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Research Paper

Implementation of Dynamic Dual Input Multiple Output Logic Gates via Enhanced Logical Resonance in Non-Locally Coupled Duffing Oscillators

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Employing second sub-harmonic or super-harmonic resonant force, in the presence of weak driven force, to the first site of N non-locally coupled double well Duffing oscillators, can induce the resonance phenomenon in the first site of coupled systems. In this paper, we report that under appropriate coupling strength and coupling radius, the second sub-harmonic or super-harmonic force coherently drives and enhances the amplitude of the weak driven force throughout the remaining sites of non-locally coupled network which are free from resonant and driven forces and it is termed as 'Enhanced Resonance' (ER). Also we illustrate that under this enhanced resonance condition, if any digital signal having amplitude greater than or equal to the certain threshold value, is given to the first site of the coupled systems, the signal will propagate throughout the entire coupled network without any attenuation. On the other hand, if we give two digital signals with asymmetric amplitudes instead of single digital signal, both the digital signals are logically added and enhanced at the output of each of the N coupled systems. This phenomenon is termed as 'Enhanced Logical Resonance' (ELR) which mimics dual input multiple output (DIMO) logic gates. Later, we demonstrate that the output of DIMO logic gate is independent of the way of giving the two inputs either simultaneously to any one of the site or separately to any two of the sites of the N coupled systems and thus paved the way to design dynamic dual input multiple output (DDIMO) logic gates such as AND/NAND, OR/NOR gates via enhanced resonance behaviour. It has also been found that the asymmetric amplitude of the two logical inputs decide the various logical behaviours rather than altering the system parameters.

Keywords: Dynamic Computing; Resonance; Coupled Systems; Logic Gates

Introduction

Now a day, investigation on designing the sequential logic circuit and combinational logic circuit using dynamical systems has become a challenging, interesting and innovative idea in the modern digital technology. Since various dynamical behaviours of the dynamical system can be exploited for implementing fundamental digital elements, especially basic logic gates which will be the fundamental building block of both sequential as well as combinational logic circuits. During the past two decades, different dynamical systems exhibiting different nonlinear phenomena, especially discrete maps exhibiting chaotic behaviour (Sinha and Ditto, 1998; Munakata *et al*., 2002; Ditto *et al*., 2010; Murali *et al*., 2009a, 2005), coupled chaotic maps or array of chaotic maps (Sinha *et al*., 1998, 2002b), continuous nonlinear dynamical systems exhibiting phenomena such as chaos, synchronization, and stochastic resonance (Sinha and Ditto, 1998; Munakata *et al*., 2002; Ditto and Sinha, 2015; Sinha and Ditto, 1999; Murali *et al*., 2009c), piece-wise linear systems exhibiting chaos, synchronization, vibrational resonance and strange non-chaotic attractor phenomena (Murali *et al*., 2003a; Murali and Sinha, 2007; Venkatesh *et al*., 2016a, 2016b, 2017a) and coupled dynamical systems exhibiting sub-harmonic and super-harmonic resonance behaviours (Venkatesh *et al*., 2017b) can be employed for this purpose. The exploitation of the various behaviours of the nonlinear dynamical system for dynamic computing starts with the pioneer work of Sinha and Ditto (1998, 1999). Then Munakata *et al*. (2002) successfully extended the threshold

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mechanism for building AND, OR, NOT, XOR & NAND operations and termed it as 'Chaogate'. Further, Sinha *et al*. (2002a, 2002b) emulated the concept of parallel computing architecture using low dimensional system as well as high dimensional system. Following this, Murali *et al*. (2003b, 2005) demonstrated an experimental module for logistic map to implement all the logic gate operations. Then, they further realized this 'Chaos computing' using the cooperative phenomena, namely, chaos synchronization and stochastic resonance (Murali *et al*., 2007, 2009b, 2009c). Recently, the present author along with Venkatesan and Lakshmanan (2016a) obtained the analytical solutions for chaos synchronization phenomenon in uni-directionally coupled Murali-Lakshmanan-Chua circuit system based on which fundamental logic gates using analytical equations have been demonstrated. To an extent, Ditto *et al*. (2010) demonstrated that the dynamical evolution of Chaogate could yield half adder/subtractor and full adder/subtractor operations.

