Research Paper

Need for Variable Frequency Control in DC-DC Switching Converters—Challenges and Opportunities using Digital Implementation

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Introduction

Low power portable battery-operated devices, such as mobile phones, laptops, PDA, etc., have been gaining growing popularity. Each device requires multiple digital processors, in which the power management unit plays the central role in efficient DC power processing for extending the battery life. This primarily consists of one or more DC-DC converters with the objectives (i) to regulate the output voltage under varying input voltage and load current conditions, and (ii) to achieve high efficiency with fast transient performance. Linear regulators are often used for low power applications, in which a power transistor is used to adjust the voltage drop across the input-output terminals for precise voltage regulation by varying its on-time resistance. While linear regulators are simple and free from switching noise, their theoretical efficiency is limited by the voltage conversion ratio. Thus, such regulators are inefficient for the power requirements beyond a few hundreds of mW. Inductor-based switching DC-DC converters are rather more efficient because of using power switching devices which operate either in fully on-state or off-state mode. Irrespective of the voltage conversion ratio, their theoretical efficiency can reach up to 100% if the ideal switches and passive components are used. However, practical parasitic may incur conduction and switching losses which can be reduced by suitably choosing power circuit devices, converter topologies, such as considering zero-current switching (ZCS), zero-voltage switching (ZVS) (Lakshminarasamma N and Ramanarayanan V, 2007), as well as other control techniques. A major issue in switching converters is the problem of electromagnetic interference (EMI) which is primarily

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due to hard switching in DC-DC converters and there exists various techniques for EMI reduction (Hsieh et al., 2008; Tse K K et al., 2002; Kapat S, 2016). Nevertheless, switching DC-DC converters are predominantly used for efficient power management solutions. When the required output voltage is lower than the supply (may be a battery) voltage, a step-down (or a buck) converter is used, whereas a step-up (or a boost) converter is used when the output voltage requirement is higher than the available input DC voltage. Fixed-frequency voltage mode control (VMC) and current mode control (CMC) techniques are the two popular pulse-width modulated (PWM) feedback control techniques that are often used in various commercial products. A CMC technique offers several performance benefits over a VMC technique; however, it suffers from sub-harmonic instability for the steady-state duty ratio higher than 50% and often requires a compensating ramp. Moreover, a higher ramp slope may degrade the closed loop bandwidth. Variable frequency CMC techniques, such as constant on-time, off-time modulation schemes, etc., can overcome current-loop stability problems; however, they suffer from a variation in the steady-state switching frequency and often require an extra phase-lock-loop (PLL) for the frequency regulation. This paper highlights the benefits and challenges of using variable frequency digital control techniques, and their design insights for future power management solutions.

This Paper is Organized as Follows

Section II discusses the various fixed-frequency PWM control methods as well as variable frequency control methods. Their benefits and shortcomings are highlighted in Section III. Section IV presents the implementation of mixed-signal variable-frequency CMC using uniform sampling and its shortcomings. Section V presents the benefits of using event-based sampling approach in variable-frequency digital CMC. Section VI describes the conclusions of the work.

Fixed-frequency and Variable-frequency Control

Figure 1 shows the schematic of a synchronous buck and boost DC-DC converters, where the gate signal ‘u’ is taken as the control variable. Under a PWM technique, the control technique adjusts the duty-ratio $d$ in order to regulate the output voltage of a DC-DC converter to the desired value. Using the inductor volt-

second balance, the respective voltage gains of the buck and boost converters can be shown to be $d$ and $1/(1-d)$ while operating under continuous conduction mode (CCM).

Fixed-frequency Control

A fixed-frequency PWM-based approach remains a prevalent choice as it simplifies the design of the input filter. In a PWM scheme, the (switching) time period is fixed, and the on-time is accordingly adjusted by the feedback control loop. The voltage-mode and current-mode are the two popular PWM control schemes. Figure 2(A) shows a synchronous buck converter operating under a PWM voltage-mode control (VMC) scheme, and the related control waveforms are shown in Fig. 3. It uses a single-loop control architecture. The voltage controller $G_v$ computes the control signal $v_c$ by taking the error voltage $v_e$ as input, where $v_e = v_{ref} - v_o$. The comparator output $u_c$ is generated by comparing $v_c$ with the periodic saw-tooth waveform with a time period $T_s$. The comparator follows a S-R latch circuit which is set by the rising edge of switching frequency clock $F_s$ and reset by the comparator output $u_c$. A dead-time circuit is used to avoid the shoot-through current using a synchronous configuration. Despite its simplicity and improved load regulation, a VMC technique suffers from poor line regulation and phase margin. An input voltage feed-forward can be used to improve the line regulation.

