ABSORPTION OF ELECTROMAGNETIC WAVES IN THE EARTH'S ATMOSPHERE.

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INTRODUCTION.

The problem of absorption of radio waves in the upper atmosphere has been attacked by several workers. Actual numerical calculations of the values of refractive indices and absorption coefficients for the two magneto-ionic components have been carried out by Goubau (1935) for Munich and by Taylor (1934) for Slough. Martyn (1934) has done the same for magnetic latitudes of 0°, 45° and 90°. Analytically the problem has been examined by Booker (1935), Toshniwal (1936) and others (1936); but these authors have restricted their treatments to values of θ (the angle between the direction of propagation and the Earth's magnetic field) between certain limits. The object of the present paper is to give an analytical treatment of general applicability and to show that for certain latitudes results contrary to those expected before may be obtained.

The notations used are

$$p_h = \frac{eh}{mc}, \text{ Larmor frequency}$$

$$p_{x,y,z} = \frac{eh_{x,y,z}}{mc}, \text{ components of Larmor frequency}$$

p = pulsatance of electromagnetic waves

N = number of electrons per cu. cm.

$$p_o^2 = \frac{4\pi Ne^2}{m}$$

ν = collisional frequency, *i.e.* the number of times an electron collides with a neutral particle per second.

$$\omega = \frac{p_h}{p}$$

$$\omega_{x,y,z} = \frac{p_{h(x,y,z)}}{p}$$

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$$r = \frac{p_o^2}{p^2}$$

$$\delta = \frac{\nu}{p}$$

$$\beta = 1 - i\delta$$

$q = \mu - \dfrac{ick}{p}$, complex refractive index

k = absorption coefficient

e = charge on an electron

m = mass of an electron

c = velocity of light in vacuo.

The suffixes o and x denote quantities pertaining to the ordinary and the extraordinary waves respectively. We start from equation (3.1) of paper 1 (1938), but we shall make the result more general by putting

$$\phi = p\left(t \pm \frac{1}{c} \int q\, dz\right)$$

We are using q in place of μ of the previous paper.

We get from equations (2.14) of the same paper

$$\begin{aligned} qA &= D \\ qB &= -C \\ qC &= -iLA - K_2 B \\ qD &= K_1 A - iLB \end{aligned}$$

exactly as in relations (3.2) of paper 1, and the remaining treatment is identical. We get, corresponding to (3.5),

$$\left.\begin{aligned} q_o^2 &= K_{11} - L\rho_o \\ q_x^2 &= K_{22} - L\rho_x \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

where
$$K_{11} = 1 - r\,\frac{\beta^2 - r\beta - \omega_x^2}{C'}$$

$$K_{22} = 1 - r\,\frac{\beta^2 - r\beta}{C'}$$

$$L = \frac{r\omega_z\,(r-\beta)}{C'}$$

$$C' = \beta(\beta^2 - \omega^2) - r(\beta^2 - \omega_z^2)$$

$$\rho_o = f\left[1 - \sqrt{1 + \frac{1}{f^2}}\,\right]$$

$$\rho_x = f\left[1+\sqrt{1+\frac{1}{f^2}}\;\right]$$

$$f = \frac{K_{11}-K_{22}}{2L}.$$

It can be easily shown that the refractive indices for the two magnetoionic components are given by

$$\left.\begin{aligned} q_o^2 &= \frac{(r-\beta)(\omega^2+r\beta-\beta^2)-\frac{r}{2}\,\omega_x^2\left[1-\sqrt{1+\left\{\frac{2(r-\beta)\,\omega_z}{\omega_x^2}\right\}^2}\right]}{\beta(\beta^2-\omega^2)-r(\beta^2-\omega_z^2)} \\ q_x^2 &= \frac{(r-\beta)(\omega^2+r\beta-\beta^2)-\frac{r}{2}\,\omega_x^2\left[1+\sqrt{1+\left\{\frac{2(r-\beta)\,\omega_z}{\omega_x^2}\right\}^2}\right]}{\beta(\beta^2-\omega^2)-r(\beta^2-\omega_z^2)} \end{aligned}\right\} \quad \ldots \quad (2)$$

These expressions have been deduced in different ways by Appleton (1932), Hartree (1929) and Försterling and Lassen (1933). They can also be put as

$$\left.\begin{aligned} q_o^2 &= 1+\frac{2r}{-2\beta-\frac{\omega_x^2}{r-\beta}+\sqrt{\frac{\omega_x^4}{(r-\beta)^2}+4\omega_z^2}} \\ q_x^2 &= 1+\frac{2r}{-2\beta-\frac{\omega_x^2}{(r-\beta)}-\sqrt{\frac{\omega_x^4}{(r-\beta)^2}+4\omega_z^2}} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (2)$$

and this is the form in which they have been generally discussed.

Since $\beta = 1-i\,\frac{\nu}{p}$, we can split up q_o^2 and q_x^2 into real and imaginary parts. We may put q in the form

$$q = \mu - i\,\frac{ck}{p}$$

so that the wave is given by

$$y = Ae^{-\int k dz}\, e^{ip(t-\frac{1}{c}\int \mu dz)}.$$

Now the general expressions for k and μ are rather complicated. We first try some simple cases and hence derive results of general application.

