REPORT ON THE PRESENT STATE OF OUR KNOWLEDGE OF THE IONOSPHERE.

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Contents.

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§ I. INTRODUCTION.

The National Institute of Sciences of India organized a symposium on the Problems of the Ionosphere which the author of this report had the honour of opening. Being engaged in conducting investigations on this subject for the past few years, he availed himself of the opportunity thus offered to write out in a connected and concise form the main results, both theoretical and experimental, of investigations concerning the ionosphere for the use of those who wanted to undertake research in this fascinating subject.

In his attempt to bring the information up to date the author has been compelled to compress a large volume of matter into a small space, and in doing so has, of necessity, omitted or curtailed accounts of investigations which are of less importance from the fundamental point of view. Copious references to original literature have, however, been given; these are all collected in Appendix I, arranged under different articles.

To help the reader a list, with explanations, of the symbols used has been given at the end of the report (Appendix II).

It is the aim of the author to publish further editions of this report from time to time embodying the latest advances made in ionospheric investigations. He would therefore be glad to receive from research workers in this subject reprints of their publications.

§ II. Ionosphere—Historical.

1. *Ionosphere.*

It has now been proved beyond doubt that there are vast stretches of ionized regions in the upper atmosphere. These regions are known collectively as the Ionosphere. The lower boundary of the ionosphere is normally at a height of about 90–100 km., the upper boundary is not well defined but has been proved to extend beyond 400–500 km. In Fig. 1 an attempt has been made to give a rough picture of the region occupied by the ionosphere in the

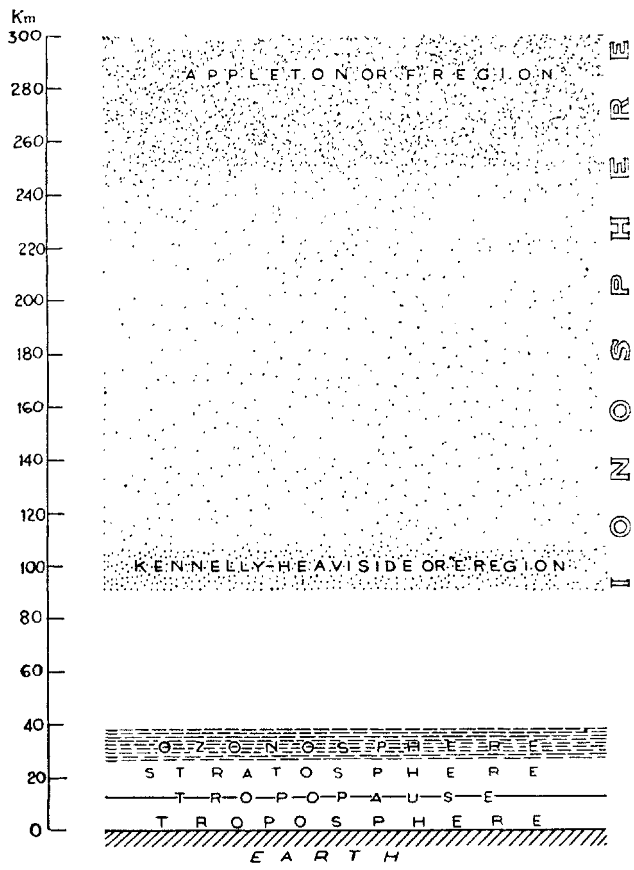

Fig. 1. Illustrating the divisions of the atmosphere. The *troposphere* is the region of weather. The temperature here falls with height. The *stratosphere* is the region of constant temperature. The dividing surface between the two is the *tropopause*. The *ozonosphere* is the region where ozone is formed by the action of the sun's rays. The *ionosphere* is divided into two main regions of maximum ionization called *E* and *F*. In the *ozonosphere* and in the upper region of the ionosphere the temperature may increase with height.

upper atmosphere in relation to other regions at lower levels, the existence of which is inferred from meteorological and spectroscopic observations.

The term *Ionosphere* connoting the whole of the ionized upper atmosphere is due to Watson Watt (see ref. 1, § VIII).

2. *Historical.*

The success of Marconi in 1901 in sending wireless signals from Cornwall to Newfoundland was the source of considerable discussion in scientific circles as to the mode of propagation of wireless waves round the curved surface of the Atlantic. Since wireless waves are nothing but electromagnetic waves as constitute radiant heat and light, it was natural to suppose that the phenomenon was one of diffraction and physicists and mathematicians like Macdonald,[1] Rayleigh,[2] Poincaré[3] and others,[4] directed their attention towards solving the problem of the diffraction of the waves by a spherical surface like that of the earth. Calculations, however, showed that the diffraction effect was quite inadequate to explain the observed bending of the waves round the curved surface of the earth.

In 1902, A. E. Kennelly[5] in America and O. Heaviside[6] in England almost simultaneously postulated the existence of a conducting layer in the upper regions of the earth's atmosphere and suggested that this medium might deflect the radio waves and force them to follow the curvature of the earth. Heaviside further made the suggestion that the conductivity of this medium is due to the presence of positive and negative ions produced most probably by the ionizing action of solar radiation. It should, however, be mentioned here that as far back as 1878, Balfour Stewart[7] postulated the existence of conducting strata in the upper atmosphere in connection with his theory of the daily variation of the earth's magnetic field.

The actual manner in which the charged particles affect the propagation of radio waves was first pointed out by Eccles[8] in 1912. Some essential details which were lacking and for which Eccles' theory lay undeveloped were supplied by Larmor[9] in 1924. The Eccles-Larmor theory is now regarded as the basic theory of radio wave propagation in the ionosphere and has been supplemented by the so-called magneto-ionic theory developed by Appleton, Hartree, Goldstein and others.

The conducting upper atmosphere, though postulated by Kennelly and Heaviside in 1902, lacked direct evidence as to its existence till 1924. Agreement of quantitative measurements on long wave field strength with Watson-Eckersley formula and the many peculiar features of short wave propagation discovered since 1921, pointed towards the existence of such a layer. But the evidence of these was rather of an indirect nature. The first direct evidence came in 1925 when Appleton and Barnett[10] by comparing the intensities of fading of signals received simultaneously on a loop and on a vertical aerial proved the existence of indirect or sky waves coming down after reflection from the Heaviside layer. Immediately afterwards Smith-Rose and Barfield[11] detected the presence of down-coming waves by employing direction finding apparatus of their own design. These experiments not only gave direct evidence of the reflecting layer but also disclosed some of its properties, e.g., its height, reflecting power, etc. But perhaps the most striking evidence was that furnished by experiments by Breit and Tuve.[12] These authors showed that wave packets of small duration sent out from the transmitter produced two (sometimes more) impulses instead of one in a receiver placed a few kilometres from the transmitter. The obvious conclusion was that the impulse coming first was due to the direct wave travelling along the ground while the others coming down later were due to indirect waves or echoes successively reflected from the ionosphere. This method—the so-called echo or group-retardation method—developed and improved by various workers is now the most powerful method of investigating the ionosphere. It has already yielded a large number of informations regarding the constitution of the ionosphere. We now know, for instance, that the ionosphere is stratified into various layers or regions of different ionic densities. The heights as well as the maximum ionic densities of these regions are also known to us. The intensity of the earth's magnetic field and the number of collisions experienced by the ions in the various ionospheric regions have been measured. Observations in various latitudes and over prolonged periods have yielded information regarding the nature of the ionizing agencies at work.

The radio waves by means of which these informations have been gathered are the only means at our disposal for studying such high regions as hundreds of kilometres above the surface of the earth. The famous Piccard balloon ascended up to 16 km., the sounding balloons sent by meteorologists may reach up to 40 km. But the radio waves alone can ascend beyond these heights and, when they come down 'reflected' by the ionospheric layers, they bring with them indelible messages regarding their physical condition which are quite as useful as those recorded by instruments carried by sounding balloons. For interpreting the results of experiments it is therefore necessary to make a close study of the mode of propagation of electromagnetic waves through the ionosphere. This has been done not only in the simple case of propagation in a medium containing ions and electrons, but also in the more complicated case, when the ionized medium is traversed by a steady magnetic field such as that due to the Earth.

§ III. Propagation of Electromagnetic Waves in the Ionized Atmosphere of the Earth.

As mentioned above, the radio wave is the only physical agency at our disposal for experimental investigation of the ionosphere. It is therefore necessary to have a clear understanding of the mode of propagation of such waves in an ionized medium, particularly when the medium is influenced by an external magnetic field. In what follows we will first study the case of simple propagation after Eccles[1] and Larmor,[2] in the absence of any external field, and then proceed to discuss the more complicated case of propagation in the presence of a magnetic field.

1. *Theory of Eccles and Larmor—Propagation in the absence of magnetic field.*

Let the ionized medium contain N ions per c.c., each of charge e and mass m. When an electromagnetic wave passes through such a medium the ions react to the electric force developed by the passing wave. For simplicity, let us suppose the wave front to be plane and the wave polarized. The ion will then find itself in a field E of fluctuating intensity given by $E=E_0 \sin pt$, E_0 being the amplitude of E and p the angular frequency of the wave. The ions will vibrate under the action of this force and if, for the moment, we neglect the collision of the vibrating ions with the neighbouring neutral atoms and molecules, the equation of motion of the ion is given by

$$m\frac{d\xi}{dt}=E_0 e \sin pt,$$

where ξ is the instantaneous velocity in the direction of the field. Integrating we have,

$$\xi=-\frac{E_0 e}{mp}\cos pt.$$

The integration constant has been made equal to zero by a proper choice of the phase. This motion of the ion produces a convection current, the density of which is given by

$$Ne\xi=-\frac{E_0 Ne^2}{mp}\cos pt.$$

Now, at the point under consideration there is also a displacement current due to the fluctuation of the field intensity given by

$$\frac{K}{4\pi}\frac{\partial E}{\partial t}=\frac{K}{4\pi}E_0 p \cos pt,$$

K being the dielectric constant of the medium itself without the ions. The total current is therefore given by

$$\left(K-\frac{4\pi Ne^2}{mp^2}\right)\frac{E_0 p}{4\pi}\cos pt.$$

The presence of the ions has thus the effect of reducing the effective dielectric constant of the medium from K to $K-\dfrac{4\pi Ne^2}{mp^2}$. If we consider free space where $K=1$, the reduction of the dielectric constant has the effect of altering the velocity c of propagation of the electromagnetic waves to a value

$$u=\frac{c}{1-\dfrac{4\pi Ne^2}{mp^2}}.$$

The above is an outline of Eccles' arguments. Larmor[2] pointed out that if the ionized medium contains particles of electronic charge and mass, then even a very moderate density of electrons is sufficient to cause a considerable change of velocity for the frequencies commonly employed in wireless communication. As an example Larmor shows that an electronic density gradient of only ·3 electrons per c.c. per cm. is sufficient to cause such bending of the course of radio waves of 1000 m. length as to make the corresponding ray follow the curvature of the earth. It may be noted here that in most of the experimental investigations on the ionosphere, exploring wave groups are sent out from the surface of the earth to meet and enter into ionospheric regions. These wave groups travel with the so-called group velocity v in the ionized medium, which is always less than the velocity c of the wave in free space, and is related to the latter and the phase velocity u by the equation $u \times v = c^2$. It is interesting to calculate the amplitude attained by a vibrating electron under a typical condition. If the field strength in the ionosphere due to waves sent out from the earth's surface be 100 m.v./m. and the wavelength 1000 m. the amplitude attained by the electrons will be 5×10^{-1} cm. Collision of the vibrating electrons with the neighbouring atoms and molecules is therefore possible. If this is taken into account, then it can be shown that the effective dielectric constant is given by

$$1 - \frac{4\pi N e^2}{m(p^2 + \nu^2)},$$

where ν is the number of collisions per sec. experienced by the ion.

It should be mentioned here that the expression for the modified value of the dielectric constant derived above, due to the presence of free electrons, can be easily deduced from the well-known classical dispersion formula of Lorentz,[3] where it is assumed that the dispersive medium contains quasi-elastically bound electrons having natural vibration frequencies of their own.

Lorentz's formula may be thus written

$$\left(\mu - i\frac{ck}{p}\right)^2 = 1 + \frac{1}{\alpha + i\beta},$$

where $$\alpha = \left\{ m\frac{(\omega^2 - p^2)}{4\pi N e^2} - \frac{1}{3} \right\},$$

and $$\beta = p\,\frac{g}{4\pi N e^2}.$$

μ refractive index.
k absorption index.
ω natural angular frequency of the bound electrons.
g $2m\nu$.
ν collisional frequency.

On separating the real and the imaginary and neglecting the square of the term $\left(\frac{ck}{p}\right)$ we have

$$\mu^2=1+\frac{\alpha}{\alpha^2+\beta^2}$$

$$\text{and } \frac{2ck}{p}\mu=\frac{\beta}{\alpha^2+\beta^2},$$

giving the values of the refractive index μ and absorption index k in terms of N, p, ν, e, m, and ω.

Now, for the case of the ionosphere the electrons are free, i.e. $\omega=0$ and also $g=m\nu$ instead of $2m\nu$.

If we omit the term $\frac{1}{3}$ known as Lorentz polarization term in the expression for α we have

$$\mu^2=1-\frac{4\pi Ne^2}{m(p^2+\nu^2)}$$

as before. We further note that the absorption index k is given by

$$k=\frac{\nu}{2c\mu}\cdot\frac{4\pi Ne^2}{m(p^2+\nu^2)}.$$

2. *The Magneto-Ionic Theory—Appleton-Hartree Formula.*

The theory of propagation of electromagnetic waves through a medium containing charged particles in the presence of an external magnetic field is known as the magneto-ionic theory. Lorentz was the first to treat theoretically the propagation phenomenon in two particular cases, viz., when the direction of propagation of the wave is (*a*) along, and (*b*) at right angles to the direction of the magnetic field.

Appleton in 1925 [4] first applied his theory to explain the phenomenon of propagation of radio waves through the upper ionized regions of the earth's atmosphere. He pointed out that the quantity $\frac{He}{mc}$, the gyromagnetic frequency of the electron about the earth's magnetic field, is comparable with radio frequencies. In 1927 [5] he generalized Lorentz's treatment and derived formulæ for dispersion, absorption and polarization for any angle of inclination of the wave normal with the magnetic field. At about the same time Goldstein [6] by a different method arrived at the same formula.

In Lorentz's treatment the scattering centres are bound electrons having natural frequencies of their own. In the present case, however, the electrons are assumed to be free, i.e., they all have only one natural frequency in the absence of the magnetic field, the value of which is zero. The same formula was later derived by Hartree [7] and the so-called Appleton-Hartree formula is :—

$$M^2=\left(\mu-\frac{ick}{p}\right)^2$$

$$=1+\frac{2}{2(\alpha+i\beta)-\dfrac{\gamma_T^{\,2}}{1+\alpha+i\beta}\pm\sqrt{\dfrac{\gamma_T^{\,4}}{(1+\alpha+i\beta)^2}+4\gamma_L^{\,2}}} \quad .. \quad (1)$$

$$R=\frac{h_z}{h_y}=+\frac{i}{\gamma_L}\left\{\frac{1}{M^2-1}-(\alpha+i\beta)\right\} \quad .. \quad .. \quad .. \quad .. \quad (2)$$

μ the refractive index.
k the index of attenuation.
c the velocity of electromagnetic waves in free space.
m and e mass and charge of electron.
N electron density.
H imposed magnetic field in gauss.
H_L component of the magnetic field along the direction of propagation.
H_T component of the magnetic field transverse to the direction of propagation.
p angular frequency of the wave.
p_0^2 $4\pi Ne^2/m$.
p_H He/mc.
$p_{L,T}$ $H_{L,T}e/mc$.
ν collisional frequency.
α $-p^2/p_0^2$.
β $p\nu/p_0^2$.
γ pp_H/p_0^2.
$\gamma_{L,T}$ $pp_{L,T}/p_0^2$.

[In the original formula a term l, called the polarization or the Lorentz term, was added to the expression for α. Darwin [8] has, however, shown that under ionospheric conditions the polarization term cannot occur.] The direction of wave propagation is along the positive direction of the x-axis of a right-handed co-ordinate system, the external magnetic field being in the XZ-plane. h_y, h_z are the components of the magnetic vector of the wave itself and R represents the state of polarization.

The two formulæ (1) and (2) give us all the desired information regarding the mode of propagation of radio waves in the ionosphere. The first formula gives us, on the one hand, the nature of the dispersion, i.e., the relation between the incident wave frequency and the refractive indices and, on the other hand, the absorption indices of the medium for the various waves into which the incident wave is split up. The second formula gives the nature of the polarization of the split waves.

3. *Simplified Appleton-Hartree Formula.*

The formula is undoubtedly complicated; in order to understand its significance it is therefore advisable to start from simple cases of propagation. We would, therefore, write out the formula in a form more suitable for graphical representation by neglecting the frictional term ν and discuss it after

Ratcliffe[9]. In a later section we will take this factor into account. Thus we have, putting $\beta=0$,

$$\mu^2=1-\frac{2x(1-x)}{2(1-x)-y_T^2\mp\sqrt{y_T^4+4y_L^2(1-x)^2}} \quad \cdots \quad \cdots \quad (3)$$

where

$$x=p_0^2/p^2=4\pi Ne^2/mp^2,$$
$$y_L=p_L/p,$$
$$y_T=p_T/p.$$

We have also to use

$$y=p_H/p=\lambda_0/\lambda_H,$$

where

$$\lambda_0=2\pi c/p,$$

and $\lambda_H=2\pi c/p_H$.

(*a*) Dispersion.

The nature of the dispersion may be understood by drawing graphs between μ^2 and x for certain typical cases. It should be remembered that x is proportional to λ^2 and also to N. The graph will therefore delineate the relation between μ^2 and N (λ^2 constant).

(i) No external magnetic field :—

In this case,

$$y_L=y_T=0.$$

Thus,

$$\mu^2=1-x \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (4)$$
$$=1-\frac{4\pi Ne^2}{mp^2}$$

and the formula reduces to the one deduced by Eccles as given in § III, sec. 1. The graph connecting μ^2 and x becomes a single straight line (Fig. 2) starting from $\mu^2=1$ at $x=0$ and gradually diminishing in value as x increases.

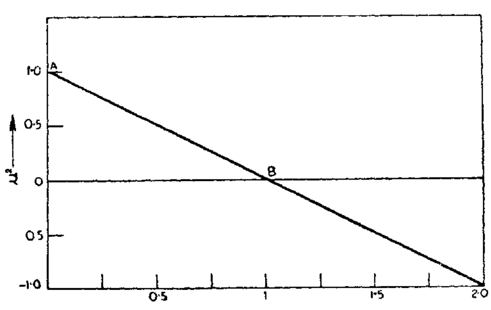

FIG. 2. Illustrating the variation of refractive index (μ^2) with $x=\frac{4\pi Ne^2}{mp^2}$. Case of no magnetic field. We may regard the curve as showing the relation between μ^2 and N (p^2 constant).

(ii) Magnetic field along the direction of propagation :—

In this case $y_L = y$, $y_T = 0$.

We thus have

$$\mu^2 = 1 - \frac{x}{1 \mp y} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The expression for μ^2 shows that for any value of x we have two values of μ. Remembering that y is the ratio between the incident wavelength and the wavelength corresponding to the gyro-frequency He/mc it will be convenient to consider the two cases, first when y is less than 1 and secondly when y is greater than 1.

Dispersion curves corresponding to the upper sign lie between AO and AB for values of y less than 1 and between AE and AD ($AD // OB$) for y greater than 1 (Fig. 3). For the lower sign the dispersion curves lie always

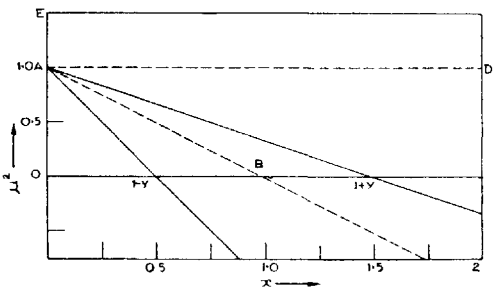

Fig. 3. Illustrating the variation of μ^2 with $x = \frac{4\pi Ne^2}{mp^2}$ when the magnetic field is along the direction of propagation. The two broken lines AB and AD show the limiting positions of the curves for zero and infinite values of the magnetic field respectively.

between AB and AD. It is thus seen that for y less than 1 it is possible for both the values of μ^2 to be zero or less than 1, while for values of y greater than 1 only one value of μ^2 can be negative. These considerations are important in the experimental study of the magnetically split waves from the ionosphere.

The value λ_H corresponding to the magnetic field at Calcutta is approximately 235·7 m.

(iii) Magnetic field at right angles to the direction of propagation :—

In this case $y_L = o$ and $y_T = y$. Hence we have two equations

$$\mu^2 = 1 - x \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6a)$$

for the lower or the positive sign, and

$$\mu^2 = 1 - \frac{x(1-x)}{1 - x - y_T^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6b)$$

for the upper or the negative sign.

We see that (6a) gives a straight line as in the case considered under no magnetic field. (Broken line AB in Figs. 4 and 5.) The wave corresponding to this refractive index is called the ordinary wave.

Equation (6*b*), as before, is conveniently considered for two values of y, viz., y less than and y greater than 1. For the first case the dispersion curve (full line) is given in Fig. 4. It is to be noticed that the dispersion curve

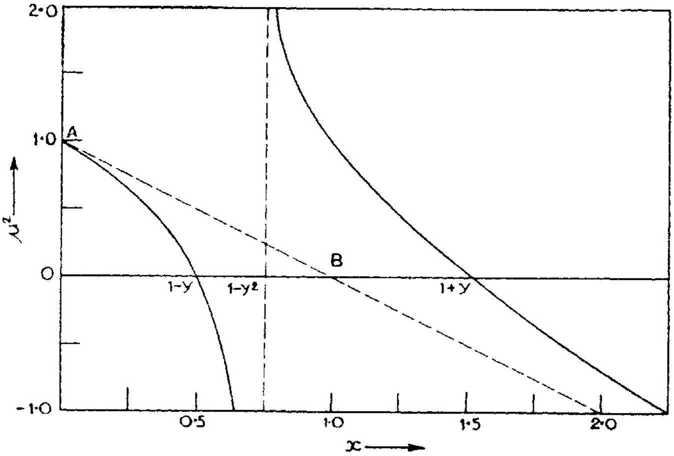

FIG. 4. Illustrating the variation of μ^2 with $x=\frac{4\pi Ne^2}{mp^2}$ when the magnetic field is perpendicular to the direction of propagation. Case of $y=\frac{p_H}{p}<1$.

has two branches (asymptotic to the vertical broken line) and it considerably differs from the curve of the longitudinal case. But, the points where the curve cuts the x axis, i.e. where $\mu^2=0$ are identical with those for the longitudinal case.

