*Research Paper*

# A Numerical Analysis of Fluid-Structure Interaction Problem with a Flow Channel Embedded in a Structural Material

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In the present work, numerical simulations have been performed on a model problem, representing a class of fluid-structure interaction problems. The model problem consists of a thin plate with a flow channel embedded in it. The governing equations for flow in the channel and displacement in plate structure are discretized using a finite volume procedure on unstructured meshes, and are solved in a one-way coupled manner with the flow in the channel influencing the stress field in the structure. The problem is presented in a generalized manner, in terms of the relevant dimensionless parameters obtained as part of the analysis. A parametric study is performed for the cases of isothermal and with heating of the fluid. The data from the parametric simulations are used to explain the stress field behavior in the solid plate, in response to independent dimensionless parameters. The overall methodology is presented in a manner that will be useful in analyzing any specific case of the class of fluid-structure interaction represented by the model problem.

**Keywords: Flow Induced Stress; Fluid-Structure Interaction; One-Way Coupling; Stress Behavior**

## Introduction

In thermo-fluid systems such as cooling of gas turbine blades, compact heat exchangers, electronics cooling, etc., a flowing fluid exerts forces (due to the pressure and viscous stresses) on the walls of the channels that are embedded within the structures, deforming them and inducing a stress field in the confining solid. Additionally, thermal stress can be induced in the solid in presence of temperature gradients. In most of the aforementioned thermo-fluid systems, the deformation of channel walls is small to be neglected, and hence the flow field can be assumed to be independent of the solid-body deformation. However, stress in the solid can be high, and depends on the properties of the fluid and flow conditions in addition to the properties of solid material. Therefore, while designing (from the strength point of view) these systems, fluid properties and the flow conditions should also be considered in addition to the properties of solid material. For good/optimal design of such systems, it is important to be able to predict the stress behavior of structural materials in response to different flow conditions in a generalized manner, i.e., in terms of dimensionless parameters. This is the aim of present work. A model fluid-structure interaction problem consisting of a U-bend channel embedded in a thin plate is analyzed. The dimensionless parameters are derived and their effects on maximum value of dimensionless effective stress in the plate are studied and important conclusions are drawn. Two cases, excluding and including fluid heating, are considered.

An extensive literature survey of fluid-structure interaction problems did not show similar kind of work. Several of the reported studies involving interaction of a fluid with a solid focused on the solution approach such as partitioned method (Piperno *et al*., 1995; Park *et al*., 2001; Matthies and Steindorf, 2003; Degroote *et al*., 2009; von Scheven and Ramm, 2011; Breuer

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*et al.*, 2012), monolithic method (Hubner *et al.*, 2004; Etienne *et al.*, 2006; Ryzhakov *et al.*, 2010; Jog and Pal, 2011), unified single solution method (Greenshields and Weller 2005; Papadakis 2008), Arbitrary Lagrangian Eulerian (ALE) method (Hirt *et al.*, 1974), Deforming Spatial Domain (DSD)/Stabilized Space Time (SST) method (Tezduyar *et al.*, 1992; Tezduyar and Sathe, 2007; Takizawa and Tezduyar, 2011), Fictitious Domain (FD) (van Loon *et al.*, 2007), Immersed Boundary Method (IBM) (Peskin, 1972; Peskin, 2002), method using interpolation operator (Gretarsson *et al.*, 2011), full Eulerian fixed grid method (Takagi *et al.*, 2012), and hybrid mesh free - Cartesian grid method (Yu *et al.*, 2011). Other studies focused on the procedures employed for specific applications such as crystal growth process (Scheafer *et al.*, 2002), fracture mechanics (Greenshields *et al.*, 2002), blast wave mitigation (Peng *et al.*, 2011), effect of pipe wall visco-elasticity during water hammer (Keramat *et al.*, 2012), flow analysis of a ribbed helix lip seal (Wen *et al.*, 2011), analysis of head-stack assembly in hard-disk drive (Kazemi, 2009), solution of an aero-elastic model describing the interactions of air flow with vocal folds (Svacek, 2011), dynamics of a cantilevered pipe aspirating fluid (Giacobbi *et al.*, 2012), analysis of flexible turbo-machinery (Campbell and Paterson, 2011), analysis of flexible flapping airfoil (Unger *et al.*, 2012), bio-mechanics and bioengineering applications (De Hart *et al.*, 2003; Tang *et al.*, 2003; Cheng *et al.*, 2004).

In view of the fact that while the situation described in the model problem considered for the present study is relevant from practical applications mentioned earlier, and since there does not seem to be any such investigation reported in the literature, this work can be considered as a fundamental engineering analysis for such applications.

## Dimensionless Parameters of the Study

The schematic of the problem is shown in Fig. 1. For the purpose of illustration, the plate mateiral is assumed to be elastic and following the von Mises yield criterion. However, it is noted that no special importance should be attached to this choice of model for computing limiting stress.

The equivalent stress (the von Mises stress) at a point in an elastic solid due to various forces acting on it is defined as
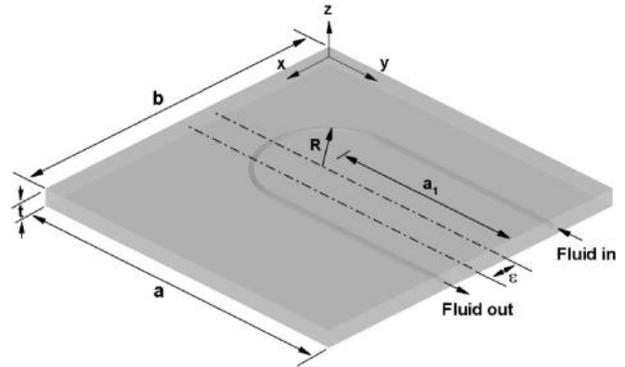


**Fig. 1: Schematic of the flow in a channel embedded in a plate**

$$\sigma_{eff} = \sqrt{0.5\left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2\right] + 3\left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2\right)}, \quad (1)$$

where $\sigma_{xx}$, $\sigma_{xy}$, $\sigma_{xz}$, etc. are the components of the stress tensor $\sigma_{ij}$. A material starts yielding at a point where $\sigma_{eff}$ reaches the yield strength. Hence, it is important to evaluate the maximum value of $\sigma_{eff}$ ($\sigma_{eff,max}$) and its location in a solid to know whether the material will yield or not under a given load condition, and the point of start of the yield.