Case I.

$$\left[\frac{2(r-\beta)\omega_z}{\omega_x^2}\right]^2 << 1 \qquad \text{(Quasi-transverse).}$$

Under the above condition, equations (2) reduce to

$$q_o^2 = 1-\frac{r}{\beta}$$

$$q_x^2 = 1-\frac{r}{\beta-\frac{\omega_x^2}{\beta-r}}.$$

We rewrite them as

$$\left.\begin{aligned} q_o^2 &= 1-\frac{r}{1+\delta^2}-i\,\frac{r\delta}{1+\delta^2} \\ q_x^2 &= 1-\frac{R}{1+D^2}-i\,\frac{RD}{1+D^2} \end{aligned}\right\} \quad \cdots \quad \cdots \quad \cdots \quad (3)$$

where

$$R = \frac{r}{1+\dfrac{(r-1)\omega_x^2}{(r-1)^2+\delta^2}}, \quad D = \delta\,\frac{1+\dfrac{\omega_x^2}{(r-1)^2+\delta^2}}{1+\dfrac{\omega_x^2(r-1)}{(r-1)^2+\delta^2}}.$$

From (3) it can be easily deduced that

$$\mu_o^2 = \frac{1}{2}\left\{\sqrt{\left(1-\frac{r}{1+\delta^2}\right)^2+\left(\frac{r\delta}{1+\delta^2}\right)^2}+\left(1-\frac{r}{1+\delta^2}\right)\right\} \quad \cdots \quad (4)$$

$$\frac{c^2k_o^2}{p^2} = \frac{1}{2}\left\{\sqrt{\left(1-\frac{r}{1+\delta^2}\right)^2+\left(\frac{r\delta}{1+\delta^2}\right)^2}-\left(1-\frac{r}{1+\delta^2}\right)\right\} \quad \cdots \quad (5)$$

$$\mu_x^2 = \frac{1}{2}\left\{\sqrt{\left(1-\frac{R}{1+D^2}\right)^2+\left(\frac{RD}{1+D^2}\right)^2}+\left(1-\frac{R}{1+D^2}\right)\right\} \cdots \quad (6)$$

$$\frac{c^2k_x^2}{p^2} = \frac{1}{2}\left\{\sqrt{\left(1-\frac{R}{1+D^2}\right)^2+\left(\frac{RD}{1+D^2}\right)^2}-\left(1-\frac{R}{1+D^2}\right)\right\} \cdots \quad (7)$$

We easily notice that the above expressions for $\mu_{o,x}$ and $k_{o,x}$ are respectively of the forms

$$\mu^2_{o,x} = \tfrac{1}{2}\left\{\sqrt{X^2_{o,x}+Y^2_{o,x}}+X_{o,x}\right\}$$

$$\frac{c^2\,k^2_{o,x}}{p^2} = \tfrac{1}{2}\left\{\sqrt{X^2_{o,x}+Y^2_{o,x}}-X_{o,x}\right\}$$

where

$$X_o = 1-\frac{r}{1+\delta^2} \qquad Y_o = \frac{r\delta}{1+\delta^2}$$

$$X_x = 1-\frac{R}{1+D^2} \qquad Y_x = \frac{RD}{1+D^2}.$$

These forms clearly show that neither $\mu^2_{o,x}$ nor $\dfrac{c^2\,k^2_{o,x}}{p^2}$ can ever be zero, unless δ is zero. We, of course, neglect the case of $r=0$, when we will have $k^2_{o,x}=0$ and $\mu_{o,x}=1$.

When $\delta=0$, $k_{o,x}=0$, $\mu_o^2=1-r$ and $\mu_x^2=1-R$.

But as long as δ is greater than zero these expressions have definite positive values. We also observe that so long as X is positive

$$\mu^2_{o,x} > \frac{c^2\,k^2_{o,x}}{p^2}$$

but when X becomes negative

$$\mu^2_{o,\,x} < \frac{c^2\, k^2_{o,\,x}}{p^2}\,.$$

Case II.

$$\left[\frac{2\,(r-\beta)\,\omega_z}{\omega_x{}^2}\right]^2 >> 1 \qquad \text{(Quasi-longitudinal).}$$

Equations (2), under this condition, reduce to

$$q_o{}^2 = 1 - \frac{r}{\beta + |\omega_z|}$$

$$q_x{}^2 = 1 - \frac{r}{\beta - |\omega_L|}\,.$$

We rewrite the above equations as

$$q_o{}^2 = 1 - \frac{r(1+|\omega_z|)}{(1+|\omega_z|)^2+\delta^2} - i\,\frac{r\delta}{(1+|\omega_z|)^2+\delta^2} \quad .. \quad (8)$$

$$q_x{}^2 = 1 - \frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2} - i\,\frac{r\delta}{(1-|\omega_z|)^2+\delta^2} \quad .. \quad (9)$$

Hence

$$\mu_o{}^2 = \frac{1}{2}\left\{\sqrt{\left[1-\frac{r(1+|\omega_z|)}{(1+|\omega_z|^2+\delta^2}\right]^2+\left[\frac{r\delta}{(1+|\omega_z|)^2+\delta^2}\right]^2} + \left[1-\frac{r(1+|\omega_z|)}{(1+|\omega_z|)^2+\delta^2}\right]\right\} \quad .. \quad (10)$$

$$\frac{c^2\,k_o{}^2}{p^2} = \frac{1}{2}\left\{\sqrt{\left[1-\frac{r(1+|\omega_z|)}{(1+|\omega_z|)^2+\delta^2}\right]^2+\left[\frac{r\delta}{(1+|\omega_z|)^2+\delta^2}\right]^2} - \left[1-\frac{r(1+|\omega_z|)}{(1+|\omega_z|)^2+\delta^2}\right]\right\} \quad .. \quad (11)$$

$$\mu_x{}^2 = \frac{1}{2}\left\{\sqrt{\left[1-\frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2}\right]^2+\left[\frac{r\delta}{(1-|\omega_z|)^2+\delta^2}\right]^2} + \left[1-\frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2}\right]\right\} \quad .. \quad (12)$$

$$\frac{c^2\,k_x{}^2}{p^2} = \frac{1}{2}\left\{\sqrt{\left[1-\frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2}\right]^2+\left[\frac{r\delta}{(1-|\omega_z|)^2+\delta^2}\right]^2} - \left[1-\frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2}\right]\right\} \quad .. \quad (13)$$