Figure 2(B) shows the schematic of a PWM CMC scheme which consists of two feedback loops. Here, the outer voltage-loop generates the reference command $v_c$ for the inner current-loop. It can be broadly categorized as peak CMC, valley CMC, and average CMC. Figure 4 shows the control waveforms using the peak and valley CMC schemes. In the former [shown in Fig. 4(a)], the control signal $v_c$ is compared with the peak value of the inductor current, whereas
the valley point of the inductor current is compared with $v_c$ in the valley CMC as shown in Fig. 4(B). Any change in the input voltage $v_{in}$ would be immediately reflected via the inductor current; thereby, it can achieve superior line transient response and line regulation. However, it suffers from sub-harmonic instability when the duty-ratio $D > 0.5$ and often requires an external ramp compensation for stabilization.

**Variable-frequency Control**

A variable-frequency control technique finds widespread applications in point-of-load converters because of its simplicity in implementation, fast transient response, and improved light-load efficiency (Redl W R and Jian S, 2009). Based on their modulation, they can be broadly categorized as constant on-time, constant off-time, and hysteretic control modulations.

**Constant on/off-time Control**

Figure 5(A) shows the schematic diagram of CMC constant off-time modulation and (B) hysteretic modulation.
mono-shot timer expires. The switch turns-off when $i_L$ crosses $v_c$ [shown in Fig. 6(A)] and the mono-shot timer is triggered.

Similarly, the operation of a constant on-time scheme can be explained using Fig. 6(B). Here, the mono-shot timer is triggered when $v_c$ crosses $i_L$ and the controllable switch turns-on. Once the on-time expires, the high-side switch $S$ turns-off. This modulation scheme can be useful to improve the light-load efficiency (Redl W R and Jian S, 2009).

**Hysteric Control**

Figure 5(B) shows the schematic diagram of a current-mode hysteretic modulator, and the relevant control waveforms are shown in Fig. 7. In this modulation, the inductor current is kept within a specified band using the controller output and a hysteresis band. The output of the voltage controller $v_c$ is used as the peak current reference, whereas the valley reference current is generated using $v_c$ and the hysteretic band $\Delta i_H$ as shown in Fig. 7. When the inductor current $i_L$ crosses $v_c$, the high-side switch turns-off, and it turns-on when $i_L$ falls below the valley reference current ($v_c - \Delta i_H$).

**Advantages and Shortcomings in Variable-frequency Control**

The fixed-frequency peak CMC suffers from sub-harmonic instability when the operating duty-ratio $D > 0.5$ and it often requires an external ramp compensation for stabilization. This degrades the achievable closed-loop bandwidth. On the other hand, the current-loop of variable frequency CMC is inherently stable and shows improved closed-loop bandwidth; however, suffer from varying switching frequency.

**Sub-harmonic Instability**

**Fixed-frequency Control**: Figure 8 shows the control waveforms under the PWM peak CMC scheme. The solid and dotted lines indicate the steady-state and perturbed waveforms. Let $i_n$ and $i_{n+1}$ be initial and final values of the inductor current during the $n^{th}$ cycle. Referring to Fig. 8, the current dynamics can be written as

$$i_{n+1} = i_n + m_1 DT_s - m_2 (1-D) T_s$$  \hspace{1cm} (3.1)

where, $m_1$ and $m_2$ are the respective rising and falling slopes of the inductor current, and D is the steady-state duty ratio. From (3.1), the perturbed current dynamics can be written as

$$\Delta i_{n+1} = - \left( \frac{m_2}{m_1} \right) \Delta i_n.$$  \hspace{1cm} (3.2)

It can be shown from (3.2) that the current-loop under PWM peak CMC remains stable if the steady-state duty ratio $D < 0.5$. This fact is verified using test results in Fig. 9 for a buck converter using the parameter set: $v_{in} = 6V$, $v_o = 3.3V$, $L = 10\mu H$, $C = 150\mu F$, $i_o = 1.5A$ and $F_s = 200kHz$. Here, a PWM peak CMC technique was implemented with the closed inner-loop and opened outer-loop and the steady-state duty-ratio was found to be $D = 0.55$. Under this condition, Fig. 9(i) demonstrates the existence of sub-
harmonic instability using the peak CMC, which is evident from the FFT trace with a sharp peak at 100–kHz for the switching frequency of 200–kHz.

