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RESEARCH PAPER

An optimal inventory policy with ramp type demand rate and time varying holding cost in two-warehousing environment

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Abstract
In presenting paper, an inventory model for deteriorating items has been developed by considering time varying deterioration and ramp type demand rate. Shortages are allowed with partially backlogged. This work is mainly focused on two-warehouse storage policy with linearly depended holding cost. It is assumed that holding cost and fare of rented warehouse is higher than the holding cost payable at owned warehouse. To make the model more general, other relevant parameters have also been considered related to inventory control system such as trade credit policy. Total cost function has been verified through numerical and graphical representation. A sensitivity analysis is also done to check the sensitivity of the model. This study further can be modified by inculcating other parameters of inventory control system.

Keywords Ramp type demand · Preservation · Trade credit · Two-warehouse

Literature survey

The effect of global warming can be easily observed over international and national market scenario. The market conditions are directly or indirectly related with different inventory factors like nature of demand, quality of product, environment conditions etc. So, it is required to do deep study of research literature which will form a solid base to develop a perfect inventory model. We have done a literature survey based on three main factors of inventory theory, mentioned in the below Table 1.

Through above thorough study of different inventory models, we have analyzed the available research work and market conditions. So according to requirement of market scenario, we have developed an inventory model for deteriorating items. In this demand function is taken as ramp type, to fulfill the demand the items are stored in two-warehouse system. The present article is sectioned in four sections, in the following section assumptions and notations are described, which are used in mathematical model formulations. In third and fourth section formulation and numerical illustration is well described. In the end we have concluded the results and implications.

Assumptions and notations

In this section we have elaborated the assumptions and notations which are inculcated in mathematical model formulation.

Assumptions

- Demand rate $D(t)$ is a ramp type function: $D(t) = Ae^{b[t-(t-\mu)H(t-\mu)]}$, where $A > 0$, $b > 0$ represent the initial demand rate and a constant governing the exponential demand rate respectively. The Heaviside's function $H(t-\mu)$ is defined as:

$$H(t-\mu) = \begin{cases} 1, t \geq \mu \\ 0, t < \mu \end{cases}$$

- The holding cost per unit per time unit is as follows: $h_i(t) = h_i + h_i\ t$, i = w (for OW), R (for RW) such that $h_2 > h_1 > 0$;
- To maintain profitability and feasibility $p_i < s < a_1/a_2$;
- The items are stored at two-warehouse system. The owned warehouse (OW) have limited storage capacity.

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Table 1 Literature survey

Researchers	Ramp type demand	Two-warehouse system	Preservation technology investment
Mandal and Pal (1998)	√	√	
Wu et al. (1999)	√		
Manna and Chaudhuri (2006)	√		
Agrawal et al. (2013)	√		
Dye (2013)			√
Saha (2014)	√		
Singh and Rathore (2014), Singh et al. (2014)		√	√
(20)	√		
Jaggi et al. (2015)	√		
	√		
Yang et al. (2015)	√		
Alfares and Ghaithan (2016)			√
Singh and Rathore (2016)		√	√
Zhang et al. (2016)			√
Luo (2017)	√		
Patro et al. (2017)			√
Mandal and Giri (2017)			√
Huang et al. (2017)			√
			√
Patro et al. (2017)			√
Shah et al. (2017)		√	
Chakrabarty et al. (2018)			√
Chung et al. (2018)			√
Tejpal and Rathore (2023)			√
Sharma and Rathore (2023)			√
Bhawaria and Rathore (2023)			√
Choudhury and Mahata (2024)			
San-José et al. (2024)			
Yadav et al. (2024)		√	√
Present Article	√	√	√

The rented warehouse (RW) with unlimited storage area, is hired at high holding cost then the holding cost of OW.

- The preservation technologies are considered to preserve the stock. The function is as follows: $m(\xi) = (1 - e^{-\mu\xi})$; where $\mu > 0$;
- The inflationary environment is put under consideration;

Notations

- D(t): The demand rate;
- Q: The initial inventory level;
- I_m: The maximum order level;
- W_1: The capacity of OW;
- W_2; The maximum stock level at RW;
- h_{wi}: The holding cost parameter at OW, i = 1, 2;
- h_{Ri}: The holding cost parameter at RW, i = 1, 2;
- C: The order cost;
- C_p: The unit purchase cost;
- ξ: The preservation technology cost;
- Θ: The constant rate of deterioration;
- τ_θ: The reduced deterioration rate, as follows: $\tau_\theta = \theta - m(\xi)$;
- r: The inflation rate;
- T: The total cycle length;
- T_1: The time moment at which stock level at RW ceases to zero level;
- $I_1(t)$: Inventory level at any time t in RW during [0, μ];
- $I_2(t)$: Inventory level at any time t in RW during [μ, T_1];
- $I_3(t)$: Inventory level at any time t in OW during [0, T_1];