These expressions also are of the same form as that of the expressions derived in the previous case and we can write them as

$$\mu^2_{o,\,x} = \frac{1}{2}\left\{\sqrt{P^2_{o,\,x}+Q^2_{o,\,x}} + P_{o,\,x}\right\}$$

$$\frac{c^2k^2_{o,x}}{p^2}=\frac{1}{2}\left\{\sqrt{P^2_{o,x}+Q^2_{o,x}}-P_{o,x}\right\}$$

where

$$P_o=1-\frac{r(1+|\omega_z|)}{(1+|\omega_z|)^2+\delta^2} \qquad Q_o=\frac{r\delta}{(1+|\omega_z|)^2+\delta^2}$$

$$P_x=1-\frac{r(1-|\omega_z|)}{(1-|\omega_z|)^2+\delta^2} \qquad Q_x=\frac{r\delta}{(1-|\omega_z|)^2+\delta^2}.$$

Obviously, $k_{o,x}=0$ only when either $r=0$ or $\delta=0$. In the former case $(r=0)$, $\mu_{o,x}$ become unity while in the latter case $(\delta=0)$

$$\mu_o^2=1-\frac{r}{1+|\omega_z|}$$

$$\mu_x^2=1-\frac{r}{1-|\omega_z|}.$$

We are however interested in the propagation of radio waves in the ionosphere where neither r is zero nor δ is zero. Hence under these conditions none of the four quantities $\mu_{o,x}$ and $k_{o,x}$ ever becomes zero; so long as p is positive

$$\mu^2_{o,x}>\frac{c^2k^2_{o,x}}{p^2}$$

but when p becomes negative

$$\mu^2_{o,x}<\frac{c^2k^2_{o,x}}{p^2}.$$

§ 2.

We shall divide absorption into two regions, viz., the non-reflecting lower region (where the density of electrons is very small) and the reflecting region where the value of the refractive index becomes very small. We have now to find out whether the propagation of waves in the above two regions is governed by Case I or Case II and how it depends on θ, the magnetic latitude.

The condition

$$\left[\frac{2(r-\beta)\omega_z}{\omega_x^2}\right]^2=1$$

which divides the two cases may be put in a more convenient form.

Let us put

$$p_\theta^{2*}=\frac{1}{4}\frac{p_x^4}{p_z^2}.$$

* This p_θ is the critical collisional frequency first pointed out by Appleton and Builder (1933). Its value at Allahabad is 3×10^6. The quantity under the square root in the expressions for $q^2_{o,x}$ is

Then the above condition becomes

$$\left[\frac{r-\beta}{p_\theta}\right]^2 = 1.$$

Since $r-\beta = r-1+i\delta$, a complex quantity, we can put it in the form

$$\left[\frac{(r-1)^2+\delta^2}{p_\theta^2} e^{2i\phi}\right] = 1$$

where $\phi = \tan^{-1}\dfrac{\delta}{r-1}$.

We can now take the above condition to be

$$(r-1)^2+\delta^2 = p_\theta^2$$

or

$$(p^2-p_o^2)^2+p^2\nu^2 = p^2 p_\theta^2 \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

This condition is the same as that given by Booker (*loc. cit.*, eqn. (31)).

Note on the Curves.

Fig. (1) * gives the assumed variation of p_o, N and ν with height. It is only a crude picture of the atmosphere, but is expected to give fairly approximate results for the E-region. Figs. (2) and (3) represent curves given by equation (14). In drawing these curves the values of p_o and ν have been read from fig. (1). p_h has been assumed to be 8×10^6. Fig. (2) is a representative of the class of frequencies greater than the gyromagnetic frequency, while fig. (3) represents frequencies below the above-mentioned frequency. They have been actually drawn for $p = 2{\cdot}5\times10^7$ ($\lambda = 75{\cdot}36m.$) and $p = 5\times10^6$ ($\lambda = 376{\cdot}8m$) respectively. Fig. (4) is another representative of medium waves and is drawn for $p = 7\times10^6$ ($\lambda = 264{\cdot}14m$). Fig. (5) is drawn for $p = 10^6$ ($\lambda = 1884m$) and thus represents propagation of long waves. Fig. (6) is drawn for $p = 5\times10^7$ ($\lambda = 37{\cdot}68m$) on the assumption that collision frequency remains constant at 10^3. It is meant to give an approximate idea of the conditions of propagation in the F-region. The curves hold for both the hemispheres and it is evident from them that,

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$$1+\left[\frac{2(r-\beta)\omega_z}{\omega_x^2}\right]^2 = 1+\left[\frac{r-\beta}{p_\theta}\right]^2 = 1+\frac{(r-1)^2-2i\delta(r-1)-\delta^2}{p_\theta^2}$$

and this becomes zero when $r=1$ and $r = p_\theta$, so that q_o becomes equal to q_x. But this condition holds only under a severe limitation, for we must have a region where $r=1$, and $\nu = p_\theta$.

* It is the same as assumed by Booker (*loc. cit.*).

in general, quasi-transverse type of propagation is of importance only at medium frequencies (except of course a very narrow region near about the magnetic equator where it holds at all frequencies) and that it is the quasi-longitudinal type which predominates at long and short waves.

We further observe that so far as the non-reflecting region is concerned (where the value of electron density is small), we can divide the whole surface of the earth into two regions, viz., (*a*) places not very near the magnetic equator and (*b*) places very near the magnetic equator. It is easy to see that in the former region both the *o*- and the *x*-waves of every frequency are propagated according to Case II but that in the latter region their propagation is governed by Case I. Again when we consider propagation in the reflecting region three cases arise. A concise description of the features of the various cases is given in Table 1 and their applicability has been discussed afterwards.

TABLE 1.

Name of Case	Definition
Case I (Quasi-transverse)	$(p^2-p_o^2)^2+p^2\nu^2<<p^2p_\theta^2$
Case II. (Quasi-longitudinal)	$(p^2-p_o^2)^2+p^2\nu^2>>p^2p_\theta^2$
Case A*	Propagation of both the *o*- and the *x*-wave governed by Case I.
Case B*	Propagation of *o*-wave governed by Case I but that of *x*-wave by Case II.
Case C*	Propagation of both the *o*- and the *x*-wave governed by Case II.

Case A.—Propagation of both the magneto-ionic components is governed by Case I. On short and long waves this holds for rather small values of θ, the magnetic latitude or the angle of dip, but on medium waves it may hold even up to 25° or so as is evident from figs. (3) and (4). We shall, however, term this region as the magnetic equatorial latitudes and for evaluating k_o and k_x in these latitudes we shall make use of eqns. (5) and (7) respectively.

