ELASTIC CONSTANTS OF SOME CUBIC NITRATES.

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1. INTRODUCTION.

For the past several years, this Laboratory has been engaged in investigations relating to crystal elasticity. The starting point for these studies was the development of a new method known as the wedge method (Bhagavantam and Bhimasenachar, 1944) which permits us to handle small crystals. This method has been applied to several single crystals and rocks. Recently, a study of the elastic constants of the isomorphous series of alums has been made by Sundara Rao (1947 and 1948). In continuation of this work, a study of the elastic behaviour of the isomorphous series of nitrates of barium, lead and strontium has been undertaken by the authors and the results are given in the present paper.

2. STRUCTURE AND FORM OF NITRATES.

The nitrates of barium, lead and strontium, belong to the cubic system. It is not yet established decisively whether they belong to the space group T or T_h^6. The crystallographic, the spectrographic and the X-ray investigations give results which do not agree with one another. Early crystallographic investigations (Groth, 1908) indicate that they belong to the class T, while the X-ray data (Wyckoff, 1931) confirm the space group T_h^6 and reveal that these nitrates are cubic with four molecules in the unit cell, the edges a_0 of the unit cube being: barium nitrate 8·11 A.U., lead nitrate 7·84 A.U., strontium nitrate 7·81 A.U.

The powder data show that the nitrate ion is not planar while the Raman effect data (Padmanabhan, 1948) show that it is planar.

This difference, however, does not affect the elastic measurements, since the number of independent elastic constants for all crystal classes in the cubic system is three.

Single crystals about 1 to 1·5 cm. long and broad and about 1 cm. thick were grown from aqueous solutions by slow evaporation. The predominantly developed faces on these crystals were (001) and (111). From these crystals sections parallel to (001) and (111) were prepared by grinding the crystals on a ground glass plate using water as lubricant.

3. DETERMINATION OF THE ELASTIC CONSTANTS.

Sections parallel to the well developed faces (001) and (111) have been employed for a determination of the velocity of propagation of the longitudinal or shear waves along directions perpendicular to the above faces. If f_l or f_t is the fundamental of natural frequency of thickness longitudinal or thickness shear vibrations of a crystal plate of thickness d then the velocity of propagation of longitudinal or shear waves along the thickness of the plate is given by $v_l = 2df_l$ and $v_t = 2df_t$. These velocities are related with the elastic constants C_{11}, C_{12} and C_{44} through the solutions of Christoffel's equations for propagation of sound along these directions; $v^2\rho = C'$ where ρ is the density and C' is the effective longitudinal or shear constant of elasticity. These solutions are included in Tables I, II and III along with other experimental data. The independent elastic constants C_{11}, C_{12} and C_{44} are calculated from these relations.

TABLE I.

Measurements on Barium Nitrate.

Density = 3·240 gm./cm.^{3}.

Plate No.	Thickness mm.	Section.	Transmission fundamental Mcs.	Mode.	C' Expression.	Value.
1	1·585	(001)	1·356	L*	C_{11}	5·99†
			0·601	T	C_{44}	1·18
2	1·096	(001)	1·940	L	C_{11}	5·86
			0·896	T	C_{44}	1·23
3	2·033	(111)	0·949	L	$\frac{C_{11}+2C_{12}+4C_{44}}{3}$	4·82
			0·570	T	$\frac{C_{11}-C_{12}+C_{44}}{3}$	1·74
4	1·093	(111)	1·772	L	$\frac{C_{11}+2C_{12}+4C_{44}}{3}$	4·86
			1·064	T	$\frac{C_{11}-C_{12}+C_{44}}{3}$	1·75

From the above readings, we have $C_{11} = 5{\cdot}93$, $C_{12} = 1{\cdot}89$ and $C_{44} = 1{\cdot}21$ as average values.

TABLE II.

Measurements on Lead Nitrate.

Density = 4·525 gm./cm.^{3}

Plate No.	Thickness in mm.	Section.	Transmission fundamental Mcs.	Mode.	C' Expression.	Value.
1	2·160	(001)	0·740	L	C_{11}	4·62
			0·400	T	C_{44}	1·35
2	1·970	(001)	0·807	L	C_{11}	4·58
			0·442	T	C_{44}	1·37
3	1·453	(001)	1·084	L	C_{11}	4·49
			0·602	T	C_{44}	1·39
4	1·705	(111)	1·014	L	$\frac{(C_{11}+2C_{12}+4C_{44})}{3}$	5·41
			0·440	T	$\frac{(C_{11}-C_{12}+C_{44})}{3}$	1·02
5	1·560	(111)	1·116	L	$\frac{(C_{11}+2C_{12}+4C_{44})}{3}$	5·48
			0·455	T	$\frac{(C_{11}-C_{12}+C_{44})}{3}$	0·91

* Here and in subsequent Tables L—Longitudinal, T—Torsional.

† Here and in subsequent Tables the values of C_{11}, C_{12} and C_{44} are given in units of 10^{11} dynes per sq. cm.