### *Dimensionless Representation of the Governing Equations and the Boundary Conditions*

In order to understand the origin and significance of the dimensionless parameters of the problem, governing equations for the fluid flow and the plate, interface condition and the boundary conditions are expressed in dimensionless form as follows.

### *Governing Equations*

Assuming steady incompressible flow, the equations governing the transfer of mass and momentum and energy for a fluid flow, in Cartesian coordinate system, can be written as

$$\frac{\partial u_j}{\partial x_j} = 0, \quad (2)$$

$$\frac{\partial\left(\rho_f u_j u_i\right)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu_f \frac{\partial u_i}{\partial x_j}\right), \quad (3)$$

$$\frac{\partial\left(\rho_f c_{p,f} u_j T\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(k_f \frac{\partial T}{\partial x_j}\right), \quad (4)$$

where $u_i$ is the component of the velocity vector $u_j$ in the direction - i, $\rho_f$ is the density of the fluid, $\mu_f$ is the fluid dynamic viscosity, $p$ stands for the fluid pressure, $k_f$ is the thermal conductivity of the fluid, $c_{p,f}$ is the specific heat of the fluid, and $T$ is the temperature of the fluid. Assuming $D_h$, $\overline{u}$, and $\rho_f \overline{u}^2$ to be the characteristic length, velocity and pressure scales respectively and the fluid properties to be constant, Eqs. (2), (3), and (4) can be written in dimensionless form as

$$\frac{\partial u_j^*}{\partial x_j^*} = 0, \tag{5}$$

$$\frac{\partial \left(u_j^* u_i^*\right)}{\partial x_j^*} = -\frac{\partial \left(p/\rho_f \overline{u}^2\right)}{\partial x_i^*} + \frac{1}{Re}\frac{\partial^2 u_i^*}{\partial x_j^{*2}}, \tag{6}$$

$$\frac{\partial \left(u_j^* T^*\right)}{\partial x_j^*} = \frac{1}{Pe}\frac{\partial^2 T^*}{\partial x_j^{*2}}. \tag{7}$$

Here, $\overline{u}$ is the average velocity of the fluid in the channel, $D_h = 4A_c/P$ is the hydraulic diameter of the channel with $A_c$ representing the area of cross section of the channel and $P$ representing the perimeter of the channel cross section. Moreover, $Re = \rho_f \overline{u} D_h/\mu_f$ is the flow Reynolds number and $Pe = \rho_f c_{p,f} \overline{u} D_h/k_f$ is the Peclet number.

Governing steady state energy equation for a solid can be written as

$$\frac{\partial}{\partial x_j}\left(k_s \frac{\partial T}{\partial x_j}\right) = 0. \tag{8}$$

Here, $T$ is the temperature of the solid and $k_s$ is the thermal conductivity of the solid. Assuming the thermal conductivity to be constant, Eq. (7) can be written in the dimensionless form as

$$\frac{\partial^2 T^*}{\partial x_j^{*2}} = 0. \tag{9}$$

Neglecting the body force, the governing equilibrium equation for an elastic solid can be written as

$$\frac{\partial \left(\sigma_{ij,s}\right)}{\partial x_j} = \frac{\partial}{\partial x_j}$$

$$\left[\mu_s\left(\frac{\partial(\delta u_i)}{\partial x_j} + \frac{\partial(\delta u_j)}{\partial x_i}\right) + \lambda_s \frac{\partial(\delta u_k)}{\partial x_k}\delta_{ij} - \left(2\mu_s + 3\lambda_s\right)\alpha_s T \delta_{ij}\right] = 0 \tag{10}$$

where $\delta u_i$ is the component of the displacement vector $\delta u_j$ in the direction - i, $\mu$ is the shear modulus, $\lambda$ is the first Lame's constant, $\delta_{ij}$ is the Kronecker delta tensor, $\alpha$ is the coefficient of thermal expansion of the solid, $\sigma_{ij}$ is the stress tensor, and the subscript $s$ $\sigma_{ij}$ stands for the solid. Assuming $D_h$ to be the characteristic length scale, as well as, a reference scale for dimensionless representation of the displacement vector, and dividing Eq. (10) throughout by $\rho_f \overline{u}^2$, it can be written in the dimensionless form as:

$$\frac{\partial}{\partial x_j^*}\left[\frac{\mu_s}{\rho_f \overline{u}^2}\left(\frac{\partial(\delta u_i^*)}{\partial x_j^*} + \frac{\partial(\delta u_j^*)}{\partial x_i^*}\right) + \frac{\lambda_s}{\rho_f \overline{u}^2}\frac{\partial(\delta u_k^*)}{\partial x_k^*}\delta_{ij} - \frac{(2\mu_s + 3\lambda_s)}{\rho_f \overline{u}^2}\frac{\alpha_s}{\left(k_f/q_w D_h\right)}T^*\delta_{ij}\right] = 0, \tag{11}$$

where, $q_w$ is the heat flux supplied at the surface of the substrate.

### Interface Condition

At/across the fluid-solid interface there is continuity of temperature, the heat flux, the traction (force per unit area) and the velocity. The condition of the continuity of heat flux and temperature can be written as:

$$k_f\left(\frac{\partial T_i}{\partial n}\right)_f = k_s\left(\frac{\partial T_i}{\partial n}\right)_s \tag{12}$$

where, $T_i$ is the temperature of the interface and $\partial()/\partial n$ stands for the normal gradient. Subscripts $f$ and $s$ stand for the fluid and the solid domains respectively.