Later on, the present author along with Venkatesan (2016b) demonstrated the fundamental logic gates using the analytical solutions of the vibrationally resonanced Murali-Lakshmanan-Chua (MLC) system. Following this, the present author along with Venkatesan and Lakshmanan (2017a) analytically discussed dynamic logic gates and R-S flip-flop circuit using the quasi-periodically driven MLC system. In the mean time, they employed the resonance behaviours, namely, sub-harmonic and super-harmonic resonances in globally coupled Duffing oscillators for implementing dual input multiple output dynamic logic gates which can meet out 'fan out limit' in statically wired hardware (Venkatesh *et al*., 2017b). In all the earlier studies of the present author, the idea of dynamic logic gates has been successfully explained by imposing constraints on the two logical inputs rather than altering system parameters. In an overview, dynamic computing can be realized in systems ranging from physical maps or systems (Sinha and Ditto, 1998, 1999; Munakata *et al*., 2002; Venkatesh *et al*., 2017a) to electronic circuits (Murali *et al*., 2003a, 2009a, 2005, 2003b; Ditto *et al*., 2010), then extended to nano mechanical system (Guerra *et al*., 2010), chemical system (Sinha *et al*., 2009), optical system (Sinha *et al*., 2002a, 2002b; Singh and Sinha, 2011, Perrone *et al*., 2012; Chlouverakis and Adams, 2005) and gene network (Dari *et al*., 2011a, 2011b, 2011c; Kia *et al*., 2015; Sharma *et al*., 2014).

The aim of this paper is to demonstrate how the non-locally coupled double well Duffing oscillators can be effectively utilized in designing dual input multiple output dynamic logic gate (DIMO) via enhanced resonance phenomenon, namely, sub-harmonic and super-harmonic resonances which have been discussed previously by the present author for globally coupled system (Venkatesh *et al*., 2017b). The purpose of choosing this DIMO dynamic logic gate using resonanced non-locally coupled system is an alternative to DIMO dynamic logic gate by resonanced globally coupled system. Our system has the following advantages, namely, (i) additional control parameter namely, coupling radius r which can also be used to control the logic gate behaviours, (ii) availability of precise digital signal propagation region only for a certain value of coupling radius r, (iii) increasing the chance of getting DIMO logic gate behaviour at lower value of system parameters say resonant force amplitude f_2, coupling strength Δ and threshold amplitude of digital signal F_T compared to DIMO by globally coupled system and (iv) increasing the chance of getting DIMO logic gate behaviour region for a fixed coupling strength Δ while changing the coupling radius r value. Also it has been demonstrated that the logic behaviour is independent of the way of giving the inputs either simultaneously to any one of the sites or separately to any two of the sites in the N coupled system provided that the input amplitude should be greater than or equal to the greatest threshold value of the N sites.

Non-locally Coupled Double well Duffing Oscillators

We considered a simple N non-locally coupled double well Duffing oscillators represented as,

$$\ddot{x}_i + \alpha \dot{x}_i + \omega_0^2 x_i + \beta x_i^3 = \delta_1 (f_1 \sin \Omega_1 t + f_2 \sin \Omega_2 t) + \frac{\Delta}{2P} \sum_{j=i-P}^{j=i+P} \left(x_j - x_i \right), \rightarrow \quad (1)$$

where i = 1,2,....N & N represents the total number of oscillators; α is the damping constant, $|\omega_0/2\pi|$ is the natural frequency of the oscillator, β is the strength of the non-linearity of the oscillator, f_1 is the strength of the driving force with frequency Ω_1, f_2 is the strength of the second external force with high or low frequency Ω_2 compared to first driving force, Δ is the coupling strength, and P specifies the number of neighbors in each direction on a ring so that the coupling radius r = P/N. Here $x_i(t)$ represents the state vector of the i^{th} oscillator. δ_1 decides the site to which the bi-harmonic forces is to be given. Here the bi-harmonic forces are given only to the first site alone and hence $\delta_1 = 1$ for i = 1 and for all other i‡1, $\delta_1 = 0$. For numerical discussion, the parameters are fixed as N = 10, $\alpha = 0.5$, $\omega_0^2 = -1$, $\beta = 1$, $f_1 = 0.05$ and $\Omega_1 =$ 1. The dynamics of the above non-locally coupled systems are seem to be identical to that of the globally coupled network in the absence of coupling strength (i.e., $\Delta = 0$) and external forces (i.e., $f_1 = f_2 = 0$) (Venkatesh *et al*., 2017b). Further, the effect of external forces (f_1 and f_2) and coupling strength (Δ) make the non-locally coupled systems to exhibit a similar enhanced resonance phenomenon as that of the globally coupled systems for certain coupling radius r (Venkatesh *et al*., 2017b).