From the above readings the elastic constants for lead nitrate are $C_{11} = 4{\cdot}56$; $C_{12} = 3{\cdot}09$; $C_{44} = 1{\cdot}37$.

TABLE III.

Measurements on Strontium Nitrate.

Density = 3·023 gm./cm.^{3}

Plate No.	Thickness in mm.	Section.	Transmission fundamental Mcs.	Mode.	C' Expression.	Value.
1	1·893	(001)	1·042	L	C_{11}	4·70
			0·574	T	C_{44}	1·43
2	1·880	(001)	1·067	L	C_{11}	4·87
			0·579	T	C_{44}	1·43
3	1·350	(001)	1·470	L	C_{11}	4·76
			0·882	T	C_{44}	1·49
4	1·204	(001)	1·620	L	C_{11}	4·60
			0·923	T	C_{44}	1·49
5	1·917	(111)	1·061	L	$\frac{(C_{11}+2C_{12}+4C_{44})}{3}$	5·00
			0·547	T	$\frac{(C_{11}-C_{12}+C_{44})}{3}$	1·33
6	1·867	(111)	1·084	L	$\frac{(C_{11}+2C_{12}+4C_{44})}{3}$	4·95
			0·566	T	$\frac{(C_{11}-C_{12}+C_{44})}{3}$	1·35

From these readings, the elastic constants of strontium nitrate obtained are

$$C_{11} = 4{\cdot}73;\ C_{12} = 2{\cdot}18 \text{ and } C_{44} = 1{\cdot}46.$$

The values of the elastic constants of the nitrates are collected together and presented in Table IV. The last column gives the Bulk modulus k calculated from the relation,

$$k = \frac{C_{11}+2C_{12}}{3}.$$

TABLE IV.

Elastic Constants of Cubic Nitrates.

	Density.	C_{11}	C_{12}	C_{44}	k
Lead Nitrate ..	4·525	4·56	3·09	1·37	3·580
Barium Nitrate ..	3·240	5·93	1·89	1·21	3·237
Strontium Nitrate ..	3·023	4·73	2·18	1·46	3·030

4. DISCUSSION OF RESULTS.

The elastic constants of barium nitrate have been determined by Sundara Rao (1948) previously in this Laboratory. His values are $C_{11} = 6{\cdot}02$, $C_{12} = 1{\cdot}86$ and $C_{44} = 1{\cdot}21$. The author's values given in Table IV are in excellent agreement with these values.

In the case of lead nitrate, no elastic data are available in the literature. These are determined here for the first time.

For strontium nitrate, Bridgmann has given the value of cubic compressibility for compressed powder-cake. This value is taken from Landolt-Bornstein Tabellen (1935). The value of Bulk modulus calculated from this data comes out as $3{\cdot}02 \times 10^{11}$ dynes/cm.[2] The elastic constants of strontium nitrate determined by the authors lead to a Bulk modulus of $3{\cdot}03 \times 10^{11}$ which compares very well with Bridgmann's value.

In this series of isomorphous substances also the Bulk modulus k increases with density. Since there are only three substances no serious attempt can be made to determine the dependence of the Bulk modulus on density.

5. Elasticity Surfaces.

Surfaces of elasticity, one for the bending modulus and the other for the rigidity modulus, the radii vectors in the two cases being proportional to s'_{33} and $2\ (s'_{44}+s'_{55})$ respectively are drawn for each crystal.

Sections parallel to (001) of the elasticity surfaces of the different crystals are given in Figs. 1, 2 and 3. In order to be able to draw these surfaces the C's are converted into the s's using the well-known relations. The values of the elastic coefficients (s's) for the three crystals are consolidated in Table V for ready reference.

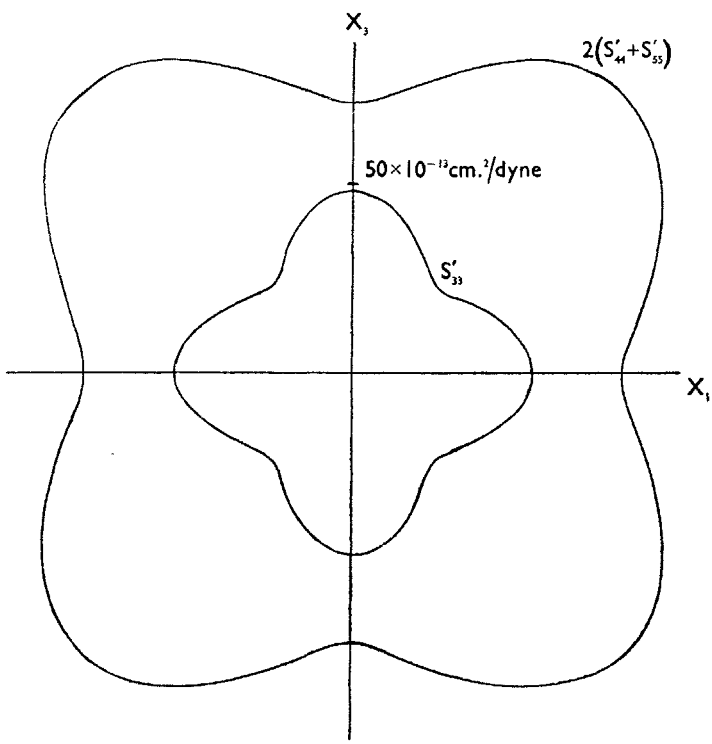

Fig. 1. Sections parallel to (001) of the Elasticity surfaces of Lead Nitrate.