For y greater than 1 the dispersion curve is shown in Fig. 5. In this case,

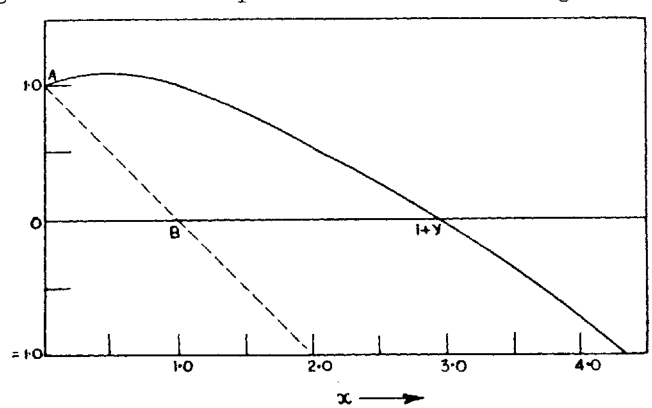

FIG. 5. Same as Fig. 4. Case of $y=\frac{p_H}{p}>1$.

for equation (6*b*), there is only one branch of the curve because the other branch which lies on the side of the negative value of x has no physical significance. Comparison of the two curves shows the interesting fact that in

Fig. 4 there might be three values of x for which μ^2 becomes zero, while in Fig. 5 there are only two such points.

(iv) General case :—Magnetic field inclined at any angle to the direction of propagation :—

Let the direction of propagation make an angle θ with the positive direction of the magnetic field. We can deduce the general nature of the dispersion curves if we plot a number of salient points corresponding, for instance, to the values of μ^2 when $x=0$ or 1, and to the values of x when $\mu^2=1$, 0 or ∞. As before we will consider the two cases of $y<1$ and $y>1$ separately.

$y<1$.

In equation (3) taking the upper negative sign of the denominator, we find the following values:

x	μ^2
0	1
$1-y$	0
$\dfrac{1-y^2}{1-y^2\cos^2\theta}$	$\mp\infty$
1	1
$1+y$	0
∞	$-\infty$

The curve corresponding to these values is given in Fig. 6.

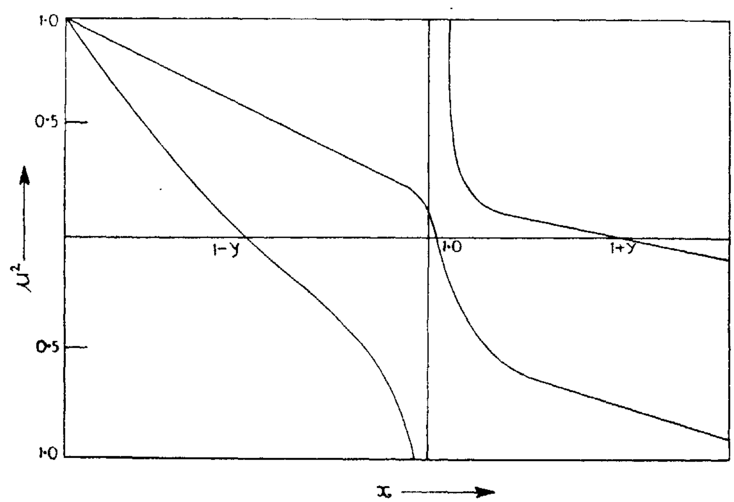

FIG. 6. Illustrating the variation of μ^2 with x, when the wave frequency is greater than the gyro-frequency. Intermediate direction of propagation.

Taking the lower sign we find the following values :

x	μ^2
0	1
1	0
∞	$-\infty$.

In this case, if intermediate values are also calculated, it is found that the curve is not a straight line passing through points A, B as in Fig. 4, but is bent as shown by the central curve in Fig. 6. At infinity the curve is parallel to the line

$$\mu^2-1=-\frac{x}{1-y\cos\theta},$$

i.e., if produced backward it would pass through $\mu^2=0$ at $x=1-y\cos\theta$. This latter curve has $\mu^2=0$ when $x=1$, i.e. at that value of x for which $\mu^2=0$ when no magnetic field is present. In other words the zero value of μ^2 is independent of the magnetic field. For this reason, the wave corresponding to this curve is called the ordinary wave.

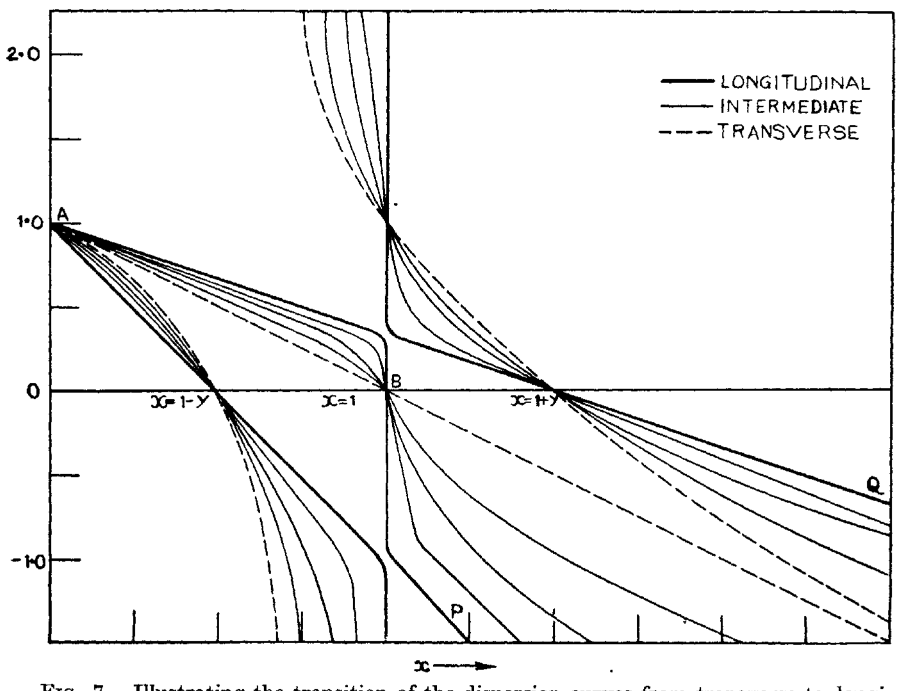

FIG. 7. Illustrating the transition of the dispersion curves from transverse to longitudinal case and *vice versa*. Case of $y<1$. Note that in the strictly longitudinal case the lines run unbroken from A to P through $1-y$ and from A to Q through $1+y$. The three broken line curves for strictly transverse case gradually bend and form the thick line curves for longitudinal case.

The dispersion curve ($y<1$) for the general case for any inclination of the wave normal with the magnetic field (Fig. 6) should gradually transform itself to the straight lines in Fig. 3 when the inclination is 0° and to the curves in Fig. 4 when the inclination is 90°. It is very interesting to trace the sequence of changes from the one extreme to the other ; this is evident from Fig. 7.

The transition from the transverse case to any intermediate case is easy to follow. But the transition to the longitudinal case is not so evident. The ordinary curve (central line Fig. 6) and the two branches of the extraordinary curve (the two lines running to infinity Fig. 6) so adjust themselves that a portion of the ordinary curve and a portion of one branch of the extraordinary curve join up to form one of the straight lines of Fig. 3. Similarly, a portion of the other branch of the extraordinary curve, together with the remaining portion of the ordinary curve, join up to form the other straight line of Fig. 3. This curious nature of the longitudinal dispersion curves is not noticeable in Fig. 3, which has been drawn from the general dispersion formula by making y_T zero, i.e., by making the wave normal actually coincide with the direction of the magnetic field. The peculiarity becomes evident if, instead of using the simplified equation (5), we use the general equation (3) and make y_T not zero but a quantity very small compared with y_L. The curve then nearly approaches the curved lines of the limiting case.

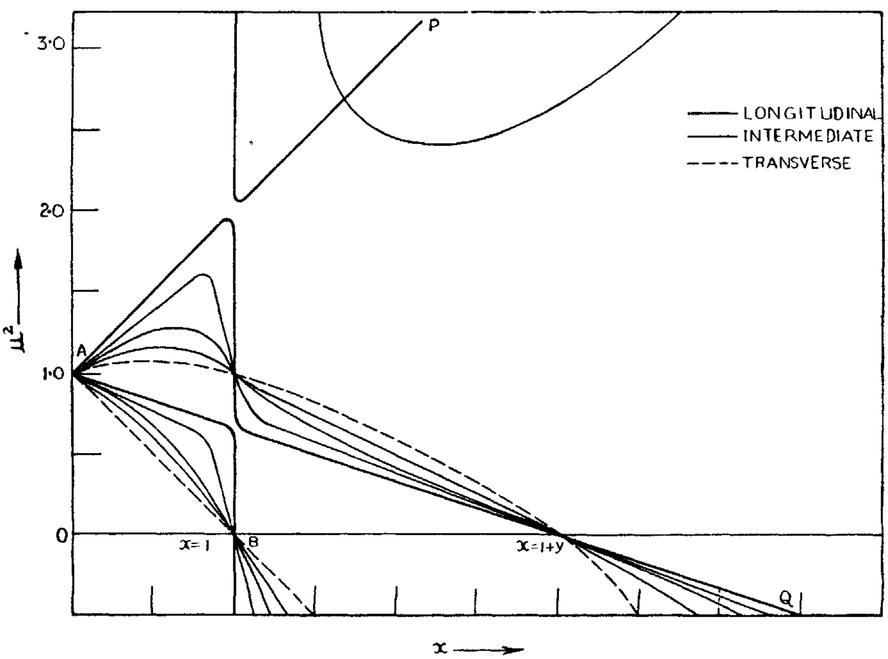

Fig. 8. Illustrating the transition of dispersion curves from longitudinal to transverse case and *vice versa* (case $y>1$). For explanatory note see Fig. 7.

$y>1$.

In this case the nature of the dispersion curve for any inclination between the wave normal and the magnetic field will be evident from Fig. 8, which also shows the gradual transition from transverse to the longitudinal case. As in the case of $y<1$ the two straight lines for the longitudinal case, when the wave normal actually coincides with the magnetic field, may be considered as made up partly of the dispersion curve of the extraordinary and partly of the ordinary.

(*b*) Polarization.

We have the polarization

$$R=\frac{h_z}{h_y}=+\frac{1}{\gamma_L}\left\{\frac{1}{M^2-1}-\alpha\right\}=f\left\{1\pm\sqrt{1+\frac{1}{f^2}}\right\} \quad \cdots \quad \cdots \quad (7)$$

where $$f=g\cdot\frac{1}{1-x};\ g=\frac{y}{2}\cdot\frac{\sin^2\theta}{\cos\theta}\ \text{(neglecting collisional term)}.$$

R thus signifies the ratio between the components of the magnetic vector of the wave in and at right angles to the magnetic plane, i.e., the plane containing the wave normal and the direction of the magnetic field (Fig. 9). Thus when R is zero, h_z is zero and the wave is plane polarized with its magnetic vector perpendicular to the magnetic plane. When R is infinite, h_y is zero and the wave is again plane polarized but with its magnetic vector in the magnetic plane. When R is unity, $h_z=h_y$ and the wave is circularly polarized. Without going into details of analysis we will give here the nature of polarization of the various types of waves as are propagated in the ionosphere. We will first deal with two especial cases, namely the longitudinal and the transverse, and then indicate in a general manner the nature of polarization for any direction of propagation.

(i) Longitudinal case :—

In this case θ being equal to zero, $R=1$: the two waves into which an incident wave is split up are both circularly polarized but in the opposite senses. If the wave is propagated along the positive direction of the magnetic field, then the polarization of the wave with a higher refractive index is left-handed when looking along the direction of propagation, and the corresponding ray is called the ordinary ray. For the same condition of propagation, the polarization of the wave with a lower refractive index is right-handed and the corresponding ray is called the extraordinary ray.

We shall see when considering the general case, that for a particular wave the sense of polarization changes from right-handed to left-handed or *vice versa*, when the ray crosses the point $x=1$, i.e., where $\mu^2=0$ for the case of no magnetic field. Remembering this we can explain the singularity in Fig. 7. If we follow the upper line, which is for the ordinary ray, it should change its sense of polarization on crossing $x=1$. This it does by coming down at $x=1$ and continuing to the right. Similarly the extraordinary ray changes its sense of polarization when it crosses the line $x=1$ and goes over to the top. The net result of these changes is that if we disregard the discontinuities and consider the lines AQ and AP as continuous, the polarizations of the two waves throughout their passage into regions of increasing ionic density remain the same. This consideration helps us to understand the singular nature of the transition of the mode of propagation from an oblique direction to the longitudinal direction.

(ii) Transverse case :—

Here putting $\theta=90°$, we find :

$$R_o=\frac{h_z}{h_y}=0,$$

$$R_e=\frac{h_y}{h_z}=0.$$

These are easily interpreted as states of plane polarization. For transverse propagation, therefore, the two component waves, in contradistinction to the longitudinal case, are plane polarized. The ordinary one has its magnetic vector perpendicular to the magnetic plane and the extraordinary one parallel to the same.

(iii) General case :—

The state of polarization for any direction of propagation other than longitudinal or transverse is in general elliptic. The two ellipses have the same ratio of the major to the minor axis. They are, however, oriented at right angles to one another and their senses of rotation are opposite (Fig. 9).

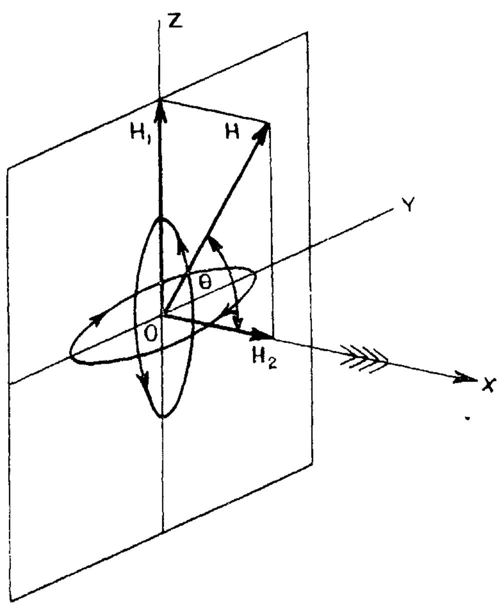

FIG. 9. Illustrating elliptic polarization of magnetically split components.

In equation (7) for polarization we have

$$R_0=\frac{h_z}{h_y}=f(1-\sqrt{1+1/f^2}),$$

$$R_e=\frac{h_z}{h_y}=f(1+\sqrt{1+1/f^2}).$$

The polarization is thus a function of f, which again is a function of y, i.e., the ratio of the gyro-frequency to the wave frequency, of θ the inclination of the wave normal to the direction of the magnetic field and of x which is proportional to the electron content N (for a particular wave frequency).

In order to make an estimate of polarization, we have to take all these factors into account. This may be done by first plotting graphs for $g\left(=\frac{y}{2}\frac{\sin^2\theta}{\cos\theta}\right)$ and θ for various values of y (Fig. 10), and then graphs for R and x for various values of g (Fig. 11).

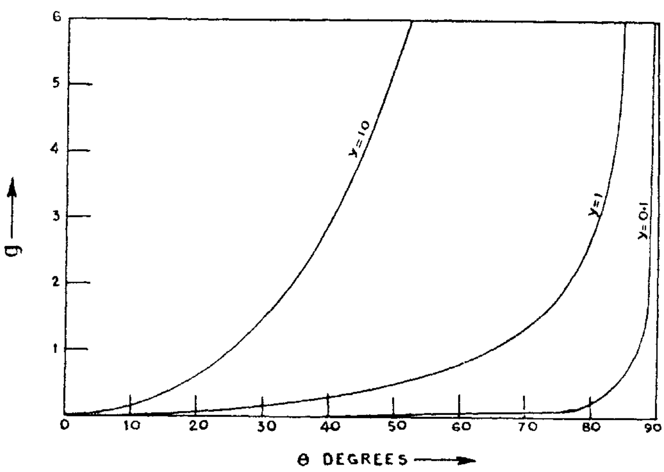

FIG. 10. (g, θ) curves for different values of y.

The state of polarization of a 150 m. wave travelling vertically upward at Calcutta may thus be deduced. Here $y=\frac{150}{235\cdot7}=\cdot6$; the magnetic dip being 31° 45′, $\theta=58°\ 15'$. g is therefore equal to ·436. The wave passing through an ionized layer of constant electronic density $N=10^5$ per c.c. will have the value $\frac{h_z}{h_y}$ for ordinary $+\cdot654$, and for extraordinary $-1\cdot500$.

In actual experimental investigation by the echo method, as is described later, the polarization phenomenon is complicated by the fact that the wave moving upward meets gradually increasing ionization. The state of polarization therefore continually changes, as is evident from Fig. 11, where R is plotted against x which is proportional to N (λ constant). If the

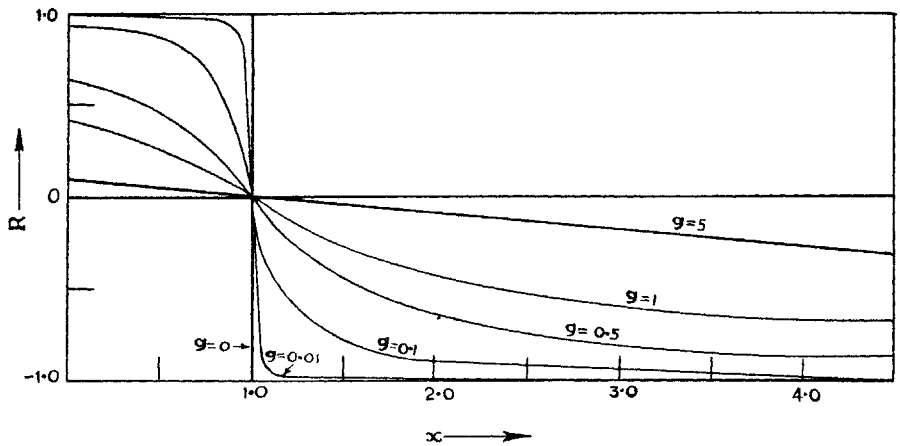

FIG. 11. Illustrating the changes in polarization as the wave of a particular frequency enters regions of increasing ionic density, i.e. increasing x. R is the ratio between the major and minor axis of the polarization ellipse. Note that for small values of y, i.e. for short wave, the limiting polarization, i.e. polarization for very small value of ionic density is circular.

exploring wave reaches the region of reflection of the ordinary wave where $N=\frac{mp^2}{4\pi e^2}$, i.e. $x=1$, the state of polarization changes sign, i.e., what was previously right-handed becomes left-handed and *vice versa*. At the point $x=1$, $R=0$, i.e., the wave is plane polarized. This sudden change of polarization we have already referred to when dealing with the longitudinal case

To trace the sequence of changes of the polarization of the wave for the case given above one may follow the curve of Fig. 12 corresponding to $g=\cdot 436$. Immediately before entering into the ionosphere, the wave may be considered as split up into two circularly polarized waves. As the two components enter into regions of greater and greater N, the polarization changes. On crossing $x=1$, the polarization changes sign. At this point both are plane polarized ($R=0$ and ∞).

4. *Complete Appleton-Hartree Formula including collisional term ν.*

We have till now discussed the propagation of the waves in the ionosphere without taking into account the effect of the collisions of the electrons and the ions with the neutral gas molecules and atoms. If ν is taken into account, then, under certain circumstances, its presence, in addition to causing absorption of the wave, profoundly affects the dispersion and the state of polarization of the wave.

The evaluation of μ, k and R from the complete complex equations (1) and (2) for the case of propagation in any direction with respect to the direction of the magnetic field is extremely complicated. Elaborate

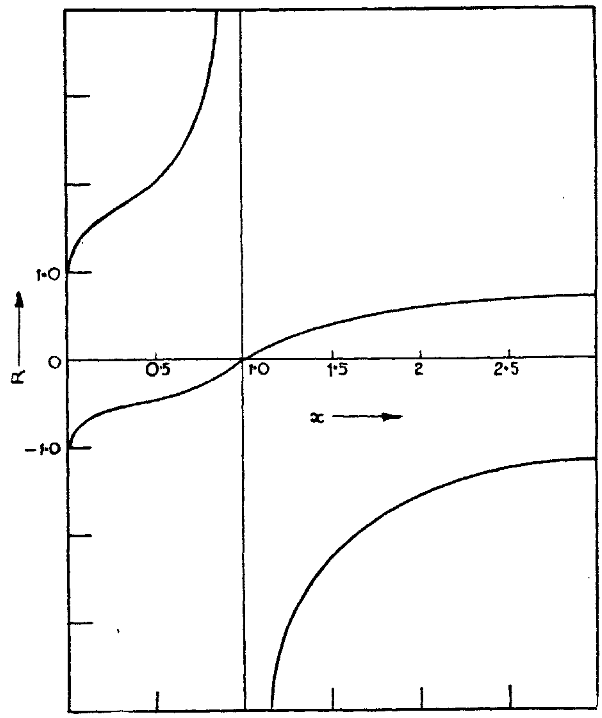

FIG. 12. Polarization curve for 150 m. wave, $g = \cdot 436$ (Calcutta).

discussions of these equations have been given by Mary Taylor[10], who has published a large number of graphs connecting N with μ, k and R. The derivation of these curves is laborious. Professor V. A. Bailey[11] has also discussed these equations and developed an elegant graphical method of delineating these curves. By utilising this method D. F. Martyn[12] has drawn dispersion, absorption and polarization curves for the particular value of the magnetic field in the southern hemisphere in Australia.

In the subsequent sections we will first deal with the nature of μ, k and R curves for the particular case of propagation along the magnetic field and then give a general description of the nature of the curves for propagation in any direction. For fuller details the original papers of Mary Taylor, Bailey and Martyn, and also one of Booker[13] may be consulted.

(*a*) Dispersion.