Equation (12) can be written in dimensionless form as:

$$\left(\frac{\partial T_i^*}{\partial n^*}\right)_f = \frac{k_s}{k_f}\left(\frac{\partial T_i^*}{\partial n^*}\right)_s \tag{13}$$

Assume that the fluid is heated by supplying a constant heat flux of $q_w$ at the two surfaces of the plate and the edges of the plate are in contact with an ambient air at a temperature $T_a$ and the heat transfer coefficient for the convective heat transfer to the air is $h_a$. Therefore,

$$\frac{k_s}{k_f}\left(\frac{\partial\left(T/\left(q_w D_h/k_f\right)\right)}{\partial n^*}\right)_B = 1, \tag{24}$$

and

$$\frac{k_s}{k_f}\left(\frac{\partial\left(T/\left(q_w D_h/k_f\right)\right)}{\partial n^*}\right)_B = \frac{h_a}{\left(k_f/D_h\right)}\frac{\left(T_B - T_a\right)}{\left(q_w D_h/k_f\right)}. \tag{25}$$

From the interface condition (Eq. (17),) it can be observed that the stress in the plate is induced due to three types of forces at the interface explained as follows. Considering the limiting case of the flow velocity tending to zero, there will be uniform pressure distribution equal to $p_{out}/\rho_f\overline{u}^2$ throughout the channel. This will cause a static force acting on the channel wall inducing a stress $\sigma_{ij,s}/\rho_f\overline{u}^2$, which will be a function of $p_{out}/\rho_f\overline{u}^2$ alone. Now, as the fluid starts flowing through the channel, the uniform pressure $p_{out}/\rho_f\overline{u}^2$ will be appended with a pressure distribution corresponding to $\Delta p/\rho_f\overline{u}^2$, which is a function of $Re$. Now, the appended pressure distribution and the viscous stress $(1/Re)$ $\left(\partial u_i^*/\partial x_j^* + \partial u_j^*/\partial x_i^*\right)$ will induce an additional stress in the plate, which will be a function of $Re$ as well as $p_{out}/\rho_f\overline{u}^2$. From the foregoing analysis, for a given geometry/dimensions of the plate and that of the channel, the dimensionless stress solution in the plate can be expressed as:

$$\frac{\sigma_{ij,s}}{\rho_f\overline{u}^2} =$$

$$f\left(\frac{E}{\rho_f\overline{u}^2},\nu,\frac{p_{out}}{\rho_f\overline{u}^2},\frac{\Delta p}{\rho_f\overline{u}^2},Re,\frac{k_s}{k_f},Pe,\frac{\alpha_s}{\left(k_f/q_w D_h\right)},\frac{T_a - T_{f,in}}{\left(q_w D_h/k_f\right)},\frac{h_a}{\left(k_f/D_h\right)}\right) \tag{26}$$

Moreover, we can write (Jagad *et al.*, 2015)

$$\frac{\Delta p}{\rho_f\overline{u}^2} = f_1(Re). \tag{27}$$

Hence, Eq. (27) can be rewritten as:

$$\frac{\sigma_{ij,s}}{\rho_f\overline{u}^2} =$$

$$f_2\left(\frac{E}{\rho_f\overline{u}^2},\nu,\frac{p_{out}}{\rho_f\overline{u}^2},Re,\frac{k_s}{k_f},Pe,\frac{\alpha_s}{\left(k_f/q_w D_h\right)},\frac{T_a - T_{f,in}}{\left(q_w D_h/k_f\right)},\frac{h_a}{\left(k_f/D_h\right)}\right) \tag{28}$$

**Parametric Study Excluding Heating of the Fluid**

Consider the model problem shown in Fig. 1. The plate is assumed to be of the dimensions $a = b =$ 50mm, $t = 2.5$mm. The cross-section of channel is assumed to be 0.5 mm x 0.5 mm. The channel is assumed to be placed symmetrically ($\varepsilon = 0$) in the plate. The dimensions $R = 10$mm and $a_1 = 30$ mm are assumed. When heating is excluded, Eq. (28) can be rewritten as:

$$\frac{\sigma_{ij,s}}{\rho_f\overline{u}^2} = f_2\left(\frac{E}{\rho_f\overline{u}^2},\nu,\frac{p_{out}}{\rho_f\overline{u}^2},Re\right). \tag{29}$$

The functional relationship given by Eqs. (29) is studied. The values of the parameters employed for the study are summarized in Table 1. The Poisson's ratio $\nu = 0.17$ represents silicon, $\nu = 0.3$ represents metal such as steel, and $\nu = 0.37$ represents polymer material Polycarbonate. Fairly wide range of $E/\rho_f\overline{u}^2$ and $p_{out}/\rho_f\overline{u}^2$ is employed.

For simulations, the symmetry in $z$-direction (along the thickness) is considered. The unstructured mesh finite volume procedure (Jagad *et al.*, 2011) is employed for solving the problem. The mesh employed is shown in Fig. 2. The flow domain through the channel is discretized using a hexahedral mesh and the domain in the plate is discretized using a hybrid

**Table 1: Values of the parameters for stress behavior study when heating of the fluid is excluded**

| Parameter | Values |
| --- | --- |
| Re | 292, 380, 468, 585, 877, 1170, 1460, 1750, 2050 |
| $E/\rho_f \bar{u}^2$ | $1.88 \times 10^5$, $2.56 \times 10^5$, $3.68 \times 10^5$, $5.75 \times 10^5$, $1.02 \times 10^6$, $2.3 \times 10^6$, $9.2 \times 10^6$ |
| $\nu$ | 0.17, 0.3, 0.37 |
| $p_{out}/\rho_f \bar{u}^2$ | 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 200 300, 500, 1000, 2000, 3000, 4000, 5000 |
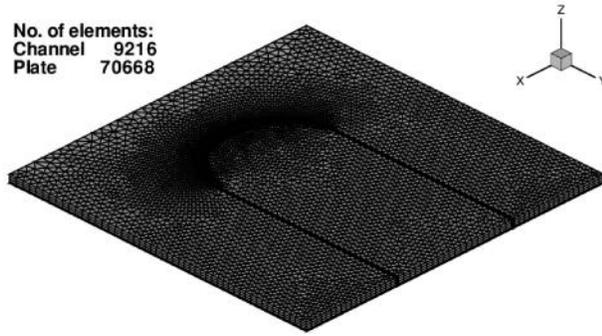


**Fig. 2: The mesh employed for the simulations for stress behavior study**

mesh consisting of hexahedral and triangular prismatic cells.