Enhanced Resonance in Non-Locally Coupled Double Well Duffing Oscillators

Optimal amplitude of the high/sub-harmonic as well as low/super-harmonic frequency second harmonic driving applied to the first coupled system enhances the response of all the other coupled systems to the first harmonic weak signal. To illustrate the occurrence of resonance phenomenon in the first site and its extension to all the other coupled sites, a second low as well as high frequency harmonic force is included only to the first oscillator.

The response of all the non-locally coupled systems to the high as well as low frequency input driving signal $f_2 \mathrm{Sin}\Omega_2 t$ can be evaluated by calculating the corresponding sine B^i_s and cosine components B^i_c respectively of the coordinate variations $x_i(t)$ (output signal of i^{th} oscillator). For the i^{th} oscillator, the sine constituent of the output signal (Venkatesh *et al*., 2017b),

$$B_s^i = \frac{2}{nT}\int_0^{nT} x_i(t)\sin\Omega_1 t dt, \rightarrow \quad (2)$$

and the cosine constituent of the output signal (Venkatesh *et al*., 2017b),

$$B_c^i = \frac{2}{nT}\int_0^{nT} x_i(t)\cos\Omega_1 t dt, \rightarrow \quad (3)$$

For the frequency $\Omega_1 = 1.0$ with n = 20, the above sine and cosine constituents are evaluated. Then, we finally obtain the dependencies on f_2 of both the response amplitude Q_i as,

$$Q_i = \frac{\sqrt{B_s^{i2} + B_c^{i2}}}{f_1}, \rightarrow \quad (4)$$

where $T = 2\pi/\Omega_1$ and n is an integer (Venkatesh *et al*., 2017b).

Numerical simulation of the system (1) is performed using Runge-Kutta fourth order (RK IV order) algorithm. The response of the entire systems to second periodic driving forces is analyzed by varying the amplitude f_2 in steps of 0.1 for the time period of 20T using the Eqs. (2), (3) and (4). It has been observed that for an appropriate low or high frequency second harmonic force applied to the first site of the non-locally coupled systems, one can induce a maximum response to a particular second harmonic force amplitude f_2, namely, resonance phenomenon in the first driven site and thus enhance the resonance phenomenon to all the N oscillators for an appropriate coupling strength and coupling radius. This resonance phenomenon can be studied by measuring the response amplitude Q_i for all the N sites using the Eqs. (2), (3) and (4). For analysis, the various parameters have been chosen as N = 10, $\alpha = 0.5$, $\omega_0^2 = -1$, $\beta = 1$, $f_1 = 0.05$, $\Omega_1 = 1$, $\Omega_2 = \Omega_1/2$ for low frequency response, $\Omega_2 = 2\Omega_1$ for high frequency response, r = 0.2, 0.3, 0.4, 0.5 and n=20T.

If the entire N non-locally coupled oscillators are not driven by the bi-harmonic forces f_1 and f_2, then they will all be in stable equilibrium point. As and when the bi-harmonic signals $f_1 = 0.05$ and $0 < f_2 < 50$ are applied to the first site alone with zero coupling strength, the bi-harmonically driven first site alone exhibits resonance phenomenon and the remaining sites do not show the resonance behaviour since for all the sites $i > 1$ exhibit stable equilibrium point. With the gradual increase of the coupling strength from $\Delta = 0$, some coupled sites may exhibit coherent resonance phenomenon and other may exhibit incoherent resonance behaviour. Beyond an optimal coupling strength, the entire N coupled sites show a coherent resonance and it is termed as 'Enhanced Resonance'. Figs. 1 and 2 show the resulting response of the non-locally coupled system (1) to the weak periodic signal $f_1 = 0.05$, as influenced by the high and low frequency force f_2 in the range $0 < f_2 < 50$ for the coupling strength range $0 \leq \Delta \leq 15$ with coupling radius r = 0.2, 0.3, 0.4, 0.5 respectively. Thus occurrence and enhancement of resonance from the first site to the remaining sites shows the propagation of the weak signal throughout the coupled network in biological or network systems. In addition to the enhancement of weak signal f_1 under resonance condition, any other digital signal with appropriate amplitude given to any one of the sites of this coupled system should also be enhanced or mimicked at the output of all the sites of the coupled network. This confirms the propagation of single digital signal or logically added two digital signals throughout the enhanced sub/super-harmonic resonanced coupled system and this phenomenon is termed as 'Enhanced Logical Resonance' which mimics dual input multiple output (DIMO) logic gate.

