ON THE THEORY OF A SYSTEM OF RECEDING PARTICLES HAVING A TENDENCY TO APPROACH THE CENTRAL MASS.

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(Communicated by Prof. G. S. Mahajani, Ph.D.)

(*Read March 11, 1940.*)

I. Introduction and formulation of the new equations of motion.

Milne has given a theory of a system of particles moving within a sphere of radius $r = ct$ at an epoch t such that the speed of any individual particle increases with the distance. The space (3-dimensional) in which the particles are supposed to move is 'flat', and it is also supposed to be devoid of any irregularities as to its structure, it being assumed that the material contents are swept away to $r = ct$. The space within the sphere $r = ct$, therefore, is homogeneous but not, in general, isotropic; it will be shown below that it is isotropic only for $G(\xi) = -1$. Hence, such a smoothed out universe is called a 'substratum'. This has been accepted to be a possible form of a world-model. The theory associated with such a model has been termed the Kinematical Theory of Relativity, and it centres round the following path-equations for the system of particles having a motion of recession, viz.

$$\frac{dV}{dt} = (P - Vt)\frac{Y}{X}G(\xi) \qquad \text{(1)}$$

where

$$X = t^2 - P^2/c^2,\ Y = 1 - V^2/c^2,\ Z = t - \frac{P \cdot V}{c^2},\ \xi = \frac{Z^2}{XY},\ P = \sqrt{(x^2+y^2+z^2)},$$

$V = (u, v, w)$, $\frac{dx}{dt} = u$, etc., the notation being the same as that given by Milne. These are the equations of motion for the kinematical system regarded as a statistical system with a distribution function $\frac{Btdxdydz}{c^3(t^2 - P^2/c^2)^2}$, the particles moving away from the observer (who is supposed to be situated at the centre of the nuclear cluster) with speed increasing with the distance. Here, the explicit assumption is that the attraction of the nuclear cluster on this receding swarm of point masses is negligible in comparison to the force tending the outward motion. This uniformly increasing outward motion in every direction gives an idea of the homogeneity of structure for the smoothed out universe; but the isotropy of pressure is not, in general, satisfied by this model as can be shown by having recourse to a general line-element for

Milne's case. This line-element has been shown by Walker[1] to be of the form

$$ds^2 = \{F(X)\}^2\left[dt^2 - \frac{1}{c^2}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)\right] \quad .. \quad .. \quad (2)$$

where $X = t^2 - \frac{r^2}{c^2}\cdot$ The condition of isotropy, viz.

$$B^r_{ihk} = b\left(a_{ik}\,\delta^r_h - a_{ih}\,\delta^r_k\right) \quad .. \quad .. \quad .. \quad (3)$$

where b is the Riemannian curvature, and the δ's are the Krönecker deltas, and $a_{11} = -\frac{1}{c^2}$, etc., in this case reduce to equations of the form

$$B^j_{iji} = B^k_{iki},\ j \neq k \quad .. \quad .. \quad .. \quad .. \quad (4)$$

which, in turn, leads to the following differential equation for $F(X)$, viz.

$$F\frac{d^2F}{dX^2} - 2\left(\frac{dF}{dX}\right)^2 = 0. \quad .. \quad .. \quad .. \quad (5)$$

The complete integral of (5) is

$$F = (aX + b)^{-1} \quad .. \quad .. \quad .. \quad .. \quad (6)$$

where a and b are arbitrary constants. If in (6) we put $b = 0$ and $a = t_0^{-2}$, (2) then gives a line-element for the case $G(\xi) = -1$. This line-element has been termed by Gilbert[2] as the 'Metric of the Substratum'.

Although, in reality, the condition of isotropy and homogeneity do not exactly hold good, this assumption is not very far from the true state. As it is, it should be remembered that (1) does not actually represent the motion of a system of particles in a slightly inhomogeneous material distribution in a possible form of a world-model. The slight irregularities here and there due to the non-homogeneity of the model under consideration are not supposed to destroy the property of an isotropic pressure to any appreciable extent for the particular case $G(\xi) = -1$. Thus, as a matter of fact, the receding particles are attracted by other masses (including the nuclear cluster) which are excluded from the 'domain' forming the statistical system. This 'extraneous' influence would naturally tend to hinder the outward motion and the recession of particles cannot 'go on' indefinitely. For the purpose of simplicity, let us assume that the space $r = ct$ is smoothed out of any inhomogeneous matter except the nuclear cluster so that the particles are attracted towards the central mass only. Let R be the radius of the model under consideration at any epoch t, so that $R = ct$. The law of recession at such a great distance is

