conditions on the mesh convergence in the FE analysis are investigated. Based on which a ratio of the perimeter of the circular cylindrical shell to mesh size along the circumference ($2\pi R/s_r$) and a ratio of the height to mesh size along the height (H/s_H) are estimated. These two ratios along circumferential and longitudinal directions of the cylindrical shell give estimate on number of elements to subdivide the FE domain in the respective directions. The effect of different boundary conditions, such as clamped - clamped (C-C), clamped - free (C-F), and simply-supported - simply-supported (S-S), height or length to radius (H/R) ratios, and thickness to radius (h/R) ratios on the modal frequencies of the shell are investigated.

The geometrical and material properties of the steel shell considered are: $R = 1$ m, $\rho = 7{,}850$ kg/m^3, $E = 210$ GPa, and $\nu = 0.3$. Variations in the modal frequencies of the shell are studied for different height to radius (H/R) ratios varying from 2 to 20 at an interval of 1, and thickness to radius (h/R) ratio varied as 1/200, 1/100, 1/50, 1/20 and 1/10.

Mesh Convergence Study

In the numerical analysis using FE approach, coarse mesh is employed to commence with and subsequently mesh refinement studies are performed to obtain solution of problems within an acceptable accuracy. Therefore, it becomes necessary to determine a good initial mesh density for conversed results to obtain with reduced computational efforts. A set of mesh convergence study is performed for different H/R ratios, based on which a ratio of perimeter of cylindrical shell ($2\pi R$) to mesh size along the circumference (s_r) and height or length (H) to mesh size along the height (s_H) are proposed.

For this study, $R = 1$ m, 2 m, and 3 m; $H/R = 2$, 10, 15, and 20 are considered for all h/R ratios and boundary conditions. The corresponding fundamental frequencies are evaluated using block Lanczos technique. The mesh size along the radius and height of the shell is varied and the converged value of fundamental frequency is considered for all the H/R ratios.

Figure 3 shows the variation of $2\pi R/s_r$ ratio with respect to the H/R ratio for the shell. It can be observed that as the H/R ratio increases the required mesh size along the radius for convergence requirement decreases, which is independent of the boundary condition and h/R ratio of the thin circular cylindrical shell. For higher H/R ratio = 15 and 20, the fundamental mode number is (1, 2) whereas for H/R ratio = 2 it is (1, 5) for shell with C-C boundary condition.