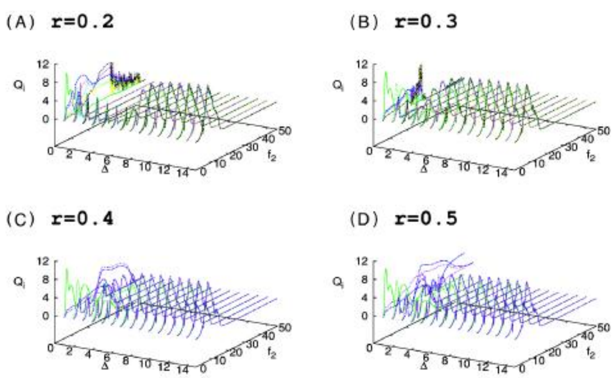

Fig. 1: Response amplitude diagram for ten non-locally coupled Duffing oscillators resonated by high frequency second harmonic force f_2 of frequency $\Omega_2 = 2\ \Omega_1 = 2$ with different amplitudes for different coupling strengths. (A) Corresponds to r = 0.2 and (B) corresponds to r = 0.3. In (A) and (B), green line (solid line) represents i = 1; blue line (long dash line) represents i = 2, 10; pink line (short dash line) represents i = 3, 9; sky blue line (dotted line) represents i = 4, 8; yellow line (long dash with dotted line) represents i = 5, 7; black line (short dash with dotted line) represents i = 6. (C) Corresponds to r = 0.4 and (D) corresponds to r = 0.5. In (C) and (D), green line (solid line) represents i = 1; blue line (long dash line) represents i = 2, 3, 4, 5, 7, 8, 9, 10; pink line (short dash line) represents i = 6

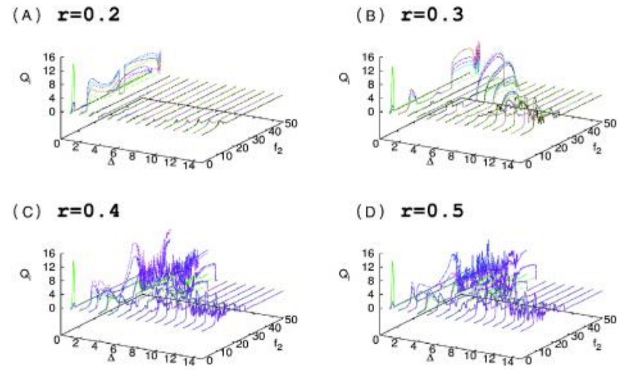

Fig. 2: Same as in Fig. 1 but with the system (1) is resonated by a low frequency second harmonic force f_2 of frequency $\Omega_2 = \Omega_1/2 = 0.5$

Implementation of Dynamic Dual Input Multiple Output (DDIMO) Logic Gate via Enhanced Resonance

For some situations, the output of the experimental logic gate can only drive a finite number (called fan-out limit) of inputs to other gates. Also to get different logical operations, the parameter of the system or rewiring the circuit or circuit elements are to be carried out. Taking account of these two limitations, a threshold mechanism to the logical inputs to obtain dynamic DIMO logic cell or gate is proposed (Venkatesh *et al.*, 2017b). Under enhanced resonanced state (i.e., either sub-harmonic or super-harmonic resonance), system (1) can act as a multiple output logic cell and thus performs different logical operations, say, AND/NAND and OR/NOR. Before performing various logical operations, one has to check whether the coupled system (1) can be able to admit/propagate a square wave or digital signal throughout its sites (i=1 to 10). For that purpose, the system equation (1) is modified as

$$\ddot{x}_i + \alpha \dot{x}_i + \omega_0^2 x_i + \beta x_i^3 = \delta_1$$

$$(f_1 \ \sin \Omega_1 t + f_2 \ \sin \Omega_2 t) + \delta_2 f(t)$$

$$+ \frac{\Delta}{2P} \sum_{j=i-P}^{j=i+P} \left(x_j - x_i \right) \ , \ \rightarrow \qquad (5)$$

