A NEW CLASSICAL THEORY OF THE PHOTON AND THE ELECTRON.

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The triumphs of Quantum Mechanics, specially in the field of spectroscopy, are too well-known to require enumeration, but there are difficulties as well. One great drawback has been the lack of a model either of the photon or of the electron. In fact Dirac holds that it is no business of Quantum Mechanics to provide a picture.

A photon is sometimes described as a wave packet, but no attempt has been made to derive therefrom its fundamental properties. In this connection, occasional mention is made of the δ-function defined as follows:

$$\delta(x) = 0, \qquad \text{when } x \neq 0, \qquad \int_{-\infty}^{\infty} \delta(x)\, dx = 1.$$

It is obvious that $\delta(x)$ is a non-analytic function and is not integrable. Even if we extend the definition and make $\delta(x) = 0$ when x does not lie between $\pm\epsilon$ and then make $\epsilon \to 0$, it is easily seen that the integral is discontinuous at the origin and the function cannot exist there. The δ-function is therefore a myth.

Further any particle is equivalent to its de Broglie waves, but the reason has not been enquired into. The holding of the charge on an electron has not yet been explained and the problem of infinite energy is still a mystery. In fact the fundamental properties of the photon and the electron are separately accepted as postulates.

Again a little critical consideration shows that the Einstein relation $W = h\nu$ associating a definite quantum of energy with every train of electromagnetic waves of frequency ν is an impossible proposition and must be regarded as an over-generalisation. For the finite breadth of the spectral lines and the existence of the continuous spectrum would make the radiated energy of a finite mass of excited atoms infinite. Nor is it possible to explain away this difficulty by replacing the continuous spectrum by a large but finite number of lines, as this would falsify the Schrödinger equation, which admits of continuous as well as discrete eigenvalues.

It is proposed here to provide a model for the photon as well as the electron, which may serve as the foundation of a theory of matter. The fundamental properties have also been derived with the minimum number of assumptions. The mathematics is classical with occasional appeals to the Relativity Theory. There is no conflict with Quantum Mechanics regarded from a statistical point of view, though there is a certain amount of divergence.

1. Starting from the Maxwell equations, we show that a linear photon which presents the Doppler effect but is otherwise invariant against the Lorentz transformation must be given by equations of the form

$$E_y = H_z = A \sin k(x-ct)/(x-ct), \qquad E_x = E_z = H_x = H_y = 0.$$

For, the general solution of the Maxwell equations in rectangular Cartesian coordinates is $E_y = H_z = f(x-ct)$, the other components being zero. If this is to present the Doppler effect, f must involve a factor $\sin k(x-ct)$.

We write, therefore,

$$f(x-ct) = \phi(x-ct) \sin k(x-ct).$$

The Lorentz transformation for an observer O' moving with velocity v parallel to the x-axis

$$x = \beta(x'+vt'), \quad y = y', \quad z = z', \quad t = \beta(t'+vx'/c^2), \quad \beta = (1-v^2/c^2)^{-\frac{1}{2}}$$

with

$$x-ct = \beta(1-v/c)(x'-ct')$$

must conserve the electric and magnetic components, where

$$E_{y'} = \beta(E_y - v/cH_z) = \beta(1-v/c)f(x-ct).$$

(Tolman, 1934.)

Sin $k(x-ct)$ transforms into sin $k'(x'-ct')$, where

$$k' = k\beta(1-v/c) = k\sqrt{(1-v/c)/(1+v/c)}.$$

We have thus

$$\nu' = \nu\sqrt{(1-v/c)/(1+v/c)},$$

where $\nu = kc/2\pi$. This, of course, is the Doppler effect.

We must also have the functional equation

$$\phi(x'-ct') = \beta(1-v/c)\phi(x-ct).$$

One solution is $\phi(x-ct) = 1/(x-ct)$. We write, therefore, the general solution as

$$\phi(x-ct) = \chi(x-ct)/(x-ct),$$

where χ is invariant against the transformation. But since $x-ct$ is a covariant of the transformation, χ can only be a constant. We have, therefore

$$f(x-ct) = A \sin k(x-ct)/(x-ct),$$

where A is a universal constant. The conclusion is therefore irresistible that if the Maxwell equations hold as well as the special Relativity Theory, a linear photon must have an expression of this form.

