# MESON INTENSITY IN THE SUBSTRATOSPHERE II.

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In two previous communications (1945 *a*, *b*) we have reported measurements of the vertical intensity of mesons in the substratosphere. The vertical intensity of some component or group of cosmic rays as a function of altitude, which we shall again call for brevity the altitude intensity curve of that component, was measured at Bangalore, magnetic latitude 3° N. to an altitude corresponding to a pressure of 275 millibars (35,000 feet) for rays penetrating 20 cm. of lead absorber, and to an altitude corresponding to a pressure of 530 millibars for rays penetrating 30 and 5·25 cm. of lead. The latter measurement was made using Bhabha's method (1944) for eliminating the soft component and was therefore a measure of the number of mesons penetrating 5·25 cm. of lead. This work has been continued, and another flight was made from Bangalore on the 21st July, 1945, to an altitude of 40,000 feet during which the altitude intensity curves for mesons penetrating 10 and 30 cm. of lead absorber were measured by two independent quadruple coincidence counter telescopes down to a pressure of 180 millibars. The purpose of this paper is to report the results of the measurement of rays penetrating 30 cm. of lead made on this flight. The measurements with the 10 cm. set will be reported in another paper after further data has been collected. The measurements were made as far as circumstances permitted in accordance with the criteria given in the previous paper. The weather conditions were, however, more difficult than on the previous flights due to the fact that the latter were made in December during the dry weather, whereas this flight was made in July during the Monsoon or rainy season. The humidity of the air was much higher and there were clouds all the way up to the greatest heights reached.

## THE MEASUREMENTS.

The counters were similar to those mentioned in the previous paper (1945*b*) and the geometry of the telescope was the same as that shown there * by Figs. 2 and 3, except that there were two counters in parallel at each level. The disposition of lead was the same as in Figs. 2 and 3, with an additional lead block of 10 cm. on top. No single particle traversing the telescope would make an angle with the vertical greater than 22°, and its length of path in the atmosphere and absorber was not more than 8% greater than that of a vertical ray. The two counters at each level were separated by sponge rubber to minimise damage due to vibration or shock. This had the disadvantage that a ray passing through, say, the top left counter and the bottom right counter had a considerable chance of passing through the rubber *between* the two counters at the intermediate levels and thus failing to produce a quadruple coincidence. The counting rate of the telescope was therefore a little more than double that of a simple telescope, and not four times, as might have been expected. The experimental arrangements were the same as on the previous flights except that this time the telescope was mounted separately in its