where f(t) is a random square wave or digital signal of amplitude +F to -F and δ_2 decides the site to which the digital signal is given as an input. For example, if f(t) is given to the first site (i=1), then $\delta_2 = 1$ and for all other i‡1, δ_2 will be 0. In general, $\delta_2 = 1$ when f(t) is given to the n^{th} site (i.e., i = n) and for all other i ‡ n, δ_2 will be 0. For un-damped propagation, one must operate the above non-locally coupled oscillators in the resonance region by fixing $f_1 = 0.05$, $f_2 = 7.4$, $\Omega_2 = \Omega_1/2$ and $\Delta = 15$ to get coherent behaviour for a low frequency second harmonic force. It has been numerically found that for propagating a random digital signal throughout the N oscillators, the amplitude of the signal given to the first site must meet certain necessary condition. In other words, only the random digital signal given to the first site i = 1 having an amplitude greater than or equal to the threshold value $F_T = 2.4005$ (see Fig. 3A) alone shows significant propagation. For digital signal amplitude $F \leq -F_T$, all the N oscillators will show output $x_i < 0$ which will be treated as logic 0 and for the amplitude $F \geq F_T$, the output will be $x_i > 0$, treated as logic 1. This shows that to change the attractor from negative potential well to positive well, a threshold value of $F \geq F_T$ is to be needed and for the reverse case, a threshold value of $F \leq -F_T$ is to be employed. Thus, by giving the digital signal to the various other sites i = 2 to 10 individually, the corresponding threshold value/ amplitude of F_T is numerically evaluated and plotted as shown in Fig. 3A. Similarly for the high frequency second harmonic signal, the threshold value of the digital signal that has been given as an input to various sites (i = 1 to 10) separately, is numerically evaluated and plotted as shown in Fig. 3B.

From the Figs. 3A and 3B, it has been found that for propagating the digital signal throughout all the N sites by giving the digital input to any one of the sites irrespectively, the threshold amplitude F_T of the input signal should be greater than or equal to the maximum of the ten sites. In other words, for better digital signal propagation irrespective of the input given to any one of the sites, the amplitude of the input digital signal should be greater than or equal to the greatest peak value of the curve i.e., $F_T \geq 2.4125$ (see Fig. 3A) for low frequency second harmonic force and $F_T \geq 2.816$ (see Fig. 3B) for high frequency second harmonic force.

According to this scheme of threshold mechanism (Venkatesh *et al*., 2017b), to construct dynamic DIMO logic gate, the non-locally coupled N oscillators exhibiting coherent resonance mimics the logic gate or cell with N more or less coherent outputs is considered here. For any logical operations, two input signals are needed. Thus, here two random square waves are used as two input signals instead of f(t) in Eq. (5). With these input signals, the four possible input combinations (0 or -F, 0 or -F), (0 or -F, 1 or +F), (1 or +F, 0 or -F), (1 or +F, 1 or +F) merge into three different input combinations (0,0), (0,1)/(1,0) and (1,1). The low input is taken as υ_1 for logical 0 and the high input as υ_2 for logical 1. From the earlier discussion in this section, for AND or OR operation, the two inputs are logically added and the resultant digital signal should be such that for the logical 0 state the amplitude must be less than or equal to $-F_T$ and for the logical 1 state the amplitude must be greater than or equal to F_T. In addition to that, this logically added signal should propagate throughout the N oscillators without damping in its amplitude. This is