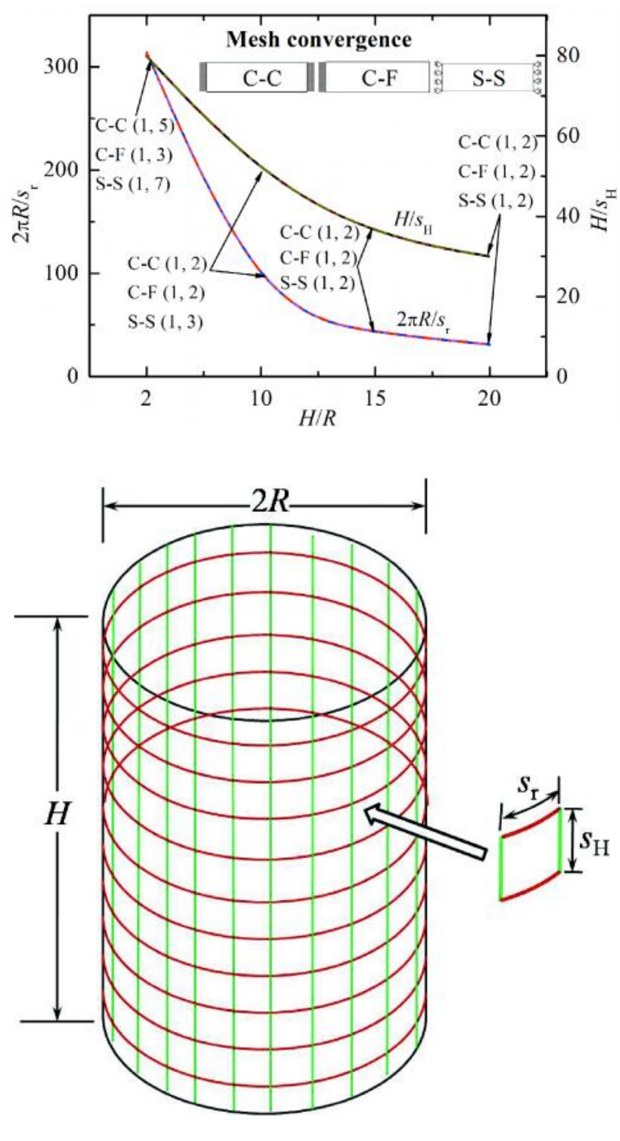


Fig. 3: Mesh convergence study of shell for H/R ratios = 2, 10, 15, and 20

Figure 3 also shows the variation of the H/s_H ratio with respect to the H/R ratio for the shell. This ratio decreases with increase in the H/R ratio of the shell. The values of H/s_H ratio are much lesser than $2\pi R/s_r$ ratio, as more elements are required along the circumferential direction as compared to the longitudinal direction of the shell for lower H/R ratio and vice-versa. This is attributed to the fact that, for slender thin circular cylindrical shell (higher H/R ratio), finer FE mesh is required along the height (decreased s_H) and relatively coarser mesh suffice along the circumference for convergence requirement, which is independent of the boundary condition and the shell thickness. The aspect ratio of the FE mesh (s_r/s_H) for cylindrical shell can be estimated based on the present mesh convergence study. The mesh convergence study gives an estimate for selecting the mesh size for the FE analysis of the circular cylindrical shell. This avoids the rigorous mesh convergence study to start FE computations and hence reduces the computational efforts.

Geometric and Boundary Conditions Study

The effects of different boundary conditions, H/R, and h/R ratios on the modal frequencies of the shell are investigated. Table 2 shows the first (lowest) ten modal frequencies for $H/R = 2$ for C-C, C-F, and S-S boundary conditions of the shells for all h/R ratios considered herein. It can be observed that maximum frequency for $H/R = 2$ corresponds to $h/R = 1/10$, for the increased thickness of the shell.

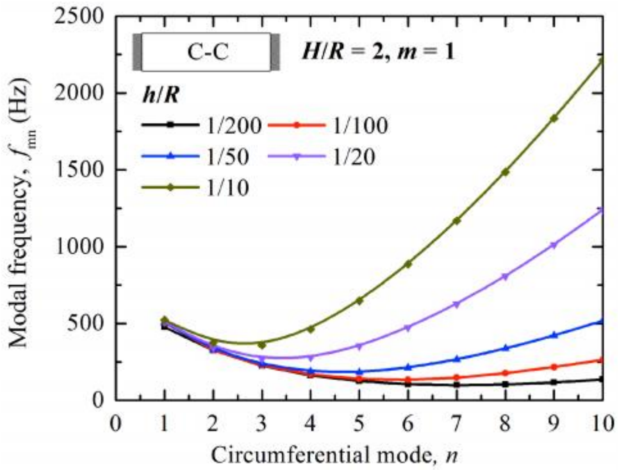
Figures 4(A), 4(B), and 4(C) show the modal frequencies for the shell with C-C boundary condition considering the first axial mode ($m = 1$), $H/R = 2$, 10, and 20, and for all considered h/R ratios, respectively. It can be seen that with increase in the circumferential mode (n) for $m = 1$, the modal frequency, f_{mn} increases for higher H/R ratios (10 and 20) however not for the $H/R = 2$ for all considered h/R ratios. As the h/R ratio increases the modal frequencies increase in the circumferential mode.

Figure 4(D) shows the frequency for different circumferential modes with $m = 1$ and for different H/R ratios considering $h/R = 1/10$ with the C-C boundary condition of the shell. It is seen that for all the n values up to $H/R = 10$ there is decrease in the frequency with increase in the H/R ratio. Beyond that, it remains almost constant for larger H/R ratios. For the first three wave numbers ($n = 1$, 2, and 3) the variation in the modal frequencies is more with respect to the H/R ratio.

