ELASTIC CONSTANTS OF CORUNDUM.

By J. BHIMASENACHAR, *D.Sc.*, *N.I.S. Senior Research Fellow*, *Department of Physics, Andhra University, Waltair.*

(Communicated by Prof. S. Bhagavantam, F.N.I.)

(*Received October 13, 1949 ; read March 3, 1950.*)

1. INTRODUCTION.

Corundum is an important naturally occurring crystal. It is a highly priced gemstone. Its industrial value as an abrasive is very great as its hardness is next to that of diamond. Therefore it is a substance whose physical properties have been studied very thoroughly. Recently Sundara Rao (1949) has reported the elastic behaviour of a single crystal of synthetic alumina. As this crystal belongs to the D_{3d} class of the trigonal system its elastic behaviour is characterised by 6 independent constants C_{11}, C_{12}, C_{13}, C_{14}, C_{33} and C_{44}. In order to determine these constants 6 independent observations are necessary on crystal plates cut with suitable orientations. In his investigations Sundara Rao has used a synthetic specimen in the form of a semicylindrical boule with the optic axis of the crystal making an angle of about 14° to the axis of the boule. Since natural crystal faces were not developed on this boule he had to use crystal plates cut with odd orientations and was therefore forced to determine the important constant C_{33} by a sum and difference method which invariably magnifies small errors and vitiates accuracy.

Thanks to Prof. C. Mahadevan, M.A., D.Sc., F.A.Sc., F.N.I., the Head of the Geology Department of this University, a natural crystal of good size was available to the author. This crystal was opaque, brownish in colour, about 1″ in diameter and 1·5″ in length, and had its basal plane (0001) and prism faces $(11\bar{2}0)$ well developed. In the present investigation suitable crystal sections were cut from this specimen to an accuracy of less than 1° of arc. Results of investigations on this crystal are presented here.

2. CHOICE OF CRYSTAL SECTIONS.

It has already been said that this crystal has six independent elastic constants and therefore six independent observations are necessary for their determination. Crystal sections employed for this purpose are tabulated in Table I along with the θ, ϕ co-ordinates as well as the values of the direction cosines of the normals to these sections.

The velocities of propagation of longitudinal and shear waves along these normals are determined by the values of C'_{33} and C'_{44} for each particular plate. In general

$$\begin{aligned} C'_{33} = {} & C_{11}(1-\alpha^2_{33})^2+C_{33}\alpha^4_{33}+(4C_{44}+2C_{13})\alpha^2_{33}(1-\alpha^2_{33}) \\ & +4C_{14}\alpha_{23}\alpha_{33}\,(3\alpha^2_{13}-\alpha^2_{23}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1) \end{aligned}$$

$$\begin{aligned} C'_{44} = {} & C_{11}(\alpha^2_{12}\alpha^2_{13}+\alpha^2_{22}\alpha^2_{23})+2C_{12}\alpha_{12}\alpha_{13}\alpha_{22}\alpha_{23} \\ & +2C_{13}\alpha_{32}\alpha_{33}(\alpha_{12}\alpha_{13}+\alpha_{22}\alpha_{23})+C_{14}[4\alpha_{12}\alpha_{13}(\alpha_{22}\alpha_{33}+\alpha_{23}\alpha_{32}) \\ & +2(\alpha^2_{12}\alpha_{23}\alpha_{33}+\alpha^2_{12}\alpha_{22}\alpha_{32}-\alpha^2_{22}\alpha_{23}\alpha_{33}-\alpha_{22}\alpha^2_{23}\alpha_{32})] \\ & +C_{33}(\alpha^2_{32}\alpha^2_{33})+C_{44}[(\alpha_{22}\alpha_{33}+\alpha_{23}\alpha_{32})^2 \\ & +(\alpha_{12}\alpha_{33}+\alpha_{13}\alpha_{32})^2]+C_{66}(\alpha_{12}\alpha_{23}+\alpha_{13}\alpha_{22})^2 \quad \ldots \quad \ldots \quad (2) \end{aligned}$$

where the α's are directions cosines between the three principal axes X_1, X_2 and X_3 of the crystal and a set of new axes in which the normal to the crystal plate is regarded as the X'_3 axes so that the direction cosines of X'_3 are α_{13}, α_{23} and α_{33}. Particular solutions corresponding to each one of the plates are also given in Table I.

TABLE I.

Description of Crystal Sections.

No.	Cut of the plate.	θ	ϕ	α_{13}	α_{23}	α_{33}	C'_{33}(L)	C'_{44}(T)
1	Z	0	ϕ	0	0	1	C_{33}	C_{44}
2	X	90°	0	1	0	0	C_{11}	C_{44}, C_{66}
3	Y	90°	90°	0	1	0	C_{11}	C_{44}, C_{66}
4	45° XZ	45°	0°	$\sqrt{\frac{1}{2}}$	0	$\sqrt{\frac{1}{2}}$	$\frac{1}{4}(C_{11}+C_{33}+4C_{44}+2C_{13})$	..
5	135° XZ	135°	0°	$\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$	$\frac{1}{4}(C_{11}+C_{33}+4C_{44}+2C_{13})$	..
6	45° YZ	45°	90°	0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	$\frac{1}{4}(C_{11}+C_{33}+4C_{44}+2C_{13}-4C_{14})$	..