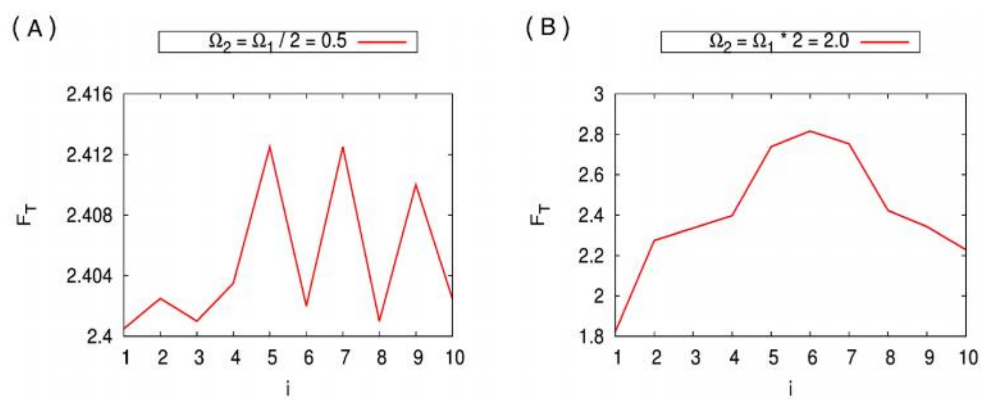

Fig. 3: Threshold amplitude of digital input f(t) given to each individual ten non-locally coupled Duffing oscillators (5) resonated by (A) low frequency second harmonic force f_2 of frequency $\Omega_2 = \Omega_1/2 = 0.5$ with the system parameter values $f_1 = 0.05$, $f_2 = 7.4$, r = 0.3, $\Delta = 15$ and (B) high frequency second harmonic force f_2 of frequency $\Omega_2 = 2\ \Omega_1 = 2$ with the system parameter values $f_1 = 0.05$, $f_2 = 9.8$, r = 0.3, $\Delta = 13$

possible only if all the non-locally coupled oscillators are set in either sub-harmonic or super-harmonic resonance state. Thus the logical low 0 output of the entire N coupled sites are considered as $x_i < 0$ and the logical high 1 outputs are taken as $x_i > 0$ with i = 1, 2...10. For NAND/NOR operations, the logical low 0 output is taken as $x_i > 0$ and the logical high 1 output is taken as $x_i < 0$ with i = 1,2...10. The three possible input combinations with the corresponding outputs for AND and OR gates are represented (Venkatesh *et al*., 2017b) as

AND Gate:

$$\upsilon_1 + \upsilon_1 \leq -F_T, \quad \rightarrow \tag{6a}$$

$$\upsilon_1 + \upsilon_2 \leq -F_T, \quad \rightarrow \tag{6b}$$

$$\upsilon_2 + \upsilon_2 \geq F_T. \quad \rightarrow \tag{6c}$$

From Eq. (6c), $\upsilon_2 \geq F_T/2$. On substituting in Eq. (6b), $\upsilon_1 \leq -3F_T/2$ and also satisfies Eq .(6a). Thus, logic 0 and logic 1 value of the inputs for AND gate should be

$$\upsilon_1 \leq \frac{-3F_T}{2},$$

$$\upsilon_2 \geq \frac{F_T}{2}. \quad \rightarrow \tag{7}$$

OR Gate:

$$\upsilon_1 + \upsilon_1 \leq -F_T, \quad \rightarrow \tag{8a}$$

$$\upsilon_1 + \upsilon_2 \geq F_T, \quad \rightarrow \tag{8b}$$

$$\upsilon_2 + \upsilon_2 \geq F_T. \quad \rightarrow \tag{8c}$$

With Eq. (8a), $\upsilon_1 \geq -F_T/2$. On substituting in Eq. (8b), $\upsilon_2 \leq 3F_T/2$ and also satisfies Eq. (8c). Thus for OR gate, the logic 0 and logic 1 of the inputs should be

$$\upsilon_1 \leq \frac{-F_T}{2},$$

$$\upsilon_2 \geq \frac{3F_T}{2}. \quad \rightarrow \tag{9}$$

For numerical simulations, the value of F_T is chosen to be greater than or equal to 1.8225 (see Fig. 3B and choose $F_T = 3.0$ here) corresponding to the system parameters $f_1 = 0.05$, $f_2 = 9.8$, $\Omega_2 = 2\Omega_1 = 2$ and $\Delta = 13$ with the two logical inputs given simultaneously to the first site (i = 1) of the ten non-locally coupled Duffing oscillators. For AND operation, the logic 0 and logic 1 of the inputs are evaluated from the above Eq. (7) and are found to be $\upsilon_1 = -4.5$ and $\upsilon_2 = 1.5$. The AND response of all the ten oscillators for the various random combinations of the two digital inputs are shown in Fig. 4. Similarly, one can get an identical AND response for low frequency second harmonic force f_2 of frequency $\Omega_2 = \Omega_1/2 = 0.5$ with system parameters $f_1 = 0.05$, $f_2 = 7.4$, $\Delta = 15$ and $F_T \geq 2.4005$ (say here $F_T = 3.0$).