Using an additional compensating ramp with a slope \( m_c \), the stability of the inner current-loop can be enhanced; the condition of current-loop stability can be derived as

\[
\Delta i_{n+1} = \left( \frac{m_c - m_e}{m_c + m_e} \right) \Delta i_n. \tag{3}
\]

Using the above condition, Fig. 10 shows that the peak CMC technique can be stabilized using ramp compensation in a synchronous buck converter under the same operating condition as in Fig. 9. The FFT trace in Fig. 10 clearly shows that there is no sub-harmonic component in the power spectrum. Comparing the steady-state performances using the unstable (in Fig. 9) and the stable (in Fig. 10) cases, one can infer that a sub-harmonic instability leads to a higher RMS value of the inductor current, which would result in higher conduction losses. Also the output voltage ripple significantly increases for the former compared to its nominal value as shown in the latter. Similarly for PWM valley CMC, it can be shown that the current stability can be ensured for the steady-state duty ratio \( D > 0.5 \), which would also require a ramp compensation for stabilization when the operating duty-ratio \( D < 0.5 \).

Variable-frequency Control: Referring to the waveforms using the current-mode constant off-time control technique as shown in Fig. 6(A), the inductor current at the end of \( n \)th cycle can be written as

\[
i_{n+1} = v_c - m_c T_{off} \tag{3.4}
\]

Applying small perturbations, the perturbed
Current dynamics can be obtained as $i_{n+1} = \hat{v}_c$. This implies that the inner current-loop is inherently stable irrespective of the steady-state duty ratio and the overall closed-loop stability is determined by the outer voltage loop. Figure 12 shows the test results using the current-mode constant off-time modulator with the opened outer voltage-loop using the same test conditions as in Fig. 9. Figure 12 shows that the stable periodic behavior can be achieved for $D > 0.5$ without the need of a ramp compensation using the current-mode constant off-time modulation. It is also evident from the FFT trace of the inductor current. Stability of the inner current-loop can be retained for any arbitrary duty-ratio without using a compensating ramp. Similar stability results can be shown for a current-mode constant on-time control scheme.

**Frequency Regulation**

While a variable frequency CMC technique offers inherent current-loop stability over a PWM CMC technique, a variation in steady-state switching frequency remains a concern for the former, which makes it difficult to design an input filter using the
former (Redl W R and Jian S, 2009). Figure 13 shows the effect of input voltage variation on switching frequency for a constant on-time controlled buck converter. This technique is very sensitive to ambient noise (Redl W R and Jian S, 2009; Fang C C and Redl R, 2014; Fang C C, 2012). It can be noted from Fig. 13 that using the constant on-time, the steady-state switching frequency increases (or decreases) as the input voltage decreases (or increases). There exist various techniques to regulate the steady-state switching frequency either by adaptively varying the steady-state on-time (Qian T, 2013; Sahu B and Rincon-Mora G A, 2007; Lin H C et al., 2008) for constant on-time control, or by varying the off-time for constant off-time control or by varying the hysteretic band (Fu W et al., 2015) under hysteretic modulation. The method of constant on-time adaptation requires precise input voltage sensing either by direct feed-forward (LM5017, 2015) or by estimation. The relation between the constant on-time \( T_{on} \) and the input-voltage \( v_{in} \) for constant on-time modulated buck converter is given as

\[
T_{on} = \left( \frac{v_o}{v_{in}} \right) T_s \quad \Rightarrow \frac{\partial v_{in}}{\partial v_{in}} \approx \frac{1}{v_{in}^2}
\]

However, for the sake of simplified analog implementation (LM5017, 2015), a piecewise linear approximation is used to relate the on-time and the input voltage to achieve a quasi-fixed frequency. However, this approximation leads to a wider frequency variation while operating over a wide range. There exist various methods for frequency regulation at steady-state, such as using analog phase-locked loop (PLL), delay-line PLL, digital PLL, etc. (Jing X and Mok P K T, 2013; Li P et al., 2011). Such methods do not require any input voltage sensing, but can precisely regulate the switching frequency using an additional frequency-loop. These methods of frequency regulation require more hardware resources and exhibits slower locking time.