(i) Longitudinal case :—

Here $\gamma_T = 0$; so from equation (1) we have

$$\left(\mu - \frac{ick}{p}\right)^2 = 1 + \frac{1}{\alpha \pm \gamma_L + i\beta} \qquad \ldots \quad \ldots \quad \ldots \quad (8)$$

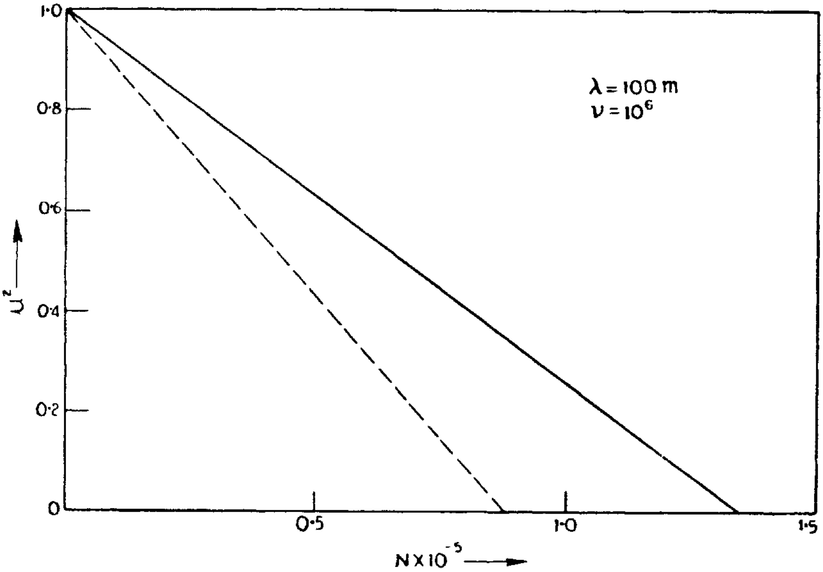

FIG. 13. μ^2 and x curve for $\lambda = 100$ m. and $\nu = 10^6$; longitudinal case $y < 1$.

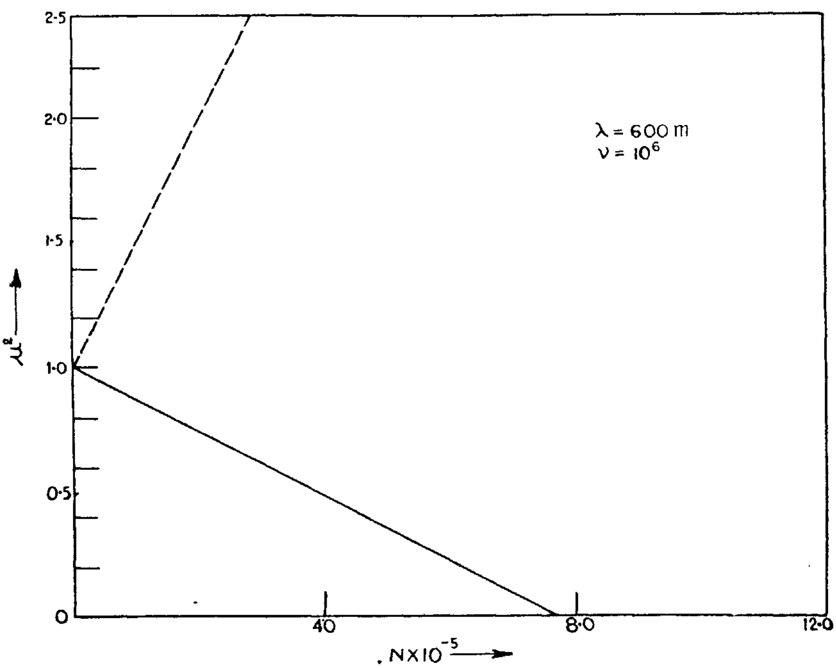

FIG. 14. μ^2 and x curve for $\lambda = 600$ m. and $\nu = 10^6$; longitudinal case $y > 1$.

On separating the real and the imaginary and supposing the absorption to be small enough to allow us to neglect the square terms of k for a first order of approximation,

$$\mu^2=1-\frac{p_0^2(p\mp p_L)}{p[(p\mp p_L)^2+\nu^2]}.$$

The dispersion curves for two values of $\frac{p_H}{p}$ less and greater than unity are shown in Figs. 13 and 14.

(ii) Propagation in any direction :—

The general effects of collision on the dispersion curves may be thus summarised :—

1. For values of ν up to about 10^6 per sec. the infinities of the dispersion curves given in Fig. 6 are removed, otherwise the nature of the curves remains unaltered (Fig. 15).

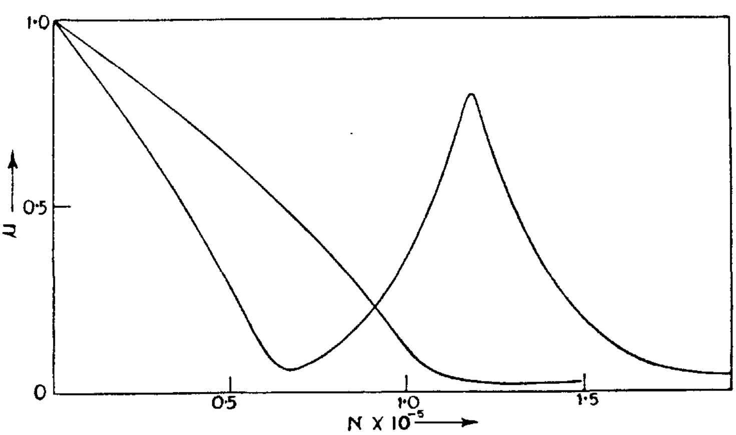

FIG. 15. μ and N curve when $\nu=10^6$. Intermediate direction of propagation.

2. If ν is very great then the sharp maxima in the dispersion curves are removed and the curves for different frequencies become more or less similar.

There is a critical frequency ν_c above and below which the nature of dispersion curves undergoes marked change ;

$$\nu_c=\frac{p_T^2}{2p_L} \quad \text{or} \quad \frac{H_T^2}{2H_L}\cdot\frac{e}{mc}.$$

For Calcutta $\nu_c=5{\cdot}4\times10^6$ per sec.

3. When $\nu<\nu_c$ the ordinary wave has its $\mu^2=0$ when $p_0^2=p^2$ and the extraordinary wave has its $\mu^2=0$ when $p_0^2=p^2\pm pp_H$.

4. When $\nu>\nu_c$ the dispersion curve tends towards the shape of Fig. 3, i.e. to dispersion curve for longitudinal propagation. The two waves have their $\mu^2=0$ at points near about $p_0^2=p^2\pm pp_H$.

(*b*) Absorption.

The calculation of absorption from the complex equation (1) is quite as complicated as that of dispersion and can only be made by graphical methods. The reader may consult papers by Mary Taylor, Bailey and others already referred to.

We will deal here only with the comparatively simple case of absorption for the longitudinal propagation which will indicate in a qualitative way the nature of the differential absorption of the two magnetically split waves.

From equation (8), the absorption term is given by the relation

$$2\mu\frac{ck}{p}=\frac{\beta}{(\alpha\pm\gamma_L)^2+\beta^2}.$$

For having a rough idea as to the nature of the differential absorption of the split waves we give an approximate formula by making some simplifying assumptions:

$$k=\frac{\nu}{2c}\cdot\frac{p_0^{\,2}}{(p\mp p_L)^2}.$$

When the direction of propagation is along the positive direction of the magnetic field the extraordinary component will be more strongly attenuated than the ordinary component and *vice versa*. In Fig. 16 the absorption curves

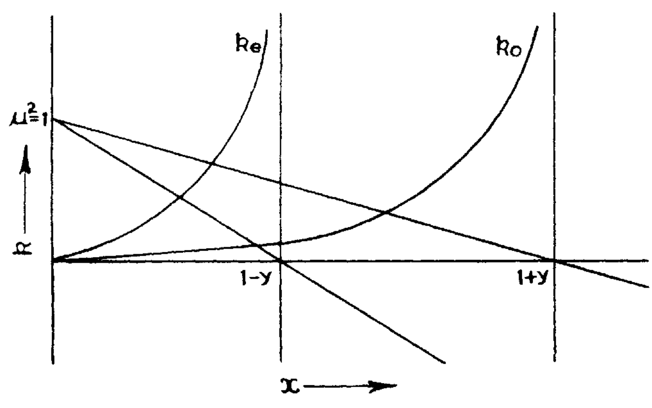

FIG. 16. Illustrating the variation of the coefficient of absorption k with N for the longitudinal case of propagation.

for the two split waves are given. It will be seen that k tends to infinity as μ^2 tends to zero.

(*c*) Polarization.

The effect of ν on polarization is to rotate the principal axes of the polarization ellipse. The major axes of the two ellipses remain perpendicular to each other as before. Generally, if ν is small the rotation is small.

It is instructive to trace the changes in the polarization as the wave enters the ionosphere into regions of greater and greater ionic density. The nature of the change is different for values of ν less and greater than the critical collisional frequency ν_c.

(i) $\nu<\nu_c$:—

Immediately before entering the ionosphere where $N=0$, we may consider the linearly polarized incident wave to be split up into two circular components of equal amplitude and of opposite senses. On entering the ionosphere the two components become elliptically polarized. As the waves enter deeper N increases and the ellipticities also increase. At the point where N corresponds to the value of $p_\circ^2=p^2$ the elliptic polarization degenerates to linear polarization. After crossing this value of N the characteristic polarizations change sign. They gradually become less and less elliptic and become circular for large values of N. Thus for values of ν less than ν_c the nature of polarization and its changes are very little affected.

(ii) $\nu>\nu_c$:—

When the collisional frequency exceeds the critical value ν_c, the nature of the polarizations of the split components is more or less circular for all values of N and they never become linear at the point $p_\circ^2=p^2$.

(*d*) Quasi longitudinal and quasi transverse types of propagation.

It is important to distinguish between two types of propagation when collision is present. One of these approximates to the longitudinal type and the other to the transverse type irrespective of the angle which the direction of propagation makes with the magnetic field.

We have seen that in the absence of collision, when the direction of propagation coincides with that of the magnetic field, μ^2 becomes zero for *two* distinct values of N given by $p_\circ^2=p^2\pm pp_H$. In this case the two waves are circularly polarized with opposite senses of rotation. This we call the *longitudinal* type of propagation.

For all other directions μ^2 becomes equal to zero for *three* distinct values of N given by $p_\circ^2=p^2$ and $p_\circ^2=p^2\pm pp_H$. The nature of polarization is independent of N and is linear, when the direction of propagation is at right angles to the magnetic field, and gradually changes over to elliptic and finally to circular for directions less and less inclined to the magnetic field. The above statements are true even in the presence of collision provided the collisional frequency is less than the critical value ν_c. From the fact that for all directions (except strictly along the magnetic field), there is a common characteristic, namely of the occurrence of *three* values of N for $\mu^2=0$, all these propagations can be called *transverse* mode of propagation.

Thus the change over from *transverse* to the *longitudinal* type in the absence of collision occurs when the angle between direction of propagation and that of the magnetic field becomes equal to zero. Hence we say that propagation without collisional friction and also with friction when ν is less than ν_c, is, in general (except when the direction of propagation actually coincides with the magnetic field), of the transverse type and may be called *quasi-transverse* type of propagation.

If, however, ν is greater than ν_c, then μ^2 is zero for only two values of N, not only for the longitudinal case but also for any other direction. From the fact that this type of propagation in any direction, except strictly perpendicular to the magnetic field, possesses the longitudinal characteristic of having two values of N for $\mu^2=0$ it is called *quasi-longitudinal* propagation. It is to be noted, however, that in this case μ^2 is not zero exactly at points given by $p_0^2=p^2 \pm pp_H$ but at points somewhat beyond these. The nature of the polarization is approximately circular for all values of N.

(*e*) Limiting polarization.

In all measurements of polarization of downcoming waves the nature of polarization studied is that which the waves possess immediately on their emergence from the ionosphere. It is therefore important to consider what would be the polarization of the magnetically split components when they leave the ionosphere.

Immediately before entering the ionosphere a plane polarized wave might be considered as composed of two circularly polarized waves with opposite senses of rotation. Each of these on entering into the ionosphere undergoes change of polarization as it gets into regions of greater and greater ionic density. Arriving at the region where the value of N is such as to send back the wave the polarization becomes, in general, linear (except in the case of longitudinal and quasi-longitudinal propagation, when its sense of rotation is altered). If we consider the particular case of vertical incidence the component waves retrace their original paths and on emergence will have that polarization which they possessed on entry near the boundary of the ionosphere. This is because, at a point immediately outside the boundary, the electric wave disturbance produced will be that due to the radiation from the periodically moving ions and electrons of the boundary in accordance with Huygen's principles. The polarization of each component will therefore be determined by equation (2) applied to the physical conditions of the ionosphere near the boundary.

If it were possible for the two component waves to travel with the same velocity then no matter what sort of change of polarization is undergone by one of them, exactly conjugate changes would be undergone by the other, and the two again would combine on emergence to give plane polarized wave. This condition is partially fulfilled in the case of very long wave propagation. In this case the two components are propagated with nearly, though not exactly, equal velocity and on emergence they are sufficiently close together to recombine and the effect of slight difference in phase due to difference in velocity is to turn the plane of polarization of the emergent wave a little.

For medium waves the two components are widely separated and on emergence each is found to be elliptically polarized.

For short waves below 100 metres polarization is approximately circular.

5. *Summary.*

We give here a summary of the theory and its results on wave propagation in the ionosphere discussed in this article, bearing in mind that these are to be applied for interpreting the results of actual experimental investigations on the ionosphere. The ionospheric boundary is supposed to be horizontal and the ionic density to increase with height from the boundary.

Taking first the simple theory, when no magnetic field is present, let a radio wave be incident on the boundary at an angle θ. The wave will suffer refraction and penetrate the boundary. If, as is generally the case, the ionic density gradually increases with height from the boundary, then the ray corresponding to the wave will be bent more and more away from the normal and ultimately become horizontal and be totally reflected. The condition for reflection is thus easily found to be

$$\sin^2\theta=\mu^2=1-\frac{4\pi\ Ne^2}{mp^2}\,.$$

If the incidence is vertical, as is in the case for study by the group-retardation method (§ IV, sec. 3), the condition for reflection is

$$\mu^2 \overline{\lesssim} 0,$$

$$\text{i.e. } \frac{4\pi\ Ne^2}{mp^2} \overline{\gtrsim} 1\,.$$

If magnetic field is present, then the single incident wave will be split up into two waves and these, as they proceed upward to meet layers of gradually increasing density, may meet the condition for reflection $\mu^2=0$. Two cases may be distinguished according to whether the gyrofrequency He/mc of the electron, due to the steady magnetic field of the earth, is less or greater than p, the angular frequency of the wave.

(i) $He/mc<p$:—The condition for reflection will be satisfied for three different values of N at three different heights. These are given by the equations

$$\frac{4\pi\ Ne^2}{mp^2}=1-\frac{He}{mcp},\ 1,\ \text{or}\ 1+\frac{He}{mcp}\,.$$

(ii) $He/mc>p$:—In this case there will, in general, be two values of N at two different heights for $\mu^2=0$. These are given by

$$\frac{4\pi\ Ne^2}{mp^2}=1\ \text{or}\ 1+\frac{He}{mcp}\,.$$

A remarkable fact about these equations is that the condition for reflection is independent of the direction of the magnetic field, except when the direction of propagation actually coincides with the magnetic field, in which case there are only two values for the first case

$$\left(\frac{4\pi\ Ne^2}{mp^2}=1\mp\frac{He}{mcp}\right),$$

and one value for the second case

$$\left(\frac{4\pi\ Ne^2}{mp^2}=1\right).$$

These conditions hold strictly for the case of no collision between electrons and ions and neutral gas molecules or atoms, but they also hold when the collision is small, less than about 10^5 per sec.

The wavelength corresponding to the critical gyrofrequency for Calcutta is 235·7 *m*.

The two magnetically split waves are, on entering into the ionosphere, in general, elliptically polarized in opposite senses. The ratio of the axes are the same for the two but the axes themselves are at right angles to one another. The left-handed component (looking along the direction of propagation) is reflected, on entrance into regions of increasing density, at the point where $4\pi Ne^2/mp^2=1$, and the right-handed component where $4\pi Ne^2/mp^2=1\mp He/mcp$. The state of polarization continually changes as the waves enter further and further into the ionosphere. Both the waves would be plane polarized at the region of density given by $4\pi Ne^2/mp^2=1$. Actually, however, the right-handed component may not reach this region because its condition for reflection will be attained at a lower level where $4\pi Ne^2/mp^2=1-He/mcp$. It may also be noted in this connection that though there are two values of N for the reflection of the right-handed wave, it has very little chance of reaching the higher level of greater ionic density given by $4\pi Ne^2/mp^2=1+He/mcp$.

It is important to remember the change of the state of polarization on reflection of any one of the split waves. Taking the right-handed, for instance, if one looks upward towards the ionosphere, he will observe the magnetic vector rotating in the right-handed sense both for the direct up-going and for the indirect reflected down-coming wave. From our definition, however, the up-going wave is polarized in the right-handed sense and the down-coming wave, looking down from above along the direction of propagation, is polarized in the left-handed sense.

The actual state of polarization of the down-coming wave on emergence from the ionosphere, as observed near the ground, depends upon the conditions near the boundary, that is, on the ion content and the angle between the wave normal and the magnetic field.

If we take collision into account then, depending upon its value, it will affect the dispersion and the state of polarization and also cause absorption.

There is a critical collisional frequency ν_c given by $\nu_c = \frac{H_T^2}{2H_L} \cdot \frac{e}{mc}$, near which the dispersion and the polarization undergo marked changes. These are summarised in sections (3*a*) (ii), and (3*b*) (i) and (ii).

The collision causes, what may be called, differential absorption. An important consequence of this is that in the northern hemisphere the right-handed component of the down-coming wave (looking along the direction of propagation) is more absorbed than the left-handed component and *vice versa* in the southern hemisphere.

§ IV. Equivalent Heights of the Ionospheric Regions.

The methods for determining the heights at which the ionized regions are situated can be divided under three heads :—

1. Angle of incidence method.
2. Wavelength change method.
3. Group-retardation or echo method.

In all these methods for investigating the ionosphere, we utilise the phenomenon of reflection of the radio waves at the ionized regions. The conditions under which reflection may occur have been discussed in the preceding article.

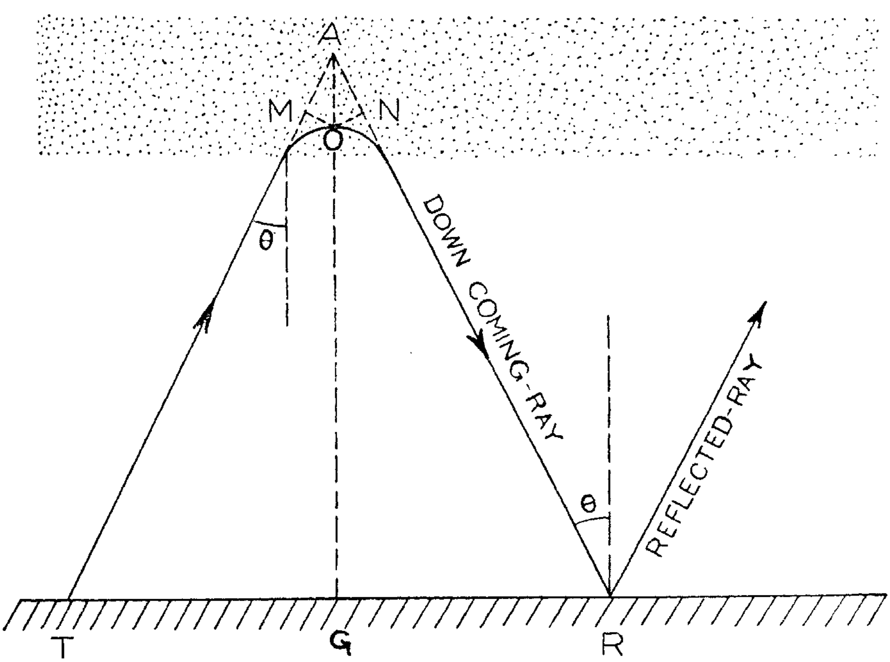

Fig. 17. Illustrating angle of incidence method of height measurement.

1. *Angle of incidence method.*

If we can measure the angle at which the down-coming ray meets the surface of the earth, the height of the layer can be determined by simple triangulation, Fig. 17. Appleton and Barnett [1] have carried out this measurement by comparing the signal intensity received on a vertical with that received on a loop aerial.

The principle of the method is that the fading effect on the vertical aerial is due to the vertical component of the electric vector and the fading effect on the loop aerial is caused by the horizontal component of the magnetic vector, which is at right angles to the plane of the loop, when it is set parallel to the plane of propagation.

Considering the earth as a perfect reflector and supposing the fading effect to be small, it can be shown[2] that to a first approximation the vertical aerial will be acted on by an e.m.f. of amplitude proportional to E_A given by

$$E_A = E_o + 2E \cos\phi \sin\theta,$$

where ϕ is the phase difference between E_o and E, Fig. 18.

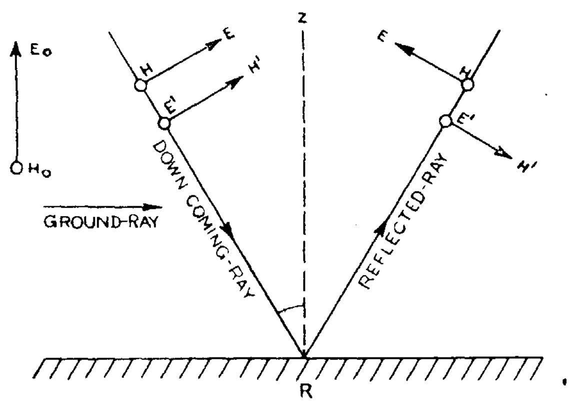

FIG. 18. Illustrating normal and abnormal components of electric and magnetic vectors in down-coming and earth-reflected ray.

Similarly the loop aerial in the plane of propagation will be acted on by an e.m.f. proportional to $\frac{d}{dt}(H_x)$; but since in an electromagnetic field the electric and magnetic forces are proportional, this e.m.f. will be proportional to E_L given by

$$E_L = E_o + 2E \cos\phi.$$

If the signal variations are due to changes in E or ϕ or both, we get in each case

$$\frac{\Delta E_A}{\Delta E_L} = \sin\theta.$$

If the two aerials are coupled to two receivers which linearly amplify the input signals and rectify the amplified ones by detectors having square law characteristics, then

$$\frac{\Delta A}{\Delta L}=\frac{K_1}{K_2}\cdot\frac{\Delta E_A}{\Delta E_L}=\frac{K_1}{K_2}\cdot\sin\theta,$$

where ΔA and ΔL are the changes from the mean values of the detector currents corresponding to the two aerials, and K_1, K_2 are constants depending on the aerial systems, on the amplification factors and detector characteristics of the corresponding receivers.