Figure 5 illustrates the logical OR operation for two random input signals with logic 0 as $\upsilon_1 = -1.5$ and logic 1 as $\upsilon_2 = 4.5$ (using Eq. (9)) corresponds to $F_T = 3.0$ ($F_T \geq 1.8225$ for high frequency force f_2 and $F_T \geq 2.4005$ for low frequency force f_2) for the ten non-locally coupled system (5). From Figs. 3A and 3B, the threshold value of the digital signal that has been given as input to various sites of the ten non-locally coupled Duffing oscillators is not same. In order

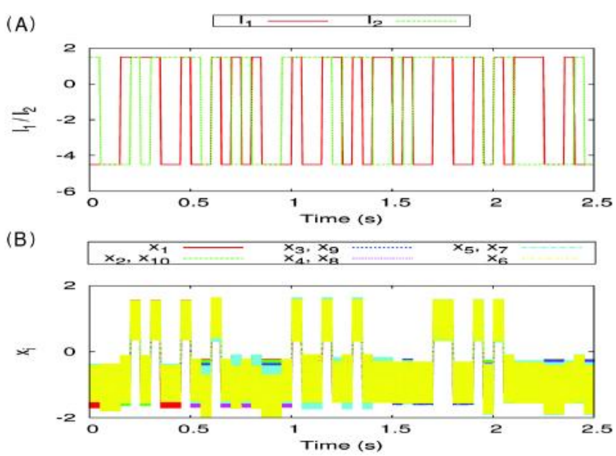

Fig. 4: Logical AND operation for two random inputs and its persistence throughout ten non-locally coupled Duffing oscillators resonated by a high frequency second harmonic force f_2 of frequency $\Omega_2 = 2\,\Omega_1 = 2$. (A) Two random input signals (Green and Red) given to the first site (i.e., i = 1) of the coupled systems and (B) 2-D plot between Time in seconds and output states x_i of N=10 oscillators

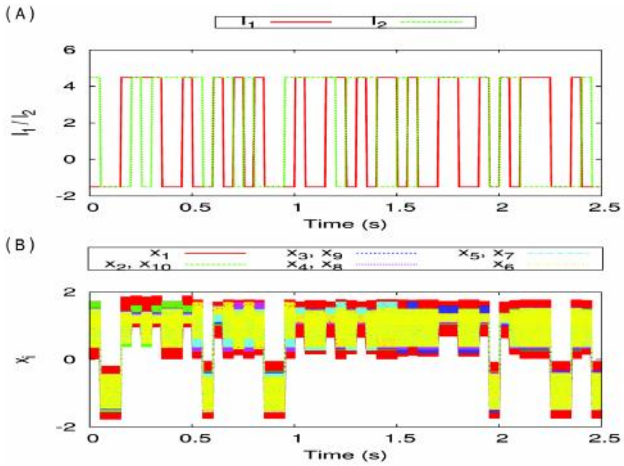

Fig. 5: Logical OR operation for two random inputs and its persistence throughout ten non-locally coupled Duffing oscillators resonated by a low frequency second harmonic force f_2 of frequency $\Omega_2 = \Omega_1/2 = 0.5$. (A) Two random input signals (Green and Red) given to the first site (i.e., i = 1) of the coupled systems and (B) 2-D plot between Time in seconds and output states x_i of N = 10 oscillators

to give the two inputs in a dynamic way such that the inputs are given either simultaneously or separately to any of the sites irrespective of their order/number (i), the value of the F_T should be greater than or equal to the maximum of the all the ten sites (For high frequency resonant system $F_T \geq 2.816$ (see Fig. 3B) and for low frequency resonant system $F_T \geq 2.4125$ (see Fig. 3A) respectively, here $F_T = 3.0$ for both low as well as high frequency resonant system). Thus for AND operation, the logic 0 and logic 1 of the input signal becomes $\upsilon_1 = -4.5$ and $\upsilon_2 = 1.5$ respectively. Fig. 6 demonstrates the dynamic dual input multiple (say ten outputs) output AND logic gate corresponds to high frequency resonant system (5) for the system parameters $f_1 = 0.05$, $f_2 = 9.8$, $\Omega_2 = 2\,\Omega_1 = 2$ and $\Delta = 13$. Here, the logic 0 and logic 1 states of the input signals shown in Fig. 6A are obtained based on the random number function which generates a random number between 0 and 1 with uniform distribution and having the mean converges to 0.5. At any instant of time, if the random number is less than or equal to 0.5 (mean value), then the input is taken as logic 0 ($I_1 = I_2 = -4.5$), otherwise the input is taken as logic 1 ($I_1 = I_2 = 1.5$). Then the overall statistical analysis of switching behaviours especially, probability of occurrence of '1' state ($I_1 = I_2 = 1.5$) and '0' state ($I_1 = I_2 = -4.5$) corresponding to the inputs in Fig. 6A are to be analysed to check whether the switching is uniform or not. In order to analyse this switching behaviour, the total number of events (including '0' state and '1' state of the input signal) occurred during the time period 0 to 1 sec is to be evaluated. Here, it is found to be 201000. Then the probability of occurrence of '0' state is calculated as P(0 or –4.5) = 0.602 and that of '1' state is P(1 or 1.5) = 0.398 for the first input signal (I_1) (red solid line in Fig. 6A). Similarly, for the second input signal (I_2) (green dotted line in Fig. 6A) the probability P(0 or –4.5) and P(1 or 1.5) are found to be 0.448 and 0.552 respectively. It has been found that for uniform switching, the probability of occurrence of 0 and 1 states should converge to 0.5 when the total number of events