(L = Longitudinal; T = Torsional, constants.)

In plates 4, 5 and 6 since the torsions are not observed their solutions are not given. It can be seen from this table that each constant is determined uniquely and there is no need of solving any simultaneous equations.

We may remark even here that Sundara Rao appears to have made, inadvertently, a mistake in describing his plate No. I in his Table I. Direction cosines for this plate must be $\alpha_{13} = 1$, $\alpha_{23} = 0$ and $\alpha_{33} = 0$ if the plate is to lead to determinations of C_{11}, C_{44} and C_{66}. According to his description, his plate I gives $C'_{33} = C_{33}$ and $C'_{44} = C_{44}$.

3. EXPERIMENTAL TECHNIQUE AND RESULTS.

The method employed for determining the normal modes of the crystal sections is the well-known wedge Method (Bhagavantam and Bhimasenachar, 1944) developed in these Laboratories. The driving circuit for the wedge is a variable frequency oscillator rigged up using a Taylor T_{55} valve and operating in the region 2 to 20 Mcs.

Experimental Results are given in Table II.

TABLE II.

Elastic Constants of Corundum.

Density 3·81.

No.	Section.	Thickness in mm.	Frequency in Mcs.	Mode.	Effective Elastic Constant C'.	
					Expression.	Value.
1	Z	2·58	2·356	L	C_{33}	56·3
2	,,	2·58	1·515	T	C_{44}	23·2
3	X	2·13	2·588	L	C_{11}	46·3
4	,,	2·13	1·833	T	C_{44}	23·2
5	,,	2·13	1·569	T	$C_{66} = (\frac{1}{2})(C_{11}-C_{12})$	17·0
6	Y	2·15	2·572	L	C_{11}	46·6
7	,,	2·15	1·830	T	C_{44}	23·6
8	,,	2·15	1·560	T	$C_{66} = (\frac{1}{2})(C_{11}-C_{12})$	17·1
9	45°XZ	1·67	3·590	L	$\frac{1}{4}(C_{11}+C_{33}+4C_{44}+2C_{13})$	54·8
10	135°XZ	2·67	2·250	L	,,	54·9
11	45°YZ	1·55	3·494	L	$\frac{1}{4}(C_{11}+C_{33}+4C_{44}+2C_{13}-4C_{14})$	44·7

(As usual C' are expressed in units of 10^{11} dynes/cm.^{2})

From the above table we have the following average values for the elastic constants.

$$\begin{array}{ll} C_{11} = 46{\cdot}5 & C_{12} = 12{\cdot}4 \\ C_{33} = 56{\cdot}3 & C_{13} = 11{\cdot}7 \\ C_{44} = 23{\cdot}3 & C_{14} = 10{\cdot}1 \end{array}$$

The elastic coefficients or moduli are determined using the usual relations.

$$\begin{array}{ll} s_{11} = 46{\cdot}5 & s_{12} = -1{\cdot}05 \\ s_{33} = 1{\cdot}93 & s_{13} = -0{\cdot}38 \\ 4s_{44} = 5{\cdot}77 & 2s_{14} = 1{\cdot}71 \end{array}$$

Here the s's are given in Wooster's notation and the units are 10^{-13} cm.^{2}/dyne.

4. Discussion of Results.

The first thing is to compare our results with those of Sundara Rao ($C_{14} = 46{\cdot}6$, $C_{33} = 50{\cdot}6$, $C_{44} = 23{\cdot}5$, $C_{12} = 12{\cdot}7$, $C_{13} = 11{\cdot}7$ and $C_{14} = 9{\cdot}4$). Excepting for a change in the value of C_{33} the results agree excellently well. As has already been remarked this may be due to the fact that Sundara Rao employs sections with odd orientations and obtains C_{33} by a sum and difference method.

The principal Young's modulus along the optic axis turns out to be 51·8 as against Auerbach's value of 53·1 (*vide* L and B Tables) and leads to a better agreement.

The cubic compressibility β turns out to be $4{\cdot}11 \times 10^{-13}$ cm.^{2}/dyne the same as that of Sundara Rao.

5. Summary.

The elastic constants of a single crystal of natural corundum are reported, the values being $C_{11} = 46{\cdot}5$, $C_{33} = 56{\cdot}3$, $C_{44} = 23{\cdot}3$, $C_{12} = 12{\cdot}4$, $C_{13} = 11{\cdot}7$ and $C_{14} = 10{\cdot}1$ in units of 10^{11} dynes/cm.^{2}

The author takes this opportunity to express his gratitude to the National Institute of Sciences of India for the award of a fellowship, to Prof. S. Bhagavantam, Hon. D.Sc., F.A.Sc., F.N.I., for the interest he took in this work and to Prof. C. Mahadevan, M.A., D.Sc., F.A.Sc., F.N.I., for supplying the beautiful specimen of corundum.

6. References.

Bhagavantam, S. and Bhimasenachar, J. (1944). Elastic Constants of Crystals. A New Method and its Application to pyrites and galena. *Proc. Ind. Acad. Sci.*, **20**, 298.

Landolt-Bornstin Tabellen (1923). Physikalisch-Chemische Tabellen. *Hw. I.*, 84.

Sundara Rao, R. V. G. (1949). Elastic Constants of Alumina. *Procd. Ind. Acad. Sci.*, **29**, 352.