$$\dot{r} = V = \frac{r}{t}. \quad .. \quad .. \quad .. \quad (7)$$

Therefore, for $r \sim ct$ we have

$$X = \epsilon\,.\,t^2,\ Y = \epsilon,\ Z = \epsilon\,.\,t,\ \xi = 1\,; \quad .. \quad .. \quad (8)$$

where $\epsilon = 1 - \frac{P^2}{c^2 t^2}$. Hence, the acceleration formula reduces to

$$\frac{dV}{dt} \sim 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

This would give zero acceleration for any particle of the system, so that a thick 'horizon' would be formed on the surface of the sphere $r = R$. So, it is natural to suppose that the motion of particles of the non-homogeneous system is reversed after they reach the limits of the model and the modified equations of motion for these particles incorporating this tendency to 'go back' can be expressed as a single vector relation

$$\frac{dV}{dt} = (P - Vt)\frac{Y}{X}G(\xi) \pm YZH(X, \xi) \qquad \ldots \quad \ldots \quad (10)$$

which is invariant for Lorentz transformations. In general, we have the vector relation, viz.

$$\frac{dV}{dt} = (P - Vt)\frac{Y}{X}G(\xi) \pm Y^m Z^n H(X, \xi) \qquad \ldots \quad \ldots \quad (11)$$

which is Lorentz-invariant provided $2m + n = 3$.

It is possible to obtain equations (10) from the Principle of Least Action. The procedure is similar to that used by Walker.[3] We have the variational integral, viz.

$$\delta \int W ds = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where $ds = Y^{\frac{1}{2}} dt$. Let $W(X, \xi) = X^{\frac{1}{2}} \xi^{\frac{1}{2}} U(X, \xi)$, then (12) can be written as

$$\delta \int (ZU) dt = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

which leads to the following variational equation

$$\frac{d}{dt}\left[\frac{\partial}{\partial V}(ZU)\right] - \frac{\partial}{\partial P}(ZU) = 0. \ldots \qquad \ldots \quad \ldots \quad (14)$$

Let us put

$$A = \frac{Z}{Y}\xi\frac{\partial U}{\partial \xi}, \quad B = \frac{Y}{Z}A + \tfrac{1}{2}U;$$

also, we have

$$\frac{dX}{dt} = 2Z, \quad \frac{dY}{dt} = -2\left\{(tY - Z)\frac{Y}{X}G(\xi) \pm \frac{V}{c^2}YZH(X, \xi)\right\},$$

$$\frac{dZ}{dt} = Y - \left\{(tZ - X)\frac{Y}{X}G(\xi) \pm \frac{P}{c^2}YZH(X, \xi)\right\},$$

$$\frac{d\xi}{dt} = \frac{2Z}{X}(1 - \xi)(1 + G) \mp 2Y\xi\left(P - V\frac{Z}{Y}\right)\frac{H(X, \xi)}{c^2}.$$

With the help of these relations, after considerable reduction, (14) reduces to

$$\left(V-\frac{Y}{Z}P\right)\left(\frac{dA}{dt}-tA\frac{Y}{X}G\right)\pm AtYH=0, \quad \ldots \quad \ldots \quad (15)$$

that is,

$$(P-Vt)\frac{dA}{dt}-tA\frac{dV}{dt}=0, \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

or,

$$\frac{d}{dt}\Big[\log\{A(P-Vt)\}\Big]=0. \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

(17) gives

$$\xi\frac{\partial U}{\partial \xi}=\frac{\alpha}{Z\left(P-V\frac{Z}{Y}\right)}, \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

where α is a constant of integration.

The relation (14) can be written in terms of A as:

$$A\frac{dU}{dt}=P\left[\frac{Y}{Z}\frac{dA}{dt}+\frac{A}{2Z}\frac{dY}{dt}\right]-V\frac{dA}{dt}, \quad \ldots \quad \ldots \quad (19)$$

or,

$$\frac{dV}{dt}=PL_1-VL_2, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where

$$L_1=\frac{Y}{AZ}\frac{dA}{dt}+\frac{1}{2Z}\frac{dY}{dt}, \quad L_2=\frac{1}{A}\frac{dA}{dt}=\frac{t\frac{dV}{dt}}{P-Vt}.$$