Fig. 19 shows the simultaneous values of the two detector currents. The

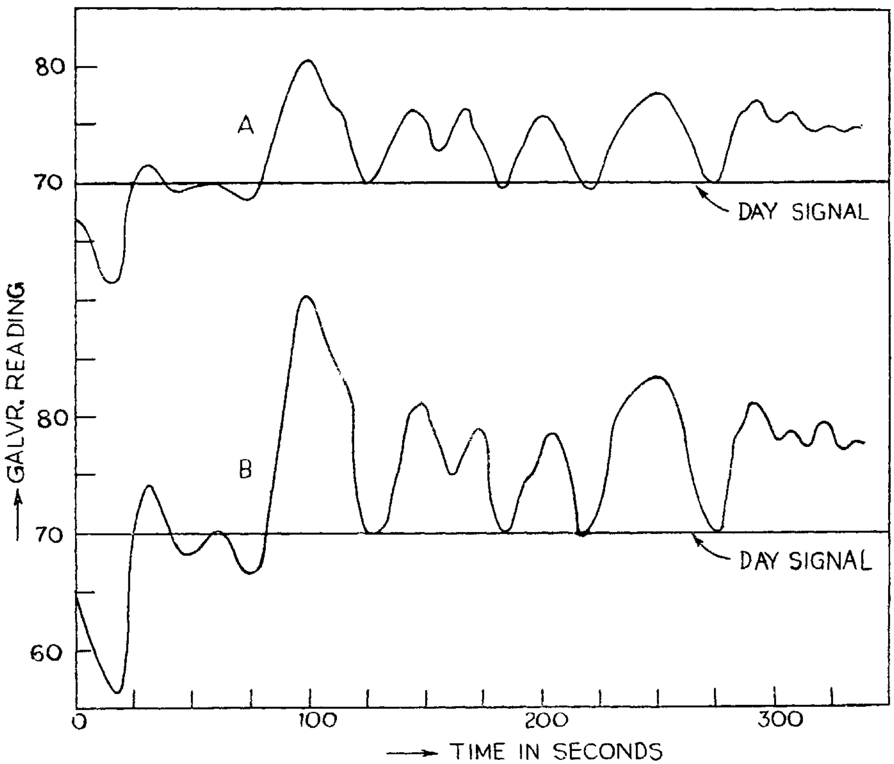

FIG. 19. Illustrating intensities of signal fading in (*A*) vertical and (*B*) loop aerial. From a comparison of the two the angle of incidence of down-coming wave may be determined.

angle θ being thus known, the height is at once found out. From Fig. 17 we see that the measured height is AG and not the actual height OG; hence the height measured is called the *equivalent height*.

2. *Wavelength change method.*

This method was first used by Appleton and Barnett.[3] The principle underlying the method is that if two sets of waves, *via* the ground and the sky, reach a receiver then the resultant signal intensity will be a maximum if they meet in the same phase and a minimum if in opposite phases. This variation in phase is artificially made by a continuous change of the transmitter wavelength through a small range and the number of signal maxima and minima thus produced is recorded.

If P be the optical path of the sky wave then the difference of path between the sky and ground waves reaching the receiver is $P-TR$, TR being the distance between the transmitter and the receiver (Fig. 17). The difference n in the number of wavelengths in the two paths is given by

$$n=\frac{P-TR}{\lambda}=f \cdot \frac{P-TR}{c} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

In actual experiments, the change Δn in n produced by a slight change Δf in f is measured. But from (1),

$$c\,\frac{\Delta n}{\Delta f}+TR=P+f \cdot \frac{dP}{df}=P' \text{ (say)},$$

$\left(c\,\frac{\Delta n}{\Delta f}+TR\right)$ is the measurable quantity and P' is therefore called the *equivalent path* of the sky wave. P' being known the *equivalent height* is found out by triangulation.

An interesting variation of the wavelength change method is the employment of a moving transmitter. Mirick and Hentschel[4] computed the equivalent height by placing the transmitter in an aeroplane and noticing the variation of signal strength in a fixed receiver placed on the ground while the aeroplane was moving.

The theory of the method is analogous to that of the frequency change method described above.

Suppose that a transmitter emitting waves of frequency f_0 is moving with a velocity V. Due to the movement of the transmitter, say towards the observer (i.e., towards the receiver), the frequency of both the direct and the indirect waves as picked up by the receiver will be changed.

For the direct waves we shall have, if f_1 is the changed frequency, since $c>>V$

$$\frac{f_1}{f_0}=\frac{c}{c-V}=\frac{c+V}{c}.$$

For the indirect waves, if these are returned by the ionosphere at an angle say θ, we shall have

$$\frac{f_2}{f_0}=\frac{c}{c-V\sin\theta}=\frac{c+V\sin\theta}{c},$$

where f_2 is the received frequency of the indirect waves.

From these two relations

$$\frac{f_1-f_2}{f_0}=\frac{V(1-\sin\theta)}{c};$$

f_1-f_2 is measured directly by observing the signal variations and, V being known, the value of $\sin\theta$ is found out.

3. *Group retardation method.*

The method involves the measurement of the time taken by a wireless signal to travel up to the ionized region and return back to the earth. Breit and Tuve [5] were the first to get this novel idea. They sent out radio-frequency pulses of very short duration and received the same at a moderate distance from the transmitter. The receiver recorded both the direct ground pulse and the indirect sky pulse (see Fig. 20) and the time interval between

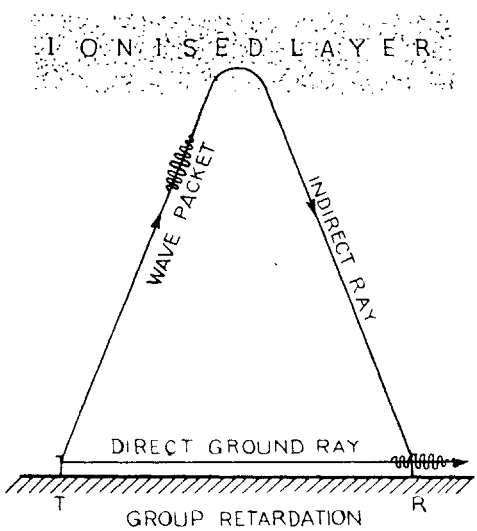

FIG. 20. Illustrating the pulse method of sounding the ionosphere. Trains of wave packets or pulses are shot upwards from the transmitter. (Typical values: 200 packets per sec., length of each packet 50 km., duration of emission of same $\frac{1}{6000}$ sec. The length of the packet is not drawn to scale in the figure.) These returned by the ionosphere reach the receiver a few milli seconds after the corresponding packets travelling directly along the ground. If the wave frequency is high they pierce through the ionosphere and fail to return. The distance between the transmitter and the receiver formerly used to be several kilometres. They need not be so separated now. The transmitting aerial can be used for receiving the echoes at absolutely normal incidence.

the two was measured. Knowing this time the *equivalent height* of the reflecting layer was easily calculated.

4. *Comparison of the three methods.*

It might be inquired at this stage whether the three methods above described measure the same quantity or not.

In the angle of incidence method if, at any point of the path within the ionized medium, the ray makes an angle θ' with the vertical then, supposing the medium to be horizontally stratified,

$$\mu \sin \theta' = \sin \theta,$$

where μ is the refractive index at the point in question. In cases where the influence of the earth's magnetic field upon the medium may be neglected, e.g. with short waves, it is easy to show that

$$v = c\mu,$$

where v is the group-velocity of the wave at the point μ. Therefore if ds is an element of the path of the sky wave, then

$$c\int \frac{ds}{v} = \int \frac{ds}{\mu} = \int \frac{dx}{\mu \sin \theta'} = \int \frac{dx}{\sin \theta} = TA + AR,$$

where dx is the projection of ds parallel to TR (Fig. 17). But $TA + AR$ is the equivalent path measured by the angle of incidence method and hence the quantity measured is the group-path

$$c\int \frac{ds}{v}.$$

In the wavelength change method, we measure

$$P' = P + f\frac{dP}{df} = \int \mu\, ds - \lambda \int \frac{\partial \mu}{\partial \lambda}\,.\, ds, \left(\because P = \int \mu\, ds\right).$$

But by using Hamilton's principle of least time it can be proved [6] that for any particular path

$$c\int \frac{ds}{v} = \int \mu\, ds - \lambda \int \frac{\partial \mu}{\partial \lambda}\,.\, ds.$$

It is evident therefore that in the wavelength change method we measure the quantity

$$c\int \frac{ds}{v}.$$

In the group-retardation method, Appleton has shown [6] that if Δt is the measured time interval between the ground and the sky pulse, then

$$c\,.\,\Delta t + TR = c\int \frac{ds}{v}.$$

From these considerations we see that the angle of incidence method (no magnetic field), the wavelength change method and the group-retardation method measure the same quantity, viz. the group path

$$c\int\frac{ds}{v}$$

for the sky waves. Thus, in cases where the influence of the earth's magnetic field upon the ionosphere (supposed to be horizontally stratified) may be neglected, the angle of incidence method and the other two methods all give the same equivalent height, viz. the height of the apex A (Fig. 17) above the ground, which is, however, always greater than the greatest height reached by the waves.

It may be mentioned that in order to test the equivalence of the quantities measured by the wavelength change and the group-retardation methods R. Naismith[7] carried out several typical sets of experiments. From a series of observations he confirmed the fact that the two methods measure the same quantity.

5. *Experimental equipments.*

(*a*) Description of a typical group-retardation equipment.

Of the three methods given above, the last one has been widely and most conveniently used in different parts of the world. We will therefore describe it in some detail.

The typical experimental equipment for the group-retardation method consists of (i) a transmitter capable of generating at regular intervals wave packets of extremely small duration, say 1/4000 second, (ii) a suitably erected aerial which can throw the waves in an upward direction, (iii) a receiver of such a design that it will not be paralyzed even when placed close to the transmitter, or at any rate will recover its sensitiveness in an interval of time small compared with the interval between the direct signal and its echo, and (iv) a suitable oscillographic recording system connected to the receiver output. These four components we proceed to describe now.

(i) *Transmitter* :—The circuit diagram of a typical transmitter is given in Fig. 21. The oscillatory circuit is of the conventional Hartley type and needs no further description. The interesting part of the circuit is the value of the leak resistance R. By suitably adjusting this resistance the negative charge accumulating on the grid during oscillations may be made so high that the oscillations stop within a very brief interval of time after their inception and only start again after the negative charge has leaked away. The number of interruptions of oscillations per second can be regulated by adjusting R and the associated capacity.

Typical values :—

Wave-frequency—4 mega cycles/sec.

Valve—100–250 watt.

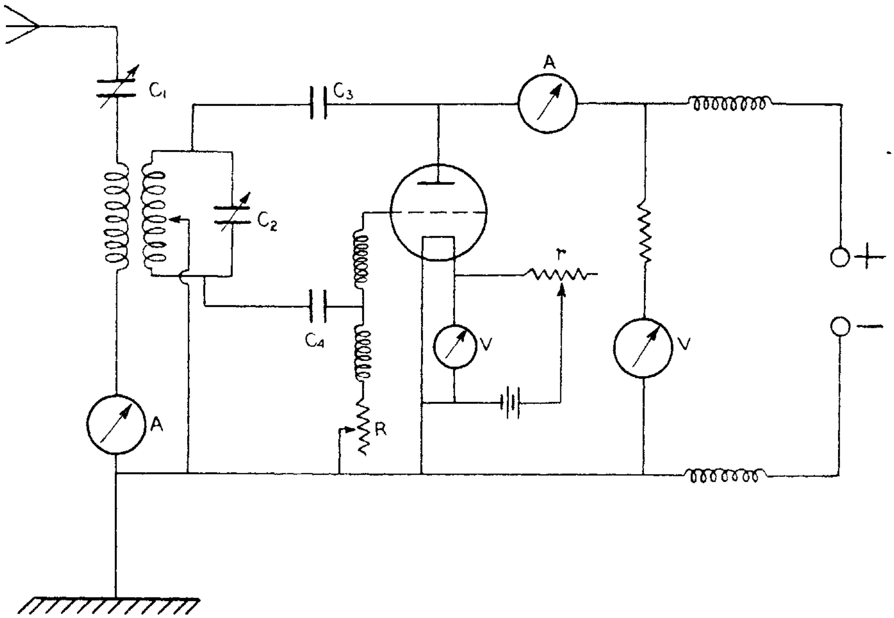

FIG. 21. Circuit diagram of a typical transmitter for group-retardation experiment. The value of the leak resistance R controls the pulse frequency. C_1, C_2 variable aerial and oscillatory circuit condensers; C_3 blocking condenser, C_4 grid condenser. A very small H.F. choke is connected between C_4 and the grid to prevent spurious oscillations.

Interruption frequency—200 per sec.
Leak resistance R—80,000 ohms.

(ii) *Aerial* :—The aerial for use with only one particular frequency may conveniently be of horizontal Hertzian dipole type fed by a Lecher system. If, however, it is desired to use a range of wavelengths the aerial may be of the inverted '*L*' type with a very long horizontal portion say about 100 metres. For medium waves, up to say 350 metres, the aerial may be loaded by suitable inductances. For short waves it may be excited in its fundamental and in its harmonics.

(iii) *Receiver* :—The receiver, as is seen from Fig. 22, is of the conventional type consisting of two stages of transformer coupled H.F., one anode bend detector with reaction and two stages of L.F. The important feature of the receiver is that there is no coupling or detecting condenser with any valve. This makes the time constant, i.e., the time of recovery of its original sensitiveness, after it has been excited by a powerful electric shock, extremely small. In the older types of receivers, where the usual grid leak and condenser were employed for detecting purposes, the time constant used to be so large that the receiver could not be used in close proximity to the transmitter owing to the paralyzing effect of the direct signal. With the type of receiver

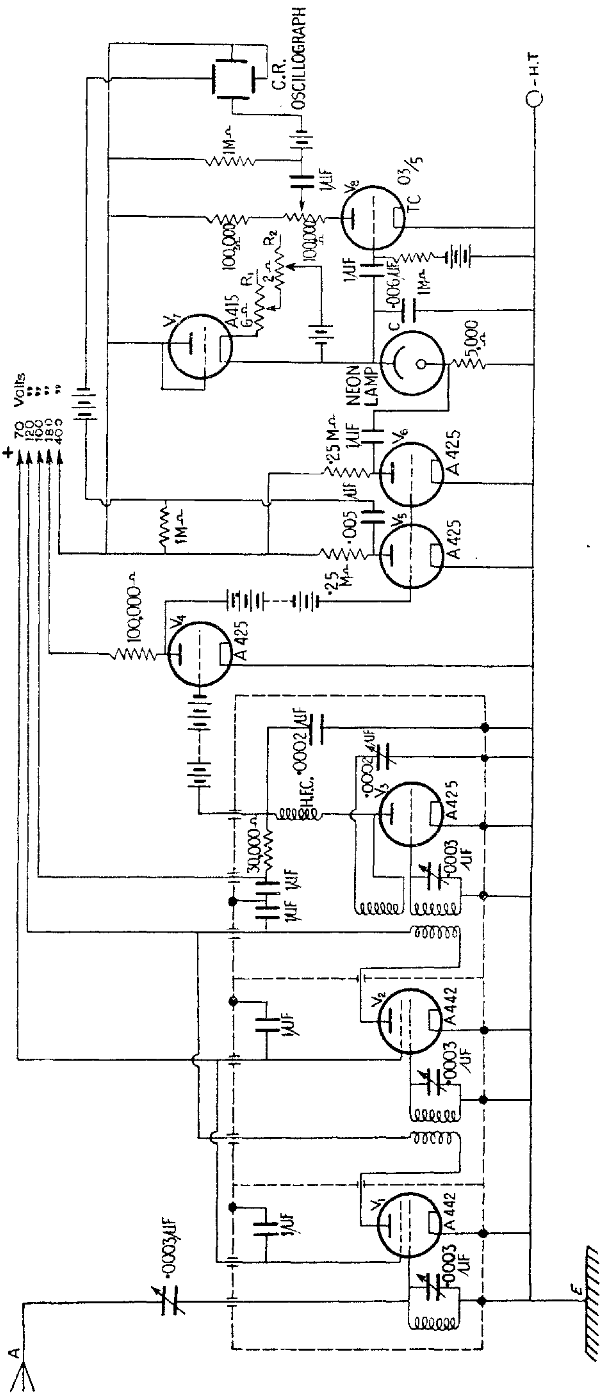

FIG. 22. Circuit diagram of a receiver and cathode ray oscillograph recorder for group-retardation experiment. V_1, V_2 H.F. valves, V_3 Det. valve, V_4, V_5 L.F. valves. V_6 serves as a buffer valve. V_7 is a saturated diode and V_8 the time base amplifier.

described above recording can be made even when the receiver is located in the same room as the transmitter.

(iv) *Recording system* :—Of the several types of recording appliances employed the cathode-ray oscillograph is the one most extensively used. The general feature of the oscillograph is well-known and we will describe here only the particular way in which it is used for recording wireless echo-signals as coming from the output of the receiver described above.

A common device is to connect one pair of the plates to a linear time base and the other pair to the output of the receiver. In the absence of a signal the cathode ray spot describes a straight line of a few cm. length starting, say from the left and moving to the end of the line with a uniform velocity, and then returning to the starting point within a time very small compared with the time of its passage from left to right. Signal voltage coming to the other pair of plates causes deflection of the cathode-ray spot in a direction at right angles to the line. The method of feeding one pair of plates from the receiver output and the other pair from the time base generating system is shown on the right side of the figure. The electric impulses for the time base are generated by the neon tube in series with a saturated diode. The pulses are amplified by the valve V_8 and then passed on to the oscillograph plates through the condenser and balancing battery. The output from the receiver is fed to the other pair of plates through another balancing battery. Part of the output energy is fed to the neon tube circuit and serves to synchronize or lock the time base with the direct or ground wave pulses coming from the receiver output.

(*b*) Various modifications.

As mentioned before the above is the description of a typical commonly used equipment. There are of course variations in its various components such as for the generation of the pulses, for the nature of the time base, etc. We give below brief descriptions of a few other methods of generating pulses and of types of time base successfully employed by other workers.

E. L. C. White,[8] by using photo-electric cell in which short flashes of light of duration ·00025 second and frequency 100 to 200 per sec. are allowed to fall by mechanical means, produced pulses of very short duration. This method of pulse generation has got the advantage over other methods, that perfect regularity of the wave trains may be maintained.

Bontch Bruewitch[9] of Leningrad generated pulses of 20 K.W. peak power with two 150-watt valves in parallel. This he did by using a condenser charged to a high voltage by a rectifier ; the condenser was discharged by a spark gap fifty times a second through the plate circuit of the oscillator valves. During the discharge, which was of very short duration, the valves were strongly forced and produced a power of 10 to 20 K.W.

Goubau and Zenneck[10] employed a novel method of generating pulses. They constructed a transmitter with a master oscillator and several stages of amplifying valves. The grid of the last valve was given such a high negative potential that under normal conditions no current flowed in the aerial. A highly saturated iron-core choke was inserted in the grid lead and the output of a 500 cycle alternator was suitably connected to it through a condenser. Owing to the saturated condition of the core the flux changed suddenly and the sudden e.m.f. thus induced neutralized the negative bias of the grid; the aerial current therefore flowed only for a correspondingly short time. By this process pulses of duration 10^{-4} second and spaced at intervals of ·002 second could be produced.

Goubau and Zenneck [10] also employed a circular time base instead of the more usual linear one. By properly adjusting sets of coils at right angles, by the side of the cathode ray oscillograph and sending a.c. through them with a phase difference of 90° between the two, the spot on the fluorescent screen could be made to describe a circle. Part of the signal voltage after amplification and rectification was allowed to pass through a low frequency amplifier and applied to the valve oscillator generating a.c. for the time base coil. The frequency of the latter was thus controlled to give exactly the same frequency as that of the pulses. On applying the rectified signal voltage to the vertically deflecting plates of the cathode-ray tube, kinks were produced in the circle. The distances between these gave the retardation time of the atmospheric pulse.

A variation in the recording arrangement has been made by Ranzi [11]. He allows his detector valve to oscillate on a frequency a little different from that of the transmitter so that beats are produced. By observing the beat curves on the oscillograms and noting their positions the time of retardation of the atmospheric pulse is determined.

A common device nowadays is to use thyratrons (mercury vapour triodes) for both generation of pulses and for cathode-ray time base. The necessary locking between the two is very conveniently secured by employing the same a.c. for actuating the two thyratrons. This has the advantage of producing an extremely steady pattern on the oscillographic screen and is very useful where continuous record is made. Various methods utilising the thyratrons for such purpose have been given by Herd, Ratcliffe and E. L. C. White, Pulley [12] and others.

A notable improvement in the method of ionospheric measurements is the employment of devices capable of making a continuous record of the echoes. Most of these devices employ cathode-ray oscillograph for this purpose but, instead of photographing the whole pattern on the screen, only a strip of the pulse pattern parallel to the base is focussed by means of a lens, the rest being covered up. The photographic paper on which the image is formed is moved continuously at right angles to the length of the time base. A continuous record is therefore obtained of the interval between the ground pulse and any of its echoes. Details of such methods for continuous record will be found in papers by Ratcliffe and E. L. C. White, Pulley, Gilliland, F. W. G. White and others.[13]

§ V. Ionic Densities of different regions of the Ionosphere—Structure of the Ionosphere.

1. *Measurement of ionization.*

An extension of the experimental method for determining the equivalent height enables one to make an estimate of the maximum ionic density of a particular ionospheric region.[1] The theory of the method is easily understood from the following consideration. Let us take the simple dispersion formula

$$\mu^2 = 1 - \frac{4\pi Ne^2}{mp^2}.$$

The condition that a wave of angular frequency p should be returned by a region of density N is that $\frac{4\pi Ne^2}{mp^2} \gtreqless 1$. If, however, the value of p is such that $\frac{4\pi Ne^2}{mp^2} < 1$, then the wave will penetrate through the region. It therefore follows that if waves of gradually increasing frequency are sent upward, then when the condition $\frac{4\pi Ne^2}{mp^2} < 1$ is attained, the wave will fail to return and no echo will be obtained. If the critical angular frequency at which the disappearance of echoes occurs be p, then obviously we have

$$N = \frac{mp^2}{4\pi e^2} = \frac{\pi m}{e^2} f^2,$$

where f is the frequency of the wave in cycles/sec.