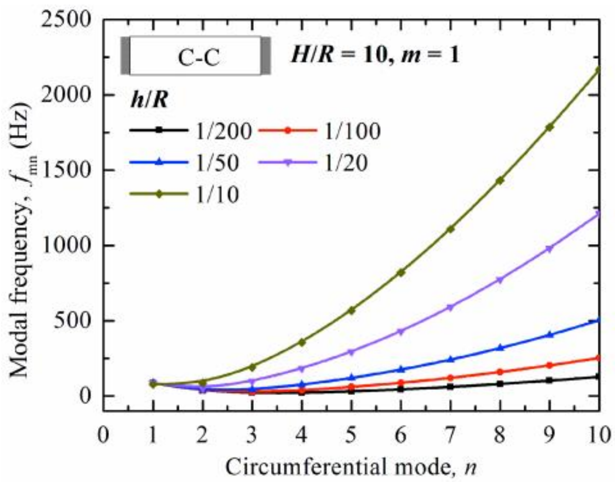
Table 2: Modal frequencies in Hz in cylindrical shell (m = 1) for H/R = 2

n	h/R=1/200	h/R=1/100	h/R=1/50	h/R=1/20	h/R=1/10
			C-C		
1	475	504	506	511	523
2	325	326	329	342	375
3	224	226	233	269	358
4	163	169	187	277	462
5	126	139	181	352	647
6	105	133	211	473	886
7	98	148	265	626	1168
8	102	176	336	807	1486
9	116	216	421	1012	1836
10	136	263	518	1241	2214
			C-F		
1	237	238	238	240	242
2	119	119	120	127	144
3	66	69	77	120	209
4	44	55	86	191	366
5	40	66	123	298	574
6	48	90	176	432	823
7	62	122	242	591	1110
8	81	160	318	773	1431
9	102	204	405	980	1782
10	127	253	503	1209	2162
			S-S		
1	503	503	504	504	506
2	324	324	325	330	346
3	223	224	228	254	328
4	162	166	182	264	437
5	125	137	177	342	629
6	104	132	208	465	872
7	100	146	262	620	1157
8	102	175	334	801	1477
9	116	215	419	1007	1828
10	136	262	516	1237	2208

Figure 4(E) shows the variation of the modal frequencies with respect to the h/R ratio for $H/R = 2$ and 10 with $(m, n) = (1, 3)$ and $(1, 7)$. It is seen that with increase in the h/R ratio, the frequency increases



A



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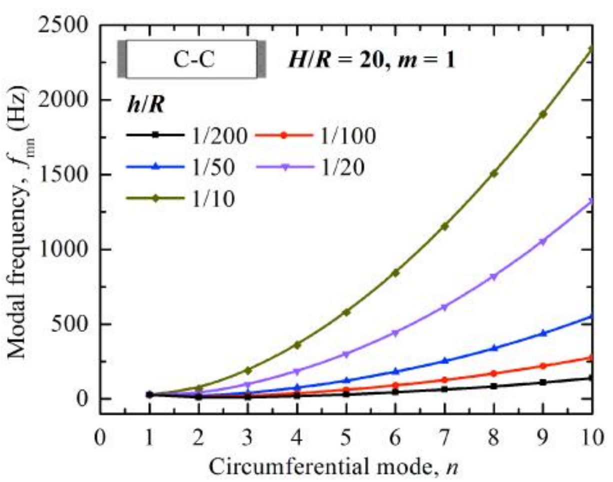
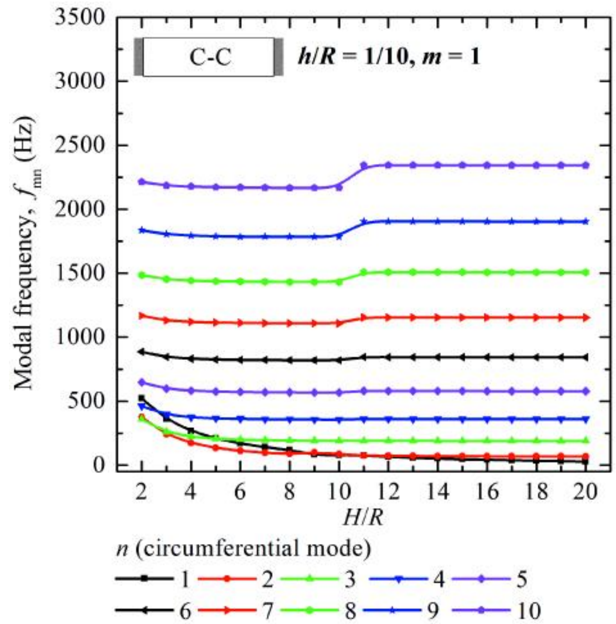
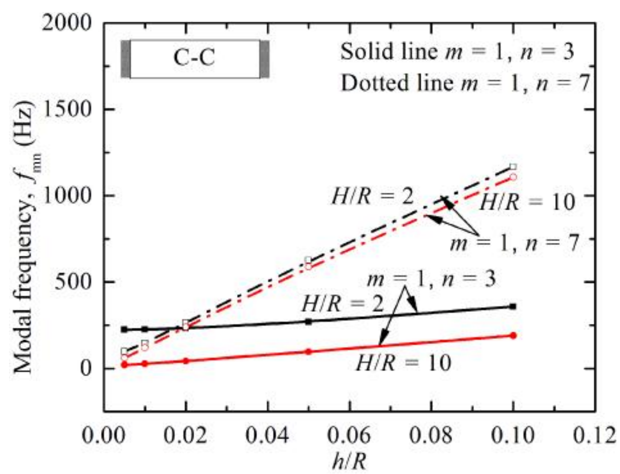