The behavior of $\sigma_{eff,max}/\rho_f \bar{u}^2$ as a function of *Re* for different values of $E/\rho_f \bar{u}^2$ is shown in Fig. 3. The collapsing plots imply redundancy of the parameter $E/\rho_f \bar{u}^2$. Figure 4 shows the behavior of $\sigma_{eff,max}/\rho_f \bar{u}^2$ as a function of $p_{out}/\rho_f \bar{u}^2$ for *Re* varying from 292 to 2050 and $\nu = 0.17$ and 0.3. It can be observed that $\sigma_{eff,max}/\rho_f \bar{u}^2$ drops with rise in *Re* since the dimensionless pressure drop $\Delta p/\rho_f \bar{u}^2$ (see Fig. 6) and the dimensionless viscous stress $(1/Re)$ $\left(\partial u_i^*/\partial x_j^* + \partial u_j^*/\partial x_i^*\right)$ at the channel wall decrease. Moreover, the effect of *Re* on $\sigma_{eff,max}/\rho_f \bar{u}^2$ is smaller at higher $p_{out}/\rho_f \bar{u}^2$ since the fluid static pressure

becomes dominant and the $\Delta p/\rho_f \bar{u}^2$ becomes insignificant. With the rise in $p_{out}/\rho_f \bar{u}^2$, force due to static pressure on the channel walls increases leading to the rise in the dimensionless stress $\sigma_{eff,max}/\rho_f \bar{u}^2$ in the plate material. In order to convey the effect of $\nu$ on $\sigma_{eff,max}/\rho_f \bar{u}^2$ clearly, an extract from Fig. 4 is shown in Fig. 5. As $\nu$ increases, $\sigma_{eff,max}/\rho_f \bar{u}^2$ decreases. As $\nu$ increases, the maximum value of the shear stress components decrease and that of the normal stress components increases. As a result, the maximum value of $\sigma_{eff,max}/\rho_f \bar{u}^2$ decreases due to the dominant contribution of shear stress components. The percentage change in the value of $\sigma_{eff,max}/\rho_f \bar{u}^2$ with $\nu$ is found to be almost constant for different values of *Re* as well as $p_{out}/\rho_f \bar{u}^2$.

A few contour plots in a plane at $z = 0.1875$ mm are shown in Figs. 7-8. Water ($\rho = 1000$ kg/m³ and $\mu = 8.55$ x $10^{-4}$ Pa–s) is assumed to be the fluid passing through the channel. From these plots, it can be observed that the point of maximum effective stress is either in the vicinity of channel inlet or near the end of left limb of the channel depending on the range of parameters, as already stated earlier.

**Parametric Study Including Heating of the Fluid**

In the present case, for given dimensions of the plate and channel, and fixed $T_a$, $T_{f,in}$ and $h_a$, Eq. (28) can be rewritten as:
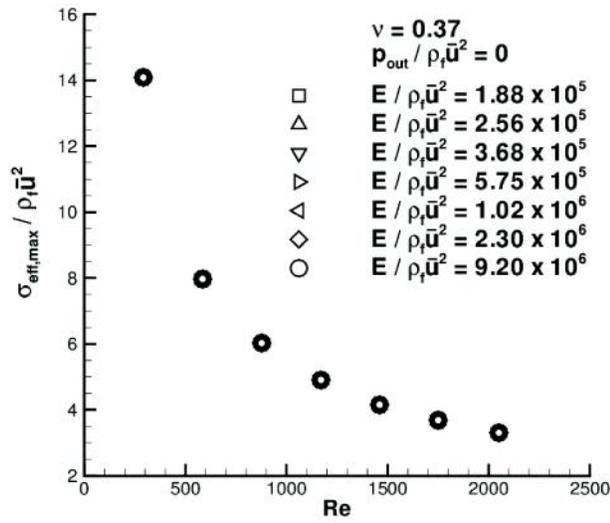
(a) $p_{out}/\rho_f \bar{u}^2 = 0$
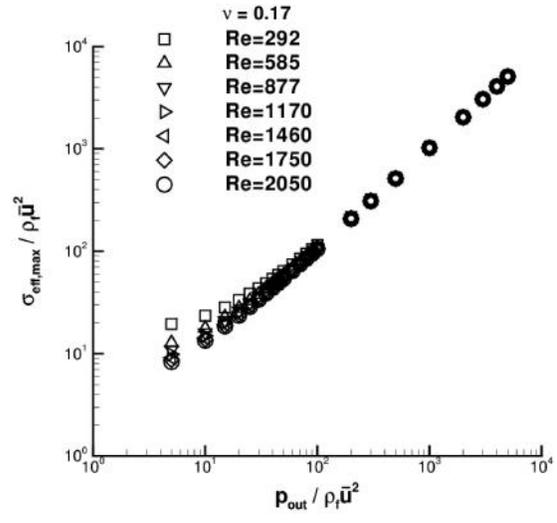


(b) For different values of $p_{out}/\rho_f \bar{u}^2$
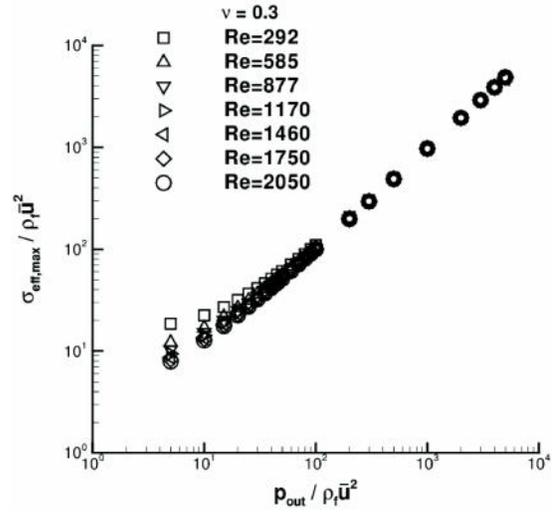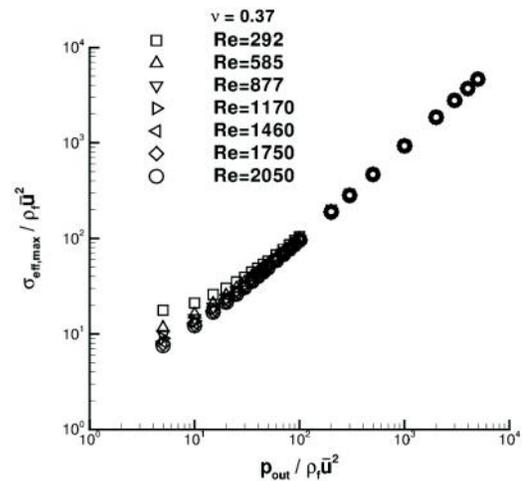
**Fig. 3:** Behavior of $\sigma_{eff,max}/\rho_f \bar{u}^2$ as a function of $Re$ for different values of $E/\rho_f \bar{u}^2$ showing redundancy of $E/\rho_f \bar{u}^2$