Therefore,

$$L_1=\frac{Y}{Z}L_2-\frac{Y}{XZ}(tY-Z)G\mp\frac{V}{c^2}YH. \quad \ldots \quad \ldots \quad (21)$$

Hence,

$$\frac{dV}{dt}=\left(P\frac{Y}{Z}-V\right)L_2-\frac{PY}{ZX}(tY-Z)G\mp\frac{PV}{c^2}YH; \quad \ldots \quad (22)$$

that is,

$$\left(1-\frac{t}{Z}\right)\frac{dV}{dt}=-\frac{PV}{c^2}\frac{Y}{ZX}(P-Vt)\mp\frac{PV}{c^2}YH \quad \ldots \quad \ldots \quad (23)$$

which gives

$$\frac{dV}{dt}=(P-Vt)\frac{Y}{X}G\pm YZH.$$

2. Mechanical properties of the new equations of motion.

It has been shown before that at a great distance the relations (8) hold good and that the acceleration of an individual particle tends to zero. For similar considerations the new equations of motion would give the following value for the acceleration at great distances:

$$\frac{dV}{dt} = \pm\frac{KG(1)}{t^3}. \qquad (24)$$

Here we have put $H(X, \xi) = KX^{-2}G(\xi)$ where K is a constant. It is not difficult to prove that $G(1) = -1$. (24) then comes out as

$$\frac{dV}{dt} = -\frac{K}{t^3}. \qquad (25)$$

It is, therefore, quite evident that for a system of particles to have a tendency to approach the central mass we must take the upper sign only for the second term on the r.h.s. of equation (10). There is, therefore, no loss of generality if in place of (10) we take the equation of motion as

$$\frac{dV}{dt} = (P-Vt)\frac{Y}{X}\,G(\xi)+\frac{KYZ}{X^2}\,G(\xi). \qquad (26)$$

If the radius of curvature of the trajectories given by (1) is ρ_0 and ρ is the radius of curvature of those given by (10), then

$$\frac{1}{\rho} = \frac{1}{\rho_0}+n, \qquad (27)$$

where

$$\frac{1}{\rho_0} = \frac{c^2YG(\xi)}{XV^3}\left[\frac{1}{c^2}(P-Vt)^2-XY(\xi-1)\right]^{\frac{1}{2}} \qquad (28)$$

and $n = n(P, V, t)$ which is positive. Hence

$$\frac{1}{\rho} > \frac{1}{\rho_0},$$

or,

$$\rho_0 > \rho \qquad (29)$$

which clearly indicates attraction towards the nuclear cluster.

From (26) we can write down the acceleration formula as [4]

$$\frac{1}{Y^{\frac{1}{2}}}\frac{d}{dt}\left(\frac{V}{Y^{\frac{1}{2}}}\right) = \left\{\left(P-V\frac{Z}{Y}\right)+\frac{K\xi}{Z}\right\}\frac{G(\xi)}{X}. \qquad (30)$$

The equation (30) contains on its r.h.s. a sum of two 'complementary' accelerations g_1' and g_2', and hence it can be expressed as

$$\frac{1}{Y^{\frac{1}{2}}}\frac{d}{dt}\left(\frac{V}{Y^{\frac{1}{2}}}\right) = g_1'+g_2', \qquad (31)$$

where

$$g_1' = -\left(P-V\frac{Z}{Y}\right)\frac{1}{X}-\frac{K\xi}{ZX} = g_1-\frac{K\xi}{ZX} \qquad (32)$$

and

$$g_2' = -\frac{C\left\{\left(P-V\frac{Z}{Y}\right)\frac{1}{X}+\frac{K\xi}{ZX}\right\}}{(\xi-1)^{\frac{3}{2}}\psi(\xi)} = g_2 - \frac{C\frac{K\xi}{ZX}}{(\xi-1)^{\frac{3}{2}}\psi(\xi)}. \quad .. \quad (33)$$

Near a condensation (not necessarily the central mass), $\xi \to 1$, $X \to t^2$, $y \to 1$, etc., and we get

$$g_2' = -\frac{P-Vt}{|P-Vt|^3}\frac{Cc^3t}{\psi(1)} - \frac{KCc^3}{|P-Yt|^3\psi(1)}. \quad .. \quad .. \quad (34)$$

The second term on the r.h.s. of (34) gives the departure from the inverse square law for the system of particles considered here. g_1 and g_2 are the values given by Milne.