TABLE V.

Elastic Coefficients of the Cubic Nitrates.

	s_{11}	s_{12}	$4s_{44}$
Lead Nitrate ..	49	−20	73
Barium Nitrate ..	20	−5	83
Strontium Nitrate ..	30	−9	69

(Units for s's in this Table are 10^{-13} cm.^{2}/dyne.)

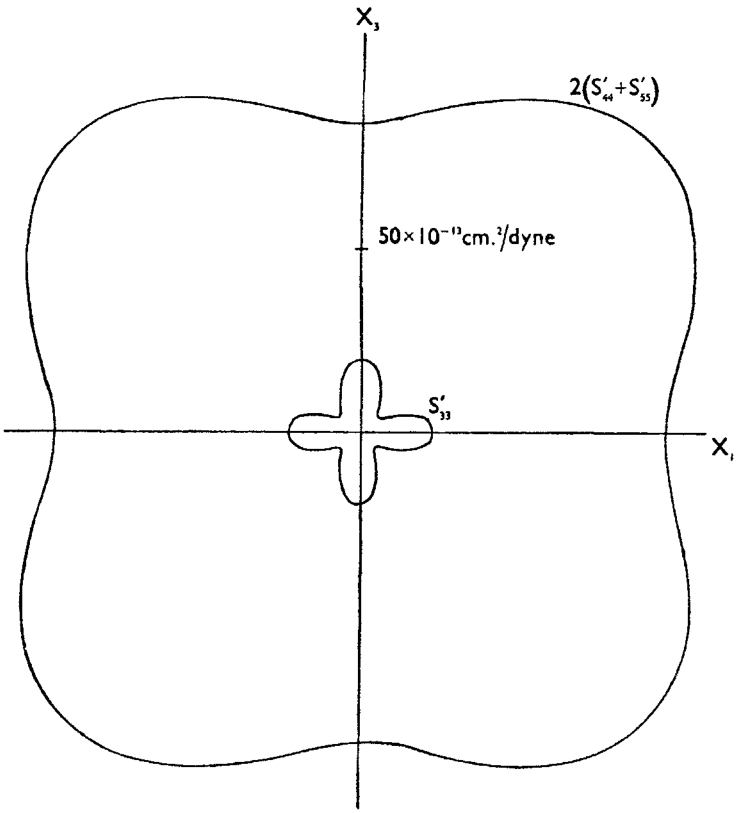

FIG. 2. Sections parallel to (001) of the Elasticity surfaces of Barium Nitrate.

6. SUMMARY.

The elastic constants of single crystals of barium nitrate, strontium nitrate and lead nitrate have been determined employing the wedge method. They are $C_{11} = 5{\cdot}93$, $C_{12} = 1{\cdot}89$, $C_{44} = 1{\cdot}21$ for barium nitrate; $C_{11} = 4{\cdot}73$, $C_{12} = 2{\cdot}18$, $C_{44} = 1{\cdot}46$ for strontium nitrate and $C_{11} = 4{\cdot}56$, $C_{12} = 3{\cdot}09$, $C_{44} = 1{\cdot}37$ for lead nitrate.

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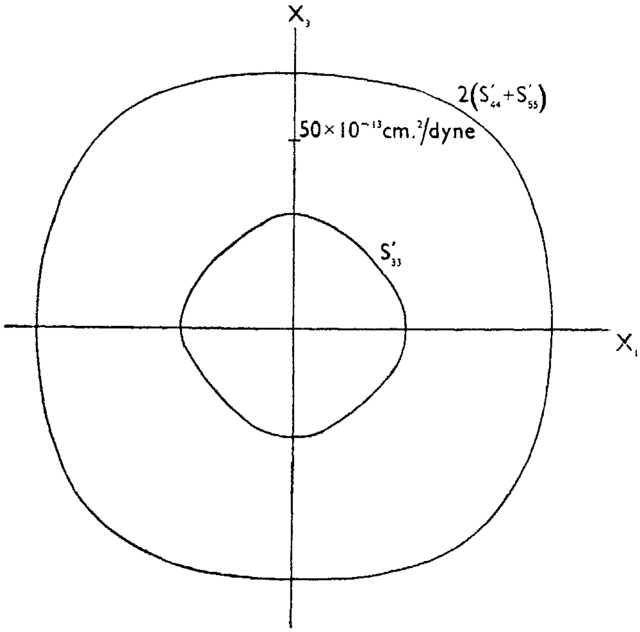

FIG. 3. Sections parallel to (001) of the Elasticity surfaces of Strontium Nitrate.

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