C



D

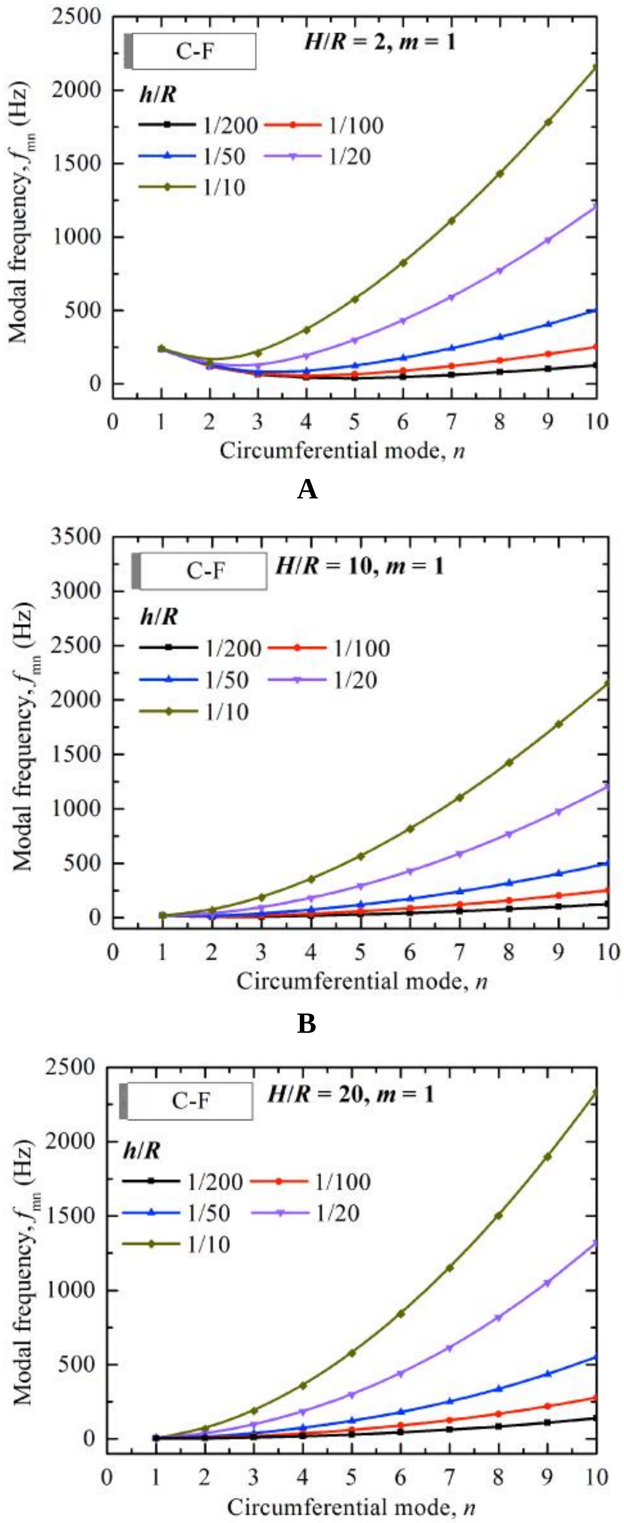


E

Fig. 4: Modal frequencies for C-C shell (m = 1), (A) For different h/R ratios, H/R = 2, (B) For different h/R ratios, H/R = 10, (C) For different h/R ratios, H/R = 20, (D) For different H/R ratios for h/R = 1/10 and (E) For different h/R ratios for H/R = 2 and 10 (1, 3), (1, 7)

in circumferential mode. For higher wave number there is no significant effect of the H/R ratio on the frequency values with respect to the h/R ratios.