**Need for Digital Implementation and Challenges**

Recently, digital controllers have emerged as a viable alternative (Maksimovic D et al., 2004; Liu Y F et al., 2009) due to their ease of integration with other digital systems and ability to implement high-performance control schemes. Using a traditional controller-tuning method based on worst-case scenario, the closed-loop performance is often
compromised during the nominal operating conditions. A real-time (auto) tuning is a viable alternative, in which the gains are adaptively varied based on the operating conditions. Suitable gains can be computed either online or offline, and a look-up-table (LUT) based method can be used for digital implementation. Further, a digital platform simplifies the frequency regulation methods using an all-digital PLL (ADPLL) with a time-to-digital converter (TDC). A TDC captures the instantaneous switching period \( T_{s,n} \) and accordingly adjusts the desired timing parameter using \( T_{c,d} = (T_c/T_{s,n}) \cdot T_{s,d} \) where \( T_c \) is the instantaneous constant-time and \( T_{s,d} \) is the desired switching period \( (1/F_s) \). This can be programmed in a digital device which also offers the flexibility of controller tuning and (constant) time adaptation (band adaptation in the case of hysteretic modulator) and speed-up the frequency locking mechanism within a few switching cycles.

**Digital Implementation**

Figure 14 shows the schematic diagram of a synchronous buck converter operating under the mixed-signal constant on-/off-time CMC scheme, where \( i_L \), \( v_o \), and \( v_{ref} \) indicate the inductor current, the output voltage, and the reference voltage, respectively. The output voltage \( v_o \) is sampled using a uniform clock \( F_{vs} \) (may be the same rate of the switching frequency \( F_s \) or higher) using an analog-to-digital converter (ADC). Using the error voltage \( v_e[n] = v_{ref} - v_o[n] \), the digital voltage controller \( G_c(z) \) computes the control signal \( v_c[n] \) at the rate of sampling clock \( F_{vs} \). In fully digital current-mode control (DCMC), the inductor current is also sampled and taken to the digital domain using a high speed ADC. Instead, in mixed-signal DCMC (Saggini S *et al.*, 2004; Trescases O *et al.*, 2011) the inductor current is placed in the analog domain and the control signal \( v_c[n] \) in digital domain is converted to analog domain using a digital-to-analog converter (DAC). The analog control signal \( v_a[n] \) is then compared with the sensed inductor current \( i_L[n] \) using a high speed analog comparator. As shown in Fig. 14, the comparator output \( u_c \) triggers the mono-shot timer for constant on-/off-time mode of operation. Followed by it is the ‘dead-time circuit’ which generates the respective gate signals for the synchronous mode of operation.

**Limit Cycle Oscillation**

Although the variable frequency control offers many advantages over fixed-frequency control, the choice of sampling frequency in a variable-frequency digital modulator still remains a challenge, which is because of the mismatch in the switching and the sampling points. Figures 15 and 16 demonstrate the experimental results of the synchronous buck converter operating under mixed-signal constant off-time CMC scheme. The converter is designed to operate with the nominal switching frequency of 200 kHz and with an output voltage of 3.3 V. The output voltage is sampled at the same rate of the switching frequency. As shown in Fig. 15, there exists a fast-scale instability which is often known as multi-limit cycle instability (Krein P T and Bass R M, 1989). This increases the inductor current ripple, thereby increasing the conduction losses. If the sampling
frequency is increased to 5 times the switching frequency \((F_{vs} = 5 \times F_s)\), the current ripple is reduced, but instability still persists as shown in Fig. 16.

Non-uniform Sampling in Variable-frequency Control

Figure 17 shows the schematic diagram of a synchronous buck converter operating under the mixed-signal current-mode constant on/off-time control using an event-based sampling approach (Hariharan K et al., 2016). The output voltage \(v_o\) is sampled using a non-uniform clock \(F_{vs}\) which uses the rising edge of the high-side gate signal \(u_H\) for the constant off-time control and the falling edge of \(u_H\) for the constant on-time control. The digital voltage controller \(G_c(z)\) takes the sampled error-voltage \(v_e\) and computes the control signal \(v_c\) using \(F_{vs}\). The rest of the operation is similar to the digital implementation part given in Section Digital Implementation.

Discrete-time Modeling

A traditional state-space averaging method fails to capture fast-scale instability as this approach ignores the ripple information. On the other hand, the discrete-time model (Vergheese G C et al., 1986) predicts the fast-scale instability and also enables one to design a controller directly in digital domain. Referring to Fig. 18, a general discrete-time model of a CCM buck converter can be written as

\[ i_{n+1} = f(i_n, v_n), \quad v_{n+1} = g(i_n, v_n), \]

where, \(i_n\) and \(v_n\) are the inductor current and the output voltage at the beginning of the \(n^{th}\) cycle, and \(i_{n+1}\) and \(v_{n+1}\) represent their respective values at the end of the same cycle. Referring to Fig. 18, the modeling is carried out in two steps: in step 1, \(i_1\) and \(v_1\) are obtained when the high-side switch is turned off. Subsequently, in step 2, \(i_{n+1}\) and \(v_{n+1}\) are obtained; thereby, the complete discrete-time model can be obtained, in which the final values \(i_{n+1}\) and \(v_{n+1}\) can be obtained in terms of their initial values \(i_n\) and \(v_n\) during the \(n^{th}\) cycle.