If g_2' be the component of local acceleration due to gravitation we get the force equation corresponding to (34) in the form

$$F' = F + \frac{MK\xi}{XZ}G(\xi) \quad .. \quad .. \quad .. \quad (35)$$

where F is the force of gravitation for Milne's case (obeying the inverse square law), and it is given by

$$F = Mg_2 + \frac{V}{Y^{\frac{1}{2}}}\cdot\frac{1}{Y^{\frac{1}{2}}}\frac{dM}{dt}, \quad M = m\xi^{\frac{1}{2}}. \quad .. \quad .. \quad (36)$$

If p' be the momentum when nuclear forces are present, and p be the momentum for Milne's case, then, we have

$$\frac{1}{Y^{\frac{1}{2}}}\frac{dp'}{dt} = M(g_1'+g_2') + \frac{V}{Y^{\frac{1}{2}}}\cdot\frac{1}{Y^{\frac{1}{2}}}\frac{dM}{dt}, \quad .. \quad .. \quad (37)$$

and

$$\frac{1}{Y^{\frac{1}{2}}}\frac{dp}{dt} = M(g_1+g_2) + \frac{V}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM}{dt}. \quad .. \quad .. \quad .. \quad (38)$$

From (37) and (38) we deduce the relation

$$\frac{1}{Y^{\frac{1}{2}}}\frac{d(\delta p)}{dt} = \frac{MK\xi}{ZX}G(\xi), \quad .. \quad .. \quad .. \quad (39)$$

where we put $p'-p = \delta p$.

3. Force and energy equations for the new system of particles.

The origin O being situated at the centre of the nuclear cluster, the acceleration of the receding particles can be written as

$$\frac{dV}{dt} = -\left\{(P-Vt)\frac{Y}{X}+\frac{KYZ}{X^2}\right\} - \frac{C\left\{(P-Vt)\frac{Y}{X}+\frac{KYZ}{X^2}\right\}}{(\xi-1)^{\frac{3}{2}}\psi(\xi)}. \quad (40)$$

If a condensation is determined by $[\psi(\xi)]_{\xi=1}$ = maximum, and, if

$$\psi(\xi) = \psi(1)e^{-\frac{(\xi-1)^2}{a^2}} \quad (a \to 0 \text{ as } \xi \to 1)$$

gives the inverse square law for the attraction, then as the departure from the inverse square law is not considerable in the present case, (40) can be written as

$$\frac{dV}{dt} = -\left\{(P-Vt)\frac{Y}{X}+\frac{KYZ}{X^2}\right\} - \frac{M\left\{(P-Vt)\frac{Y}{X}+\frac{KYZ}{X^2}\right\}}{M_0(\xi-1)^{\frac{3}{2}}} \quad .. \quad (41)$$

where M is the mass of the condensation. Let m_1 be the mass of the particle (P, V, t). Then the relation giving the interaction between m_1 and M under the influence of the nuclear forces is given by

$$\frac{1}{Y^{\frac{1}{2}}}\frac{d}{dt}\left[\frac{M_1 V}{Y^{\frac{1}{2}}}\right] = -\frac{M_1\left\{\left(P-V\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{X}$$

$$-\frac{MM_1\left\{\left(P-V\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{M_0X(\xi-1)^{\frac{3}{2}}}+\frac{V}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM_1}{dt}, \quad M_1 = m_1\xi^{\frac{1}{2}}, \quad .. \quad (42)$$

where the force of attraction on m_1 due to M is

$$F_P = -\frac{MM_1\left\{\left(P-V\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{M_0X(\xi-1)^{\frac{3}{2}}}+\frac{V}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM_1}{dt} \quad .. \quad (43)$$

and the time-component corresponding to F_P [i.e., the time-component of the four-vector (F_P, F_t)] is

$$F_t = -\frac{MM_1\left\{\left(ct-c\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{M_0X(\xi-1)^{\frac{3}{2}}}+\frac{c}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM_1}{dt}. \quad .. \quad (44)$$