* These cases refer to the reflecting region only.

Case B.—These are the latitudes where propagation of the x-wave is governed by Case II (quasi-longitudinal) while that of the o-wave by Case I (quasi-transverse). These conditions will hold good at places where the magnetic latitudes are neither too high nor too low. It is not possible to give definite values of these latitudes in any of these cases, for they depend on the frequency of the wave under consideration, as is evident from a comparison of the various figures. This region we shall call intermediate magnetic latitude. For finding out the values of k_o and k_x in these latitudes we shall use eqns. (5) and (13) respectively.

Case C.—This is the region where both the o- and x-waves propagate according to Case II. For short waves these conditions exist in the higher magnetic latitudes, i.e., places near the magnetic poles but for medium waves they may exist in magnetic latitudes as low as 30° as shown in figs. (3) and

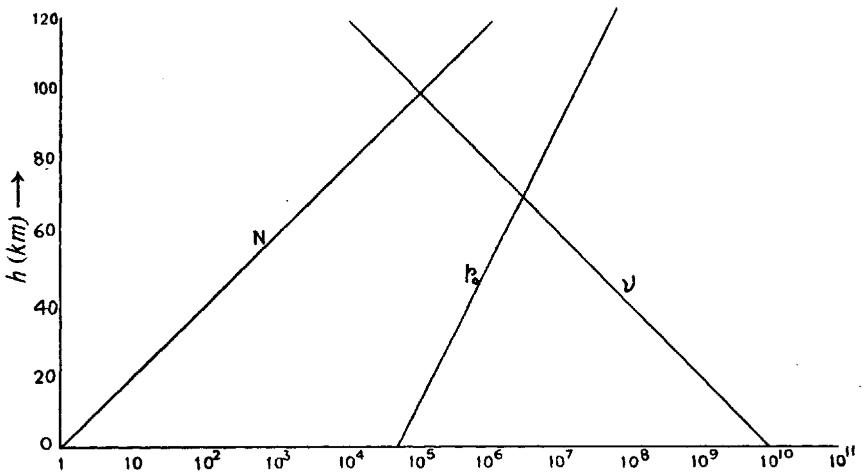

FIG. 1.

(4) and for long waves they may be fulfilled down to still lower latitudes as is evident from fig. (5).

ABSORPTION IN THE NON-REFLECTING REGION.

(a) *Places not very near the magnetic equator.*

We have already seen that in this region conditions of propagation are governed according to Case II. Hence, since r is small, we have from (11) and (13) as a first approximation

$$\frac{ck_o}{p}=\frac{1}{2}\frac{r\delta}{(1+|\omega_z|)^2+\delta^2}$$

$$\frac{ck_x}{p}=\frac{1}{2}\frac{r\delta}{(1-|\omega_z|)^2+\delta^2}.$$

Substituting for r, ω_z and δ, we get

$$k_o = \frac{2\pi e^2}{mc} \frac{N\nu}{(p+ \mid p_z \mid)^2+\nu^2} \quad \ldots \quad (15)$$

$$k_x = \frac{2\pi e^2}{mc} \frac{N\nu}{(p- \mid p_z \mid)^2+\nu^2} \quad \ldots \quad (16)$$

Hence

$$\frac{k_o}{k_x} = \frac{(p- \mid p_z \mid)^2+\nu^2}{(p+ \mid p_z \mid)^2+\nu^2} < 1 \quad \ldots \quad (17)$$

It is obvious from the above expression that at a particular level in this region the o-wave suffers less absorption than the x-wave. The difference in the relative absorption of the two waves becomes greater and greater both as we go towards the magnetic poles and as we consider frequencies approaching

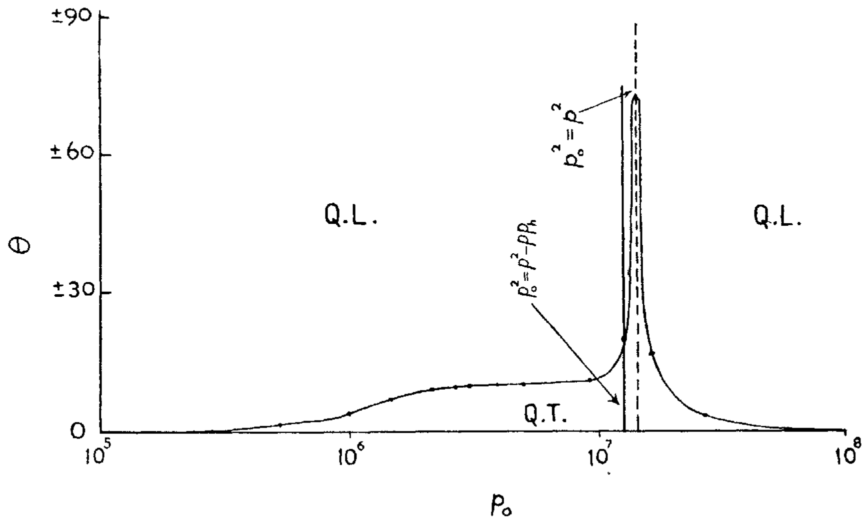

FIG. 2. Variation of Q.T. and Q.L. regions with θ for $p = 2{\cdot}5 \times 10^7$.

p_z. We can also say from (17) that in this region at places not very far from the magnetic equator the o-wave will be only slightly less absorbed than the x-wave, provided we confine our attention only to waves well away from p_z. We further observe from (15) and (16) that k_o has got a unique maximum at the level where $\nu = p + \mid p_z \mid$, while k_x has the same in the region given by $\nu = p - \mid p_z \mid$. Thus if N varies slowly, the maximum absorption suffered by these waves in the non-reflecting region will be at the above mentioned two different levels; but as we proceed towards the magnetic equator the two levels come nearer and nearer till at places not far removed from the magnetic equator, both the waves suffer maximum absorption in the non-reflecting region at approximately the same level, which tends to show that they should be almost equally absorbed—a conclusion derived before.

