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RESEARCH PAPER

Reservoir approach, Bose's derivation, and Einstein's rate equation approximation

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Abstract
Since the reservoir approach is recognised as the foundation of laser theory, it is imperative to establish a correlation between it and the electromagnetic radiation formula derivations of different scientists. In this paper, we contrast the approaches used to develop Plank's black body radiation formula by S. N. Bose, A. Einstein, W. E. Lamb Jr., and their associates. Photons have been used to characterise light ever since Plank's quantum interpretation of black body radiation, the discovery of the photoelectric phenomenon, and Einstein's interpretation. Bose's methodology produced Bose statistics and the theory of Bose–Einstein condensation, whereas Einstein's method produced lasers. Both methods are used in the reservoir approach to quantum theory of lasers, which has applications in photon statistics.

Keywords Plank's quantum interpretation · Reservoir theory · Einstein's rate equation · Black body radiation · Bose–Einstein system

Introduction

The mathematical formula developed by Max Planck (Planck 1900) was based only on the premise that the energy states of point particle oscillators were discrete rather than continuous. Planck spent many years considering the physical consequences of his mathematical assumptions and his constant h. Planck formulated his second theory by adopting a less stringent assumption of quantization, motivated by his dissatisfaction with his earlier assumption about the distribution of black body radiation. The equation included a temperature-dependent term derived from the original theory, as well as an additional term that was completely independent of temperature (Planck 1911; Kittel and Kroemer 1980; Reif 2009). Einstein formulated the equation for black body radiation, based on a collection of atoms with distinct energy levels at various frequencies, all in a state of thermal equilibrium (Einstein 1917, 1924; Pathria and Beale 2011). The formula relies on the notion of meticulous equilibrium between the phenomena of spontaneous emission, stimulated emission, and absorption. The fundamental concept behind laser operation, which Einstein did not anticipate, is likely the process of stimulating emission of radiation. S.N. Bose derived Max Planck's law for black body radiation without relying on classical electrodynamics. This was a major step in the development of quantum mechanics, shifting the understanding of light from classical electrodynamics to quantum statistics. This work laid the foundation for the development of the Bose–Einstein system of quantum mechanics (Bose 1924). These fundamentals have allowed for the investigation of reservoir engineering approaches in the past and their potential application to classical nonlinear fields, bringing comparisons to quantum systems (Poyatos et al. 1996; Tissot et al. 2024). It is noteworthy that Lamb Jr. and his colleagues adopted the reservoir approach almost fifty years later (Sargent et al. 1974) concerning the quantum theory of lasers. This approach led to the derivation of the Planck distribution law as a result of their efforts to determine the equation of motion for the average number of photons. The reservoir technique provides a theoretical basis for

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both Einstein's work, which can be approximated using rate equations, and Bose's work. This study aims to do a comparative analysis of Einstein's rate equation approximation, the reservoir approach, and Bose's work. In general, Bose's derivation is a Quantum statistical derivation, Einstein's rate equation is a Kinetic equation for transitions and reservoir theory is a general framework for system-environment interaction. Although the phenomena are fundamental, no direct correlations among them were evaluated until now. Here we are trying to analyse the characteristics and views of each phenomenon. We replicate the fundamental characteristics of the reservoir theory as presented by W. Lamb Jr. and colleagues (Sargent et al. 1974) in Sect. "Reservoir theory". Section "S.N. Bose and Planck's law" provides a concise explanation of S.N. Bose's derivation of Planck's law. Section "Comparative study" provides a comparative analysis that encompasses the Reservoir approach as well as the contributions of Bose and Einstein.

Reservoir theory

In this theory, we consider a laser cavity where coupling of a time-dependent electric field takes place with a beam of two-level atoms. Atoms initially in the upper state |a⟩ pass through the cavity at the r_a per second and atoms initially in the lower state |b⟩ pass through per second r_b. The field interacts with the atoms to reach an equilibrium temperature T that is determined by the Boltzmann distribution.

$$\frac{r_a}{r_b} = \exp\left(-\hbar\omega / k_B T\right) \tag{1}$$

The effect of the beam on the simple harmonic oscillator is to bring it to the same temperature *T*. The photon rate equation in density matrix notation is given by Bose (1924)

$$\begin{aligned}\rho_{nn}(+) = &-\left[R_a(n+1) + R_b n\right]\rho_{nn} \\ &+ R_n n \rho_{n-1,\,n-1} + R_b(n+1)\,\rho_{n+1,\,n+1}\end{aligned} \tag{2}$$