If we take the magnetic field of the earth into account, then the splitting of the wave into two components on entry into the ionosphere will have to be considered. If we consider propagation against the magnetic field, then there will be one critical value of p for and above which the left-handed component will penetrate a region of density N given by

$$N = \frac{mp^2}{4\pi e^2}.$$

For the right-handed component, as previously shown, there will be two critical values of p for and above which a region of density N will be penetrated. These are given by

$$N = \frac{m}{4\pi e^2}\left(p^2 \pm \frac{pHe}{mc}\right).$$

Remembering that in actual experiments waves of a particular frequency are sent up to meet regions of increasing ionic density, the two split waves, according to the previous formulae, may be returned from three different levels and give three echoes. This, however, is of very rare occurrence because the right-handed component will be copiously returned from the region of lower value of N at a lower height given by

$$N=\frac{m}{4\pi e^2}\left(p^2-\frac{pHe}{mc}\right),$$

and only a very small fraction of it can possibly penetrate the region of this density and return from the region of higher density given by

$$N=\frac{m}{4\pi e^2}\left(p^2+\frac{pHe}{mc}\right).$$

(Toshniwal[2] reports, however, that he has been able to detect echoes with three split components.) Thus, in the case of magnetic splitting only two components are almost always observed. To measure the maximum ionic density the exploring wave frequency is gradually increased and the highest frequency for which neither of the echoes is returned is noted. This gives the maximum density according to the simple formula

$$N=\frac{mp^2}{4\pi e^2}.$$

It may be mentioned that magnetic splitting even into two components is observable only under favourable circumstances. On account of absorption, only one echo is ordinarily obtained.

In the above formulae we must know the charge and mass of the charged particles in order that N may be calculated. If we assume that the electrons constitute the charged particles and calculate, what we may call N_e, the number of equivalent electrons, then this number of electrons is replaceable by N_i ions of mass m_i each according to the relation

$$N_e=N_i\,.\,\frac{m}{m_i}\,.$$

In anticipation of what is discussed in subsequent articles we may say that in the case of the F region, the nature of polarization of the downcoming echoes, the phenomenon of magnetic splitting and the difference between the critical frequencies for the ordinary and extraordinary rays, clearly indicate that the charged particles are mainly electrons. In the case of the E layer also, since the phenomenon of magnetic splitting is exhibited, existence of sufficient electrons may be safely inferred : but a preponderance of heavier ions is not improbable, due to the presence of molecular oxygen and nitrogen. The high ionic density (10^8 to 10^9 ions per c.c.) demanded by the 'dynamo theory' of the variation of the earth's magnetic field can be satisfied if the charged particles are ions of, say, nitrogen or oxygen.

In actual experimental determination,[3] the apparatus employed is similar to that for equivalent height measurement, excepting that the transmitting and the receiving systems are adjusted for working over a large range of frequencies. In the receiver the tuning for different wavelengths is easily made by employing a number of different sets of coils. In the transmitter one may also use the same device for making the requisite changes of wave-lengths. The aerial system is also such that it is capable of oscillating over the range of frequencies chosen, which is usually from 20 m. to about 400 m. A convenient aerial system for the purpose is an inverted '*L*' type with a very long horizontal portion. The aerial may be excited in its harmonics for the short waves. For long waves it may be loaded with inductances. An essential point for successful determination of penetration frequency is that the change of frequency of the waves must be accomplished within the shortest possible time.

In recent times great improvements in this direction have been made by various workers. Gilliland [4] in America has designed an apparatus in which the tunings of the receiver and the transmitter are simultaneously made by a mechanical arrangement. Further, the whole tuning system being driven by a synchronous motor, the recording of the echoes and registering of the penetration frequencies are made automatically every half hour or one hour as desired.

Such automatic recording systems have been developed also by Naismith [5] and by Bainbridge-Bell [6] in England, the design of which is somewhat different from that of Gilliland.

2. *Structure of the Ionosphere.*

Recording of penetration frequencies, by the methods described in the previous section, soon revealed that the ionosphere is not a region of

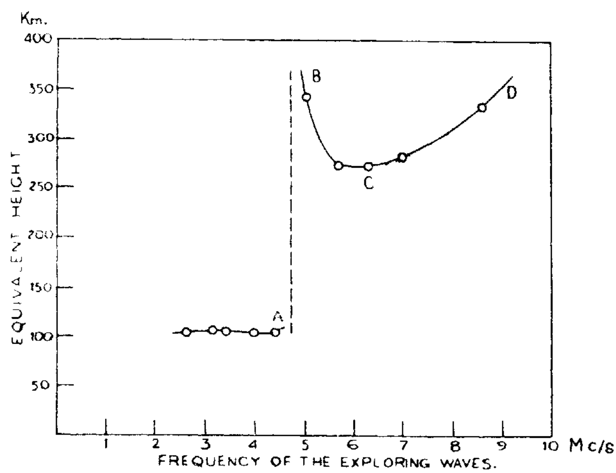

FIG. 23. Illustrating the variation of equivalent heights of the *E* and *F* layers with the change of frequency of the exploring waves. The *E* layer height does not change sensibly. The *F* layer height increases with increase of frequency. Note the abnormal increase of the equivalent *F* layer height near the critical penetration frequency of the *E* layer.

continuous ionization with a lower boundary more or less sharp as imagined by Kennelly and Heaviside, but that inside it there are regions of high and low ionic densities.

Fig. 23 which is a typical record of variation of equivalent height with variation of exploring wave frequency f shows that the recorded equivalent height increases discontinuously at a certain frequency. The obvious interpretation is that at this frequency the region reflecting the waves is penetrated and that reflections are being obtained from a higher region of greater maximum density. In Fig. 24 one notices four penetration frequencies which obviously

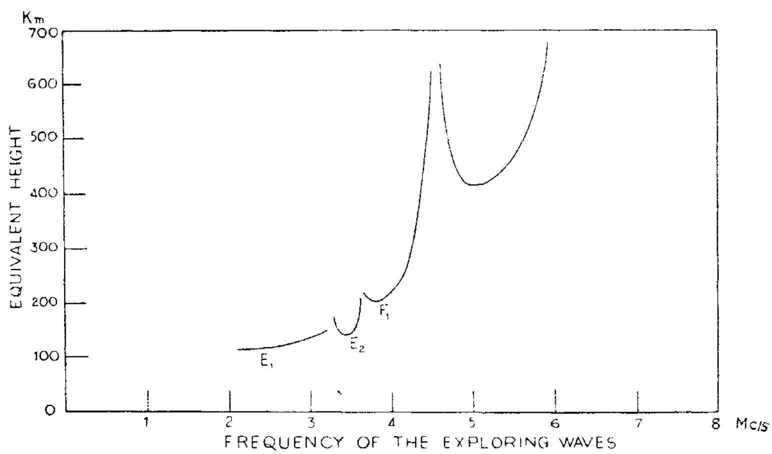

FIG. 24. Typical record by an automatic multi-frequency recorder of echoes from different ionospheric layers.

correspond to four regions of maximum density. Of these four, the lowest E_1 is what may be called the classical Kennelly-Heaviside region. The uppermost F_2, which was discovered by Appleton[7], is called the Appleton region. These two are the most persistent regions of the ionosphere and are found at all hours and in all seasons. The intermediate region E_2 and the shelf region F_1[8] are generally detected in the daytime and in summer. The average heights of the levels of maximum ionization of these four regions E_1, E_2, F_1 and F_2, are 90, 150, 180 and 250–400 km. respectively. Their electronic densities of course vary from hour to hour and also from season to season. The average values in England[8] in summer are (in equivalent electrons per c.c.) $1{\cdot}8\times10^5$, $2{\cdot}5\times10^5$, $3{\cdot}8\times10^5$, $6{\cdot}1\times10^5$ for E_1, E_2, F_1 and F_2 layers respectively.

Besides the above four regions, the existence of a lower absorbing region has been postulated by several workers.[9] Till lately, however, there had been a tendency to discredit its existence. In a recent note in *Nature* Mitra and Syam[10] report that they have been able to detect echoes from a virtual height of only about 55 km. Besides the echoes, which are of infrequent occurrence, they adduce other evidence in support of the existence in the daytime of an ionospheric region at this height. It has been found that during the daytime there is a frequency band which is reflected from the *E* region. The upper limit of the frequency band is due to penetration of the *E* layer and may be termed the 'penetration limit'. The lower limit, which may be termed the 'absorption limit', is due to absorption by the *D* layer which, on account of large collisional frequency, is strongly absorbing (Fig. 25). Mitra and Syam

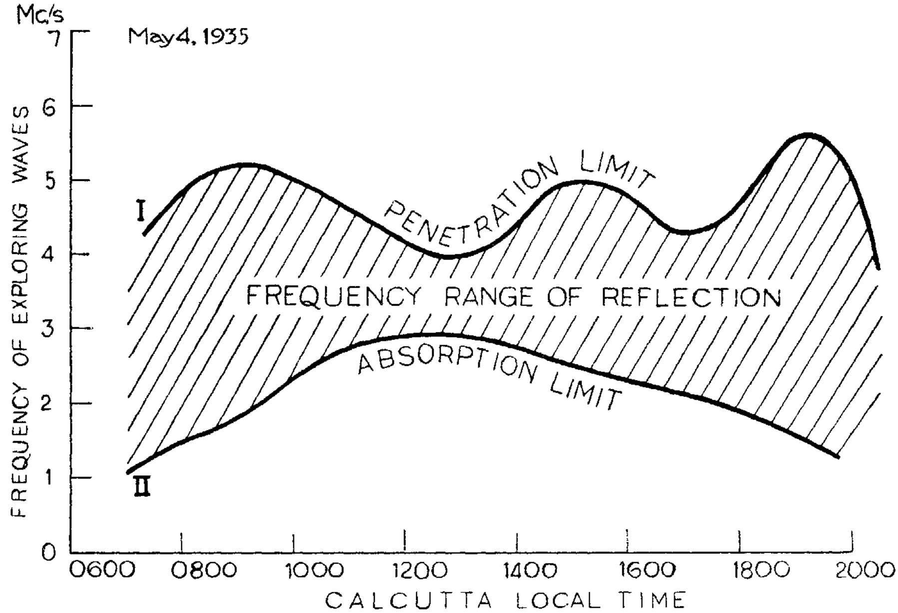

FIG. 25. Curve *I* gives the penetration frequencies of the *E* layer for various hours of the day. Curve *II* gives the frequencies for which echoes from the *E* layer first appear from the side of long waves, at different hours. The vertical distance between the two curves for any hour of the day gives the range of frequency for which echoes are obtained from the *E* layer.

also give reasons why the absorption limit cannot be due to formation of a diffuse *E* layer boundary, and lowering of the same in the daytime. It may be mentioned here that Bontch-Bruewitch in his ionospheric measurements in the polar regions observed complete cessation of echoes lasting sometimes for several hours. The picture of the echo pattern, before and after the disappearance of the echoes, was found to be the same. He suggests that this may be due to a *separate* absorbing layer produced at times below the *E* layer, probably at a height less than 65 km. (See ref. 9, § IV.)

One is tempted to locate the *D* layer in the region of the ozone layer. Recent investigations, however, show that the ozone layer is at a somewhat lower height.[11] In Fig. 26 an attempt has been made to roughly delineate the structure of the ionosphere during the daytime. It should be remembered that

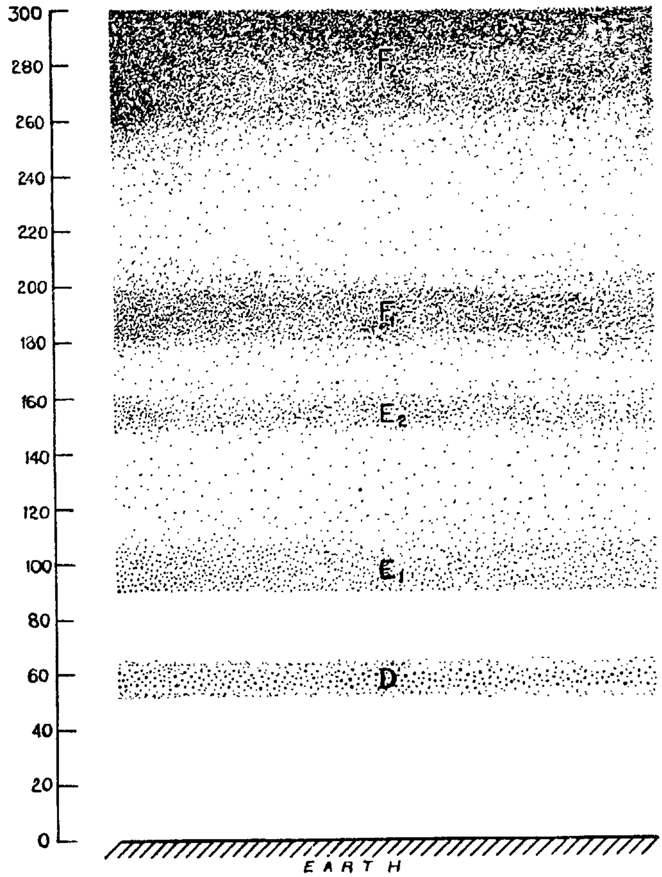

FIG. 26. Illustrating stratification of the ionosphere. E_1 and F_2 are always present. D, E_2 and F_1 are formed during the daytime.

the positions of the various layers as indicated are only approximate, and are liable to vary with the hour of the day, the season of the year and with the place of observation.

Mimno [12] reports having observed echoes from regions above F_2. These he calls G and H regions and estimates their heights as 600 km., and 1100–1800 km. respectively. These observations, however, lack confirmation from other investigators.

It should be remembered that the previously described method of ionic density measurement will fail to reveal the existence of a region of ionization the maximum density of which is lower than that of a region below it. Wave of a frequency which penetrates the lower region will obviously penetrate such a higher region as well.

In connection with density measurement, attempts have been made to estimate the thickness of the different regions. The only region regarding which the estimate can be made with any degree of definiteness is the lowermost E_1 region. Gilliland [13] in America and Ratcliffe and White [14] in England have observed a certain type of echoes from which one can make an estimate of the height of the upper boundary of the E_1 region. These echoes are interpreted to be due to the waves penetrating the E_1 region and suffering successive reflections from the bottom of F_2, top of E_1, and again bottom of F_2 and finally coming down by penetrating E_1. From a comparison of the group-retardation of this type of echo with that suffering single reflections from the bottoms of E_1 and F_2, estimate is made of the thickness of the E_1 as about 10 km.[13]

The origin of the different ionospheric strata is still not clearly understood. Besides the suggestions, which are mentioned in § X, sec. 4, on 'Sources of Ionization', other attempts have been made to explain the stratification.

Hulburt [15] suggests that after sunset, as a result of temperature diffusion and electromagnetic drift, the ionized regions separate into two banks of maximum ionization.

It has been proposed by Appleton [16] that the separation of F into F_1 and F_2 during the daytime may be due to thermal expansion and consequent upward drift of the outermost layers.

Again it has been suggested by some investigators that the distribution of ionization may really be continuous and that the mode of exploration of the ionosphere is responsible for the grouping of echoes into certain heights.

Hollingworth [17] thinks that the different orders of retardation may not indicate a stratification of the ionosphere and that the grouping of retardations into different orders may be due to inflexions on the group velocity curve.

Jonescu and Mihul [18] are of opinion that the electronic density varies with the height in a continuous manner, and that the jumping of echo may be explained on the assumptions that (*a*) the time interval between two collisions of an electron, in a given region of the atmosphere, is the same for all electrons, and (*b*) the electrons are animated with a velocity greater than that demanded by the thermal equilibrium between the electrons and the gas molecules.

These hypotheses do not, however, explain all the connected phenomena and we therefore refrain from entering into a detailed discussion of the same.

§ VI. Gradient of Ionization—Reflection Coefficient.

1. *Gradient of ionization from (P', f) curve.*

We have seen that the ionosphere is divided into various regions or layers of maximum ionic density. A question naturally arises as to the nature of the gradient of ionic density in these regions particularly near their boundaries. Experiments have been conducted for this purpose and the results obtained are interesting. For the E layer, the lower boundary has been found to be very sharp and it is also believed that the ionization rises exponentially near it. The observations on the other regions are not so complete but it has been found that the ionization gradient for F_2 is gradual.

A very simple method of estimating the gradient is to measure the equivalent heights for gradually increasing frequencies for a particular layer. If the gradient is steep, then it is found that the equivalent height remains practically constant even up to the penetration frequency. If, however, the gradient is low, then the recorded equivalent heights increase as one approaches the critical frequency. Another and a more elaborate method is to measure the reflection coefficient and also the equivalent height of the ionospheric layer for various angles of incidence of the exploring wave.

If the values of the equivalent heights are plotted against frequency then these are generally found to vary. A typical plot of the values for frequencies below and above the critical penetration frequency of the E layer is given in Fig. 23. It will be seen that for the E layer the height remains practically constant very nearly up to the penetration frequency. This is interpreted as the lower boundary of the E layer, having a very steep density gradient. The curve for the F layer (BCD) first shows a decrease in the equivalent height and then a gradual increase of the same. The portion of the curve from C to D shows that the equivalent height increases with increasing frequency. This is interpreted as the F layer, having a boundary which, unlike the E layer boundary, is very diffuse. The abnormally high values of the equivalent height, shown by the portion of the curve between C and B, are due to abnormally low group-velocity of the exploring wave packets, due to their frequencies being close to the critical penetration frequency for which the group velocity is zero in E layer.

It is possible to make a quantitative estimate of the density gradient by an extension of the above method.[1] Referring back to Fig. 17. T and R are the transmitter and the receiver respectively and TAR the equivalent path. TOR is the path followed by the wave. If the angle of incidence is greater than 30° it can be shown[2] that the corresponding optical path for TOR is equal to $TM+NR$, M and N being the feet of perpendiculars from O on TA and RA. Thus, if we can find out the corresponding optical path, we can determine the position of the point O, where the path of the ray becomes horizontal and the electronic density is given by

$$N=\frac{mp^2\cos^2\theta}{4\pi e^2}.$$

A series of such determinations for different wave frequencies will give values of N for different heights of O. The density gradient can thus be determined.

The value of the corresponding optical path $TM+NR$ can be determined by a method suggested by Appleton.[1]

2. *Reflection coefficient for typical ionization gradients.*

The experimental study of the relative intensities of the wireless echoes has been found to be a useful means of determining the nature of the gradient of ionization near the boundary and of the nature of absorption of the waves and, consequently, of collisions experienced by the vibrating electrons in the ionosphere. It may be recalled in this connection that a complete return of the incident energy from the ionospheric boundary is only possible in the case of a total reflection when the collisional friction is small and the boundary is sharp. Sharpness is here defined as the increase of the critical ionization density corresponding to the exploring wave frequency occurring within a distance from the boundary small compared with the wavelength. Since in the actual case the boundary is seldom sharp, and also loss due to collision is always present, the energy of the returned wave is always less than that of the incident wave, due partly to transmission and partly to absorption. Experiments have been carried out by F. W. G. White who has measured the relative intensities of the reflected wave at various hours of the day and studied how ρ varies with these two factors (see ref. 13, § IV).

In what follows, we first give some relations defining the reflection coefficient and also consider the behaviour of certain idealized types of ionization gradient such as linear, parabolic and exponential, and then describe the experiments regarding the measurements of reflection coefficient and consider the bearing of the results on the theoretical deductions regarding the various types of ionization gradient.

If E_1 is the intensity of the incident wave and E_0 the intensity of the reflected wave the reflection coefficient ρ is given by

$$\rho=\frac{E_0}{E_1}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

We have seen that the wireless echoes return from the ionosphere by a process of refraction rather than of reflection. But, for simplicity, we can always conceive the ionosphere as a bad reflector, and loss in energy of the reflected wave as caused by scattering by its rough surface. As a matter of fact, the above ratio can be represented as

$$\frac{E_0}{E_1}=Exp.\left[-\int kds\right] \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where k is the absorption coefficient of the medium through which the wave passes (ds is an element of its path and the integration is over the whole length of the path). So that

$$\rho=Exp.\left[-\int kds\right]=Exp.\ (-\eta), \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where η is the total attenuation of the waves.

It can be easily shown that the attenuation

$$\eta=\frac{\nu}{2c}\ (P'-P) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where P' and P are the equivalent and the optical paths of the sky wave and ν is the frequency of collision of the electrons and ions with gas molecules. From the above we find that the attenuation and the reflection coefficient are two ways of defining the same physical property.

In formula (4) we find that the reflection coefficient depends upon the group path and the optical path of the wave. These will evidently depend upon the angle of incidence, the frequency of the wave and also on the nature of the gradient. We will therefore first give the formulae for ρ for three idealized types of gradient.

(*a*) Linear gradient.

In this case

$$\left.\begin{array}{l}\log \rho = -\eta = -Ap^2\nu \cos^3\theta_0 \\ \text{and } P' = h_0 + \dfrac{mp^2 \cos^2\theta_0}{2\pi\alpha e^2}\end{array}\right\} \quad \dots \quad \dots \quad \dots \quad (5)$$

where A is a constant, ν is the collisional frequency, h_0 is the height of the lower boundary of the ionized layer above the ground and θ_0 is the angle of incidence which the waves make with the lower boundary of the ionized layer. This formula has been deduced by Appleton and Ratcliffe[3] and also by Martyn[4] on the assumption that the density of ionization N varies linearly with the height h, from the lower boundary upwards, given by $N=\alpha \,.\, h$, while ν remains constant.

(*b*) Parabolic gradient.

In this case

$$\log \rho = -\eta = -Bp\nu \cos^2\theta_0$$

$$\text{and } P' = h_0 + \frac{\sqrt{\pi m}p \cos \theta_0}{4\beta e},$$

where B is another constant. In this formula N varies as h^2 being given by

$$N=\beta^2 h^2$$

and has been deduced by the above authors, with the same assumption for ν.

(*c*) Exponential gradient.