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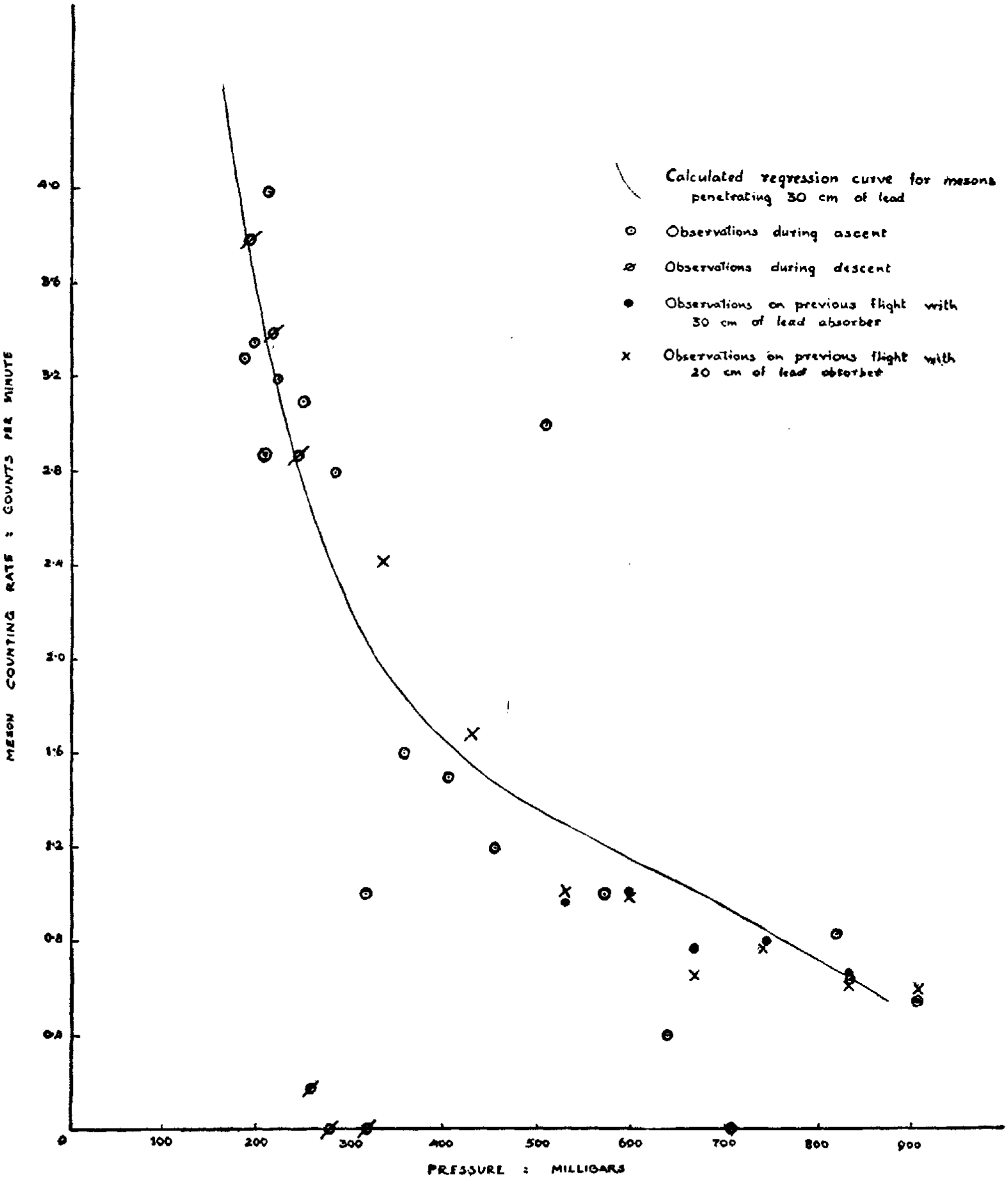
*** Figures 1 and 2 in that paper have been interchanged by accident. The captions are correct.**

own wooden box with separate provision for heating while the amplifier was mounted in a separate box, also having provision for heating. Shielded leads from the amplifier box conveyed the readings to a telephone call counter mounted on a dashboard together with a clock and a standard altimeter. This part of the equipment was the same as on the previous flight. The readings were photographically recorded at specified instants and also written down. The temperature was also noted separately.

TABLE I.

| Indicated altitude. | Pressure in millibars. | Time. | Counter reading. | Mean pressure in millibars. | Time interval in minutes. | Total counts. | Mean counting rate in counts/min. |
|---|---|---|---|---|---|---|---|
| 3,000 | | | | 906 | 120 | 65 | 0·542 |
| 3,000 | 906 | 11·45 | 8,508 | | | | |
| | | | | 820 | 14 | 12 | 0·857 |
| 7,500 | 742 | 11·59 | 8,520 | | | | |
| | | | | 709 | 5 | 0 | 0·000 |
| 10,000 | 678 | 12·04 | 8,520 | | | | |
| | | | | 639 | 5 | 2 | 0·400 |
| 12,500 | 602 | 12·09 | 8,522 | | | | |
| | | | | 571 | 4 | 4 | 1·000 |
| 15,000 | 542 | 12·13 | 8,526 | | | | |
| | | | | 510 | 7 | 21 | 3·000 |
| 17,500 | 480 | 12·20 | 8,547 | | | | |
| | | | | 454 | 5 | 6 | 1·200 |
| 20,000 | 430 | 12·25 | 8,553 | | | | |
| | | | | 404 | 4 | 6 | 1·500 |
| 22,500 | 380 | 12·29 | 8,559 | | | | |
| | | | | 357 | 5 | 8 | 1·600 |
| 25,000 | 336 | 12·34 | 8,567 | | | | |
| | | | | 316 | 6 | 6 | 1·000 |
| 27,500 | 297 | 12·40 | 8,573 | | | | |
| | | | | 281 | 5 | 14 | 2·800 |
| 30,000 | 265 | 12·45 | 8,587 | | | | |
| | | | | 248 | 9 | 28 | 3·111 |
| 32,500 | 230 | 12·54 | 8,615 | | | | |
| | | | | 221 | 14 | 45 | 3·214 |
| 34,000 | 213 | 1·08 | 8,660 | | | | |
| | | | | 213 | 5 | 20 | 4·000 |
| 34,000 | 213 | 1·13 | 8,680 | | | | |
| | | | | 208 | 17 | 49 | 2·882 |
| 35,000 | 204 | 1·30 | 8,729 | | | | |
| | | | | 197 | 30 | 101 | 3·367 |
| 36,200 | 190 | 2·00 | 8,830 | | | | |
| | | | | 186 | 7 | 23 | 3·286 |
| 36,850 | 181 | 2·07 | 8,853 | | | | |
| | | | | 192 | 5 | 19 | 3·800 |
| 35,000 | 204 | 2·12 | 8,872 | | | | |
| | | | | 217 | 5 | 17 | 3·400 |
| 32,500 | 230 | 2·17 | 8,889 | | | | |
| | | | | 243 | 8 | 23 | 2·875 |
| 30,500 | 256 | 2·25 | 8,912 | ........ | ........ | ........ | ........ |
| | | | | 256 | 23 | 4 | 0·174 |
| 30,500 | 256 | 2·48 | 8,916 | | | | |
| | | | | 277 | 2 | 0 | 0·000 |
| 27,500 | 297 | 2·50 | 8,916 | | | | |
| | | | | 316 | 4 | 0 | 0·000 |
| 25,000 | 336 | 2·54 | 8,916 | | | | |