Here,

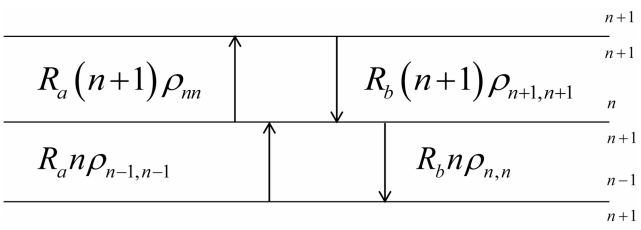


Fig.1 Diagram of photon number probability $\rho_{n.n}$. The R_a terms are due to emission and R_b terms are due to absorption. Each term is understood in terms of rates and transition probability

$$R_a = r_a g^2 T^2$$

$$R_b = r_b g^2 T^2$$

$$g = -\frac{\wp}{\hbar}\xi \sin kZ$$
$$= \text{atom field coupling constant}$$

$$\xi = \left(\frac{\hbar\Omega}{V\varepsilon_0}\right),$$

Electric field per photon.

By referring to the energy level diagram in Fig. 1, we can make sense of Eq. (2). In which the flow of probability of photon number is depicted.

The net flow between any two levels must eventually disappear for there to be equilibrium. How about we look at the two

$$R_a n \rho_{n-1,\,n-1} = R_a n \rho_{nn} \tag{3}$$

This condition for arbitrary *n* is referred to as the principle of detailed balancing. Similarly, we have

$$R_a(n+1)\,\rho_{nn} = R_a(n+1)\,\rho_{n-1,\,n-1} \tag{4}$$

We have at equilibrium,

$$\left[R_a(n+1) + R_b n\right]\rho_{nn} = R_a n \rho_{n-1,\,n-1} + R_b(n+1)\,\rho_{n+1,\,n+1}$$

From this we have

$$\left.\begin{aligned} &R_b \rho_{n+1,\,n+1} = R_a \rho_{nn} \\ &\text{and } R_b \rho_{nn} = R_a \rho_{n-1,\,n-1}\end{aligned}\right\} \tag{5}$$

Also,and hence

$$\frac{R_a}{R_b} = \frac{r_a}{r_b},$$

$$\rho_{nn} = \frac{r_a}{r_b}\rho_{n-1,\,n-1} = \exp\left(-\hbar\omega / kT\right)\rho_{n-1,\,n-1} \tag{6}$$

Iteration of this result for successively lower *n* yields the Planck's distribution for single mode

$$\begin{aligned}\rho_{nn} &= \exp\left(-\hbar\omega / kT\right)\rho_{n-1,\,n-1} \\ &= \exp\left(-\hbar\omega / kT\right)\exp\left(-\hbar\omega / kT\right)\rho_{n-2,\,n-2} \\ &= \exp\left(-\hbar\omega / kT\right)\exp\left(-\hbar\omega / kT\right)\exp\left(-\hbar\omega / kT\right)\rho_{n-3,\,n-3} \\ &= \exp\left(-\hbar\omega / kT\right)\rho_{00}\end{aligned}$$

Thus

$$\begin{aligned}\rho_{nn} &= \rho_{00}\exp\left(-n\hbar\omega/kT\right)\\ &= \rho_{00}\exp\left(-\hbar\omega/kT\right)\end{aligned} \tag{7}$$

for single mode.

To obtain ρ_{00} we use the normalization condition $\sum_n n\rho_{nn} = 1$,

And finally,

$$\rho_{nn} = \left[1-\exp\left(-\hbar\omega/k_BT\right)\right]\exp\left(-n\hbar\omega/k_BT\right) \tag{8}$$

Therefore, in the state of equilibrium, the field reaches the same temperature as the atomic beam reservoir. This technique also provides a rationale for Einstein's approximation of his rate equation in his concept of radiation (Einstein 1917). To get the radiation formula of Planck, we begin by expressing the average number of photons.

$$\langle n\left(t\right)\rangle = \sum n\rho_{nn}\left(t\right) \tag{9}$$

$$\langle n\left(t\right)\rangle = \sum n\left[1-\exp\left(-\hbar\omega/k_BT\right)\right]\exp\left(-n\hbar\omega/k_BT\right) \tag{10}$$