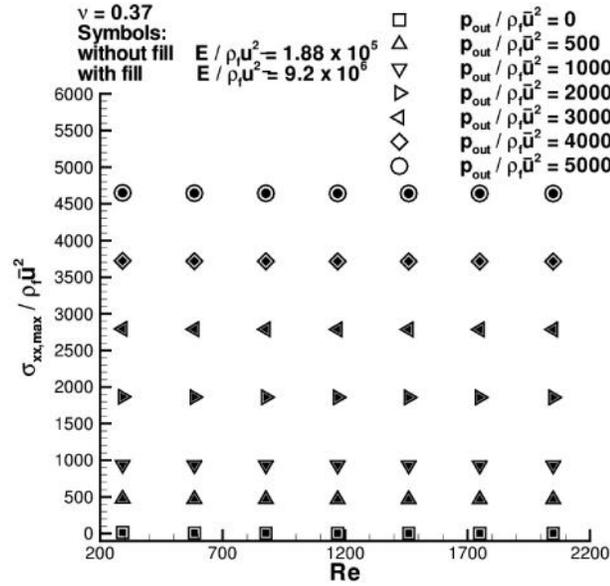$$\frac{\sigma_{eff,max}}{\rho_f \bar{u}^2} = G_1\left(\frac{E}{\rho_f \bar{u}^2}, \nu, Re, \frac{p_{out}}{\rho_f \bar{u}^2}, \frac{k_s}{k_f}, Pe, \frac{\alpha_s}{\left(k_f/q_w D_h\right)}\right).$$

(30)

The problem described in Sec. 3 is now studied assuming heating of the fluid. The functional relationship given in Eq. (30) is studied. The effects



(a) $\nu = 0.17$



(b) $\nu = 0.3$



(c) $\nu = 0.37$

**Fig. 4:** Behavior of $\sigma_{eff,max}/\rho_f \bar{u}^2$ as a function of $p_{out}/\rho_f \bar{u}^2$ for different values of $Re$ and $\nu$

of $Re$, $E/\rho_f \bar{u}^2$, $\nu$, $k_s/k_f$, $Pe$ and $\alpha_s/\left(k_f/q_w D_h\right)$ on

**Table 2: Values of the parameters for a case when heating of the fluid is considered**

| $Re$ | 292, 877, 1460, 2050 |
|---|---|
| $E/\rho_f \bar{u}^2$ | $1.88 \times 10^5$, $9.2 \times 10^6$ |
| $\nu$ | 0.3, 0.37 |
| $k_s/k_f$ | 0.3, 350 |
| $Pe$ | 200, 10000 |
| $\alpha_s/\left(k_f/q_w D_h\right)$ | $1.2 \times 10^{-7}$, $1.2 \times 10^{-5}$ |

$\sigma_{eff,max}/\rho_f \bar{u}^2$ are determined. For the present study, $p_{out}/\rho_f \bar{u}^2 = 0$ is assumed. The values of different parameters used for the study are summarized in Table 2.

The symmetry of the problem in *z*-direction (along the thickness of the plate) is considered. A uniform velocity and temperature of the fluid are assumed at the channel inlet. At the channel outlet, normal gradients of the flow variables and that of the

fluid temperature are assumed to be zero. No-slip boundary condition is employed on the channel walls. All external boundaries of the plate are assumed to be fixed. Continuity of the traction (force per unit area), velocity, temperature and heat flux are implemented at the channel walls. Uniform heat flux condition is assumed at the top and bottom surfaces of the plate. At the edges of the plate, natural convection heat transfer to the ambient air is assumed.

A solution procedure similar to (Jagad *et al.*, 2011) is used. For simultaneous determination of the fluid and solid temperature fields, the procedure described in (Jagad *et al.*, 2012) is used. The same mesh as that shown in Fig. 2 and described in Sec. 3 is used.

The effects of the aforementioned parameters on $\sigma_{eff,max}/\rho_f \bar{u}^2$ are shown in Figs. 9-10. It is found that $Re$ does not affect $\sigma_{eff,max}/\rho_f \bar{u}^2$ for larger values of $\alpha_s/\left(k_f/q_w D_h\right)$ and $E/\rho_f \bar{u}^2$ because the thermal stresses (due to the temperature distribution in the plate) are the predominant contributors and are not dependent on $Re$. At lower values of $\alpha_s/\left(k_f/q_w D_h\right)$ and $E/\rho_f \bar{u}^2$, the contribution of mechanical stresses (due to the forces on channel walls due to the fluid pressure and viscous effect) becomes equivalent and