2. It may be pointed out that the usual Maxwell field $E_y = H_z = f(x-ct)$ covering the entire space makes no discrimination between the path of the photon and any other line parallel to it. So we restrict the forces to act only at points on the x-axis. The electric field is therefore confined to the xy-plane and the magnetic field to the xz-plane. The energy of the electric field in a thin slab of thickness δ along the z-axis and breadth l along the y-axis at any instant is given by

$$l\delta \int_{-\infty}^{\infty} E_y^2/8\pi \, dx.$$

We have to make $l \rightarrow \infty$, $\delta \rightarrow 0$ independently.

We assume that the limit $l\delta$ exists and is taken as 1. The total energy W is therefore given by [putting $\xi = x-ct$]

$$\frac{A^2}{4\pi}\int_{-\infty}^{\infty} \sin^2 k\xi/\xi^2 \, d\xi = \frac{A^2}{2\pi}\left[-\frac{\sin^2 k\xi}{\xi}\Big|_0^{\infty} + \int_0^{\infty} \frac{k \sin 2k\xi}{\xi}\, d\xi\right]$$

$$= \frac{A^2 k}{2\pi}\int_0^{\infty} \frac{\sin 2k\xi}{\xi}\, d\xi = \frac{A^2 k}{4},$$

a unit factor of dimension L^2 being understood.

We now draw a graph of the function $A \sin k\xi/\xi$. It possesses a hump at $\xi = 0$ and has nodes at $\pm\pi/k$, $\pm 2\pi/k$, $\pm 3\pi/k$ .... The amplitude at $\xi = 0$ is Ak and trails off to zero on either side.

We take the wavelength $\lambda = 2\pi/k$ and therefore the frequency $\nu = kc/2\pi$. The total energy $W = \frac{1}{4} A^2 k = h\nu$, where $h = \pi A^2/2c$.

To get an expression for the impulse, we note that Poynting Vector $c\,[EH]/4\pi$ exists only on the x-axis. The impulse is therefore

$$\frac{A^2}{4\pi c}\int_{-\infty}^{\infty} \sin^2 k\xi/\xi^2 \, d\xi = \frac{A^2 k}{4c} = \frac{h\nu}{c}$$

on the same assumption of replacement of volume integral by line integral.

Taking $h = 6{\cdot}5 \times 10^{-27}$, we have $A = 1{\cdot}1 \times 10^{-8}$.

3. It is obvious that the expression

$$\sin k\xi/\xi = \int_0^k \cos k\xi \, dk$$

represents a packet of simple harmonic waves, but it shows only the length of the shortest wave in its composition. It is a part of our contention that the packet is more than a mere conglomeration of harmonic waves and possesses a distinct identity of its own.

The integral

$$A\int_0^{k_1} \cos k\xi \, dk$$

can of course be split up mathematically into

$$A\left\{\int_0^{k_2} + \int_{k_2}^{k_1} \cos k\xi \, dk\right\} = A\left\{\frac{\sin k_2\xi}{\xi} + \frac{\sin k_1\xi - \sin k_2\xi}{\xi}\right\}.$$

That this is also physically possible may be verified as follows.

Consider a photon given by the relation

$$E_{y_1} = A \sin k_1\xi/\xi$$

and superpose on it another,

$$E_{y_2} = -A \sin k_2\xi/\xi$$

in the opposite phase. The total energy of the combination is given by the expression

$$W = \frac{A^2}{4\pi}\int_{-\infty}^{\infty} \frac{(\sin k_1\xi - \sin k_2\xi)^2}{\xi^2} d\xi = \frac{A^2}{2\pi}\int_0^{\infty} \frac{(\sin^2 k_1\xi + \sin^2 k_2\xi - 2\sin k_1\xi \sin k_2\xi)}{\xi^2} d\xi$$

$$= \frac{A^2(k_1+k_2)}{4} + \frac{A^2}{2\pi}\int_0^{\infty} \frac{\cos (k_1+k_2)\xi - \cos (k_1-k_2)\xi}{\xi^2} d\xi$$

$$= \frac{A^2(k_1+k_2)}{4} + \frac{A^2}{2\pi}\left[-\frac{\cos(k_1+k_2)\xi - \cos(k_1-k_2)\xi}{\xi}\Bigg|_0^\infty - \int_0^\infty \frac{(k_1+k_2)\sin(k_1+k_2)\xi - (k_1-k_2)\sin(k_1-k_2)\xi}{\xi} d\xi \right]$$