Discrete-time Modeling Under an Event-driven Control Scheme

The state-space model of a synchronous buck converter is given by

\[ \dot{x} = A_x x + (A_x B v_m) u + A_x (1-u), \quad v_o = C_o x \]

(5.1)

where,

\[ x = \begin{bmatrix} i_L & v_{cap} \end{bmatrix}^T, \]

and

\[ A_x = \begin{bmatrix} 0 & 1 \\ -\frac{L}{C} & -\frac{1}{C} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix}, \quad A_x B = \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix}, \quad A_x (1-u) = \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix}. \]
This is evident from the inductor current FFT traces shown in Fig. 19.

For constant on-time modulation, the same stability boundary in (5.2) can be used with $T_c$ as the constant on-time. Figure 20 demonstrates the experimental results of a CCM buck converter under the mixed-signal constant on-time CMC scheme using an event-based sampling approach. The event-based method of sampling the output voltage at the falling edge of the high-side gate signal completely eliminates the limit-cycle oscillation behavior and achieves stable periodic behavior. This is evident from the inductor current FFT traces shown in Fig. 20.

**A Controller Design Case Study Using Current-mode Constant Off-time Modulation**

The feedback voltage controller using the current-mode constant on-time and/or off-time modulation techniques can be designed using discrete-time small-signal models. Consider a case study under the current-mode constant off-time modulation with the objective to achieve 45° phase margin and 10 kHz closed-loop bandwidth using a PI voltage controller. Following the modeling approach (Kapat S, 2017), the discrete-time controller gains are found to be $k_p = 8$ and $k_i = 0.3$. Figure 21 shows the transient response of the synchronous buck converter for a step change in load current from 1.5 A to 6 A using the mixed-signal current-mode constant off-time scheme. The
figure shows that 128 \( \mu s \) settling time and 600 mV (output voltage) undershoot are achieved.

The performance can be further improved by further increasing the proportional gain; however, this would increase the peak (inductor) current. While this is acceptable if the inductor current stays within its current limit, this would also increase the on-time duration of the high-side MOSFET for faster slew-up, thereby increasing the sampling interval using this event-based sampling approach. Due to the unavailability of output voltage samples during this slew-up process, the voltage controller fails to update, which may result in a complete collapse of the closed-loop system. This can be avoided by incorporating an additional uniform sampling clock which needs to be time-multiplexed with the event-based sampling clock, in which the latter is used throughout and the former would be only used when the inter-sample duration exceeds a threshold value. With this arrangement, a variable frequency digital/mixed-signal current mode controller can be designed to meet a high bandwidth demand over a wide operating range. Here, unlike in a PWM CMC technique, a ramp-compensation is not needed for current-loop stability while operating over a wide duty-ratio range; thus the benefits of current-mode control can be retained without compromising the current-loop bandwidth.

While the objectives in this paper are to highlight the benefits of using variable frequency digital current-mode control over its fixed-frequency counterpart, the relevant discussions are equally applicable for the given application needs. Then all the design guidelines are equally applicable, and the salient features would be retained. It is important to note that the benefits would be more pronounced for higher and/or lower duty-ratio operating regions. For any specific application, the following steps may be considered:

i) The power circuit needs to be designed first.

ii) Using power circuit parameters, the effective sampling frequency and the nominal timing parameters need to be designed.

iii) The digital voltage controller can then be designed to meet the steady-state and closed-loop transient performance requirements.

Conclusions

In this paper, the merits and demerits of both the fixed-frequency and the variable-frequency control techniques were highlighted for DC-DC converters. It was found that the variable-frequency control would achieve inherent current-loop stability over a wide range of duty-ratio without the need of an additional compensating ramp. The steady-state switching frequency can be precisely regulated using an all-digital PLL. Further, a digital controller can be tuned for further performance improvement. An event-driven sampling mechanism would be very useful to overcome multi-limit cycle oscillations. Thus, such techniques emerge as an attractive digital control solution for future power management applications.

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