In case of the shell with C-F boundary condition, one end is clamped due to which the modal frequencies are reduced. Nevertheless, trend of the results similar



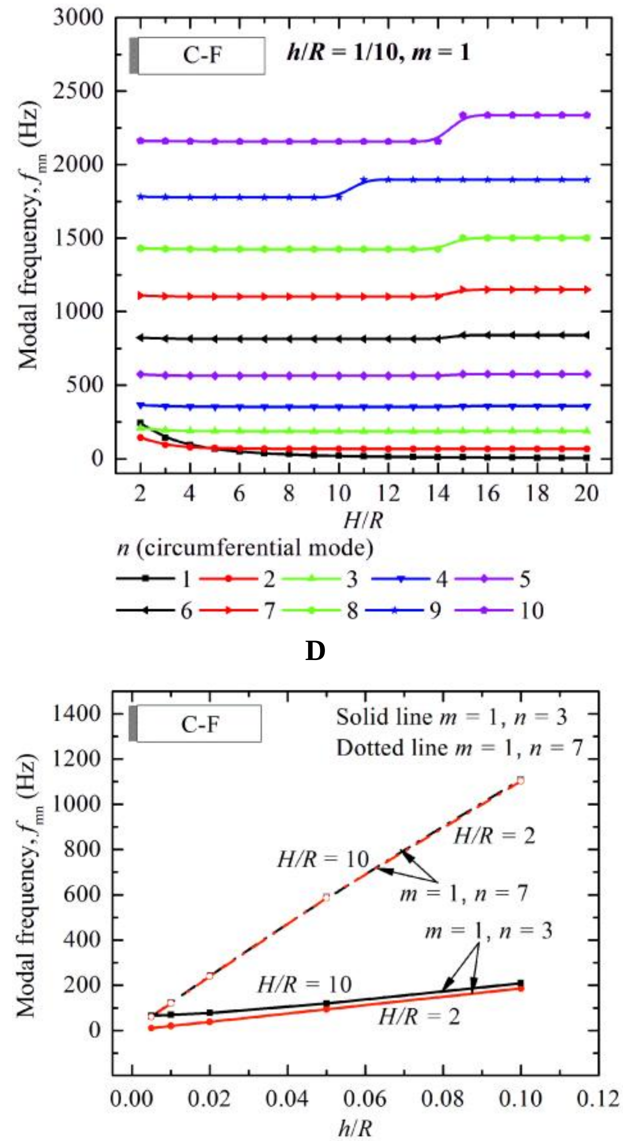


Fig. 5: Modal frequencies for C-F shell (*m* = 1), (A) For different *h*/*R* ratios, *H*/*R* = 2, (B) For different *h*/*R* ratios, *H*/*R* = 10, (C) For different *h*/*R* ratios, *H*/*R* = 20, (D) For different *H*/*R* ratios for *h*/*R* = 1/10 and (E) For different *h*/*R* ratios for *H*/*R* = 2 and 10 (1, 3), (1, 7)

to the C-C boundary condition are observed in case of the C-F boundary condition, which can be seen from Figs. 5(A), 5(B), 5(C), 5(D) and 5(E).

Figure 6 shows the comparison of modal frequencies with $m = 1$ and $n = 1$ to 10 with C-C and C-F shell boundary conditions considering $H/R = 2$