The average photon number is

$$\begin{aligned}\langle n\left(t\right)\rangle &= \sum n\rho_{nn}\left(t\right)\\ &= \frac{1}{\exp\left(\hbar\omega/k_BT\right)-1}\end{aligned}$$

The average number of photons in a steady-state value can be found using the equation (Bose–Einstein)

$$\begin{aligned}\overline{n} &= \langle n\left(\infty\right)\rangle\\ &= \sum n\rho_{nn}\left(t\right)\\ &= \frac{1}{\exp\left(\hbar\omega/k_BT\right)-1}\end{aligned} \tag{11}$$

This is a simple theoretical consequence of the reservoir approach. It is a Classical nonlinear dynamical system. Here, the information processing and memory size are limited by classical dynamics. The classical reservoir approach is widely used in machine learning applications (Felix 2023; Canady et al. 2110).

Quantum reservoir computing (QRC) is the quantum analogue of the classical reservoir approach. QRC explores the potential advantages of quantum mechanics to progress computation, particularly in processing complex time-series data. The new field of reservoir engineering has become a prominent tool for controlling quantum systems. It has recently been used in the first experiments on many-body systems, particularly to construct particle-conserving dissipation for boson-based quantum simulations. Reservoir engineering can accelerate and de-accelerate moving solitons, relax solitons to a stable form, and analyse how wave packets break up into soliton trains (Poyatos et al. 1996; Tissot et al. 2024).

It is to be mentioned that the modern application of reservoir approach extends far beyond open quantum systems. It helps to study quantum error correction, quantum information, and Computing (Fujii and Nakajima 2021; Zhong et al. 2021; Du et al. 2017). The idea of reservoir computing, which involves contracting out information processing to natural physical processes, originates from the concept of quantum reservoir computing. This concept works well in the noisy intermediate quantum era for quantum machine learning on near-term quantum devices. Quantum reservoir computing (QRC) is a connection between two different natural computing paradigms: quantum computing and reservoir computing. It provides a pedagogical introduction to QRC and related approaches, highlighting their feasibility on current quantum devices. Using the inherent parallelism of quantum dynamics and the exponential capacity of quantum systems, QRC provides a way around some of the limitations of traditional computing architectures. Time-series forecasting, pattern identification, and other machine learning applications may become more accurate and hence data processing may become more efficient. It is anticipated that quantum reservoir computing (QRC), which takes use of the information-processing powers of quantum systems, would outperform its classical counterparts in time-series forecasting problems. In theory, a great number of QRC systems have been put forth (Wang et al. 2025).

S.N. Bose and Planck's law

In 1924, S.N. Bose derived Planck's radiation formula, which looked to signal a significant shift between the old quantum theory of Planck, Einstein, Bohr, and Sommerfeld and the contemporary quantum mechanics of Heisenberg, Dirac, and Schrodinger. This shift can be attributed to the fact that Bose was able to distinguish between the two. Because of Bose's discoveries, Einstein was able to conduct research on the quantum theory of monoatomic ideal gases and gas degeneracy, which ultimately led to his prediction of the condensation events. Bose's discoveries laid the framework for Einstein's study. It was the Bose–Einstein statistics that immortalised Bose's name in the realm of physics. These statistics were able to incorporate themselves without any difficulty into the fundamentals of quantum mechanics. Bose contends that past attempts to derive Planck's law have been unsuccessful because they have not

been supported by sufficient logical basis. The combination of the light quanta hypothesis and statistical mechanics, stated to fit the constraints of quantum theory, appears to be enough to derive the law independently of classical theory. Bose aimed to justify Planck's utilisation of the Boltzmann formula.

$$S = k \ln W$$

way to obtain Einstein's findings without using Wien's displacement law. It is important to note that the connection between the Boltzmann law and quantum theory presumptions is at the heart of the various scholars' derivations of Planck's formula. Bose began by claiming that the fundamental assumption of quantum theory contradicts the principles of classical electrodynamics. All derivations before to Bose employed the relation.

$$\rho_{\upsilon} d\upsilon = \frac{8\pi\upsilon^2 d\upsilon}{3} E$$

This describes the link between an oscillator's radiation density and its mean energy. They also made assumptions regarding the other's degrees of freedom, as seen in the above calculation. This factor, however, can only be obtained by the classical theory. This is the unsatisfying element of all derivations, hence obtaining a derivation free of this logical flaw is considered necessary. Bose considered the radiation composed of light quanta or photons of total energy E enclosed in a volume V. He obtained $\beta = kT$ and the Planck's formula for the energy density of the black body radiation.