$$= \frac{A^2(k_1+k_2)}{4} - \frac{A^2}{2\pi}\{(k_1+k_2)-(k_1-k_2)\}\frac{\pi}{2} = \frac{A^2(k_1-k_2)}{4} = h(\nu_1-\nu_2),$$

where $\nu_1 = k_1c/2\pi, \quad \nu_2 = k_2c/2\pi.$

The energy of the combination, which may be called a partial photon, is thus the difference of the energy of the two photons. It is easily verified that two photons in the same phase cannot be superposed as the principle of energy is violated.

A photon of energy $h\nu$ may, therefore, split up into a photon of less energy $h\nu_1$ and a partial photon of energy $h(\nu-\nu_1)$ and the process of subdivision can be further extended. In particular if $k-k_1$ is small and equal to δk while ξ is not too large, the primary photon may split up into a photon of slightly less energy $h\nu_1$ and a train of simple harmonic electromagnetic waves given by

$$E_y = H_z = A(\sin k\xi - \sin k_1\xi)/\xi = A\delta k \cos k\xi.$$

The latter is equivalent to a train of aether waves in the language of Physical Optics and may be identified with a beam of light. It presents the familiar phenomena of reflection, refraction, interference, diffraction, etc., which are treated so simply and successfully in Physical Optics.

We have therefore two distinct entities, viz., the photon given by $A \sin k\xi/\xi$ and the partial photon $A(\sin k\xi - \sin k_1\xi)/\xi = A\delta k \cos k\xi$ to explain the particle and wave properties of light. They possess the same wavelength $2\pi/k$ and are inextricably mixed up together. In experiments involving the energy and momentum properties such as the Compton effect, the photo-electric effect, the energy distribution in the spectrum, it is the complete photons of energy $h\nu$ which preponderate, while in experiments on wave properties, the partial photons of small energy figure.

Here there is a divergence from the accepted theory that the energy of a beam of light of frequency ν is $h\nu$. It is a challenge to the Einstein law, which has been shown in the preface to be an over-generalisation. Our contention is that the law applies only to the complete photons, but not to the partial photons. If this could be verified experimentally it would have been a crucial test. But the Quantum Mechanics postulates that it is impossible to make the particle and wave properties the subject of the same experiment. The energy and frequency cannot be simultaneously determined. Logically speaking, according to Quantum Mechanics which deals only with observables and not concepts, the Einstein relation $W = h\nu$ is fundamentally irrational.

4. It may also be pointed out that the equation of the partial photon offers a simple explanation of the breadth of the spectral lines. The expression

$$A(\sin k\xi - \sin k_1\xi)/\xi = A\int_{k_1}^{k} \cos k\xi d\xi$$

embraces continuous range of simple harmonic waves and is equivalent to $A\delta k \cos k\xi$ only approximately when $k-k_1 = \delta k$ is small and ξ not too large.

It may be further observed that a beam of light can be represented by a train of simple harmonic waves only approximately. Otherwise an infinite train of waves once started would continue for ever. The expression $(\sin k\xi - \sin k_1\xi)/\xi$ solves the difficulty as it tends to zero in the limit when $\xi \to \infty$.

Again if in the train of simple harmonic waves $A\delta k \cos k\xi$ possessing energy proportional to δk, the amplitude is halved, the phase remains sensibly unaltered and the energy is also halved. This gives a complete picture of the process followed in producing interference fringes, where a beam of light is split up into two halves and made to recombine after one of them has been retarded. This ought to settle the present controversy between the wave and particle nature of light.

In contrast, the following extract from Dirac gives the Quantum Mechanical explanation of Interference:—

> 'Suppose we have a beam of light which is passed through some kind of Interferometer, so that it gets split into two components and the two components are subsequently made to interfere. We may take an incident beam consisting of only a single photon and inquire what will happen to it as it goes through the apparatus. * * * * We must now describe the photon as going partly into each of the two components into which the incident beam is split. * * * * For a photon to be in a definite state of motion it need not be associated with one single beam of light, but may be associated with two or more beams of light which are the components into which one original beam has been split.'
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> (Dirac, 1935.)

But what happens if the two components are widely separated? The Quantum of energy cannot then be present in both and we must have a beam of light without energy!