For finding out the total absorption suffered by the waves in the non-deviating region we have to integrate (15) and (16). Thus the integrated absorption coefficients k_o' and k_x' are

$$k_o' = \frac{2\pi e^2}{mc}\int_o^z \frac{N\nu}{(p+|p_z|)^2+\nu^2}dz \quad \ldots \quad (18)$$

$$k_x' = \frac{2\pi e^2}{mc}\int_o^z \frac{N\nu}{(p-|p_z|)^2+\nu^2}dz \quad \ldots \quad (19)$$

Now we have to find out as to how N and ν vary with height. In the case of stable layers, Chapman (1931) worked out a formula for variation of

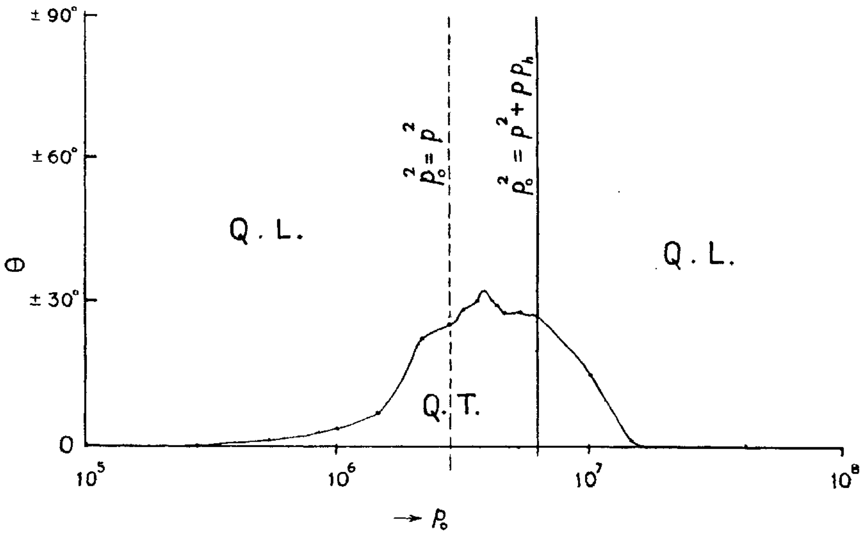

FIG. 3. Variation of Q.T. and Q.L. regions with θ for $p = 5\times10^6$.

N with height, on the supposition that electrons are produced by the absorption of monochromatic light. Saha and Rai (1938) have recently shown that this expression holds for continuous spectrum as well. We have according to these authors

$$N = N_o e^{\frac{1-z-e^{-z}\sec\chi}{2}} \quad \ldots \quad (20)$$

where

N = number of electrons per c.c.

N_o = number of electrons at the level h_o when $\chi = 0$

$$z = \frac{h-h_o}{H} \quad \ldots \quad (21)$$

$$H = \frac{kT}{m'g} \quad \cdots \quad (22)$$

χ = angle of incidence of solar radiation

k = Boltzmann's constant

T = absolute temperature

m' = mean molecular mass

g = acceleration due to gravity

h_o = the level where the ion production is maximum at the equator at equinox.

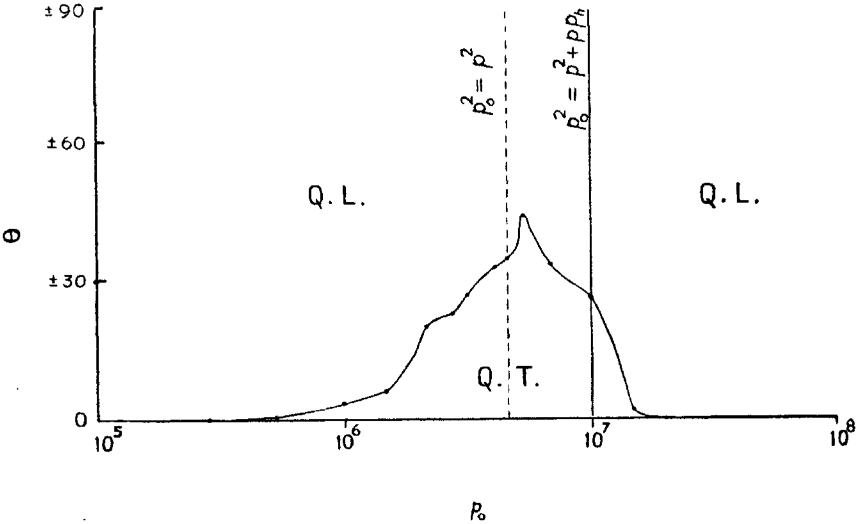

FIG. 4. Variation of Q.T. and Q.L. regions with θ for $p = 7 \times 10^6$.

As usual the height h is being measured from the datum level h_o and expressed in units of H, the height of equivalent atmosphere.

Again, we know that the frequency of collisions, ν, is given by

$$\nu = \frac{\bar{c}}{\lambda} \quad \cdots \quad (23)$$

where

$\bar{c}$ = mean velocity of the particles (electrons in our case)

λ = mean free path of an electron.

We have from the kinetic theory of gases that

$$\bar{c} = \sqrt{\frac{8kT}{m\pi}}, \quad \lambda = \frac{4}{\sigma^2 n\pi}$$

where

m = mass of an electron

n = number of particles per c.c. with which electrons collide

σ = diameter of the particles.

So that (23) becomes

$$\nu = \sqrt{\frac{\pi k \sigma^4 T}{2m}}\, n \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

Now if we assume that

$$n = n_o e^{-z}$$

we have for (24)

$$\nu = \sqrt{\frac{\pi k \sigma^4 T}{2m}}\, n_o e^{-z}$$

n_o being the number of particles per c.c. at the level h_o.