Readings were taken continuously on the way up, at *indicated* altitudes* increasing by steps of 2,500 feet from 7,500 feet till an indicated altitude of 32,500 feet was reached corresponding to a pressure of 230 millibars. Seven more readings were taken at higher altitudes. The actual readings are given in the first four columns of Table I. It will be seen from Table I that the aeroplane flew at pressures below 204 millibars for some 42 minutes, so that the highest reading is statistically



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* The real altitude may differ considerably from the indicated altitude. For example, the highest indicated altitude attained on this flight was 36,850 feet, corresponding almost exactly to an actual altitude of 40,000 feet.

the most accurate. The next four columns give the mean pressure in each interval, the time taken in traversing it, the total counts in this interval and the observed mean counting rate.

The observed mean counting rates for the telescope are shown in the figure as points with circles round them. The points taken during the descent are distinguished by a slanting line through them. For this telescope eight independent readings were taken at pressures below 256 millibars, five of these as the aeroplane ascended slowly to a height corresponding to the lowest pressure attained, namely 180 millibars, and three on the way down. All these points lie fairly close to each other and are an indication of the accuracy attained with the 30 cm. set at the highest altitude reached. While the aeroplane flew for another 23 minutes at 256 millibars and then descended further, cold air entered the boxes, water condensed on the electrical connections outside the boxes and froze into ice. In these circumstances the set stopped working, as clearly indicated in Table I and the figure by the sudden drop in the count after 2·25 p.m. The dotted line in the table marks the place below which the set ceased to work.

## DISCUSSION OF THE RESULTS.

In the figure we have also plotted as full circles the readings for rays penetrating 30 cm. of lead obtained on the previous flight up to 15,000 feet, these being fitted at ground level to allow for the difference in the two sets. These readings agree with the present observations quite well. Despite the fluctuations due to the smallness of the total number of counts at the lower altitudes the general shape of the curve is quite clear. The intensity increases approximately six times as we ascend from ground level at Bangalore (pressure 906 millibars) to a pressure of 193 millibars.

On the same figure the crosses show the intensity of mesons penetrating 20 cm. of lead measured down to a pressure of 275 millibars on the previous flights. The 20 cm. readings have been multiplied by a constant factor so as to make the ratio of the 20 cm. to the 30 cm. reading at ground level 1·086, as observed experimentally (S. V. Chandrashekhar Aiya, 1944). The 20 cm. and 30 cm. readings lie very close together down to a pressure of 400 millibars, but there is an indication that below this pressure the 20 cm. readings increase more rapidly than the 30 cm. readings.

A glance at the figure shows that certain points measured on this flight appear to lie well off any smooth curve drawn through the other points. Such are, for example, the points at 316, 511, 639 and 709 millibars. In order to investigate further whether these aberrations are merely statistical or significant, it was suggested to us by Prof. D. D. Kosambi that we should use for an analysis of our data some of the new statistical methods which are now regularly used in analysing biological and other experimental data. The methods are given in R. A. Fisher's Statistical Methods for Research Workers (9th edition, 1944) and we refer in particular to section 29.2 of the later editions. We summarise the method briefly for those who are not familiar with it.