5. Let us consider a train of electromagnetic waves coiled in a circle and placed on a sphere. Taking r, θ, ϕ for the usual spherical polar co-ordinates, the coil being a parallel of latitude, let us place the electric force E in the direction of r, the magnetic force H in the direction of θ. E has components E_r, 0, 0 and H, 0, H_θ, 0. Then the six Maxwell equations reduce to

$$\frac{1}{\varpi}\frac{\partial E_r}{\partial \phi} = -\frac{1}{c}\frac{\partial H\theta}{\partial t} \qquad -\frac{1}{\varpi}\frac{\partial H\theta}{\partial \phi} = \frac{1}{c}\frac{\partial E_r}{\partial t}$$

$$-\frac{1}{r}\frac{\partial E_r}{\partial \theta} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) = 0$$

ϖ being the perpendicular on the axis. (Love, 1906.)

It is easily seen that $E_r = H_\theta = f(\varpi\phi - ct)$ satisfy the equations along the coils. Since the coils are distinct, there is no continuity in the direction of r and θ and the differential coefficients in these directions do not exist.

6. It is proposed that all fundamental particles which bear a charge $\pm e$ are represented by a Maxwell wave $E = A \sin k\xi/\xi$ coiled in circles along the parallels of latitude on a sphere. Then the total energy is $\frac{1}{4}A^2k = h\nu$ assuming that there is no change of energy when a linear wave is coiled. This is in accordance with de Broglie's theory. In particular this photon of energy $h\nu$ may split up into a photon of slightly less energy and a beam of simple harmonic electromagnetic waves of frequency ν offering a simple explanation of the electron waves in case they do not possess finite impulse. In case they do, the electron waves are to be identified with complete linear photons.

7. Consider the above structure for an electron. The electric intensity at any point P outside the sphere is the force E_r at the point on the sphere where the join of P to the centre cuts the surface. The lines of force, therefore, radiate from the centre and their number through any extended surface varies as the inverse square

of the distance from the centre. The inverse square law of electro-statics is, therefore, a simple geometrical consequence of our model.

Turning to the magnetic intensity at the point P, we see that it is the contribution of the magnetic forces at two points on the sphere. These are the points of contact of tangents from P to the circular section by the meridian plane through P. The model, therefore, behaves as a magnetic dipole. Points on the axis are singular.

8. To find an expression for the electronic charge, we have to apply Gauss' equation. But in the present case, we cannot take the surface integral as the electrical force is continuous only along the coils. Taking the case of straight linear distribution of charge and plane coils, we have

$$4\pi e = \int N\,dS = \int_{-\infty}^{\infty} E\,ds = 2A\int_{0}^{\infty} \sin ks/s\,ds = \pi A.$$

This gives a value $\frac{1}{4}A = 2{\cdot}8\times10^{-9}$ for the electronic charge against the experimental figure $4{\cdot}8\times10^{-10}$. This is not, apart from the order of magnitude of much consequence, since the limit $l\delta$ (§1) has been taken to be unity.

We can also make a rough calculation of the size of the electron from the known value of the angular momentum $h/2\pi$ ($\sim mca$). We get the radius $a\sim2\times10^{-11}$. The value obtained on the classical theory is 2×10^{-13}, the entire mass being due to moving electric charges—a hypothesis now discarded.

Again since the charge depends only on A which has been found to be a universal constant, a ready explanation is found for the uniformity of charge on the electron, positron, proton, and meson particles of widely different masses. Further the problem of infinite energy of the electron resolves itself.

9. To explain the apparent change of mass due to motion relative to an observer, we recall the formula for the transverse Doppler effect. Consider a photon given by the expression $A \sin k(y'-ct')/(y'-ct')$, the origin O' of the dashed co-ordinates having a velocity v in the direction of the x-axis with respect to an observer at the origin O. The Lorentz transformation is

$$x' = \beta(x-vt), \quad y' = y, \quad z' = z, \quad t' = \beta(t-vx/c^2).$$

Then the trigonometrical factor which gives the frequency becomes

$$\sin k\,\{y-c\beta(t-vx/c^2)\} = \sin k(y+\beta vx/c-c\beta t).$$

The apparent frequency to the observer at O is $\nu/(1-v^2/c^2)^{\frac{1}{2}}$, ν being the frequency to the observer at O'. The longitudinal Doppler effect has already been considered in §1.