In this case

$$\log \rho = -\eta = \frac{-2\nu_1 \cos \theta_0}{c\beta}\left(\frac{\sigma\gamma^{1/2}}{p \cos \theta_0}\right)^{2\beta H}$$

$$\text{and } P' = h_0 + \frac{1}{\beta} \log \frac{4 \cos^2\theta_0 \,.\, p}{\sigma^2\gamma},$$

where $N=\gamma.\ Exp.\ (\beta h)$, γ and β are two constants, ν_1 is the value of ν near the boundary and $\sigma^2=4\pi e^2/m$. It is to be noted that in deriving formulae for the linear and parabolic cases it is assumed that ν is constant within the range of penetration of the wave. In deriving the formula for the exponential case Martyn has taken the variation of ν into account by putting

$$\nu=\nu_1 \, . \, Exp. \, (-h/H),$$

where H is the height of the homogeneous atmosphere.

From the above formulae we find that the reflection coefficient ρ increases with the angle of incidence in all the three cases. But, whereas ρ decreases with the frequency in the linear and parabolic cases, it increases slightly with frequency in the exponential case.

3. *Measurement of reflection coefficient.*

We know that ρ, the reflection coefficient for a particular angle of incidence, is the ratio of the amplitudes of the emergent and the incident waves. Complications arise in actual measurement of ρ because it is not possible to make this measurement near the ionospheric boundary. We describe below the underlying principle of the methods by which it is possible to make an estimate of the reflection coefficient by comparison of the intensity of the direct wave with that of the reflected down-coming wave, as received near the ground.[3]

Let us put

- H_1 intensity of the magnetic vector of the down-coming wave at the receiver.
- H_2 intensity of the magnetic vector of the ground wave at the receiver.
- θ angle at which the indirect ray meets the ionospheric boundary.
- d_1 distance traversed by the indirect ray in going from transmitter to receiver.
- d_2 distance between the transmitter and receiver$=d_1 \cos\phi$.
- ϕ angle with the horizontal at which the indirect ray leaves the transmitting aerial.

If the ionosphere be considered a perfect reflector, then

$$H_1=\frac{C\cos\phi}{d_1},$$

where C is a constant depending upon the effective height of the aerial, the aerial current and the wavelength of the transmitter, assuming that the directional distribution of intensity of the transmitting aerial in a vertical plane follows the cosine law.

If, however, the boundary has a reflection coefficient ρ, then

$$H_1=\frac{C\rho\cos\phi}{d_1}$$

or

$$\rho=\frac{H_1}{H_2}\,(H_2)\,\frac{d_1}{C\cos\phi}.$$

Now H_2, the intensity of the ground wave, can be measured directly, while H_1/H_2 may be measured by any of the two methods described below ; ρ can thus be calculated from a knowledge of C, the transmitting aerial constant.

(*a*) Natural fading method of measuring H_1/H_2.

If we assume that fading is occurring at the receiving station, due only to change of relative phase between the direct and the atmospheric waves, then

$$\frac{2H_1}{H_2}=\frac{H_{max}-H_{min}}{H_{max}+H_{min}},$$

where H_{max} and H_{min} are the successive maximum and minimum values respectively of the resultant intensity of the two sets of waves.[3] We have $2H_1$, here instead of H_1, because the receiver is situated at the ground and the intensity there is reinforced by reflection, and becomes double if the reflection coefficient of the ground is taken to be unity.

It is possible to find out the value of ρ even when we do not know the aerial constants. But in this case we have to calculate the attenuation factor k for the ground wave after Sommerfeld's theory.
Thus

$$H_1=\frac{C\rho\cos\phi}{d_1}$$

and

$$H_2=\frac{Ck}{d_2}$$

$$\therefore\ \rho=\frac{H_1}{H_2}\cdot\frac{d_1}{d_2}\cdot\frac{k}{\cos\phi}.$$

It is to be noted that these calculations involve a knowledge of the height of the reflecting boundary which can be determined from a separate set of experiments.

(*b*) Frequency change method of measuring H_1/H_2.

In the above there may be some doubt as to the nature of the fading observed, whether it is 'phase fading' or 'intensity fading'. This uncertainty is obviated if we use the frequency change method as described in § IV, sec. 2. Here again, by noting the maxima and minima of the resultant signal intensity, due to the gradual change of frequency of the transmitter, we have

$$\frac{H_1}{H_2}=\frac{1}{2}\,\frac{H_{max}-H_{min}}{H_{max}+H_{min}}.$$

There is another advantage of this method over the previous one as the height of the layer is determined by the same set of experiments.

(*c*) Multiple echo method.

There is another method of determining ρ, by utilising the multiple echoes. This is easily done by the group-retardation method of Breit and Tuve. If F_1 is the intensity of the once reflected wave and F_2 is that of the twice reflected wave, it can easily be shown that

$$\rho\,\rho_g=\frac{2F_2}{F_1},$$

where ρ_g is the reflection coefficient of the ground. If ρ_g be taken roughly equal to unity as before, we have

$$\rho=\frac{2F_2}{F_1}.$$

This method has the advantage that neither the characteristics of the transmitting aerial nor the attenuation due to the ground need be known, as in the above methods.

4. *Ionization gradient from measurement of reflection coefficient.*

Measurements of the coefficient of reflection have been carried out in England by Appleton and Ratcliffe[3] by fading and by frequency change methods given above. Working on a wavelength of 491 m. for the first method and 400 m. for the second, they observed that the reflection coefficient did not show the expected increase in accordance with a linear, a parabolic or an exponential gradient, when the distance between the transmitter and the receiver was increased. As an explanation of this discrepancy they postulated the existence of a low-lying absorbing region, the so-called '*D*' region. It is obvious that for an oblique incidence the wave will have to traverse a greater distance through the '*D*' region and will therefore be more absorbed and may not show the increased intensity due to increased coefficient of reflection of the *E* layer boundary.

Martyn[4] has, however, recently collected the available measurements of intensities of the indirect waves over a range of wavelengths, and for distances from 25 to 1000 km., and has shown that these measurements do show a variation of reflection coefficient with distance. These variations, according to him, can be explained if the gradient of ionization is taken to be of exponential form. He has deduced an empirical formula for the reflection coefficient given by

$$\rho=3{\cdot}3\ \mathit{Exp.}\ [-\cos^2(\theta-35^\circ)\log_{10}\lambda].$$

He further shows that the observed variation of ρ with θ is incompatible with a linear or a parabolic gradient and that the assumption of an absorbing '*D*' region below the *E* layer does not remove the incompatibility.

It is seen here that the increase of coefficient of reflection with angle of incidence agrees with that derived from theory. We have seen in sec. 2 in the discussion of the various types of ionization gradients, that the equivalent height, as measured by the height measuring methods, should decrease with increase of ϕ for all the types of gradients described. In fact it is possible, at least theoretically, to determine the nature of the gradient from observations on the variation of the equivalent heights. Experiments have been conducted with this end in view by Appleton and Ratcliffe[3] and by Martyn, Cherry and Green.[5] None of the authors has, however, been able to find any marked change of equivalent height when the angle of incidence is varied. Appleton and Ratcliffe working on wavelengths 212, 330 and 400 metres, and for a range of θ from 0° to 30°, report only a very slight increase which is contrary to the expected result. Martyn, Cherry and Green working on a wavelength of 212 m. and for a range of θ from 0° to 72° could not detect any change. Appleton and Ratcliffe explain the slight increase observed by them as due to the intervention of a low-lying absorbing layer (D layer). As the angle of incidence increases, the ray has to pass through this region more and more obliquely and the equivalent path increases in value. Martyn, Cherry and Green being unable to detect any increase discredit the D layer hypothesis of Appleton and Ratcliffe. The constancy of the observed equivalent heights is explained as due to the extremely steep gradient near the boundary of the reflecting E layer. The gradient is certainly not linear or parabolic, but may be exponential. This view receives support from the theoretical deductions of Mary Taylor,[6] who shows that the collisional frequency ν is so high at the lower boundary of the E layer that, unless the density gradient is very steep, the waves are likely to suffer considerable absorption if they cross the boundary and enter appreciably into the ionized region before being reflected down.

It should be mentioned here that all the above observations were carried out at night when the existence of the D layer is extremely hypothetical.

§ VII. Collisional frequency of electrons and ions with neutral atoms and molecules in the ionosphere.

1. *From kinetic theory considerations.*

Since the absorption of radio waves passing through or returned by the ionosphere depends upon the collisional frequency of the electrons and ions, it is of interest to determine the value of collisional frequency at different ionospheric levels.

If we assume that the pressure of the atmosphere diminishes exponentially with height then it is possible to calculate ν from kinetic theory.[1]

Thus, the mean free-path l of an electron, when the gas pressure is P, is according to many authorities equal to $3{\cdot}2\times10^{-2}\frac{1}{P}$, where P is in mm. of mercury.

The pressure at any altitude h being given by $P = P_0 \cdot Exp. \left(-\frac{h}{H}\right)$, [$P_0$ is pressure at the surface of the earth, and H is height of the homogeneous atmosphere], it is easy to calculate the collisional frequency ν from the expression

$\nu = \frac{v}{l}$, where v is thermic mean velocity of the electrons or ions.

ν as calculated from this method has the value of the order of 10^5 per sec. at an altitude of 100 km.

We will see in the next section that determinations by the radio method also yield values of the same order.

2. *From the phenomenon of interaction of radio waves,—lower region.*

The recently discovered phenomenon of cross-modulation [2] as a result of interaction of radio waves offers a very interesting method of estimating the collisional frequency of electrons in the E layer of the ionosphere.

Cross-modulation may be described as the modulation of the carrier wave of one frequency impressing itself upon the carrier wave of another frequency. The effect was accidentally discovered by amateurs in 1933. It was found that while a receiver was tuned to a particular station the modulation of the carrier wave of another station of a different wavelength was heard in the background. The phenomenon has since been studied by various workers. Tellegen [2] seems to have been the first to suggest the ionospheric origin of the phenomenon. The explanation now generally accepted is that given by Bailey and Martyn [3] and is as follows :—

When an electromagnetic wave passes through the ionosphere the normal agitation of the charged particles, due to thermic velocity, will be increased by the action of the external electric forces impressed by the wave. The amplitude of this agitation and consequently the collisional frequency will depend upon the amplitude of the wave. If this wave is modulated the collisional frequency and the absorption coefficient of the region through which the wave is passing will undergo variations in accordance with the frequency and coefficient of modulation.

Now, if the carrier wave of another wavelength passes through this ionospheric region then, obviously, owing to the variation of absorption due to the action of the first wave, the coefficient of reflection of this other wave will undergo similar variations. In other words the second carrier will be modulated to the modulation frequency of the first carrier.

Detailed calculations by Bailey and Martyn, taking into account the effect of the earth's magnetic field, show that the coefficient of modulation M' produced by the unwanted carrier wave is given by

$$M' = \frac{4 \cdot 6 \times 10^{16} (p^2 + \omega^2) C_0 \nu^2 Z^2 M}{(p^2 - \omega^2) \sqrt{f^2 + G^2 \nu^2 + CQZ^2}} .$$

p angular carrier frequency of the unwanted signal.
f angular modulation frequency of the unwanted signal.
Z R.M.S. value of the electric intensity in the ionosphere due to unwanted signal.
M coefficient of modulation of the carrier of the unwanted station.

$$\omega=\frac{H_p e}{m},$$

where H_p is the component of the earth's magnetic field perpendicular to both the electric vector and the direction of propagation of the wave from the unwanted station.

ν collisional frequency in the absence of external electric forces.

$$\left.\begin{array}{l} G=26\times 10^{-4} \\ C=1{\cdot}2\times 10^{31} \end{array}\right\}\text{ constants.}$$

C_0 a factor depending on the frequency of the wave. the length of the path, and the density and gradient of ionization in the ionosphere.
Q a factor depending on p, ω and ν.

The formula shows, if CQZ^2 is neglected in comparison with $(f^2+G^2\nu^2)$, that the impressed modulation is approximately inversely proportional to f.

Experimental observations carried out by van der Pol[4] show that the hypothesis explained above is substantially correct.

A receiver was tuned to the carrier of a particular station and the frequency of modulation of another station which was to act on the carrier of the first was gradually varied. The coefficient of the impressed modulation when plotted against the frequency of the modulation of the unwanted station gave the curve as shown in Fig. 27.

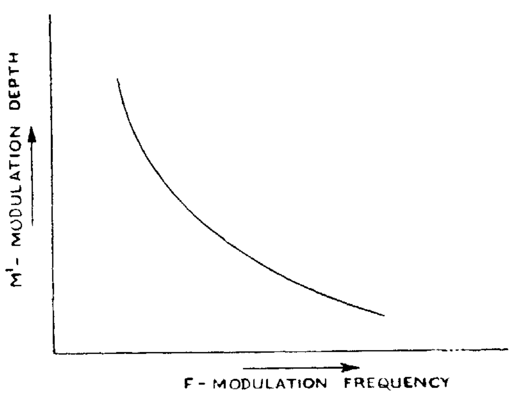

Fig. 27. Cross-modulation by ionosphere. The curve shows relation between modulation frequency of unwanted signal and coefficient of modulation impressed on the carrier of the wanted signal to which the receiver is tuned.

Results of an actual experiment from which ν was determined by Bailey and Martyn[5] are given below :—

Modulation impressed on the wanted signal (which had been sent unmodulated) at the modulation frequency of 100 cycles per sec. of the unwanted signal was found to be 7·5%.

Modulation depth of the wanted signal as measured at the receiving station at the modulation frequency of 800 cycles per sec. of the unwanted signal was found to be 1·2%.

From these results we get

$$\frac{1{\cdot}2^2}{7{\cdot}5^2}=\frac{(2\pi\times 100)^2+6{\cdot}8\times 10^{-6}\nu^2}{(2\pi+800)^2+6{\cdot}8\times 10^{-6}\nu^2}$$

from which ν is found to be 2×10^5 collisions per sec.

The value of ν thus obtained agrees with that found from the kinetic theory. For further details the original papers may be consulted.

3. *From measurement of reflection coefficient and (P′, f) curve,—upper region.*

It is possible to measure the collisional frequency in the F layer from observations on the reflection coefficient of the waves returned from the layer. Eckersley[6] has carried out the measurements and has obtained a value of $3{\cdot}6\times 10^3$ at an estimated height of about 265 km.

The method consists in measuring simultaneously the equivalent height and the relative intensity of the reflected wave at a time when the former is changing.

If we assume that the absorption of the wave depends upon the time spent by the wave in the ionosphere and upon the total number of collisions of the electrons during this time, then remembering that the reflection coefficient ρ is the ratio between the intensities of the reflected and the incident wave we have

$$\rho = Exp.\left[-\frac{t\nu}{\beta}\right] \text{ where } \beta \text{ is a constant.}$$

If we take magnetic splitting into account then for the extraordinary ray which is the one on which observations are made,

$$\rho = Exp.\left[-\frac{t\nu}{\beta(1-\tau)}\right], \text{ where } \tau=\frac{eH}{2\pi m\nu}.$$

Thus

$$\log \rho = -\frac{t\nu}{\beta(1-\tau)}.$$

In other words there is a linear relation between $\log \rho$ and the measured equivalent height which is proportional to t.

If a number of measurements is made on the relative intensity and the group-retardation time t of the exploring wave when t is changing, it is possible to calculate ν.

Farmer and Ratcliffe[7] have also carried out similar experiments but the value of the collisional frequency found by them comes out to be $1{\cdot}6\times10^{3}$. This lower value is ascribed to the fact that owing to absorption by the E layer the intensity of the reflected wave as observed by Eckersley was less.

According to Appleton[8] the method of calculation as employed above is not quite rigorous. One has to take into account both the group and optical paths of the waves. We have seen in § VI, sec. 2, that if ρ is the reflection coefficient and P' and P the group and optical paths of the waves respectively then

$$\log \rho = -\frac{\nu}{2c}\,(P'-P).$$

If the values of ρ and P' are determined experimentally for two radio-frequencies f_1 and f_2 then we have

$$f_1 \log \rho_1 - f_2 \log \rho_2 = \frac{\nu}{2c}\left[\left(P'_2 f_2 - P'_1 f_1\right) - \left(P_2 f_2 - P_1 f_1\right)\right].$$

Knowing that

$$\int_{f_1}^{f_2} P'df = P_2 f_2 - P_1 f_1$$

and from the P', f curve AB we get $(P'_2 f_2 - P'_1 f_1) - (P_2 f_2 - P_1 f_1) =$ the shaded area X. (Fig. 28.)

$$\therefore \frac{\nu}{2c} = \frac{f_1 \log \rho_1 - f_2 \log \rho_2}{\text{area } X}.$$

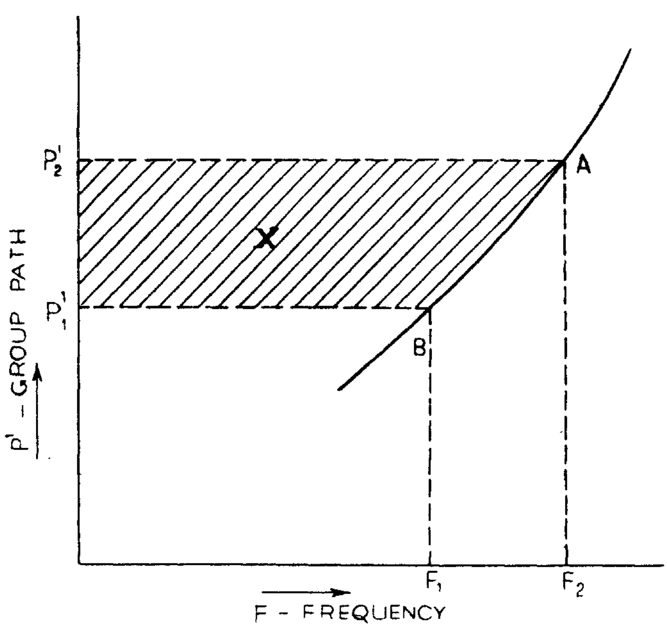

FIG. 28 Illustrating the relation between group-paths and signal frequencies.

Thus ν can be rigorously determined from the measurements of the reflection coefficients and group-paths which are proportional to group-retardation times, for two wireless frequencies.

§ VIII. Intensity of Earth's Magnetic Field in the Ionosphere. The Phenomenon of Magnetic Splitting.

We have seen in § III that in an ionized medium influenced by an external magnetic field a single wave is split up into two components travelling with different velocities. The splitting can be considered from two points of view ; first, if the medium is such that the ionic density gradually increases in the direction of propagation of the wave, then the reflection of the two split components will take place at different points of the medium depending on the values of N at these points. If the frequency of the exploring wave is greater than the gyro-frequency and the propagation is against the magnetic field the extraordinary ray will be the first and the ordinary the second to be so reflected.

Secondly, if we consider the medium to be of limited thickness and of insufficient ionic density to cause reflection of either the extraordinary or the ordinary, then of the two emergent split waves, the extraordinary will be more retarded than the ordinary. Both these types of splitting are observed in wireless echoes obtained in connection with group-retardation experiments.

Splitting of the first kind produced by variation of ionic density, from the bottom upwards, of a particular ionospheric layer E or F is known as 'stratification splitting'[1]. The splitting of the second kind produced by piercing of a lower layer by a wave group and reflection of the same by a higher layer is known as 'group-retardation splitting'. The two types of splitting can be distinguished by the polarization of the two component echoes. In the stratification case, in the northern hemisphere, the long delayed one is the ordinary component and polarized left-handed, while in the group-retardation case the short delayed one is the ordinary component and polarized left-handed. It should be mentioned that the first type of splitting is comparatively more frequent in the F layer. This is because the density gradient of the F layer is much less sharp than that of the E layer.

The phenomenon of stratification splitting has been utilised by Appleton[2] for estimating the intensity of the magnetic field in the ionosphere. The experimental observations from which the intensity of the magnetic field is deduced are generally on the F layer because, on account of the steep ionization gradient of the E layer, stratification splitting is difficult to be observed. It is to be recalled that in discussing the magneto-ionic theory we distinguished between two classes of propagation, the quasi-longitudinal and the quasi-transverse, depending upon the critical collisional frequency

$$\nu_c = \frac{p_T^2}{2p_L}.$$

The conditions under which actual experiments were carried out were such that the wave propagation in the F layer was of the quasi-transverse type.

The experiment consists in observing the disappearance of the two split components for the F layer reflection, by gradually increasing the frequency. The long delayed ordinary component is the first to penetrate the region and to cease to be reflected. On further increase of the frequency the short delayed extraordinary component also disappears. The two critical frequencies f_0 and f_e thus measured allow one to calculate the magnetic field.

Remembering that in actual experiments the exploring wave frequency is greater than the gyro-magnetic frequency

$$f_H = \frac{He}{2\pi mc}$$

the conditions for reflection of the ordinary and the extraordinary wave frequencies f_0 and f_e are given by,

$$N = \pi me^2 f_0^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(ordinary)}$$

and $$N = \pi me^2 (f_e^2 - f_e f_H) \quad \ldots \quad \ldots \quad \ldots \quad \text{(extraordinary).}$$

Equating these two expressions we have

$$f_0^2 = f_e^2 - f_e f_H$$

$$\therefore\ H = \frac{2\pi mc}{e} \frac{(f_e^2 - f_0^2)}{f_e}.$$

In England Appleton made 200 observations. The average value of H has been found to be ·42 gauss. The value of H at the ground is ·467 gauss. Thus the average value of H in the ionosphere at the height of the F region is about 10% less than its value at the ground. If the variation of H with height above the ground is taken into account after Schmidt's formula :

$$H = H_0 \left(1 - \frac{3h}{R}\right).$$

where H_0 is the intensity of the magnetic field at the ground, h the height above and R the radius of the earth. The value obtained by radio methods is found to give the right order of magnitude of H.

As already mentioned, the above equation is for the quasi-transverse type of propagation, i.e., when f is greater than the critical collisional frequency. If the propagation be of the quasi-longitudinal type, i.e., if f is less than the critical collisional frequency :

$$N = \pi me^2 (f_0^2 + f_0 f_H) \quad \ldots \quad \ldots \quad \ldots \quad \text{(ordinary)}$$

and $$N = \pi me^2 (f_e^2 - f_e f_H) \quad \ldots \quad \ldots \quad \ldots \quad \text{(extraordinary).}$$

Equating these two expressions we have

$$H = \frac{2\pi mc}{e} (f_e - f_0).$$

§ IX. Polarization Measurements.

We have already seen in § III, sec. 4 that the polarization of down-coming waves should have certain characteristic features depending upon the intensity of the earth's magnetic field and its direction with respect to the wave normal, the wave frequency and the collisional frequency of the electrons and ions in the ionosphere. Some of these characteristics can be studied experimentally and it has been found that the deductions from the Appleton-Hartree formula are in substantial agreement with the observed results.