- $I_4(t)$: Inventory level at any time t in OW during [T_1, T];
- OC: The order cost;
- PC: The purchase cost;
- HC: The holding cost;
- TC_i (T_1, T ξ): The total cost function with decision variables T_1, T and ξ;
- Note: The superscript * is shows the optimal value of respective parameters;

Mathematical model formulations

At the time $t = 0$ the inventory level is Q as the time precedes the stock level depletes continuously just because of total effect of deterioration and demand. The complete functioning of inventory is elaborated by following differential equations. Here we consider two types of cases (i) Case 1: $\mu \le T_1 \le T$, (ii) Case 2: $T_1 \le \mu \le T$.

(i) **Case 1: $\mu \le T_1 \le T$**

$$\frac{dI_1(t)}{dt} + \tau_\theta I_1(t) = -\mathrm{A}e^{bt};0 \le t \le \mu \tag{1}$$

$$\frac{dI_2(t)}{dt} + \tau_\theta I_2(t) = -\mathrm{A}e^{b\mu};\mu < t \le T_1 \tag{2}$$

$$\frac{dI_3(t)}{dt} + \tau_\theta I_3(t) = 0;0 \le t \le T_1 \tag{3}$$

$$\frac{dI_4(t)}{dt} + \tau_\theta I_4(t) = -\mathrm{A}e^{b\mu};T_1 \le t \le T \tag{4}$$

Solving (1), (2), (3) and (4) under the conditions $I_m(t=0)=Q=W_1+W_2$, $I_1(t=\mu)=I_2\ (t=\mu)$ $I_2(t=T_1)=0$, $I_3(t=T_1)=I_4(t=T_1)$ and $I_4(t=T)=0$, we get

$$e^{\tau_\theta t}I_1(t) = -A\left(t + \frac{(\tau_\theta + b)t^2}{2}\right) + W_2 \tag{5}$$

$$e^{\tau_\theta t}I_2(t) = -Ae^{b\mu}\left(t + \frac{(\tau_\theta)t^2}{2}\right) + e^{\tau_\theta \mu}\left[W_2 + A\frac{\mu^2}{2}\left(b - 2\tau_\theta + \mu\left(b - \tau_\theta\right)\right)\right] \tag{6}$$

$$e^{\tau_\theta t}I_3(t) = \mathrm{W}_1 \tag{7}$$

$$e^{\tau_\theta t}I_4(t) = -Ae^{b\mu}\left(T - t + \frac{(\tau_\theta)\left(T^2 - t^2\right)}{2}\right) \tag{8}$$

$$\mu = \frac{W_1}{b} - \frac{A}{b}e^{b\mu}\left(T - T_1 + \frac{(\tau_\theta)\left(T^2 - T_1^2\right)}{2}\right) \tag{9}$$

(ii) **Case 2: $T_1 \le \mu \le T$**

$$\frac{dI_1(t)}{dt} + \tau_\theta I_1(t) = -\mathrm{A}e^{bt};0 \le t \le T_1 \tag{10}$$

$$\frac{dI_2(t)}{dt} + \tau_\theta I_2(t) = 0;0 \le t \le T_1 \tag{11}$$

$$\frac{dI_3(t)}{dt} + \tau_\theta I_3(t) = -\mathrm{A}e^{bt};T_1 \le t \le \mu \tag{12}$$

$$\frac{dI_4(t)}{dt} + \tau_\theta I_4(t) = -\mathrm{A}e^{b\mu};\mu \le t \le \mathrm{T} \tag{13}$$

Solving (5), (6), (7) and (8) under the conditions $I_m(t=0)=Q=W_1+W_2$, $I_1(t=T_1)=0$, $I_2(t=T_1)=I_3(t=T_1)$, $I_3(t=\mu)=I_4(t=\mu)$, $I_4(t=T)=0$, $I_3(t=T_1)=I_4(t=T_1)$ and $I_4(t=T)=0$, we get

$$e^{\tau_\theta t}I_1(t) = A\left(T_1 - t + (\tau_\theta + b)\frac{\left(T_1^2 - t^2\right)}{2}\right) \tag{14}$$

$$e^{\tau_\theta t}I_2(t) = \mathrm{W}_1 \tag{15}$$

$$e^{\tau_\theta t}I_3(t) = W_1 + A\left|T_1 - t + \frac{(\tau_\theta + b)\left(T_1^2 - t^2\right)}{2}\right| \tag{16}$$

$$e^{\tau_\theta t}I_4(t) = Ae^{b\mu}\left(T - t + \frac{(\tau_\theta)\left(T^2 - t^2\right)}{2}\right) \tag{17}$$

$$\frac{W_1}{A} - e^{b\mu}\left(T - \mu + \frac{(\tau_\theta)\left(T^2 - \mu^2\right)}{2}\right) + \left|T_1 - \mu + \frac{(\tau_\theta + b)\left(T_1^2 - \mu^2\right)}{2}\right| = 0 \tag{18}$$