Suppose the number of cosmic ray counts has been recorded in $n$ different altitude intervals for varying lengths of time. We assume that the mean intensity of this component of cosmic rays as a function of altitude is given sufficiently accurately by a polynomial of degree $m<n-1$. We use this polynomial to calculate the total number of counts to be expected in each altitude interval for the actual length of time during which the observation was made. The coefficients of the polynomial are then determined so as to make the *sum* of the squares of the actual minus the expected counts in each interval a minimum. The intensity altitude curve calculated from the polynomial with its coefficients determined in this way is known as a *regression curve.* The discrepancy between observation and calculation in each interval is then measured by the corresponding value of $\chi^2$ which is defined as the

square of the difference between the observed count and the corresponding calculated total divided by the calculated total. It should be noted that as far as the value of $\chi^2$ is concerned, the discrepancy between observation and expectation may be in either direction. The probability in any particular altitude interval for a deviation from the calculated value equal to or greater than that observed is given in Table III of Fisher's book at the top of the column in which the corresponding value of $\chi^2$ appears in the *first* row of the table. Thus the probability of $\chi^2$ exceeding 3·841 in any particular altitude interval is less than 5%. The sum of the values of $\chi^2$ for all the intervals can also be used for judging the goodness of fit of the regression curve as a whole. Since the $m+1$ coefficients of the polynomial have been chosen to give the best fit possible, the number of 'degrees of freedom' left is $n-(m+1)$. The probability $P$ of deviations due to random fluctuations from the calculated curve equal to or greater than those observed can be obtained from the same table of Fisher's book by locating the column in which this total value of $\chi^2$ appears in the $(n-m-1)$th row. The probability $P$ is then given at the head of the column. For example, for $n-m-1$ equal to 9 the probability of $\chi^2$ exceeding 16·919 is only 5%, whereas the probability of its exceeding 21·666 is less than 1%. We should ordinarily consider any value of $\chi^2$ for which the probability is less than 5% as significant.

The purpose of this analysis is to investigate whether the deviations are due to random fluctuations in agreement with what is to be expected on the basis of a Poisson distribution, or if they are significant. If the total $\chi^2$ is larger than that to be expected at the 5% level of significance for the number of degrees of freedom concerned, then we must conclude that the actual observations differ significantly from the regression curve we have calculated. One may then investigate whether $\chi^2$ is abnormally large (greater than 3·841) for each of the intervals, or only for a few. If $\chi^2$ is only abnormally large for a few intervals, and the removal of these brings the total $\chi^2$ for the remaining intervals to within the value to be expected at the 5% level for the reduced number of degrees of freedom concerned, then one is faced with one of two alternatives. Either (*a*) the real altitude intensity curve differs significantly from the calculated regression line by having, for example, humps or hollows at some points, or (*b*) there is some real physical factor which causes the apparatus to give abnormal counts in the altitude interval concerned. It is interesting to note that since any regular trend in the measurements, as for example the variation of the counting rate with altitude, is eliminated by fitting a regression curve, *it is quite immaterial* whether this trend is due to a real variation of cosmic ray intensity with height or systematic instrumental effects, climatic conditions or any other cause.

It should also be mentioned that fitting the variation of intensity as a function of the logarithm of the pressure instead of the indicated altitude would here lead to the same results since correct to the second place of decimals the relation between log. millibars and altitude is linear, and this is all that is justified by the accuracy of the measurement.

In fitting the regression curve* and calculating the values of $\chi^2$ we use the fact that from 7,500 feet indicated altitude readings were taken at regular intervals of 2,500 feet till an indicated altitude of 35,000 feet, which allows Tchebysheff polynomials to be used. In the 30,000–32,500 interval the readings given in Table I taken during the descent have been added to those taken during the ascent and the same has been done in the 32,500–35,000 interval. As already mentioned, the readings taken below 30,000 feet *during the descent* (figures below the dotted line in Table I) have been omitted since the apparatus stopped working. A prolonged reading of two hours was taken at ground level (3,000 feet) and there was one reading in going from 3,000 feet to 7,500 feet. For simplicity of calculation the latter