The polarization properties which have been studied may be thus summarised :—

(1) For short waves below 200 metres, the two split components are of nearly equal intensity and are circularly polarized in opposite senses. The difference in the absorption index is very small.

(2) For medium waves the polarization is approximately circular ; there is differential absorption and only one of the two components is received, the other being more or less completely absorbed. In the northern hemisphere, where the wave normal for vertical downward propagation makes an acute angle with the magnetic field, the down-coming wave is polarized in the left-handed sense : *vice versa* in the southern hemisphere.

(3) For very long waves, the received down-coming wave is plane polarized, being the resultant of the two split components which are of nearly equal intensity and of small path difference.

All the above features are explainable by the complete Appleton-Hartree formula.

Various types of apparatus have been designed and employed for the purpose of studying the polarization of the down-coming ray. All these devices are more or less of an elaborate nature employing a number of aerials which receive the signal simultaneously. It is not possible to give detailed descriptions of these apparatus in the body of a report like this and we will content ourselves by very briefly referring to the principles on which these work.

The earliest measurements on polarization are due to Appleton and Ratcliffe.[1] The aerial system of their apparatus consisted of three loop aerials, one being in the plane of propagation and the other two making angles of 45° in either direction. The experiment consisted in changing the wavelength of the transmitter as in the wavelength change method described in § IV and recording successively the fringe systems produced in the three loops. These observations sufficed to give the constants of ellipticity. As mentioned before, the experiments ($\lambda=400$ m.) showed that the ratio between the normal and abnormal components was approximately unity, indicating that the down-coming wave was approximately circularly polarized. Further, from phase difference measurements it was found that the polarization was in the left-handed sense.

A type of radio polarimeter has been developed by the staff of the Radio

Research Station, Slough,[2] in which two similar loop aerials are fixed at right angles. The signals received by the two loops are separately amplified, without any relative change of phase, and are applied to the two pairs of deflecting plates of a cathode-ray oscillograph. It is possible to study with this apparatus separately the polarizations of the ground wave and of the echoes in group-retardation method.

An ingenuous type of polarization apparatus called *circularly polarized receiver* has been designed by Ratcliffe and White.[3] The apparatus consists of two loop aerials at right angles to one another. The tuning inductances connected with the loops are mutually coupled in a suitable manner. The resultant received signal is amplified and applied to a cathode-ray oscillograph. It is possible with this device, by suitably altering the sense of coupling of the two inductances, to accept only one of the two polarized components and reject the other. The circularly polarized receiver can be employed for studying the split echoes in group-retardation experiments.

§ X. Temporal Variation of Ionic Density—Sources of Ionization.

We have seen in § V how the maximum equivalent electron content of the ionospheric regions can be experimentally determined by a study of the (P', f) curves. Such measurements at different hours of the day and night and in different seasons yield valuable information regarding the nature of the agencies responsible for producing and maintaining ionization. Before proceeding to a detailed description of the nature of the temporal variations we may mention at once that it has been definitely established that the ultraviolet radiation from the sun is the chief agency responsible for causing ionization. It has been found that there are types of variation of the ionization which are correlated with the diurnal and the seasonal variation of the altitude of the sun, and with the long period solar activity cycle of 11 years. These variations, the periodic nature of which leaves little doubt regarding the solar origin of the ionization, we will call ' normal ' variations. Besides these there are other variations of a more or less erratic nature the origin of which may be traced to other physical agencies like thunderstorms, magnetic storms and meteoric showers. These variations we will call ' abnormal ' variations. In what follows we will first deal with the so-called normal variations and, as a first step, describe briefly the results of Chapman's theoretical investigation on the ionizing effect of monochromatic solar radiation in the earth's atmosphere.

1. *Ionization caused by monochromatic solar radiation in the earth's atmosphere.*

Chapman[1] assumes that the density of the atmosphere varies exponentially with height. A definite fraction of the radiant energy of the sun falling upon it is absorbed causing ionization. The ionized products recombine with one another and are assumed not to diffuse away. Taking into account the variation of the rate of ionization due to the rotation of the earth and assuming a constant recombination coefficient, the distribution of density of the ionized products is determined. The rate of production of ions, I per sec. per c.c. is found to be given by

$$I/I_0 = Exp.\left[1-\frac{h-h_0}{H}-Exp.\left(-\frac{h-h_0}{H}\right)\cdot f\left(\frac{a+h_0}{H}+z, \chi\right)\right].$$

χ zenithal distance of the sun.

I_0 value of I when $\chi=0$.

h height of the point of observation.

h_0 height of the level above the equator at noon, i.e. $\chi=0$, where the rate of production of ions is a maximum.

$H=\frac{KT}{mg}=8{\cdot}4$ kilometres for m corresponding to the masses of oxygen and nitrogen molecules.

T, temperature of 300° K.

K, Boltzmann's constant.

g, acceleration due to gravity.

a radius of the earth.

$$f(x, \chi)=x \sin \chi \int_{\lambda=0}^{\lambda=\chi} Exp.\left\{x(1-\sin \chi/\sin \lambda)\right\} \operatorname{cosec}^2\lambda d\lambda,$$

λ angle of incidence of the sun's rays at any point on the earth's surface and $z=\frac{h-h_0}{H}$.

Let α be the recombination coefficient of the ions and N_0 the maximum steady value of the ionic density N for I_0 the maximum value of I. Then

$$N_0=\sqrt{I_0/\alpha}.$$

The methods of obtaining numerical values of N/N_0 have been developed by Chapman[1] and by Millington.[2] We will see in the following sections that records of variation of the ionization content over 24 hours closely resemble the variation in the lower layers as calculated from Chapman's formula. Further, similar agreement is found between observed and calculated values for the variation of ionization from summer-noon to winter-noon due to variation in the altitude of the sun.

2. *Normal variations of ionization.*

(*a*) Diurnal variation.

Several authors have measured the variation of the maximum equivalent electron density over 24 hours of the lower E region on selected days in winter, summer and spring. These measurements show that the ionization rises rapidly immediately before and during the ground sunrise and then slowly till noon when the maximum of ionization is reached. It then falls off steadily till sunset and then rapidly until, a few hours later, a low value is attained which is maintained during the night. At the approach of ground sunrise the ionization again increases rapidly. The general nature of these diurnal variation curves shows marked correlation with sunrise and sunset and strongly resembles the theoretical curves drawn after Chapman. The curves in Fig. 29 depict the observed diurnal variation of the maximum

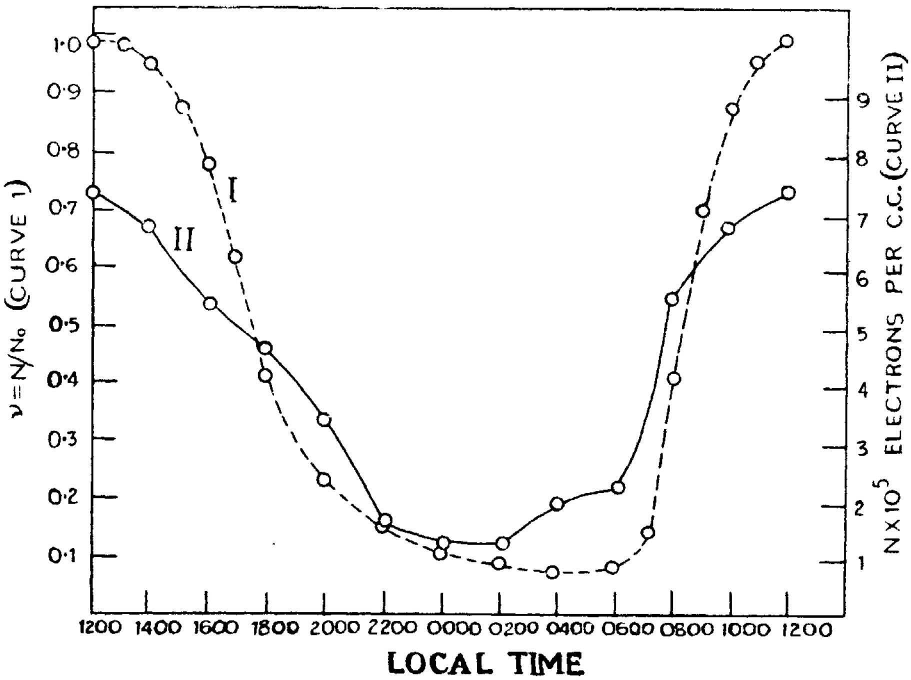

FIG. 29. Illustrating hourly variation of ionization content. Curve *I* is calculated after Chapman. Curve *II* is from actual observations. Both curves are for summer solstice, Calcutta.

equivalent electron density at Calcutta[3] and the relative variation of the same as calculated from Chapman's theory.

It should be mentioned here that there are regions on the surface of the earth above which the diurnal variation of ionization of the lower layers may not follow Chapman's theory. Such regions are those situated at high latitudes. We shall see presently that this anomaly is due to other predominant factors besides the ultraviolet radiation from the sun being at work.

The diurnal variation of ionization of the F_1 and F_2 layers has been studied by Kirby, Berkner and Stuart[4] in Washington, and by Appleton and Naismith[5] at Slough. It has been observed that the ionization of the F_1 layer follows the diurnal variation of the altitude of the sun in the same way as that of the E layer. Similar variation of the F_1 layer ionization has also been observed by T. R. Gilliland.[6] In the case of the F_2 layer the ionization varies with the altitude of the sun but the variation does not show any similarity with the predictions of Chapman's theory. The diurnal maximum is attained in summer not at noon but near about sunset. Appleton and Naismith have observed a second maximum in summer at about 1000 G.M.T. In winter, the diurnal maximum occurs near noon as in the case of the E and F_1 regions.

(*b*) Seasonal variation.

A much more interesting study of the variation of ionization is to make daily or weekly measurements over long periods at a particular hour of the day. The measurements when plotted show in a striking manner how the variations follow the variation of the sun's altitude. Recently a large number of records of such observations has been published. We reproduce below plot of such a series of observations on the E region made by Appleton and Naismith[7] in England during the years 1931-32 (Fig. 30). The curve shows

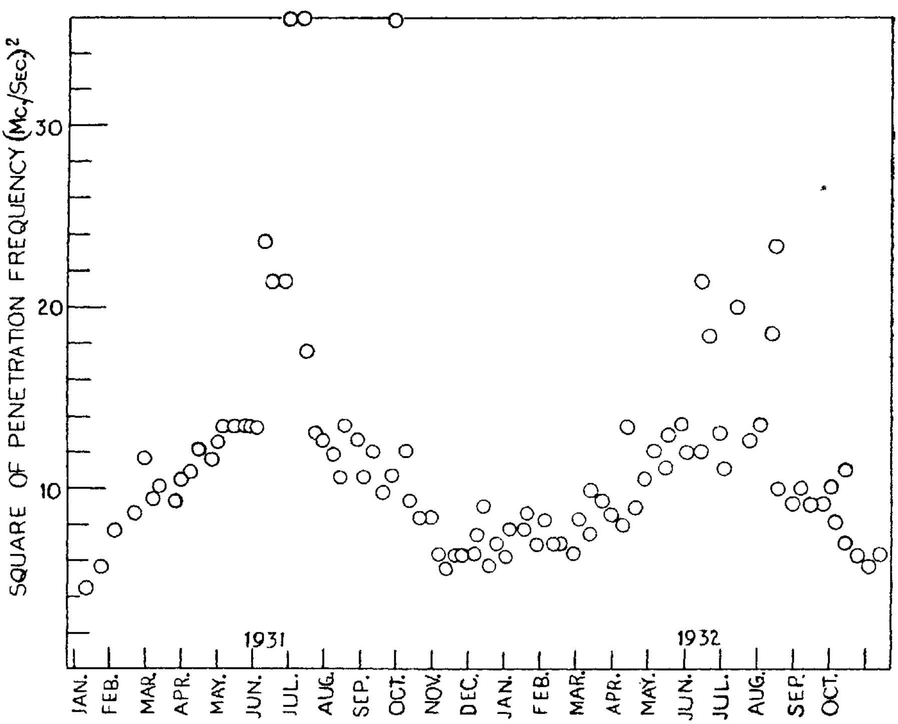

FIG. 30. Illustrating the seasonal variation of noon ionization of the E region (England).

that the ionization density is greater in summer than in winter. Disregarding the occasional abnormally high values of ionization on summer days but following the general trend of variation Appleton and Naismith[5] have deduced from their observations that the ratio of the summer-noon ionization to winter-noon ionization of the E layer is of the order of 2·2. Calculations after Chapman's theory, taking into account both the recombination of ions and the attachment of electrons to neutral atoms and molecules, show that for the case of the particular latitude at which these observations were made, the ratio must lie between 1·84 and 3·4. Of late, fresh daily observations by these authors[5] of noon-ionization over a period of one year (Dec., 1933—Nov., 1934) show that the ratio of summer-noon to winter-noon ionization of the E layer is 1·8. This indicates that the recombination of electrons with positive ions is the predominant cause of the lowering of N value and that the effect of attachment of electrons with neutral atoms and molecules must be negligible.

The seasonal variation of ionization of the F_1 layer has been systematically studied by several observers both in America and in England. The observed results show that the seasonal variation of noon-time F_1 ionization is almost the same as that for the E layer, the annual maximum being attained in summer and the minimum in winter.

The variation of noon-time ionization of the F_2 layer is depicted in Fig. 31

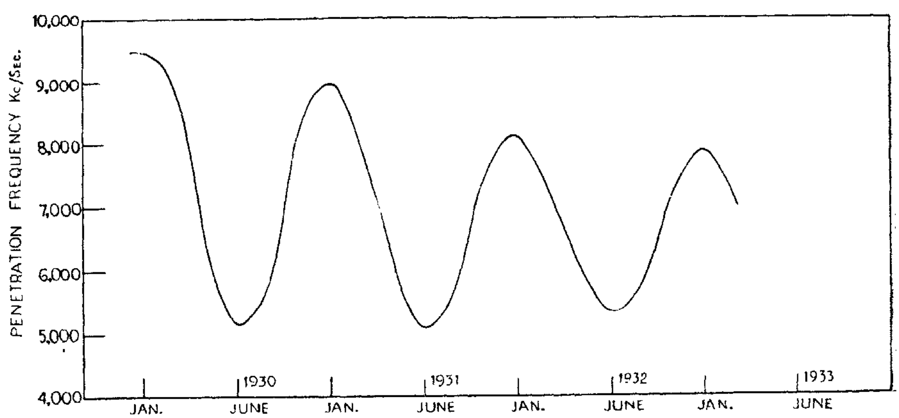

FIG. 31. Seasonal and year to year variation of the average values of F_2 layer ionization.

as obtained by Kirby, Berkner and Stuart[4] over the period 1930–1932, the measurements being made at Washington. The curve shows regular seasonal variation. The remarkable fact about the curve is that the maxima of noon-time ionization are attained not in the summer months, as in the case of E and F_1 layers, but are attained in winter when the sun's altitude is lowest. This fact is further corroborated by later observations made by Schafer and

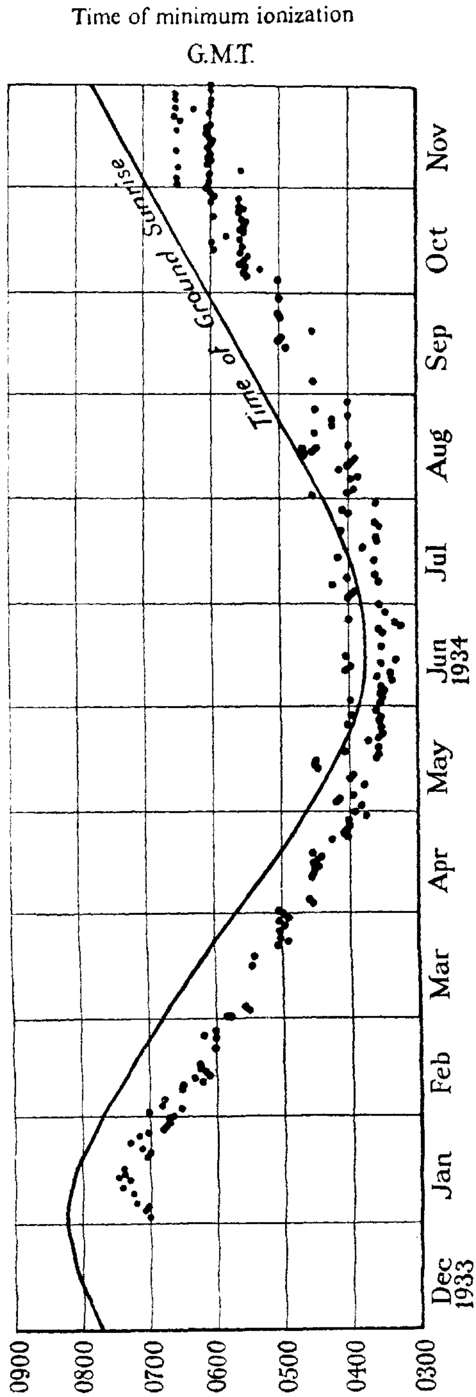

FIG. 32. Illustrating the seasonal variation of the hour of minimum ionization as compared with that of g round sunrise. Note that the minimum nearly coincides with sunrise in summer while it occurs about one hour earlier in winter.

Goodall.[7a] The correlation of the ionization with the sun's altitude, though of an inverse nature, still points to the fact that the F_2 ionization is closely related to solar radiation. There are two hypotheses to explain the attainment of maxima in winter instead of in summer. The one given by the authors of this curve is that in the summer months at noon the sun's altitude is high and the observations are vitiated by absorption, i.e. on account of absorption limitation the waves fail to be returned before the critical penetration frequency is reached, though this critical frequency is actually greater in summer than in winter. The other view urged by Appleton[8] is that this apparent abnormality is due to the fact that the molecular temperature of the outermost layers of the atmosphere is greatly increased in summer, causing a lowering of ion-production on the one hand and volume expansion on the other. The maximum ionization density is thus effectively reduced. In support of this Appleton and Naismith[5] have found from observations on the time of the early morning minimum of ionization that the minimum occurs near about ground sunrise in summer but about one hour earlier in winter (Fig. 32). This shows that the rate of increase of ionization with the approach of sunrise is greater on a winter morning than on a summer morning. The actual rate of increase of N_{max} in winter is about $2\frac{1}{2}$ times that in summer. From these anomalous variations of ionization, Appleton and Naismith conclude that the temperature of this region in summer is at least as high as $1200°K$.

(*c*) Variation with latitude.

The variation of ionization with latitude seems again to agree, from available data, with the predictions of Chapman's theory. Observations made at Tromso (lat. 69° 39·8′ *N*; long. 18° 56·9′ *E*) clearly demonstrate that the daily maximum of the ionization content for both *E* and *F* layers is less at Tromso than at London (lat. 51° 30′ *N*) under normal conditions.[9] Again Appleton and Naismith[7] have found on comparison that the daily maximum ionization of the *E* layer at Washington (lat. 38° 48′ *N*) is greater than that at London. Observations at Huancayo[10] (lat. 12° 02·7′ *S*) show that the average F_2 ionization is much greater at Huancayo than at Washington for the corresponding local summer-noon. It is evident from the above facts that ordinarily the ionization decreases as we proceed away from the equator towards higher latitudes.

(*d*) Variation with solar cycle.

It is well known that the 11-year cycle of sun-spot activity is closely associated with the terrestrial magnetic activity of the same period. It is thus natural to suppose that the average equivalent electron density of the ionosphere will also show a variation corresponding to this cycle. Though the observations as yet made are insufficient it seems fairly certain that such a correspondence does exist. Fig. 30 shows that the average summer-noon ion content of the E layer in London is 10–15 per cent. greater than in the succeeding year. This diminution may be ascribed to the general depression in the sun-spot activities in the 11-year solar cycle of which the expected minimum had been in 1933-34.

The curve in Fig. 31 obtained by Kirby, Berkner and Stuart demonstrates that the F_2 ionization distinctly falls off as this minimum is approached.

Observations by T. R. Gilliland[6] show that the average ionization at night of the F layer in September, 1934 was greater than that in September, 1933. This shows that the average ionization had perceptibly increased after the minimum sun-spot activity had been passed over.

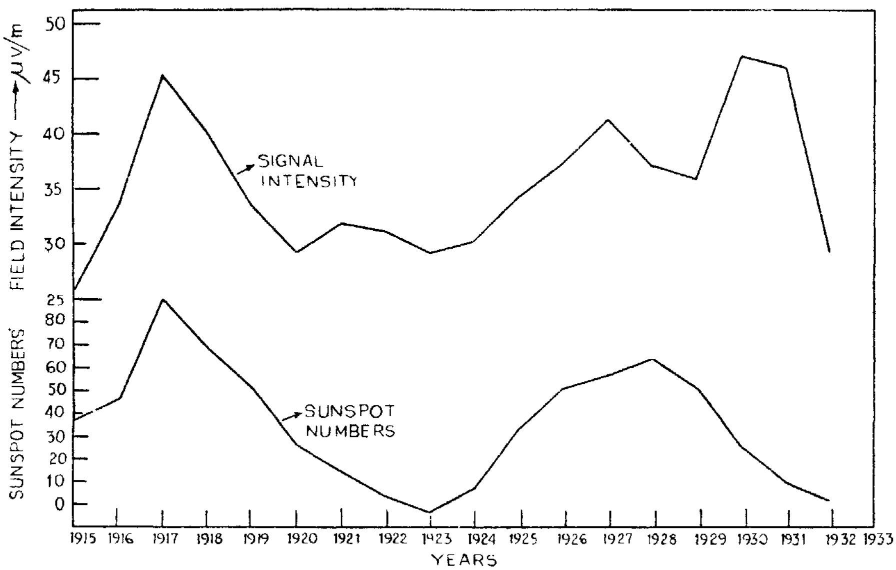

FIG. 33. Illustrating correlation between the annual average field intensities of transatlantic daylight signals and sunspot numbers.

Records of signal intensities of long distance radio reception show a correlation between the signal intensity and the sun-spot numbers. The upper curve of Fig. 33 depicts the annual average field intensities of trans-atlantic daylight signals as received at Washington[11] for the period 1915–1932. The lower one in the same figure gives the variation in the sun-spot numbers during the same period. The remarkable similarity between the two curves points to the conclusion that, since long-distance long-wave communication is essentially a phenomenon of reflection from the lower boundary of the Kennelly-Heaviside layer, the ionic condition of this layer must be closely correlated to the 11-year sun-spot cycle. It should be mentioned here that an increase of sun-spot activity does not necessarily mean an increase of the signal strength. In fact, records show that increased sun-spot activity is sometimes accompanied by a decrease in signal strength. That this is possible is easily seen from the fact that a reduction in the effective height of the reflecting layer, due to an increase in its ionization, may either strengthen or weaken the reflected signals depending upon the wavelength and the distance of the receiver from the transmitting station. The actual relationship between the signal strength and the solar activity, direct or inverse—is not of particular consequence. The fact that long-distance signal intensity is known to be connected with the upper atmospheric ionization is enough to establish the latter's correlation with the sun-spot cycle.