**(a)** $p_{out}/\rho_f \bar{u}^2 = 5$

**(b)** $p_{out}/\rho_f \bar{u}^2 = 4000$
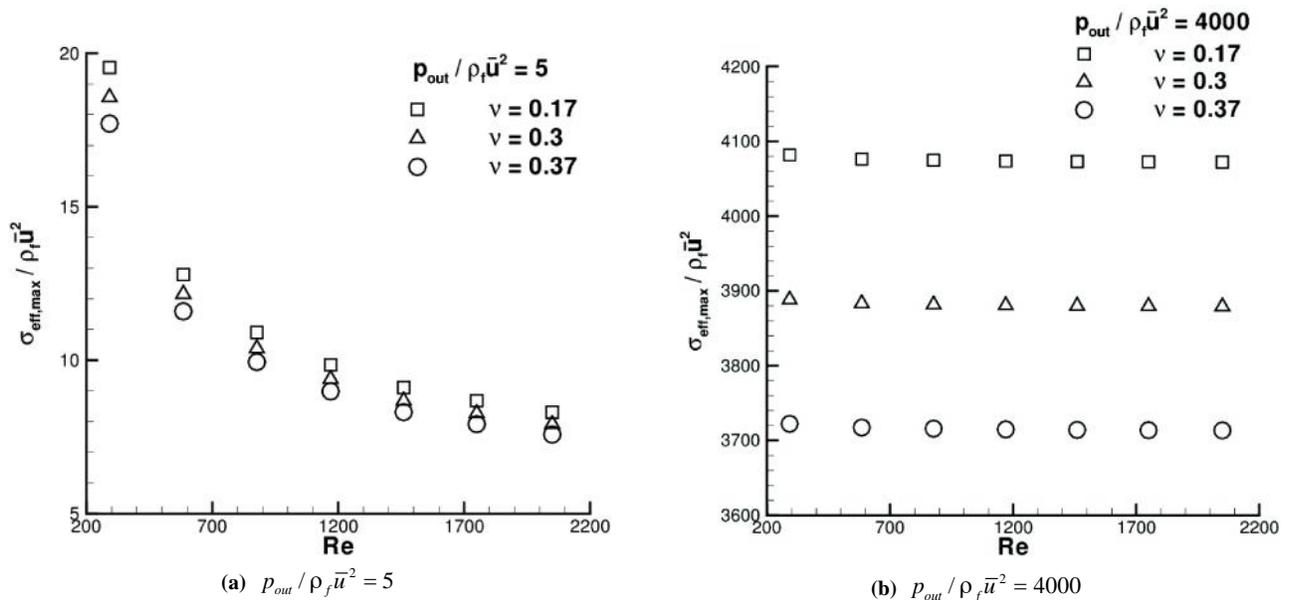
**Fig. 5: Behavior of $\sigma_{eff,max}/\rho_f \bar{u}^2$ as a function of *Re* for different values of $\nu$, showing the effect of $\nu$**
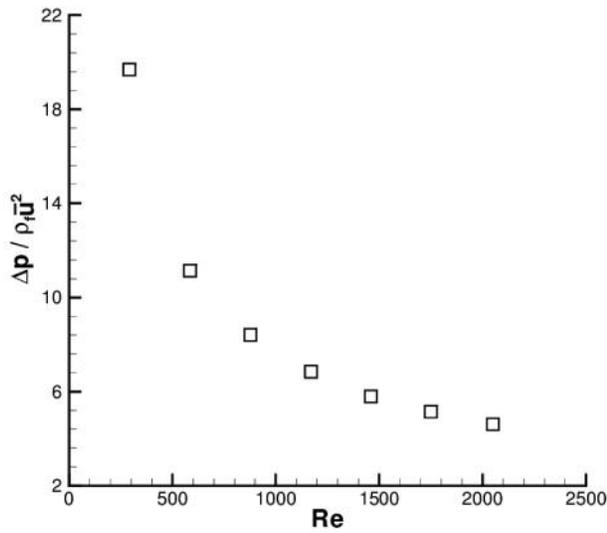
**Fig. 6:** **Dimensionless pressure drop** $\Delta p/\rho_f \bar{u}^2$ **as a function of** *Re*

$\sigma_{eff,max}/\rho_f \bar{u}^2$ slightly increases with increase in *Re*. As *Pe* or $k_s/k_f$ increases, $\sigma_{eff,max}/\rho_f \bar{u}^2$ decreases because temperature gradients in the plate decrease, hence, the thermal stress decreases. The percentage change in $\sigma_{eff,max}/\rho_f \bar{u}^2$ with *Pe* is found to be more for larger $k_s/k_f$ and $\nu$, and smaller $E/\rho_f \bar{u}^2$ and

$\alpha_s/\left(k_f/q_w D_h\right)$. Similarly, the percentage change in $\sigma_{eff,max}/\rho_f \bar{u}^2$ with $k_s/k_f$ is found to be more for larger *Pe* and $\nu$, and smaller $E/\rho_f \bar{u}^2$ and $\alpha_s/\left(k_f/q_w D_h\right)$. As $E/\rho_f \bar{u}^2$ or $\alpha_s/\left(k_f/q_w D_h\right)$ or $\nu$ increases $\sigma_{eff,max}/\rho_f \bar{u}^2$ increases because thermal stress increases in the plate.

**Conclusions**

A model problem of determination of stress behavior of the confining structure in response to different flow conditions in an internal flow situation consisting of a U-bend channel embedded in a thin plate is analyzed here. Two cases, without and with the fluid heating, are considered. The dimensionless governing parameters are derived. The effects of these parameters on maximum value of dimensionless effective stress in the plate material are studied.

In the first case, when heating of fluid is not considered, the following conclusions are derived:

- The effect of a dimensionless Young's modulus on the dimensionless maximum effective stress is found to be insignificant over a wide range of the parameters.



**(a)** $p_{out} = 1.25 \times 10^3$ Pa


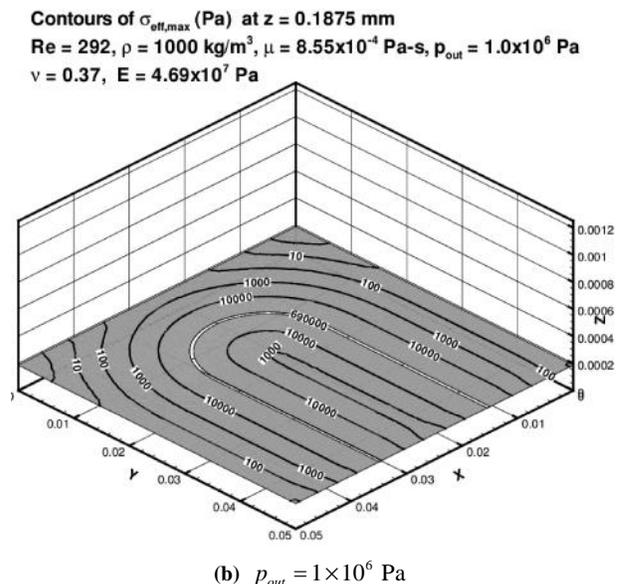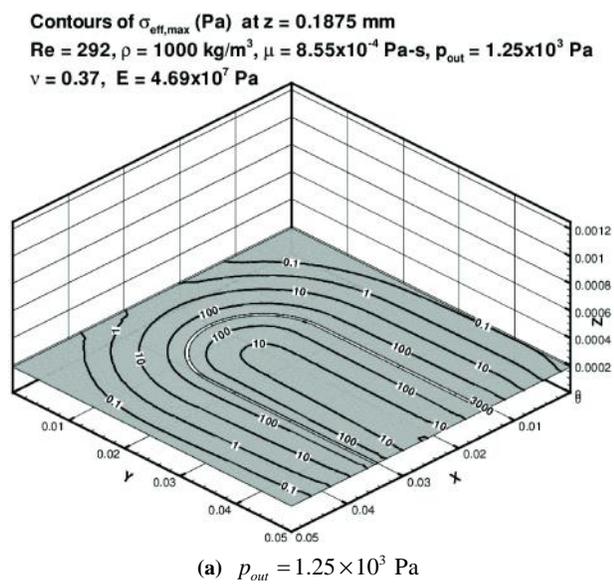
**(b)** $p_{out} = 1 \times 10^6$ Pa