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* All altitudes mentioned in the rest of this section are the *indicated* not the actual ones. The pressures, when given, are of course the actual pressures.

reading has been split into two parts in the ratio 4 : 5, and the second part has been put as occurring between 5,000 and 7,500 feet, while the first part has been added to the ground level reading and put as occurring between 2,500 and 5,000 feet. Since the meson intensity decreases slowly below 7,500 feet the error introduced by this procedure is not very great and a very considerable simplification is introduced in the calculations by having the readings at equal (indicated) altitude intervals. The data is reliable to the greatest height reached, and we have therefore observations at 13 altitude intervals from 2,500 to 35,000 feet (indicated). Fitting a cubic regression curve ($m = 3$) to this data again leaves us with 9 degrees of freedom. In addition, the plane gradually ascended from 35,000 feet to 36,800 feet, the highest altitude indicated, and then descended. The time spent above 35,000 feet was 42 minutes, and we may put the counts registered above 35,000 feet at the average altitude 35,900 feet. This point can then be used as an *independent check* on the regression curve.

The fitting of the regression curve and the calculation of $\chi^2$ has been carried out for us by Prof. D. D. Kosambi and his results are given in Table II. Only the first thirteen intervals were used in the fitting of the regression line. The last interval below the line was used as an independent check, as mentioned above. The values of $\chi^2$ in each interval are listed in the last column of the table. The four coefficients of the polynomial are given in the standard notation by $A = 1{\cdot}214852$, $B = 0{\cdot}129772$, $C = 0{\cdot}009788$ and $D = 0{\cdot}012594$. The smallness of the last two coefficients shows that the quadratic and cubic terms in the polynomial are small compared with the constant and linear terms. The sum of $\chi^2$ for the 13 intervals is 28·91. It can be seen from Table IV of Fisher and Yates (1943, p. 31) that the probability of $\chi^2$ exceeding 28 for $13-4 = 9$ degrees of freedom is less than 0·1%, showing that the overall fit of the observations with our regression line is significantly bad. On examining the individual values of $\chi^2$ it is, however, obvious that a substantial contribution to this sum comes from the value of $\chi^2$ in the 15,000–17,500

TABLE II.

| Indicated altitude interval in 100 feet. | Mean pressure in millibars. | Calculated mean rate in counts/min. | Observed mean rate in counts/min. | Time interval in minutes. | Calculated total counts. | Observed total counts. | $\chi^2$ |
|---|---|---|---|---|---|---|---|
| 25– 50 | 876 | 0.541764 | 0·555 | 126 | 79·00 | 70 | 1·0253 |
| 50– 75 | 784 | 0·758324 | 0·875 | 8 | 10·74 | 7 | 1·3024 |
| 75–100 | 709 | 0·934296 | 0·000 | 5 | 4·67 | 0 | 4·6700* |
| 100–125 | 639 | 1·082274 | 0·400 | 5 | 5·41 | 2 | 2·1494 |
| 125–150 | 571 | 1·214852 | 1·000 | 4 | 4·86 | 4 | 0·1522 |
| 150–175 | 510 | 1·344624 | 3·000 | 7 | 9·41 | 21 | 14·2750*** |
| 175–200 | 454 | 1·484184 | 1·200 | 5 | 7·42 | 6 | 0·2717 |
| 200–225 | 404 | 1·646126 | 1·500 | 4 | 6·58 | 6 | 0·0511 |
| 225–250 | 357 | 1·843044 | 1·600 | 5 | 9·22 | 8 | 0·1614 |
| 250–275 | 316 | 2·087532 | 1·000 | 6 | 12·52 | 6 | 3·3846 |
| 275–300 | 281 | 2·392184 | 2·800 | 5 | 11·96 | 14 | 0·3480 |
| 300–325 | 248 | 2·769594 | 3·000 | 17 | 59·05 | 51 | 1·0974 |
| 325–350 | 218 | 3·232356 | 3·195 | 41 | 132·53 | 131 | 0·0177 |
| | | | | | | TOTAL .. | 28·9062 |
| 350–375 | 190 | 3·793064 | .. | .. | .. | .. | .. |
| 375–400 | 162 | 4·464312 | .. | .. | .. | .. | .. |
| Independent check. | | | | | | | |
| 325–368 | 193 | 3·789789 | 3·381 | 42 | 159·17 | 143 | 1·852 |