The energy equation can now be expressed as

$$F_P\left(\frac{V}{Y^{\frac{1}{2}}}-P\frac{Y^{\frac{1}{2}}}{Z}-\frac{K}{XY^{\frac{1}{2}}}\right)-F_t\left(\frac{c}{Y^{\frac{1}{2}}}-ct\frac{Y^{\frac{1}{2}}}{Z}-\frac{K}{XY^{\frac{1}{2}}}\right)=\frac{1}{Y^{\frac{1}{2}}}\frac{d}{dt}(c^2M_1) \quad (45)$$

which with the help of (43) and (44) yields the relation, viz.,

$$\frac{c^2XY-K(c-V)}{XY^2}\frac{dM_1}{dt}=\frac{c^2MM_1}{M_0Z(\xi-1)^{\frac{3}{2}}}+\frac{2K(\mu_P-\mu_t)MM_1}{M_0X^2Y(\xi-1)^{\frac{3}{2}}}, \quad .. \quad (46)$$

$$\mu_P = P-V\frac{Z}{Y}, \quad \mu_t = ct-c\frac{Z}{Y}.$$

The gravitational potential is given by the equation

$$\frac{d}{dt}(c^2M_1+\chi) = -\frac{KZ(c-V)m_1M}{M_0X^2Y(\xi-1)^{\frac{3}{2}}}. \quad \ldots \quad \ldots \quad (47)$$

If $\chi = \chi(P, t)$, then the gravitational potential is determined by

$$\frac{\partial\chi}{\partial P} = \frac{m_1M\left\{\left(P-V\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{M_0X(\xi-1)^{\frac{3}{2}}}+\frac{V}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM_1}{dt} \quad \ldots \quad (48)$$

and

$$\frac{1}{c}\frac{\partial\chi}{\partial t} = -\frac{m_1M\left\{\left(ct-c\frac{Z}{Y}\right)+\frac{KZ}{XY}\right\}}{M_0X(\xi-1)^{\frac{3}{2}}}-\frac{c}{Y^{\frac{1}{2}}}\frac{1}{Y^{\frac{1}{2}}}\frac{dM}{dt}. \quad \ldots \quad (49)$$

Considering the condensation m as a simple point-mass just as m_1, the equation corresponding to (47) would be

$$\frac{d}{dt}[\chi+Mc^2] = -\frac{KZ(c-V)mM_1}{M_0X^2Y(\xi-1)^{\frac{3}{2}}}. \quad \ldots \quad \ldots \quad (50)$$

If we put $\chi =$ the potential energy P.E., $M_1c^2 = E_1$, $Mc^2 = E$, etc., then (47) and (50) give together the energy equation for the pair of two interacting particles m_1 and m as

$$\frac{d}{dt}[2\chi+E+E_1] = -\frac{KZ(c-V)(mE_1+m_1E)}{c^2M_0X^2Y(\xi-1)^{\frac{3}{2}}}. \quad \ldots \quad (51)^*$$

Let us suppose that m is the mass of the nuclear cluster, $m_1, m_2, \ldots, m_n$ are the masses of the receding particles; if the interaction of the particles amongst themselves be neglected, then the energy equation for this system of particles surrounding the nucleus of mass m is given by the relation, viz.,

$$\frac{d}{dt}\left[2n\chi+nE+\sum_{r=1}^{n}E_r\right] = -\frac{KZ(c-V)\left\{m\sum_{r=1}^{n}E_r+E\sum_{r=1}^{n}m_r\right\}}{c^2M_0X^2Y(\xi-1)^{\frac{3}{2}}}. \quad \ldots \quad (52)$$

If, however, the interaction between the particles themselves is comparable with that between the particles and the central mass, then the energy equation for the system can be written as:

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* It can be verified that

$$\frac{KZ(c-V)(mE_1+m_1E)}{c^2M_0X^2Y(\xi-1)^{\frac{3}{2}}}>0.$$

Similar results can be obtained for the equations (52) and (53).

$$\frac{d}{dt}\left[(n+1)\chi+E+\sum_{r=1}^{n}E_r\right]=$$

$$-\frac{(n^2-n+2)}{2n}\cdot\frac{KZ(c-V)\left\{m\sum_{r=1}^{n}E_r+E\sum_{r=1}^{n}m_r\right\}}{c^2M_0X^2Y(\xi-1)^{\frac{3}{2}}}. \quad .. \quad (53)$$

It will be seen that the total energy for a pair of two interacting particles as given by (51) is negative. Similarly, (52) and (53) give negative energy for the particular cases considered. This negative value for the total energy of the three particular cases considered is, perhaps, due to a tendency of approach towards the nuclear cluster, present in the slightly non-homogeneous system of particles in (an approximately statistical) receding motion.