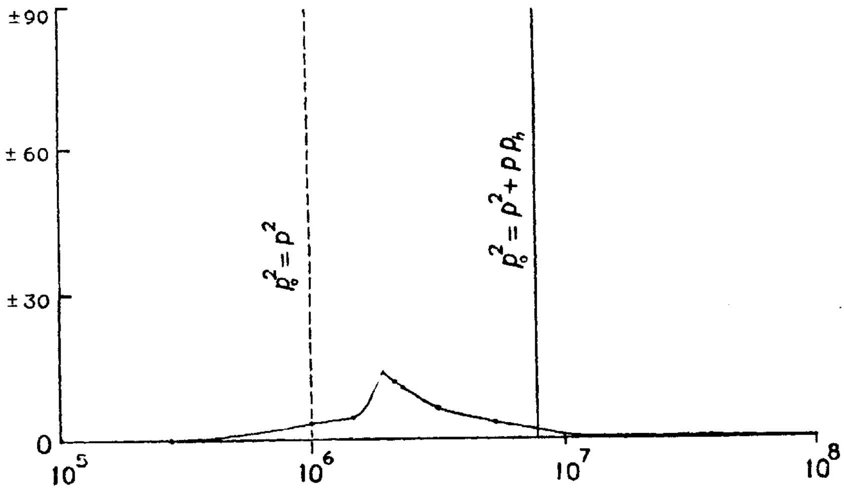

FIG. 5. Variation of Q.T. and Q.L. regions with θ for $p = 10^6$.

Putting $\quad \nu_o = n_o \sqrt{\frac{\pi k \sigma^4 T}{2m}}$, we get

$$\nu = \nu_o e^{-z} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

Utilizing (22) and (25), we have for the integrated absorption of the waves, say in the E-region, if we are considering waves reflected from the F-region,

$$k_o' = \frac{2\pi e^2 N_o \nu_o}{mc} \int_{-\infty}^{+\infty} \frac{e^{\frac{1-3z-e^{-z}\sec\chi}{z}}}{(p+|p_z|)^2+\nu_o^2 e^{-2z}}\, dz \qquad \ldots \quad \ldots \quad (26)$$

and

$$k_x' = \frac{2\pi e^2 N_o \nu_o}{mc} \int_{-\infty}^{+\infty} \frac{e^{\frac{1-3z-e^{-z}\sec\chi}{z}}}{(p - |p_z|)^2 + \nu_o^2 e^{-2z}} dz \quad \cdots \quad \cdots (27)$$

These integrals have been solved by Appleton (1938). He deduces that

$$k'_{o,x} = \frac{2\pi N_o e^2 \nu}{mc} \left(\frac{e'}{\nu_o (p \pm |p_H|)} \right)^{\frac{1}{2}} X'$$

where

$$X' = \int_0^{\infty} \frac{t^{\frac{1}{2}}}{1+t^2} e^{-at} dt$$

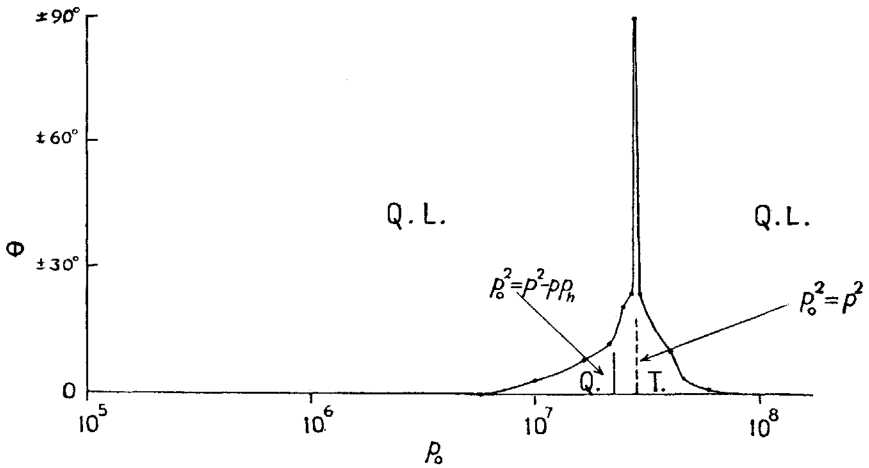

FIG. 6. Location of Q.T. and Q.L. regions with varying value of θ for $p = 5 \times 10^7$.

and $\quad a = \dfrac{p \pm |p_H|}{2\nu_o} \sec\chi$

when $\quad a >> 1$

$$X' = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

when $\quad a << 1$

$$X' = \frac{\pi}{\sqrt{2}} (\cos a + \sin a) - 2\pi^{\frac{1}{2}} a \left(1 - \frac{4}{15} a^2\right) + O(a^{\frac{5}{2}}).$$

When $a = 1$, he has found out the value of X', using integration by quadrature and has drawn a curve between X' and a.

(*b*) *Places very near the magnetic equator.*

We have already seen that at these places propagation of both the magneto-ionic components is governed by Case I (quasi-transverse). Hence we shall use equations (11) and (13) for evaluating the special absorption coefficients k_o and k_x respectively.

Since r is small, we have from the above equations as a first approximation

$$\frac{ck_o}{p} = \frac{1}{2}\frac{r\delta}{1+\delta^2} \quad \ldots \quad (28)$$

and

$$\frac{ck_x}{p} = \frac{1}{2}\frac{RD}{1+D^2} = \frac{1}{2}\frac{r\delta}{\dfrac{\left(1-\dfrac{\omega_x^2}{1+\delta^2}\right)^2}{\left(1+\dfrac{\omega_x^2}{1+\delta^2}\right)^2}+\delta^2} \quad \ldots \quad (29)$$

It is obvious from (28) and (29) that $k_o < k_x$ so that the o-wave suffers less absorption than the x-wave, but as for most of the useful waves the term

$$\left(1-\frac{\omega_x^2}{1+\delta^2}\right)^2 \Big/ \left(1+\frac{\omega_x^2}{1+\delta^2}\right)^2$$

is only slightly less than unity, the x-wave will be only slightly more absorbed than the o-wave. It will not be so for frequencies approaching p_x.