The cost parameters

The total cost function includes following cost parameters:

- The order cost (OC) = C
- The purchase cost (PC) = I_mC_p
- The holding cost (HC):

– Case 1:

$$HC_1 = \left[\int_0^{\mu} h_R I_1(t)e^{-rt}dt + \int_{\mu}^{T_1} h_R I_2(t)e^{-rt}dt + \int_0^{T_1} h_w I_3(t)e^{-rt}dt + \int_{T_1}^{T} h_w I_4(t)e^{-rt}dt\right]$$

– Case 2:

$$HC_2 = \left[\int_0^{T_1} h_R I_1(t)e^{-rt}dt + \int_{T_1}^{\mu} h_w I_3(t)e^{-rt}dt + \int_0^{T_1} h_w I_2(t)e^{-rt}dt + \int_{\mu}^{T} h_w I_4(t)e^{-rt}dt\right]$$

- The total cost function

  – Case 1: $\mu \le T_1 \le T$
  $TC_1\ (T_1, T\xi) = (1/T)[OC + PC + HC_1 + \xi]$
  – Case 2: $T_1 \le \mu \le T$
  $TC_2\ (T_1, T\xi) = (1/T)[OC + PC + HC_2 + \xi]$

The optimal conditions

Necessary and sufficient condition for the optimality of total cost function is as follows:

$$\frac{\partial TC_i(T, T_1, \xi)}{\partial T} = 0, \frac{\partial TC_i(T, T_1, \xi)}{\partial T_1} = 0,, \frac{\partial TC_i(T, T_1, \xi)}{\partial \xi} = 0;$$

Provided DET (H1) > 0, DET (H2) > 0, DET (H3) > 0, where H1, H2 H3 are the principal minor of the Hessian matrix. Here is the Hessian Matrix of the total cost function for $i = 1, 2$:

$$\begin{bmatrix} \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial T^2} & \frac{\partial^2 T(T,T_1,\xi)}{\partial T \partial T_1} & \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial T \partial \xi} \\ \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial T_1 \partial T} & \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial {T_1}^2} & \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial T_1 \partial \xi} \\ \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial \xi \partial T} & \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial \xi \partial T_1} & \frac{\partial^2 TC_i(T,T_1,\xi)}{\partial \xi^2} \end{bmatrix}$$

Numerical illustrations and sensitivity analysis

Following data is used for the numerical verification for the optimality.

Table 2 Sensitivity analysis

Parameters		ξ*	T_1*	T*	μ*	TC_1* (T, T_1, ξ)
Case 1: $\mu \le T_1 \le T$						
A	450	11.125	0.912	1.201	0..23	433.65
	500	12.412	0.944	1.244	0.25	450.76
	550	13.322	0.978	1.523	0.26	490.12
b	0.04	12.013	0.921	1.155	0.20	449.23
	0.05	12.412	0.944	1.244	0.25	450.76
	0.06	12.765	0.985	1.521	0.28	451.69
r	0.01	11.878	0.934	1.234	0.24	449.54
	0.02	12.412	0.944	1.244	0.25	450.76
	0.03	13.121	0.947	1.256	0.26	451.11
Θ	0.03	10.432	0.944	1.244	0.25	440.24
	0.05	12.412	0.944	1.244	0.25	450.76
	0.06	14.546	0.944	1.244	0.25	452.76
Case 2:$T_1 \le \mu \le T$						
A	450	11.243	0.613	1.011	0..73	433.65
	500	11.412	0.644	1.044	0.75	455.76
	550	12.745	0.723	1.589	0.76	490.12
b	0.04	11.013	0.641	1.021	0.71	450.23
	0.05	11.412	0.644	1.044	0.75	455.76
	0.06	12.135	0.743	1.098	0.85	460.69
r	0.01	10.241	0.612	1.014	0.71	449.21
	0.02	11.412	0.644	1.044	0.75	455.76
	0.03	12.121	0.678	1.087	0.77	460.15
Θ	0.03	10.231	0.644	1.044	0.75	454.45
	0.05	11.412	0.644	1.044	0.75	455.76
	0.06	14.245	0.644	1.044	0.75	458.11