Consider now an electron moving with velocity v relative to the observer at O with its axis perpendicular to the direction of v. Take a coil, its plane parallel to v. At the extremities of the transverse diameter, the Doppler effect is longitudinal and the apparent frequencies are $\nu\sqrt{(1-v/c)/(1+v/c)}$ and $\nu\sqrt{(1+v/c)/(1-v/c)}$. The mean frequency is therefore $\nu/(1-v^2/c^2)^{\frac{1}{2}}$. At the ends of the longitudinal diameter, the Doppler effect is transverse and the apparent frequency is $\nu/(1-v^2/c^2)^{\frac{1}{2}}$. At intermediate points on the coil, both effects are partially present and are complementary. The net effect is an apparent change of frequency from ν to $\nu/(1-v^2/c^2)^{\frac{1}{2}}$ and the mass from m to $m/(1-v^2/c^2)^{\frac{1}{2}}$. For an electron moving in the direction of the axis, the plane of the coils is perpendicular to v and at every point on the coil the Doppler effect is transverse. The frequency, therefore, changes from ν to $\nu/(1-v^2/c^2)^{\frac{1}{2}}$ and the mass from m to $m/(1-v^2/c^2)^{\frac{1}{2}}$. Combining these two cases we get the usual formula for the apparent change of mass due to motion relative to an observer.

10. We have thus explained the chief characteristic properties of the electron on our hypothesis. The last important difficulty is the absence of radiation from

an accelerated electron. It will be recalled that in classical electrodynamics, such radiation is deduced from a continuous space distribution of the Poynting vector. Since we have denied such distribution, the method of deduction does not apply.

It may further be observed that the Bohr theory of atomic spectra depends upon the Newtonian potential which has not got the sanction of the Relativity Theory. The radiation of energy from the corpus of the electron itself, as indicated above, seems generally to be more in accordance with the Theory of Relativity.

Why photons of certain wavelengths should have the tendency to form coils, we have no idea. Probably it depends on the cosmological constant. There must be a number of knotty points which will have to be settled before the hypothesis can be regarded as established. It is quite likely that adjustments may have to be made. But if the hypothesis can stand these tests, modern Physics will have been brought down from the clouds to the base but solid earth.

Summary.

It is proved that a linear Maxwell wave which presents the Doppler effect and is otherwise invariant against the Lorentz transformation must have an equation of the form $E_y = H_z = A \sin k(x-ct)/(x-ct)$, the other components being zero. These forces are restricted to act at points on the x-axis only and the wave packet is identified with a photon. This represents a train of waves of length $2\pi/k$ of diminishing amplitudes extending in either direction to infinity with a hump at the middle. The energy is given by a line integral and is found equal to $h\nu$ and the impulse to $h\nu/c$ on the assumption of the existence of a limit. It is pointed out that the Einstein relation $W = h\nu$ must be regarded as an over-generalization.

It is then verified that a photon can split up into a photon of less energy and a partial photon of the form $A\{\sin k(x-ct)-\sin k_1(x-ct)\}/(x-ct)$ of energy $h(\nu-\nu_1)$. The latter reduces to a simple harmonic wave $A\delta k \cos k(x-ct)$ when $k-k_1 = \delta k$ is small and $x-ct$ not too large. This explains the breadth of the spectral lines and the ultimate fading away of light. Further if the amplitude of the harmonic wave be halved the energy is also halved. This corresponds exactly with the method followed in producing Interference fringes. The current controversy between the wave and particle nature of light is therefore settled.

An electron is conceived as a photon in the form of coils along the parallels of latitude on a sphere. This agrees with de Broglie's theory and gives in particular the electron waves. The Inverse Square Law follows as a matter of elementary geometry and the problem of infinite energy resolves itself. By applying the Gauss' equation to the allied two dimensional problem, we get $2{\cdot}8 \times 10^{-9}$ e.s.u. as an approximation to the charge. The known value of angular momentum makes the radius of order 10^{-11}. Lorentz transformation leads to the formula for apparent change of mass with velocity. Other important features are (1) complete identification of mass with energy, (2) elimination of charge as a fundamental concept, (3) uniformity of charge on the elementary particles of such varying mass as electrons, positrons, protons and mesons.

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