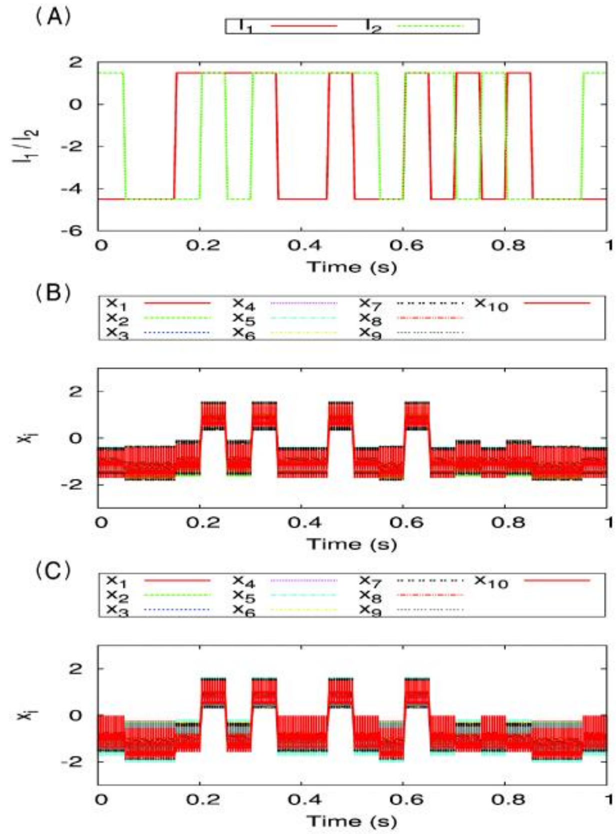

Fig. 6: Logical AND operation for two random inputs and its persistence throughout ten non-locally coupled Duffing oscillators resonated by a high frequency second harmonic force f_2 of frequency $\Omega_2 = 2\Omega_1 = 2$. (A) Two random input signals (Green and Red), (B) 2-D plot between Time in seconds and output states x_i of the entire (ten) oscillators for the two inputs in figure (A) given separately to two different oscillators such that I_1 to i=1 and I_2 to i=2 respectively, and (C) same 2-D plot for the same two inputs in figure (A) given to two different oscillators separately such that I_1 to i=5 and I_2 to i=10 respectively

increases.

Similar response can be obtained for low frequency resonant system with the system parameters $f_1 = 0.05$, $f_2 = 7.4$ and $\Delta = 15$ respectively. Similarly, one can also get the DDIMO OR behaviour with the low as well as high frequency resonant system for the various combinations of the logical inputs $\upsilon_1 = -1.5$ (logic 0) and $\upsilon_2 = 4.5$ (logic 1) respectively.

Conclusions

In a brief overview, we have shown the occurrence and coherent enhancement of sub/super harmonic resonance in ten non-locally coupled oscillator which depends on the coupling strength and coupling radius. Also, we have numerically illustrated the digital response of all the sites of the ten non-locally coupled networks for the random digital signal that too given as an input to any one of the ten sites. Further, it has been found that for the better response of the digital input or in other words mimicking the digital input at the output of N sites, the amplitude of the digital input should always be equal or greater than the threshold value F_T. To an extent, we have also enumerated the idea of implementation of DDIMO logic gate structures which is one of the significance of sub/super harmonic resonance. In particular, we have shown the direct and flexible implementation of the basic DDIMO logic gates AND/NAND and OR/NOR using the non-locally coupled oscillator system under resonanced condition by simply altering the high or positive and low or negative amplitude of the input signals without altering the system parameters. Such a scheme of implementation of basic DDIMO logic gates for parallel processing digital circuits may serve as ingredients of a general purpose device and are more flexible than statically wired hardware.

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