**Fig. 7:** **Contours of** $\sigma_{eff,max}$ **(Pa) for the stress behavior study at** $z = 0.1875$ **mm,** *Re* = 292, $\rho = 1000$ kg/m³, $\mu = 8.55$ x $10^{-4}$ Pa-s, $\nu = 0.37$, E = 4.69 x $10^7$ Pa

**(a)** $p_{out} = 6.125 \times 10^4$ Pa



**(b)** $p_{out} = 4.9 \times 10^7$ Pa
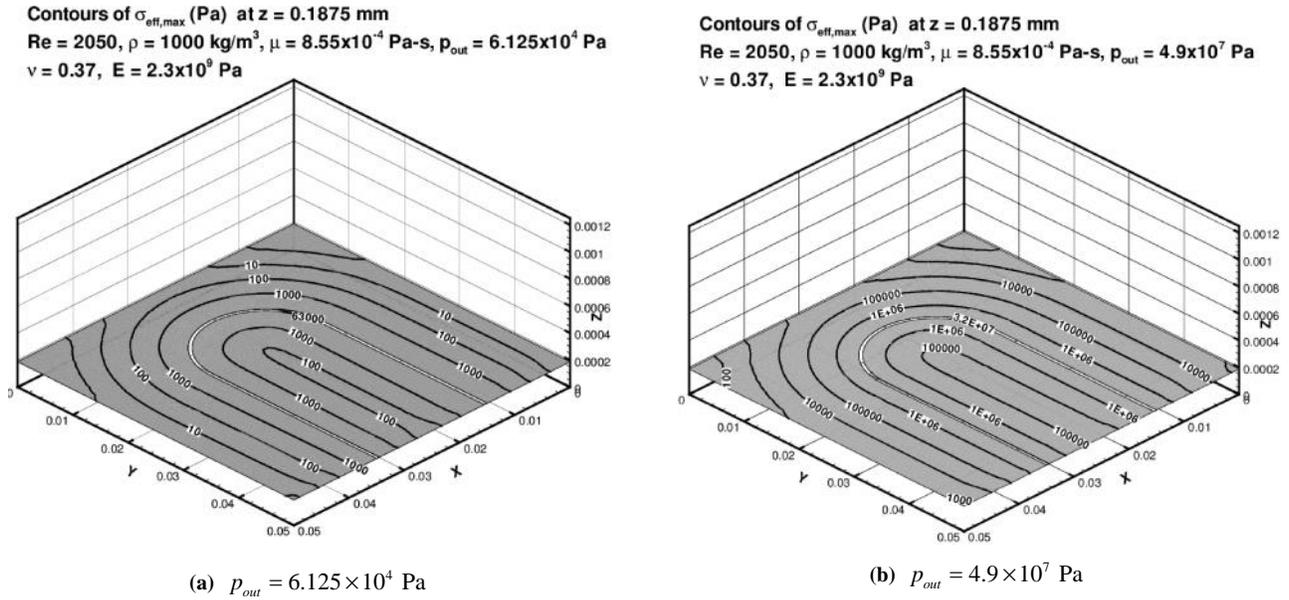
**Fig. 8:** .......... $\sigma_{eff,max}$ **(Pa) for the stress behavior study at** $z$ **= 0.1875 mm,** $Re$ **= 2050,** $\rho$ **= 1000 kg/m³,** $\mu$ **= 8.55 x 10⁻⁴  Pa-s,** $\nu$ **= 0.37, E = 4.69 x 10⁹ Pa**



**(a)** $\alpha_s/\left(k_f/q_w D_h\right) = 1.2 \times 10^{-7}$



**(b)** $\alpha/\left(k_f/q_w D_h\right) = 1.2 \times 10^{-5}$

**Fig. 9: Effects of the parameters on** $\sigma_{eff,max}/\rho_f \bar{u}^2$ **for  a  case when  fluid heating is included and** $\nu$ **= 0.3**

- The dimensionless stress is found to decrease with increase in the flow Reynolds number. The effect of the flow Reynolds number on dimensionless maximum effective stress is found to become less significant with increase in the dimensionless static pressure at the channel outlet. At high values of dimensionless static pressure at the channel outlet (say ≥ 500), dimensionless pressure drop becomes insignificant as compared to static pressure of the fluid in the channel. Hence, the effect of the flow Reynolds number also becomes insignificant.