Substituting for r, δ, ω_x, etc. in (28) and (29), we get,

$$k_o = \frac{2\pi e^2}{mc}\cdot\frac{N\nu}{p^2+\nu^2} \quad \ldots \quad (30)$$

and

$$k_x = \frac{2\pi e^2}{mc}\cdot\frac{N\nu}{p^2\dfrac{(p^2+\nu^2-p_x^2)^2}{(p^2+p_x^2+\nu^2)^2}+\nu^2} \quad \ldots \quad (31)$$

Now using (22) and (25) we get for the integrated absorption of waves in a region that they traverse through,

$$k_o'' = \frac{2\pi e^2 N_o \nu_o}{mc}\int_{-\infty}^{+\infty}\frac{e^{\frac{1-3z-e^{-z}\sec\chi}{2}}}{p^2+\nu_o^2 e^{-2z}}\,dz \quad \ldots \quad (32)$$

$$k_x'' = \frac{2\pi e^2 N_o \nu_o}{mc}\int_{-\infty}^{+\infty}\frac{e^{\frac{1-3z-e^{-z}\sec\chi}{2}}}{p^2\left[\dfrac{p^2-p_x^2+\nu_o^2 e^{-2z}}{p^2+p_x^2+\nu_o^2 e^{-2z}}\right]^2+\nu_o^2 e^{-2z}}\,dz \quad \ldots \quad (33)$$

(32) can be integrated as (28) and (29). The integration of (33) we do not propose to take up in this paper.

Absorption in the Reflecting Region.

Case A.—*Magnetic Equatorial Latitudes.*

It has been already shown that we have to use equations (5) and (7) in this case. Hence, expanding the right hand side of (5) and remembering that r is approximately unity for the o-wave in this region, we get, as a first approximation,

$$\frac{c^2 k_o^2}{p^2} = \frac{1}{2}[\delta - \delta^2] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

the powers of δ higher than the second having been neglected.

Now in these latitudes $\omega \doteq \omega_x$ * and for frequencies greater than the gyromagnetic frequency the x-wave reflecting region is given by

$$r \doteq 1 - \omega.$$

Making these substitutions and neglecting δ'^2 in comparison with ω^2 we get

$$R = \frac{r}{1 + \frac{\omega_x^2(r-1)}{(r-1)^2+\delta^2}} \doteq \frac{1-\omega}{1 - \frac{\omega\omega_x^2}{\omega^2}} \doteq 1$$

and

$$D = \delta' \frac{1 + \frac{\omega_x^2}{(r-1)^2+\delta^2}}{1 + \frac{\omega_x^2(r-1)}{(r-1)^2+\delta^2}} \doteq \delta' \frac{1 + \frac{\omega_x^2}{\omega^2}}{1 - \frac{\omega\omega_x^2}{\omega^2}} \doteq \frac{2\delta'}{1-\omega}$$

δ' refers to the level $r \doteq 1 - \omega$.

Now expanding (7) and neglecting powers of D greater than the second, we get, as a first approximation,

$$\frac{c^2 k_x^2}{p^2} = \frac{1}{2}[D - D^2] = \frac{\delta'}{1-\omega} - \frac{2\delta'^2}{(1-\omega)^2} \quad \ldots \quad \ldots \quad (35)$$

Again for $p < p_h$, we put

$$r \doteq 1 + \omega$$

so that we get, as before,

$$R \doteq 1$$

$$D \doteq \frac{2\delta''}{1+\omega}$$

δ'' referring to the level $r \doteq 1 + \omega$.

Hence, for $p < p_h$, we get,

$$\frac{c^2 k^2}{p^2} = \frac{\delta''}{1+\omega} - \frac{2\delta''^2}{(1+\omega)^2} \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

---

* This condition we can take only in the case of long and short waves, where values of θ encountered are small. When we consider medium waves, this condition no longer holds, for θ, for such waves in this case may extend even to 25° as shown in figs. (3) and (4) and we have to use general expressions.

Now for $p>p_h$, we get, from (34) and (35)

$$\frac{k_x^2}{k_o^2}=2\,\frac{\dfrac{\delta'}{1-\omega}-\dfrac{2\delta'^2}{(1-\omega)^2}}{\delta-\delta^2}$$

$$\doteqdot\frac{2}{1-\omega}\,\frac{\delta'}{\delta}\quad\text{retaining only }\delta\text{ and }\delta'$$

$$\doteqdot\frac{2p}{p-p_h}\quad\text{if we assume that }\delta'\doteqdot\delta$$

$$>1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

For $p<p_h$ (34) and (36) yield

$$\frac{k_x^2}{k_o^2}=2\,\frac{\delta''/(1-\omega)-2\delta''^2/(1-\omega)^2}{\delta-\delta^2}$$

which after making assumptions similar to the above becomes

$$\frac{k_x^2}{k_o^2}\doteqdot\frac{2p}{p+p_h}<1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Thus in the reflecting region for $p>p_h$ the o-wave is less absorbed than the x-wave, but for $p<p_h$ the reverse happens.

Wells and Berkner (1937) have carried out measurements of the intensities of the o- and the x-waves at Huancayo ($\theta = 2°\ 11'$), a place very near the magnetic equator for a frequency of 4·6 Mc./sec. They find that both the o- and the x-waves are received with almost equal intensities.

Now (38) shows that if the main absorption occurs in the reflecting region $k_x = 1{\cdot}63\ k_o$, while we have shown previously that at such latitudes in the non-reflecting region the two waves will be almost equally absorbed, the ordinary being slightly less absorbed than the extraordinary. This shows that the waves in the above-mentioned experiments suffered absorption mainly in the non-reflecting region.