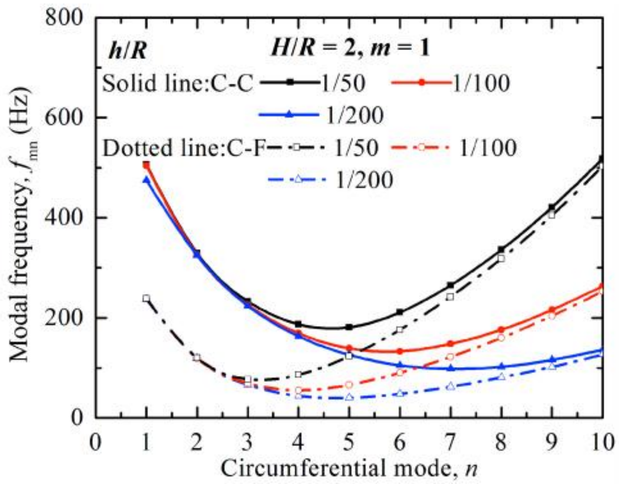
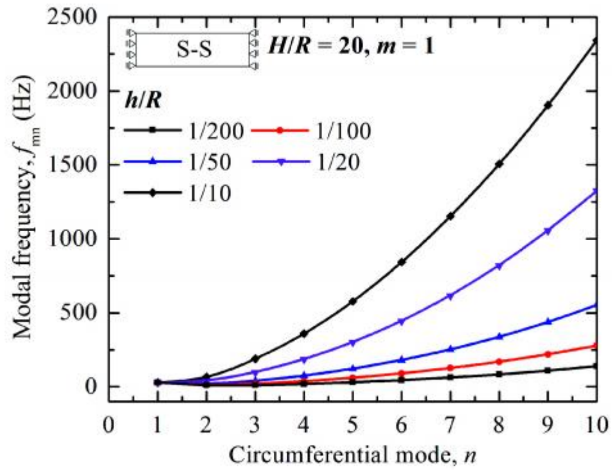
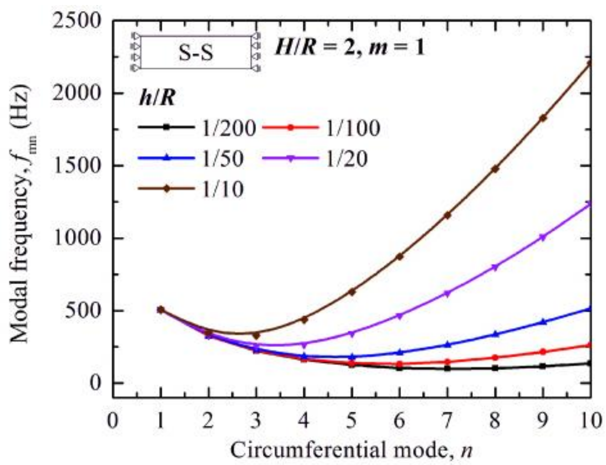


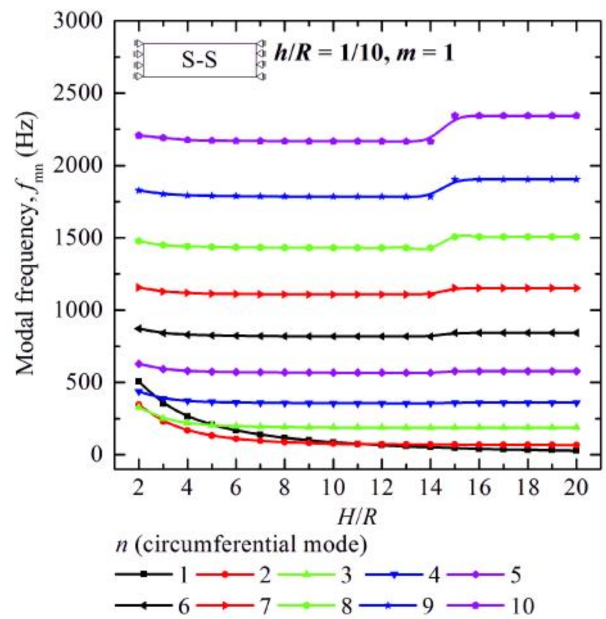
Fig. 6: Comparison of modal frequencies for C-C and C-F shells considering $H/R = 2$, and for $h/R = 1/50$, 1/100, and 1/200



C



A



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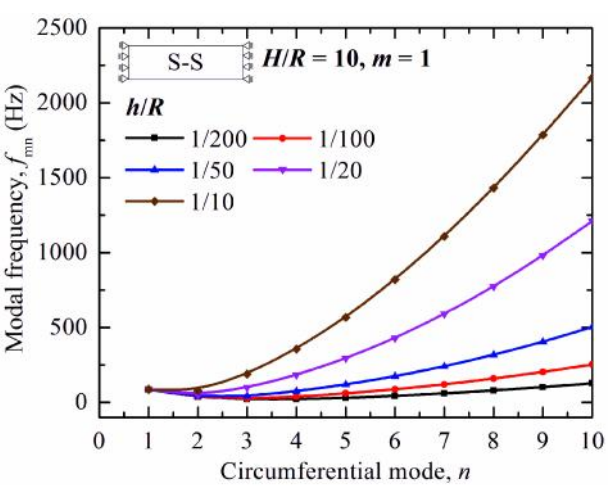