The use of Planck's technique, which divides the phase space into cells of volume h^3, is the crucial step in the derivation described above.The probability of distribution is simply defined by the frequency with which a site is occupied by photons (or energy the quanta), and the number of sites (Planck's oscillators) is contingent upon the number of cells that are present in the phase space. Bose provided an illustration of the aspects of the Planck theory that seem the most natural. In order to comprehend the reason why he was not directed to Bose's derivation, it is important to keep into consideration that Planck looked for an explanation for quantization in interacting systems rather than using free systems. It is important to remember that Bose is regarded a free quantum gas. He understood that light quanta are particles that must be handled in a relativistic manner because they do not possess any mass, their number does not remain constant, and most significantly, they are indistinguishable from that of other particles. Due to the fact that it is impossible to differentiate between them, it is necessary to regard all configurations of a specific number of phase points contained within a phase cell of volume to be identical. In the beginning, when Bose was coming up with the Planck distribution law, he was completely unaware of the differences between it and Boltzmann statistics. The immediate impact of Bose's discovery, on the other hand, was observable in the form of Bose Einstein statistics, which were included into the conventional approach to quantum mechanics.

Comparative study

In this section, we aim to provide a critical evaluation of Lamb Jr. and colleagues' reservoir approach, which leads to Planck's distribution law, in the light of Bose's deduction and Einstein's rate equation approximation. Before proceeding to do so it is useful to include all the salient features of the approaches in a tabular form as shown in Table 1. The 1917-developed Einstein technique essentially discusses both the process of absorption and the emission of an assembly of atoms. Stimulated emission was included along with the process of spontaneous emission. Einstein revealed that Planck's formula can be obtained on the basis of the ensemble of atoms with discrete energy levels of various frequencies and at thermal equilibrium with the principle of detailed balancing among the processes of stimulated emission, Spontaneous emission and Stimulated absorption. But this was the distance to which Einstein travelled. The fact that stimulating photons and stimulated photons are in the same phase was overlooked by many who attempted to observe "Stimulated Emission". The central idea of laser is presumably the concept of stimulating radiation which was not anticipated by Einstein and subsequently by others for three decades. The Eq. (3) was not derived by Einstein using Bohr's correspondence theory. The idea of stationary states, proposed by Bohr, was utilised by him. Using references to Einstein's work, Bohr first published in 1918 an explicit statement of what he would later refer to as "the correspondence" concept (Einstein 1917).

In the reservoir approach of Lamb Jr. and coworkers an F.P. laser cavity is already assumed and the active medium consisting of atoms is allowed to interact with a time-dependent electric field (simple harmonic oscillator) and bring the medium to remember that in Einstein's approach radiation of right frequency is allowed to be incident on the two-level atomic system enclosed in a volume V and actually interact resulting in stimulated absorption, spontaneous emission and stimulated emission. Einstein's rate equation approach is probably reflected in some proportion by the reservoir approach. Reservoir approach assumes a laser cavity of volume V, and in the steady state the field comes to the temperature of the atomic beam reservoir as shown by Eq. (10). The

Table 1 S.N. Bose's derivation, Einstein's rate equation approximation, and reservoir approach