$C = 200, r = .02, \theta = 0.05, h_{R1} = 0.19, h_{R2} = 0.25, h_{w1} = 0.15,$
$h_{w2} = 0.20, C_p = 6, W_1 = 100, A = 500\,\text{unit/year}, b = 0.05.$

The optimal values are as follows:

- Case 1:
  $TC_1^*\ (T, T_1, \xi) = 450.76$, $T_1^* = 0.944$ years, $T^* = 1.244$ years, $\xi = 12.412$, $\mu = 0.25$years
- Case 2:
  $TC_2^*\ (T, T_1, \xi) = 455.76$, $T_1^* = 0.644$ years, $T^* = 1.044$ years, $\xi = 11.412$, $\mu = 0.75$years

The observations mentioned in above Table 2 reflects that the slight variation in different parameters, like: demand factors, inflation rate, deterioration rate etc., fluctuate the optimal values of various decision variables and total cost value. This show that this model is very sensitive and put a very solid base for a perfect inventory system to main the all the parameters optimally.

Convexity of total cost function

See Figures 1, 2, and 3.

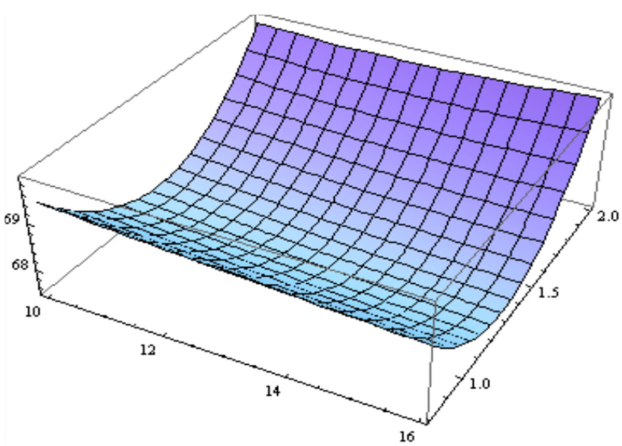

Fig. 1 Convex curve of total cost TC_1^* in regards to preservation cost ξ^* and total cycle length T^*

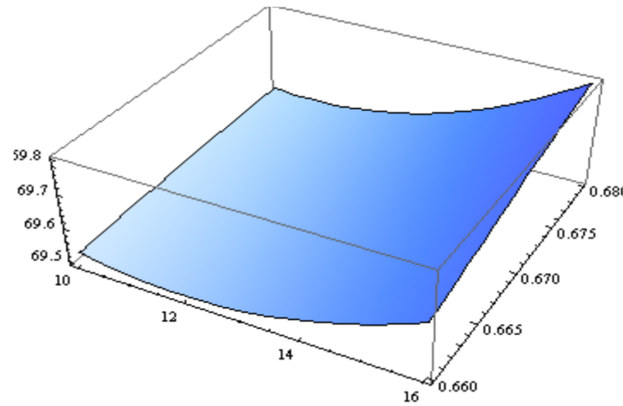

Fig. 2 Convex curve of total cost TC_2^* in regards to preservation cost ξ^* and advertising frequency T_1^*

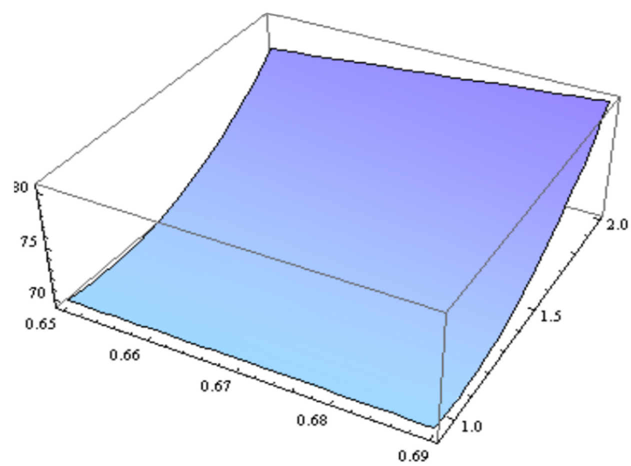

Fig. 3 Convexity of total cost TC_2^* in regard to advertising frequency T_1^* and total cycle length T^*

Conclusion

This study provides a quality of guidance for a newly launched inventory control system where demand of products increases with time up to a certain moment (μ) and then remains constant. In such cases where demand depends on time, ramp type demand pattern is more suitable. This inventory model is developed with preservation technologies in two-warehouse inventory system. Total cost function is established with convex graph. The model is numerically verified. The present study can be applied for fashionable products, modern technology-based products. In these cases, for newly launched products initially there is some hike in demand but after some time it become constant. For future modification, present study can be extended by using other parameters of inventory system.

Data availability Not applicable.

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