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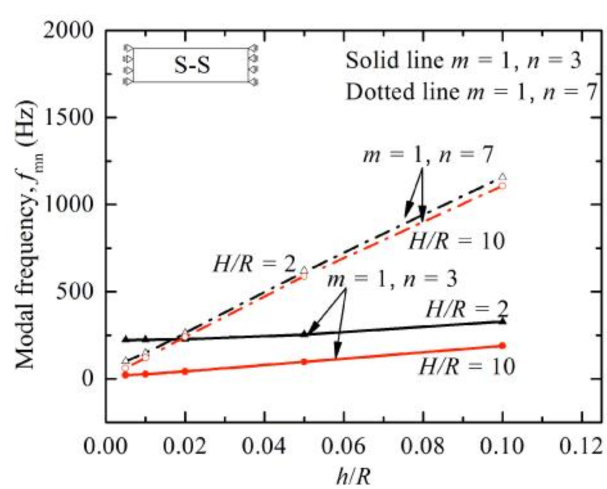

E

Fig. 7: Modal frequencies for S-S shell ($m = 1$), (A) For different h/R ratios, $H/R = 2$, (B) For different h/R ratios, $H/R = 10$, (C) For different h/R ratios, $H/R = 20$, (D) For different H/R ratios for $h/R = 1/10$, (E) For different h/R ratios for $H/R = 2$ and 10 (1, 3), (1, 7)

and h/R = 1/50, 1/100, and 1/200. Here, the first three modal frequencies are same irrespective of the h/R ratio seen for both C-C and C-F boundary conditions of the shells. The modal frequency (1, 1) for $H/R = 2$ considering the shell with C-C boundary condition is 475 Hz while in case of the shell with C-F boundary condition it is 237 Hz corresponding to h/R = 1/200. The modal frequencies are lower in case of the C-F boundary condition as compared to the C-C condition. Notably, the effect of boundary conditions on the modal frequencies diminishes at higher mode, *n*.

Figures 7(A), 7(B) and 7(C) show the modal frequencies for the shell with S-S boundary condition considering $m = 1$ (first axial mode only), H/R ratio 2, 10, and 20, respectively and for all considered h/R ratios. It can be seen that with increase in the circumferential mode (n) for $m = 1$, the f_{mn} increases for higher H/R ratios (10 and 20) however not for $H/R = 2$. The circumferential frequencies increase with the increase in the wave number; however, they are insignificantly affected by the increase in the H/R ratio from 10 to 20 as seen from Fig. 7(D). For the first three wave numbers the variation in the modal frequencies is more with respect to the H/R ratio than higher *n*.

Figure 7(E) shows the variation of the modal frequencies with respect to the h/R ratio for $H/R = 2$ and 10 with $(m, n) = (1, 3)$ and (1, 7). It is seen that for higher circumferential mode, $n = 7$ the modal frequency with varying h/R ratio for $H/R = 2$ and 10 is almost same.

As the height to radius (H/R) ratio of the circular cylindrical shell decreases, it becomes more rigid and its modal frequencies increase. The broad shell (lower H/R ratio) behaves more rigidly in longitudinal direction thereby its longitudinal frequencies (*m*) decrease and circumferential frequencies (*n*) increase.

Conclusions

The free vibration finite element (FE) analysis of thin circular cylindrical shells is conducted. As the circular cylindrical shell vibrates in different modes, its analysis becomes intricate, especially analytical/closed-form solutions may become intractable. Hence, the FE method may be relied on for conducting free vibration analysis of the circular cylindrical shells. A set of mesh convergence studies are performed for the selecting the appropriate FE mesh size criteria which is based on the perimeter and height of the circular cylindrical shell, thickness to radius (h/R) ratios, and various boundary conditions. From the results of the present study, the following conclusions are derived:

1. For slender thin circular cylindrical shell (higher H/R ratio), finer FE mesh is required along the height (decreased s_H) and relatively coarser mesh suffice along the circumference for convergence requirement, which is independent of the boundary condition and the shell thickness.
2. The modal frequency of the circular cylindrical shell is influenced mainly by the boundary conditions, H/R, and h/R ratios. The modal frequency decreases for the C-F boundary condition as compared to the C-C and S-S boundary conditions, and it also decreases with decreased shell thickness (lower h/R ratio).
3. The broad shell (lower H/R ratio) behaves more rigidly in longitudinal direction thereby its longitudinal frequencies (*m*) decrease and circumferential frequencies (*n*) increase.

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