3. *Abnormal variations of ionization.*

We have mentioned in the beginning of this article that, besides the regular variations of ionization due to the variations of the sun's altitude, there are other variations of ionization, the origin of which cannot be connected with ultraviolet solar radiation. Some of these abnormal variations show some sort of regularity in their hours of occurrence and may be traced to such agencies as thunderstorms, magnetic storms and meteoric showers. We now proceed to describe these so-called abnormal variations.

(*a*) Evening concentration.

It has been very often observed by various workers[12] that the ionization of the *E* layer, instead of gradually falling off after sunset, maintains a steady value or even increases. The maximum of this increase is very often found to occur a few hours after sunset and may be at times as much as 300 per cent. of the normal sunset value. Two explanations have been offered of this phenomenon. One is that it is connected with thunderstorms which are of most frequent occurrence in the afternoon, and the other is that the increase is due to bombardment by charged particles shot off from the sun arriving on the dark side of the earth by a curved path. The nature and effect of both these agencies will be presently discussed in the following sections.

(*b*) Increase of ionization during hours of low barometric pressure and during thunderstorms.

The ionization density of the *E* layer has been observed to increase abnormally during hours of low barometric pressure and also during and after thunderstorms which latter are well-known to be correlated with the former. This phenomenon seems rather surprising at first sight, since, it would appear unlikely that meteorological disturbances, occurring within the tropospheric region—the upper boundary of which does not exceed 12 to 14 km.—should affect conditions of the ionosphere, the lower boundary of which is at a height of about 90 km. Statistical analysis of observations by Ranzi,[12] Appleton and Naismith,[7] and Ratcliffe and White,[12] however, proves beyond doubt that there exists a strong correlation between tropospheric disturbances and the variation of ionospheric conditions. The explanation of this is to be found in the charged thunderclouds [13] which are produced during low barometric pressure. The exact nature of the influence of thunderclouds on the ionization of the *E* layer is not yet definitely understood. The probable modes by which the ionospheric conditions may be affected are as follows :—

If the upper part of a thundercloud is of positive polarity, then an electric field is produced between the cloud and the ionosphere. This field may be so intense as to exceed the critical sparking value in the region immediately below the layer, where the pressure is much lower than that near the cloud. An electric discharge produced as a consequence will cause an increase of ionization in this region.

Spark discharge sometimes occurring between the upper part of a thundercloud and the lower boundary of the *E* layer may also be responsible for an increase of ionization in an indirect way. The intense ionization current between the cloud and the ionosphere will tend to destroy the field between them. An electric field between the upper and lower part of the cloud will, however, remain and electrons within the cloud will be accelerated by this field and will shoot up from the cloud and move with tremendous velocity towards the ionosphere, where there now exists very little retarding field. These so-called run-away electrons moving in clusters will reach the ionized layer continually and produce ionization by collision. On account of the earth's magnetic field these will follow a curved path and will affect the ionization of the portion of the ionosphere not directly above the cloud but towards the side. The presence of the electrons inside the cloud may be due to the two following causes. First, they may be produced inside the cloud by local discharges and secondly, electrons of radioactive origin which are usually found at low levels due to the earth's radioactivity may also find their way inside the cloud.

(*c*) Abnormal ionization and magnetic storm.

Observations have shown that a close correlation exists between the incidence of magnetic storms and abnormal increase of ionization. In lower latitudes the correlation appears to be comparatively poor but it is very marked in high latitudes. Magnetic storms are believed to be caused by high-speed charged particles shot off from the sun. These particles penetrating the upper atmosphere produce ionization. According to the theory of magnetic storms, first propounded by Störmer, the charged particles on account of the earth's magnetic field concentrate themselves near the polar regions. These regions of the earth are, therefore, susceptible to frequent and violent magnetic storms. Investigations conducted in high latitude by Appleton, Naismith and Builder [9] have proved conclusively that the agency which causes the magnetic storms also causes simultaneous increase of the ionization. In lower latitudes this correlation, though not so marked as in high latitudes, is still sufficient to prove beyond doubt that magnetic storms are associated with increase of ionization. It may be mentioned here that Ratcliffe and White[12] have found that 67 per cent. of magnetically disturbed nights showed the so-called nocturnal '*e*' or the evening concentration of ionization.

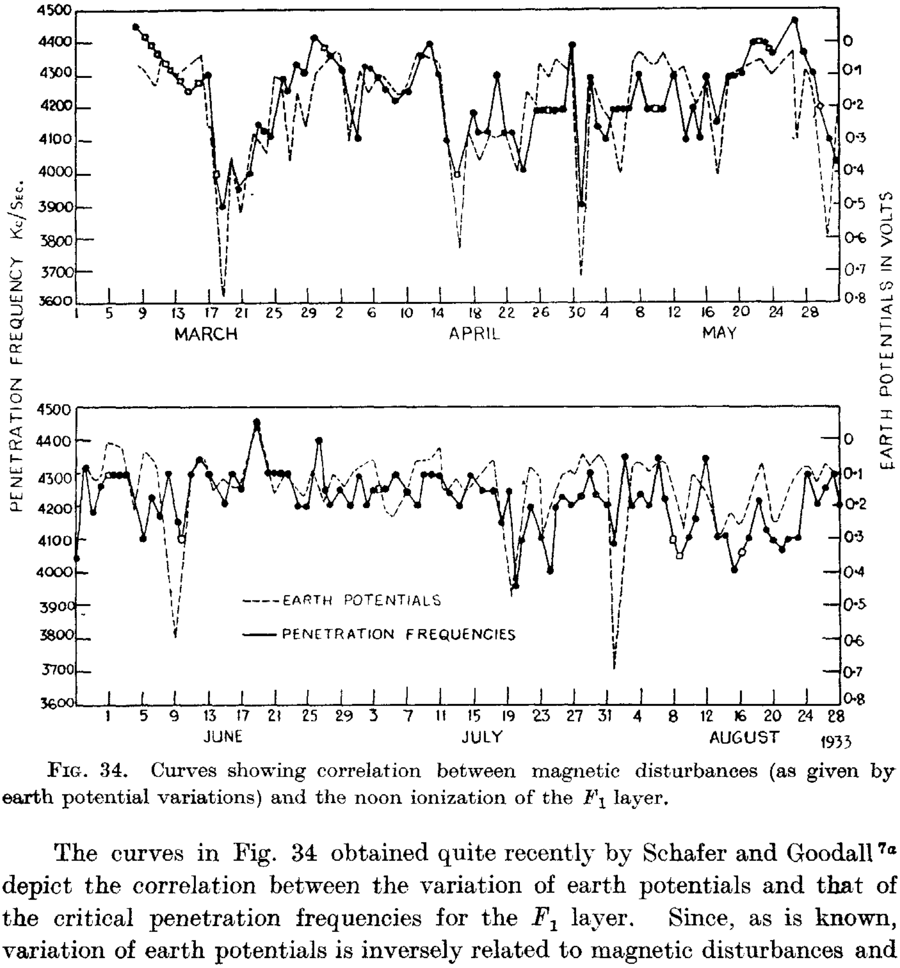

FIG. 34. Curves showing correlation between magnetic disturbances (as given by earth potential variations) and the noon ionization of the F_1 layer.

The curves in Fig. 34 obtained quite recently by Schafer and Goodall[7a] depict the correlation between the variation of earth potentials and that of the critical penetration frequencies for the F_1 layer. Since, as is known, variation of earth potentials is inversely related to magnetic disturbances and the square of the critical frequency is proportional to the maximum ion content, it is at once evident from the figure that the ionization of the F_1 layer is very closely and inversely related to magnetic disturbances.

(*d*) Increase of ionization by meteoric shower.

Various observers[14] have noticed that the ionization of the upper atmosphere is increased during a meteoric shower. That the impact of meteors may impart sufficient energy to produce ionization has been shown by Skellet[15] who finds that during a shower the energy received by the earth may be as high as $\frac{1}{14}$th of that due to the ultraviolet light of the sun. Observations by

Schafer and Goodall[14] and by Mitra, Syam and Ghose[14] during the Leonid meteoric shower of 1931 and of 1933 respectively, demonstrate that ionization increases during the period of the shower (Fig. 35). It seems probable that

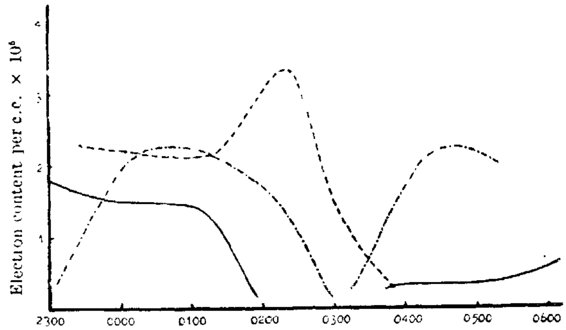

FIG. 35. Illustrating the effect of meteoric shower on the ionosphere.

the major portion of the energy of meteors is utilised in ionizing the lower regions of the ionosphere, the upper layers being rarely affected, especially when the meteors are relatively large.

(*e*) Early morning increase.

Very recently T. R. Gilliland,[6] in the course of his study of the nocturnal variation of F layer ionization at Washington, has observed a definite increase in the ionic density between 2200 and 0430 E.S.T. followed by a decrease before sunrise in winter months. It seems that this increase occurs round about the month of December. It has not yet been possible to correlate this increase of ionization with any of the ionizing agencies known so far.

4. *Sources of ionization.*

From the records of observations on the variation of ionic density of the various regions of the ionosphere as discussed above, we can draw some definite conclusions regarding the nature of the ionizing agencies.

The ultraviolet radiation of the sun causes the ionization of the E as well as the F_1 region. Besides the regular diurnal and seasonal variations of ionization of these regions, the solar eclipse experiments give the most conclusive proof of this hypothesis. It has been observed[16] that during a total solar eclipse, when the ultraviolet rays from the sun are intercepted by the moon, the ionization density falls off enormously. Further, detailed calculations by Chapman[17] show that on account of the recombination of the ions the minimum of ionization will occur about 15 minutes after the centre of the eclipse. Eclipse observations made in Japan, India, North

America and Europe during the last few years have proved that not only does the ionization decrease during a total solar eclipse but also that the minimum occurs at the predicted time.

It might be inquired at this stage what portion of the sun's spectrum is most effective in causing the ionization. The atmosphere at higher levels most probably consists of atomic and molecular oxygen and nitrogen. The sun must radiate wavelengths sufficiently short and of sufficient intensity to cause ionization of these atoms and molecules. Now, the ionization potentials of O_2, O, N_2 and N are respectively 16·1, 13·6, 16·9 and 14·48 volts, the corresponding wavelengths being 770, 910, 730 and 855 Å.U. and the sun is usually regarded as a black body radiator of temperature 6000° K. The question arises if radiation from a sun of this temperature contains wavelengths 909 Å.U. and below with sufficient energy to cause ionization of these atoms and molecules. This point has been discussed by Chapman.[18] He shows that there is sufficient energy within the wavelength range 770 to 910 to produce electron density of 4×10^6 per c.c. in the ionosphere over the equator in summer-noon. Wavelengths below 770 Å.U. do not contain sufficient energy to produce ionization of N_2, N and O_2 to any marked extent. He therefore concludes that in the upper regions of the ionosphere the gas to be ionized is chiefly atomic oxygen. In support of Chapman's calculations it may be mentioned that summer-noon ionization for English latitude is 2×10^6 electrons per c.c. for the F layer.

There is, however, a difficulty in accepting Chapman's arguments. It is obvious that the most effective portion of the solar spectrum, viz. 770–910 Å.U. will be absorbed in the upper regions of the ionosphere and the portion of the spectrum filtering below will not contain any radiation for ionizing any of the atmospheric constituents. What then is the source of ionization of the E region? As an answer to this question Chapman [17] suggests that, besides light quanta and charged particles causing magnetic storms, uncharged particles or corpuscles, as supposed by Milne,[19] are shot off from the sun. According to Chapman the F region is ionized by ultraviolet radiation and the E region by corpuscles emitted from the sun. These corpuscles being uncharged do not suffer deflection by the earth's magnetic field and are able to reach and affect the ionization in low latitudes. By taking into account the probable velocity of these corpuscles and the motions of the moon and the earth, Chapman has also calculated the hour of the corpuscular eclipse as distinguished from that of the optical eclipse. Unfortunately, however, the solar eclipse experiments point to conclusions which are just the opposite to those given by Chapman's hypotheses. As mentioned above it has been clearly established that the ultraviolet radiation definitely causes ionization of the E region. The corpuscles, if they have any effect at all, have that on the F region.[16]

In this connection it may be mentioned that Appleton [20] has made the interesting suggestion that the different ionization densities of the four component regions of the ionosphere E_1, E_2, F_1 and F_2 at different levels might be associated with different ionizing wavelengths of the solar ultraviolet radiation. Appleton is inclined to associate the four components with the four ionization potentials of oxygen and nitrogen atoms and molecules.

Charged particles emitted by the sun, which are well known to produce magnetic storms, have a marked influence on the ionization of ionospheric regions particularly in high latitudes. On account of the earth's magnetic field these particles cannot, in any number, reach low latitudes. They are mostly concentrated near the polar caps where the correlation between the incidence of magnetic storms and occurrence of abnormal increase of ionization is very close indeed.[9]

Besides the above agencies of solar origin there is a source of terrestrial origin which accounts for the abnormally high values of the E region ionization usually observed during the afternoon in summer. This source is to be found in high-velocity electrons produced during thunderstorms by the intense electric field associated with charged thunderclouds. The actual manner in which such electrons affect the E ionization has been described in sec. 3 (*b*).

Meteoric showers as one of the ionizing agencies have also been discussed in sec. 3 (*d*).

It has been sometimes suggested that radiations more penetrating than ultraviolet light, such as cosmic rays or X-rays, may contribute to the ionization of the ionosphere. For instance, E. A. W. Müller[21] has argued from a consideration of mass absorption that the ionization of the E region must be caused mainly by X-radiation from the sun and not by ultraviolet light.

§ XI. Acknowledgements.

The undernoted figures have been reproduced after being redrawn from *Proceedings of the Physical Society of London, Wireless Engineer and Experimental Wireless, Proceedings of the Institute of Radio Engineers, Nature, Report of the U.R.S.I.* and *Proceedings of the Royal Society of London.* The author expresses his indebtedness to the publishers and the writers of the respective articles for their kind permission to use the diagrams.

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APPENDIX I.

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APPENDIX II.

Symbols.

A, B—Constants, in connection with reflection coefficient; § VI, Sec. 2.

a—Radius of the earth; § X, Sec. 1.

C—Constant, radio wave interaction formula; § VII, Sec. 2.

C_0—Factor depending on wave frequency, radio wave interaction formula; § VII, Sec. 2.

c—Velocity of the electromagnetic waves in vacuo.

D—Ionospheric layer at 55 km.

d_1, d_2—Distance traversed by indirect ray from transmitter to receiver and distance between transmitter and receiver; § VI, Sec. 3.

E, E_1, E_2—Ionospheric layer at 90 km. and its subdivisions.

E_0, E_1—Electric vectors of reflected and incident waves in connection with reflection coefficient; § VI, Sec. 2.

E, E_0—Instantaneous and maximum value of the electric vector of the wave; § III, Sec. 1. Intensities of electric vector; § IV, Sec. 1.

E_A, E_L—Amplitudes of e.m.f. developed in aerials; § IV, Sec. 1.

e—Electronic charge.

F, F_1, F_2—Ionospheric layer at 250 km. and its subdivisions.

F_1, F_2—Intensities of once and twice reflected waves; § VI, Sec. 3 (*c*).

f—Wave frequency.

$$= Y/_2 \frac{\sin^2\theta}{\cos\theta} \cdot \frac{1}{1-x}; \text{ § III, Sec. 3 } (b).$$

Angular modulation frequency; § VII, Sec. 2.

f_0—Frequency of the waves emitted by the transmitter; § IV, Sec. 2.

f_0, f_e—Penetration frequencies for the ordinary and the extraordinary components of the wave ; § VIII.

f_1, f_2—Changed frequencies of the direct and the indirect waves ; § IV, Sec. 2.

G—Ionospheric layer at 600 km. ; § V, Sec. 2.
Constant, radio wave interaction formula ; § VII, Sec. 2.

g—$y/_2 \dfrac{\sin^2\theta}{\cos\theta}$; § III, Sec. 3 (*b*).
Acceleration due to gravity ; § X, Sec. 1.

H—Magnetic field of the earth.
Ionospheric layer at 1100 km. ; § V, Sec. 2.
Height of the homogeneous atmosphere $= \dfrac{KT}{mg}$; § X, Sec. 1.

H_1, H_2—Components of magnetic vector of the down-coming wave at the receiver ; § VI, Sec. 3.

H_L, H_T—Longitudinal and transverse components of the earth's magnetic field ; § III, Sec. 2.

H_p—Component of the earth's magnetic field perpendicular to the direction of propagation and also to the electric vector of the waves ; § VII, Sec. 2.

h—Height above the lower boundary of the E layer ; § VI, Sec. 2 (*a*).
Height of the point of observation ; § X, Sec. 1.

h_0—Height of the lower boundary of the E layer above the ground ; § VI.
Height of the level above the Equator at noon where the rate of production of ions is a maximum, § X, Sec. 1.

h_y, h_z—Magnetic vectors of the wave ; § III, Sec. 2.

I—Rate of production of ions ; § X, Sec. 1.

I_0—Rate of production of ions for $\chi=0$; § X, Sec. 1.

i—$\sqrt{-1}$.

K—Boltzmann's constant ; § X, Sec. 1.
Dielectric constant ; § III, Sec. 1.

K_1, K_2—Constants depending on the aerial system ; § IV, Sec. 1.

k—Coefficient of absorption.
Attenuation factor for ground wave ; § VI, Sec. 3 (*a*).

l—Mean free path of an electron ; § VII, Sec. 1.

M—Complex refractive index ; § III, Sec. 2.
Coefficient of modulation of the carrier of the unwanted signal ; § VII, Sec. 2.

M'—Coefficient of modulation produced by the unwanted carrier wave ; § VII, Sec. 2.

m—Electronic mass.
Mass of oxygen and nitrogen molecules ; § X, Sec. 1.

N—Ionic density.

N_0—Maximum value of N.

n—Number of waves by which the atmospheric ray is behind the ground ray; § IV, Sec. 2 (*a*).

P—Pressure at an altitude h; § VII, Sec. 1.

P, P_1, P_2—Optical paths; § IV, Sec. 2, § VI, Sec. 2, and § VII, Sec. 3.

P', P'_1, P'_2—Equivalent paths; § IV, Sec. 2, § VI, Sec. 2, and § VII, Sec. 3.

P_0—Pressure at the surface of the earth; § VII, Sec. 1.

p—Angular wave frequency.

$$\left.\begin{aligned} p_o &= \sqrt{\frac{4\pi N e^2}{m}} \\ p_H &= \frac{He}{mc} \\ p_L &= \frac{H_L e}{mc} \\ p_T &= \frac{H_T e}{mc} \end{aligned}\right\} ;\ \S\,\text{III, Sec. 2.}$$

Q—Factor depending on p, ω and ν, interaction of radio waves; § VII, Sec. 2.

R—Polarization $=\dfrac{h_z}{h_y}$.

T—Absolute temperature; § X, Sec. 1.

t—Time.

Time difference between the direct and the indirect ray; § IV, Sec. 3.

u, v—Phase and group velocities; § III, Sec. 1.

v—Thermic mean velocity of electrons and ions; § VII, Sec. 1.

V—Velocity of the transmitter; § IV, Sec. 2.

$$\left.\begin{aligned} x &= \frac{4\pi N e^2}{m p^2} \\ y &= \frac{\lambda}{\lambda_H} \\ y_L &= \frac{p_L}{p} \\ y_T &= \frac{p_T}{p} \end{aligned}\right\} ;\ \S\,\text{III, Sec. 3.}$$

Z—R. M. S. value of the electric intensity in the ionosphere due to unwanted signal; § VII, Sec. 2.

$$z = \frac{h - h_0}{H};\ \S\,\text{X, Sec. 1.}$$

α—Recombination coefficient; § X, Sec. 1.

α, β, γ—Constants; § VI, Sec. 2 (*a*), (*b*), (*c*).

$$\left.\begin{aligned}\alpha&=\frac{p^2m}{4\pi Ne^2}\\ \beta&=\frac{pm\nu}{4\pi Ne^2}\\ \gamma&=\frac{pp_H}{p_0^2}\\ \gamma_L&=\frac{pp_L}{p_0^2}\\ \gamma_T&=\frac{pp_T}{p_0^2}\end{aligned}\right\}\text{; § III, Sec. 2.}$$

ΔA, ΔL—Changes from the mean values of the galvanometer readings as recorded on the vertical and on the loop aerial in connection with angle of incidence measurement; § IV, Sec. 1.

η_1 η_0—Attenuation of wave; § VI, Sec. 2.

θ_1 θ_0—Angles of incidence at the ionosphere; § IV, Sec. 1 and § VI, Sec. 2 (*a*).

θ—Angle between the direction of propagation and the magnetic field; § III, Sec. 3 (*b*).

λ—Wavelength.

Angle of incidence of the sun's rays at any point on the earth's surface; § X, Sec. 1.

μ—Refractive index of ionized region.

ν—Frequency of collision of the electrons.

ν_c—Critical collisional frequency.

ξ—Velocity of ions; § III, Sec. 1.

ρ—Coefficient of reflection of ionized region.

ρ_g—Coefficient of reflection of the ground.

$$\sigma=\sqrt{\frac{4\pi e^2}{m}}.$$

$$\tau=\frac{eH}{2\pi m\nu}.$$

ϕ—Angle with the horizontal at which the indirect ray leaves the aerial; § VI, Sec. 3.

Phase difference between the ground wave and the down-coming wave; § IV, Sec. 1.

χ—Zenithal distance of the sun.

ω—Natural frequency of bound electrons; § III, Sec. 1.

$=\frac{H_p e}{m}$; § VII, Sec. 2.