Case B.—*Intermediate Magnetic Latitudes.*

For the o-wave we have as before from equation (5)

$$\frac{c^2k_o^2}{p^2}=\frac{1}{2}[\delta-\delta^2] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

For the x-wave we use equation (13) and substitute

$$r-1=-\omega \qquad \text{for } p>p_h$$

$$\text{and}\quad r-1=\omega \qquad \text{for } p<p_h$$

After simplification, we get, for $p>p_h$

$$\frac{c^2k_x^2}{p^2}=\frac{1}{2}\left\{\frac{(1-|\omega_z|)(|\omega_z|-\omega)-\delta'^2}{(1-|\omega_z|)^2+\delta'^2}+\sqrt{\frac{(\omega-|\omega_z|)^2+\delta'^2}{(1-|\omega_z|)^2+\delta'^2}}\right\} \quad \ldots \quad (39)$$

Expanding and taking first approximation, we get from (39),

$$\frac{c^2k_x^2}{p^2}=\frac{1}{2}\left\{\frac{|\omega_z|-\omega}{1-|\omega_z|}\left[1-\frac{\delta'^2}{(1-|\omega_z|)(|\omega_z|-\omega)}\right]\left[1-\frac{\delta'^2}{(1-|\omega_z|)^2}\right]\right.$$
$$\left.+\frac{\omega-|\omega_z|}{1-|\omega_z|}\left[1+\frac{1}{2}\frac{\delta'^2}{(\omega-|\omega_z|)^2}\right]\left[1-\frac{1}{2}\frac{\delta'^2}{(1-|\omega_z|)^2}\right]\right\}$$
$$=\frac{1}{4}\frac{\delta'^2(1-\omega)^2}{(1-|\omega_z|)^3(\omega-|\omega_z|)} \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

Similarly for $p<p_h$ we obtain,

$$\frac{c^2k_x^2}{p^2}=\frac{1}{4}\frac{\delta''^2(1+\omega)^2}{(|\omega_z|-1)^3(\omega+|\omega_z|)} \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

Comparing (34) and (40), we get, for $p>p_h$

$$\frac{k_x^2}{k_o^2}=\frac{1}{2}\frac{\delta'^2(1-\omega)^2}{(1-|\omega_z|)^3(\omega-|\omega_z|)(\delta-\delta^2)}$$

which after neglecting δ^2 in comparison to δ and assuming that

$$\delta \doteq \delta'$$

becomes

$$\frac{k_x^2}{k_o^2}\doteq\frac{1}{2}\frac{\delta(1-\omega)^2}{(1-|\omega_z|)^3(\omega-|\omega_z|)}<1 \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Similarly, from (34) and (41), we get, for $p<p_h$

$$\frac{k_x^2}{k_o^2}\doteq\frac{1}{2}\frac{\delta''^2(1-\omega)^2}{(1+|\omega_z|)^3(\omega+|\omega_z|)(\delta-\delta^2)}$$
$$\doteq\frac{1}{2}\frac{\delta''(1-\omega)^2}{(1+|\omega_z|)^3(\omega+|\omega_z|)}<1 \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

From fig. (2) it is clear that for $p = 25\times10^6$ we can consider a place for which $\theta = 30°$ in this case. For Calcutta where $\theta = 31° \ 45'$ we have

$$k_x = \cdot 7428\times10^{-3}\sqrt{\nu}\, k_o.$$

Even if we assume the value of $\nu = 10^6$ for the level from which the x-wave is reflected we have

$$k_x = \cdot 7428 k_o$$

which shows that the extraordinary wave will be less absorbed than the ordinary wave.

CASE C.—*High Magnetic Latitudes.*

We have already seen that propagation in this region is governed by Case II and we have to use equations (11) and (13) for the o- and the x-wave respectively.

Simplifying (11) as in Case B, we get, for the o-wave

$$\frac{c^2k_o^2}{p^2}=\frac{1}{4}\frac{\delta^2}{\omega_z(1+|\omega_z|)^3} \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

Similarly, from (13) we get for the x-wave for $p>p_h$

$$\frac{c^2k_x^{\,2}}{p^2}=\frac{1}{4}\,\frac{\delta'^2(1-\omega)^2}{(1-|\,\omega_z\,|)^3(\omega-|\,\omega_z\,|)} \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

So that for $p>p_h$

$$\frac{k_x^{\,2}}{k_o^{\,2}}=\frac{|\,\omega_z\,|(1-\omega)^2(1+|\,\omega_z\,|)^3}{(1-|\,\omega_z\,|)^3(\omega-|\,\omega_z\,|)}\,\frac{\delta'^2}{\delta^2}$$

$$\doteqdot\frac{|\,\omega_z\,|(1-\omega)^2(1+|\,\omega_z\,|)^3}{(1-|\,\omega_z\,|)^3(\omega-|\,\omega_z\,|)} \quad \ldots \quad \ldots \quad \ldots \quad (46)$$

$$>1.$$

Again for $p<p_h$ we get for the x-wave, from (13)

$$\frac{c^2k_x^{\,2}}{p^2}=\frac{1}{4}\,\frac{\delta''^2(1+\omega)^2}{(|\,\omega_z\,|-1)^3(\omega+|\,\omega_z\,|)} \quad \ldots \quad \ldots \quad \ldots \quad (47)$$

so that we get, from (48) and (44), for $p<p_h$

$$\frac{k_x^{\,2}}{k_o^{\,2}}=\frac{|\,\omega_z\,|\,(1+|\,\omega_z\,|)^3(1+\omega)^2}{(|\,\omega_z\,|-1)^3(\omega+|\,\omega_z\,|)}\,\frac{\delta''^2}{\delta^2}$$

$$\doteqdot\frac{|\,\omega_z\,|(1+|\,\omega_z\,|)^3(1+\omega)^2}{(|\,\omega_z\,|-1)^3(\omega+|\,\omega_z\,|)} \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

$$>1.$$

(46), (47) and (48) clearly show that in these latitudes the extraordinary wave suffers greater absorption than the o-wave in the reflecting region.

All our results are summarized in the following table :—

Regions	Cases	Sub-cases	Results
Non-Reflecting Region	(*a*)		$k_o<k_x$, $k_o\doteqdot k_x$ as we approach the magnetic equator.
	(*b*)		$k_o<k_x$. But $k_o\doteqdot k_x$
Reflecting Region	A	$p>p_h$	$k_o<k_x$
		$p<p_h$	$k_o>k_x$
	B	$p>p_h$	$k_o>k_x$
		$p<p_h$	$k_o>k_x$
	C	$p>p_h$	$k_o<k_x$
		$p<p_h$	$k_o<k_x$

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