interval, where it is as large as 14·28. Subtracting this, the total $\chi^2$ falls to 14·63 for 8 degrees of freedom, the probability of which is greater than 5% and therefore not significant. The value of $\chi^2$ is very highly significant for the indicated altitude interval 15,000 to 17,500 feet with a mean pressure of 511 millibars. In fact, the probability that due to a statistical fluctuation the value of $\chi^2$ should exceed 14·27 is far smaller than 0·1%. The observed count of 21 in this interval is far in excess of the expected count of 9·41. It is important to note that the telescope with 10 cm. of lead absorber also showed an abnormally high count in precisely this interval. Both telescopes were electrically shielded and the only electrical connection between them was the common low voltage supply from the aeroplane and the common high voltage supply for the counters from a battery which was also enclosed in a separate box with provision for heating. The probability that due to a random fluctuation both telescopes should have shown such an abnormally high count in this interval is negligibly small. *The conclusion seems inescapable that there is either a real anomaly, such as a hump, at this altitude in the intensity curve of mesons, or that there is some real physical condition at this level of the atmosphere in the tropics which causes the sets to register an abnormally high count.* A slight but definite hump in the total intensity at pressures between 400 and 500 millibars has also been noted by Pfotzer (1936) and others. The hump in the meson intensity is at a lower level and appears to be more pronounced in certain circumstances. It is our intention to investigate this point more carefully in future experiments.

The two points at 639 and 709 millibars both appear to lie well below the calculated regression curve. The value of $\chi^2$ for the former is 2·15 which is not significant while that for the latter is 4·67 which is significant. The sum of the two values of $\chi^2$ is 6·82, which is still significant for *two* degrees of freedom, the probability of this value being below 5%. There is therefore a definite indication from this data alone of a depression in the altitude intensity curve between 639 and 709 millibars. This should be taken in conjunction with the fact pointed out in our previous paper that for all three curves giving the intensity of mesons penetrating 5·25, 20 and 30 cm. of lead the observed readings at 10,000 feet (670 millibars) lay markedly below the smooth curves drawn through the other points. All the measurements therefore agree in indicating an anomalous depression in the altitude intensity curve of mesons between 650 and 700 millibars. This is also clear from the figure.

The reading at 316 millibars also lies below the regression curve, but the $\chi^2$ test shows that this is not significant and could be due to a random fluctuation.

Coming finally to the independent check at the mean indicated altitude 35,900 feet, we have calculated the expected count at the altitude accurately from our regression curve and given it in the last line of Table II. The observed count agrees with the calculated remarkably well, $\chi^2$ being only 1·852. Thus the regression we have calculated fits the extra observation very well for the most reliable and important portion of the flight. It is worth remarking that the regression curve has been distorted upwards in the pressure range between 350 and 700 millibars by the very large aberration of the point at 511 millibars mentioned above. A glance at the figure shows that the true altitude intensity curve probably lies a little lower in this pressure range.

To sum up, our measurements show that the vertical intensity of mesons penetrating 30 cm. of lead increases roughly 6 times in going from ground level (906 millibars) to an altitude corresponding to a pressure of 193 millibars. There is definite indication of an anomalously high count at a pressure of 511 millibars, and a depression between 650 and 700 millibars in the altitude intensity curves of mesons.

## ACKNOWLEDGMENT.

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## SUMMARY.

The vertical intensity of mesons penetrating 30 cm. of lead has been measured by a quadruple coincidence counter telescope at Bangalore, magnetic altitude 3·3 degrees north from ground level (906 millibars) to an altitude of 40,000 feet (186 millibars). In ascending from ground level to the highest altitude reached, the intensity of mesons penetrating 30 cm. of lead increased roughly sixfold. There is indication of an anomalously high count in the altitude intensity curve at a pressure of roughly 511 millibars (indicated altitude interval 15,000–17,500 feet).

A comparison with the altitude intensity curve for mesons penetrating 20 cm. of lead measured on a previous flight shows that both curves coincide within the accuracy of the measurements to an altitude corresponding to a pressure of 400 millibars, but there is an indication that at higher altitudes the 20 cm. curve rises more rapidly.

The present measurements agree with those made on the previous flights for mesons penetrating 5·25, 20 and 30 cm. of lead in indicating a depression in the altitude intensity curve of mesons between 650 and 700 millibars (about 10,000 feet).

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