4. Some consequences resulting from the new equations of motion.

As an example of the tendency to approach towards the central mass exhibited by the system of particles considered here, let us take the system of particles given by $G(\xi)+1 \sim 0$, $X \sim t^2$, $Y \sim 1$, $Z \sim t$, so that (26) is simplified to the form, viz.,

$$\frac{dV}{dt} \sim -\frac{P-Vt}{t^2}-\frac{K}{t^3}. \quad .. \quad .. \quad .. \quad (54)$$

This system of particles is constrained to move under the influence of the central mass and the velocity of the constrained particle is now $\left(V-\frac{P}{t}-\frac{K}{2t^2}\right)$ instead of $\left(V-\frac{P}{t}\right)$. It is interesting to study the electrical state of this system of particles. If m is the mass of each particle and e its charge, then the equation of motion of a particle of the system takes the form:

$$m\frac{dU}{dt}=eF-\mu U, \quad .. \quad .. \quad .. \quad .. \quad (55)^5$$

where $F=(X, Y, Z)$ is the electric force, U the velocity of the constrained particle and μ is a constant. In the present case, $U=V-\frac{P}{t}-\frac{K}{2t^2}$ (55) now reduces to

$$m\frac{d}{dt}\left(V-\frac{P}{t}-\frac{K}{2t^2}\right)=eF-\mu\left(V-\frac{P}{t}-\frac{K}{2t^2}\right). \quad .. \quad (56)$$

Therefore,

$$\rho=\text{div } F=\frac{m+\mu t}{et}\text{div } V_1, \quad V_1=V-\frac{P}{t}, \quad .. \quad .. \quad (57)$$

where ρ is the density of the electric charge. When ρ is not a function of position the equation of continuity [6] gives:

$$\frac{d\rho}{dt}+\rho \operatorname{div} V_1 = 0. \quad \ldots \quad (58)$$

From (57) and (58) we get

$$\frac{d\rho}{dt}+\frac{et}{m+\mu t}\rho^2 = 0 \quad \ldots \quad (59)$$

which intergrates into

$$\rho^{-1}-\rho_0^{-1} = \frac{e(\mu t+m \log m)}{\mu^2} - \frac{em}{\mu^2}\log(m+\mu t), \quad \ldots \quad (60)$$

where ρ_0 is the initial density of charge. The magnetic force $B = (\alpha, \beta, \gamma)$ is given by

$$curlB = \rho V_2 - \frac{mV_1}{et^2}+\frac{2Km}{et^4}, \quad V_2 = V-\frac{P}{t}-\frac{K}{2t^2}, \quad \ldots \quad (61)$$

the other equation remaining the same as in an earlier paper by the author. The mechanical stress can be expressed as

$$(P, Q, R) = \rho\left\{\frac{(m+\mu t)V_2}{et} - \frac{mK}{2et^3}+[B \,.\, V_2]\right\}. \quad \ldots \quad (62)$$

The rate at which work is done by the mechanical forces and the flow of energy per unit area (Poynting flux) are respectively given by

$$\frac{dW}{dt} = \rho V_2\left\{\frac{m+\mu t}{et}V_2 - \frac{mK}{2et^3}\right\} \quad \ldots \quad (63)$$

and

$$\pi = \frac{m+\mu t}{et}[B \,.\, V_2] - \frac{mKB}{2et^3}. \quad \ldots \quad (64)$$

The points of difference between the system of particles considered here and the fundamental system (i.e., the simple system) considered in a former paper can be clearly brought out from the above relations. Two points of interest are: (1) the density of electric charge does not tend to zero as $t \to \infty$; and (2) the rate of change of the magnetic vector per unit time is the same for these two systems, although its magnitude is different for the two systems.

It is a great pleasure to record my grateful thanks to Dr. G. S. Mahajani for his keen interest in my work.

5. Summary.

In this paper we have extended Milne's Kinematical Theory to a system of receding particles in a slightly inhomogeneous space with a dense cluster at the centre of the sphere $r = ct$. This system has, therefore, a tendency to approach the central mass due to the mutual attraction. Equations of motion

are obtained for this new system of particles from the principle of least action. These equations are given by (26). The attraction towards the nuclear cluster is also evident from the relation $\rho_0 > \rho$, where ρ is the radius of curvature of the space for the new system and ρ_0 is that for Milne's case. Consideration of the energy equation for this system leads to a negative value for the energy-function from which it can be surmised that this must be due to the 'tendency of approach' towards the central cluster. As an example, a system of charged particles conforming to the extended theory is considered and it is compared to the simple system of charged particles referred to above.

References.

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September 1, 1939.
Revised: April 20th, 1940.