S.N. Bose's derivation	Rate equation approximation	Reservoir theory
S.N. Bose (1924)	A. Einstein (1917)	W.E. Lamb Jr. (1974) and Coworker
Bose considered the radiation composed of light quanta of total energy E enclosed in a volume V. Energy density $\rho = E/V$	Einstein assumed an ensemble of atoms with discrete energy levels at an equilibrium with temperature T^oK and photon density $\rho(\omega)$	A Fabry–Perot laser cavity is assumed. Single mode electric field is allowed to interact with two level atoms which pass through the cavity. As a result of interaction, the atoms come to the temperature T^oK at equilibrium
Bose wanted to show that Planck's use of the Boltzmann formula $S = k \ln W$ is justified	Boltzmann distribution is assumed in thermal equilibrium $n_a = n_b \exp(-\hbar\omega/kT)$	Boltzmann distribution is assumed.$r_a/r_b = \exp(-\hbar\omega/kT)$
The Bose's approach has given birth to Bose statistics and the theory of the Bose–Einstein condensation and the universal Boson particle	Stimulated emission has given birth to laser	Reservoir approach includes both Bose's approach and rate equation approximation. It also gives the equation of motion of the average photon number
Bose considered photon gas particles, massless, the number is not conserved, the light quanta must be treated as indistinguishable	At equilibrium the population is constant and accordingly $\dot{n_a} = \dot{n_b} = 0$	At equilibrium $\dot{\rho_{nn}} = 0$
Bose obtained for the indistinguishable photon gas the energy density of the black body radiator as $\rho = \frac{E}{V} = \sum \frac{8\pi h\upsilon^3}{c^3} \frac{1}{\exp(h\upsilon/kT) - 1}$ The important method is the Planck's method of dividing the phase space into cells of volume h^3(no of Planck's oscillators) and enquire how often the cells are occupied by photons	Solution of the rate equation in equilibrium combined with Boltzmann distribution and $h\upsilon = E_2 - E$, results in the energy density $\rho(\omega) = \frac{A}{B} \frac{1}{\exp(\hbar\omega/kT)-1}$ with $\frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3}$	Solution of the rate equation $R_a n \rho_{n-1,\,n-1} = R_b n \rho_{nn}$ using Boltzmann distribution yields Planck's distribution for single mode $\rho_{nn} = [1 - \exp(-\hbar\omega/kT)]\exp(-n\hbar\omega/kT)$ The average photon number is $\langle h(t)\rangle = \frac{1}{\exp(\hbar\omega/kT)-1}$ In equilibrium the average photon number is given by $B - E$ distribution

Bose–Einstein expression provides the steady state value of average photon number given by Eq. (13). Thus, we observe that there is no mention of Planck's law but Bose–Einstein distribution is given more emphasis. We consider it is important to point out that the reservoir approach, which Lamb Jr. and coworkers adopted about three decades after Planck's derivation, captures the fundamental physics of the issue. It consists of Einstein's rate equation approach, Bose's idea of indistinguishability of photon gas particles and equation of motion of the average photon number. It is more crucial to note that the reservoir approach explains the fundamental physics of how photon numbers build up from non-zero values. The black body cavity of Planck, Volume of Einstein's rate equation approach, Volume of photon gases of S.N. Bose and laser cavity (F.P) of volume V of Lamb Jr. may be considered as analogous terms. The Boltzmann law assumption is inevitable in all the approaches. The fundamental premise of quantum mechanics is Planck's quantum theory. Planck was the first to say (and prove through his radiation law of the black body) that a body or matter radiates energy or absorbs energy in the smallest bundles or packets known as quantum. In every area where quantum mechanics is employed, Planck's quantum theory is applicable. Einstein's approaches have already found applications as a laser. Bose statistics and the theory of the Bose–Einstein condensation are fruits of Bose's approaches.

Concluding remarks

We may appropriately conclude this review with some comments on the reservoir theory of laser using photon statistics being dealt in the book Laser Physics by Sargent et al. (1974) rigorously and also in detail. We acknowledge the reservoir approach to be the core of the laser theory, so we considered it necessary to correlate it with various workers (Anderson et al. 1995; Dalfovo et al. 1999; Pethick and Smith 2008) derivations of the radiation formula. Understanding what a reservoir means is crucial. In the treatment of quantum theory of radiation, we are involved primarily in entire systems such as the electromagnetic field in a cavity. In most areas of quantum optics, only a portion of the entire system is of interest to us. Specifically, in problems associated with the lasers we want to know the field but are not particularly interested to know the fate of the atoms. The system of primary interest should therefore be distinguished from that of secondary interest, with the former being referred to simply as system and the latter as "reservoir". One can eliminate the reservoir by using the reduced density operator method in the Schrödinger or interaction picture. It is interesting to note that the average photon number, which is given by the B.E. distribution (Einstein 1917), is also produced by the reservoir theory. Bose's approach has given

birth to Bose statistics and the theory of Bose–Einstein condensation. Einstein's approach has given birth to laser in the name of stimulated emission although Einstein himself was unaware of it. The reservoir approach includes both Einstein's and Bose's approaches taken together. Boltzmann's law is invariably present in all of the methods, which we believe is critical to notice. It is worthy to remark that the three approaches are concerned primarily with the properties of photons from different angles. Therefore, such comparative study may help us to know more about photon and photonics in general.

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