(a) $\alpha_s/\left(k_f/q_wD_h\right) = 1.2 \times 10^{-5}$

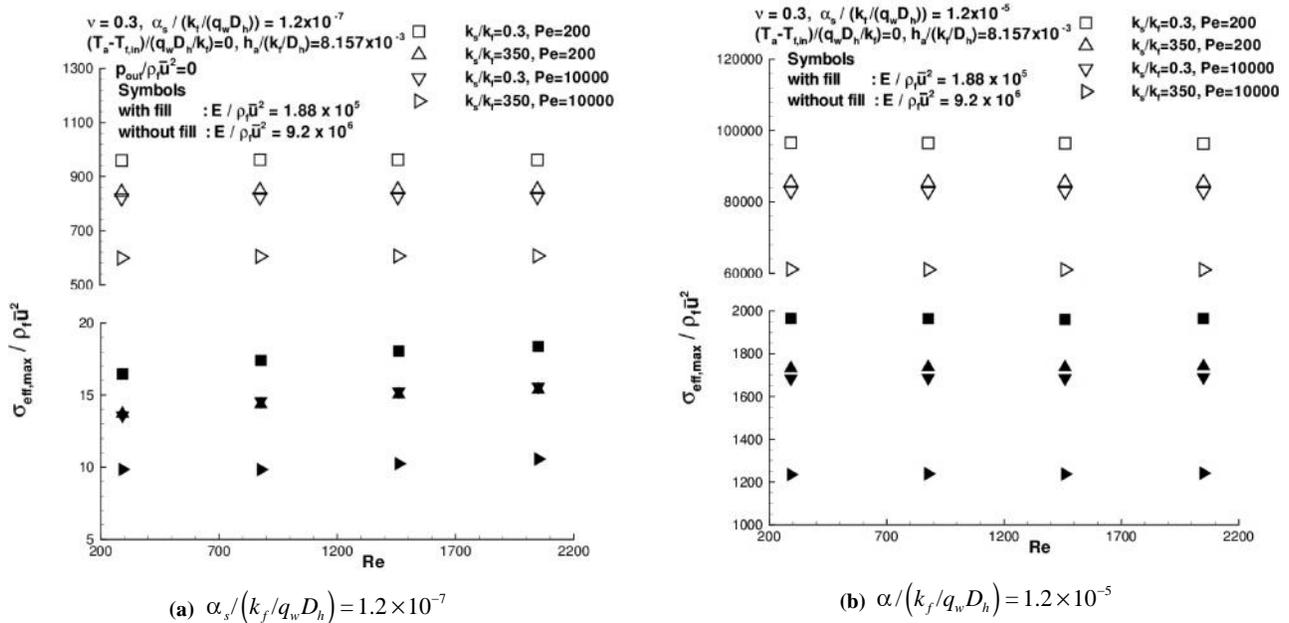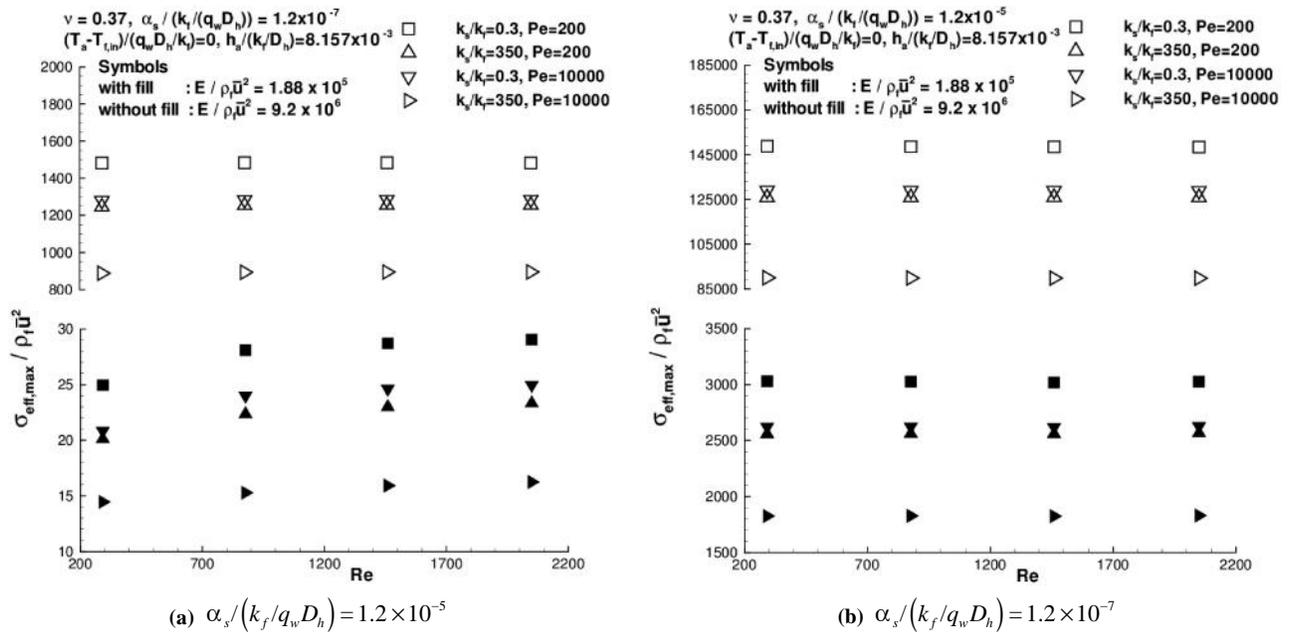(b) $\alpha_s/\left(k_f/q_wD_h\right) = 1.2 \times 10^{-7}$

**Fig. 10: Effects of the parameters on $\sigma_{eff,max}/\rho_f\overline{u}^2$ for a case when fluid heating is included and $\nu = 0.37$**

- The dimensionless maximum effective stress is found to decrease with increase in the Poisson's ratio.

  For the second case, when heating of the fluid is considered the conclusions derived are as follows:

- At higher values of dimensionless coefficient of thermal expansion and dimensionless Young's modulus the thermal stress is found to be predominant over mechanical stress. Hence, the effect of the flow Reynolds number on dimensionless maximum effective stress is found to be insignificant at higher values of the dimensionless coefficient of thermal expansion and the dimensionless Young's modulus.

- The dimensionless maximum effective stress is found to decrease with increase in the flow Peclet number or the conductivity ratio due to the decrease in temperature gradients in the plate.

- Unlike in the first case, in the second case the dimensionless Young's modulus is found to affect the dimensionless maximum effective stress. With increase in the dimensionless Young's modulus or the dimensionless coefficient of thermal expansion or Poisson's ratio the dimensionless maximum effective stress is found to increase due to increase in the thermal stress.

  While the foregoing conclusions are specific to the model problem analyzed in this work, an overall methodology is presented in a general framework, and is considered to be useful for the class of fluid-structure situations exemplified by the model problem.

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