A NOTE ON THE OSCILLATIONS OF AN INFINITE CYLINDER SUBJECT TO RADIAL MAGNETIC FIELD

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I. INTRODUCTION

In a recent paper Ferraro and Memory (1952) have discussed the problem of oscillation of a star in its own magnetic field. Following them, we discuss here a case of torsional and radial oscillations of a cylindrical mass of highly conducting incompressible fluid in the presence of a permanent infinite axial magnetic line-pole of strength m/unit length. In numerical illustration of the problem we have considered the same values for radius, density and surface magnetic field as used by them. We have also compared the period of oscillations for the first two harmonics with those obtained by Ferraro and Memory.

II. FUNDAMENTAL EQUATIONS

The motion of incompressible fluid with the density ρ, the electrical conductivity σ, and the permeability μ (= 1), placed in a magnetic field $\mathbf{H}_0$ is described by Maxwell's equations in e.m. units

$$\text{curl}\,\mathbf{H} = 4\pi\mathbf{j} \quad \ldots \quad (1)$$

(neglecting displacement current)

$$\text{curl}\,\mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} \quad \ldots \quad (2)$$

$$\text{div}\,\mathbf{H} = 0 \quad \ldots \quad (3)$$

$$\mathbf{j} = \sigma(\mathbf{E}+\mathbf{V}\times\mathbf{H}) \quad \ldots \quad (4)$$

and the hydrodynamic equations

$$\rho\frac{d\mathbf{V}}{dt} = \mathbf{j}\times\mathbf{H} - \text{grad}\,p \quad \ldots \quad (5)$$

$$\text{div}\,\mathbf{V} = 0. \quad \ldots \quad (6)$$

Here $\mathbf{V}$ denotes the material velocity, $\mathbf{H}$ and $\mathbf{E}$ the magnetic and electric field strength respectively, $\mathbf{j}$ the current density and p the pressure. For $\sigma = \infty$, it follows from (4) that

$$\mathbf{E} = -\mathbf{V}\times\mathbf{H}. \quad \ldots \quad (7)$$

Outside the cylinder, $\mathbf{j} = 0$ and the field equations are the same as (1) to (3) with $\mathbf{j} = 0$ in (1).

Using (1), (5) may be written

$$\rho\frac{d\mathbf{V}}{dt} = \frac{1}{4\pi}(\text{curl}\,\mathbf{H})\times\mathbf{H} - \text{grad}\,p. \quad \ldots \quad (8)$$

Substituting (7) in (2), this becomes

$$\frac{\partial \mathbf{H}}{\partial t} = \text{curl } (\mathbf{V} \times \mathbf{H}) \quad \ldots \quad (9)$$

which expresses the fact that the magnetic lines of force move with the fluid.

To form the equations of small oscillations, we write

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h} \quad \text{and} \quad p = P + p' \quad \ldots \quad (10)$$

where $\mathbf{H}_0$ is the permanent field, neglecting squares and products of quantities of the first order $\mathbf{V}$, $\mathbf{h}$ and p' (8) and (9) become approximately

$$\rho \frac{\partial \mathbf{V}}{\partial t} = \frac{1}{4\pi} (\text{curl } \mathbf{h}) \times \mathbf{H}_0 - \text{grad } p' \quad \ldots \quad (11)$$

$$\frac{\partial \mathbf{h}}{\partial t} = \text{curl } (\mathbf{V} \times \mathbf{H}_0). \quad \ldots \quad (12)$$

Operating (11) with a curl we get

$$4\pi\rho \frac{\partial}{\partial t} (\text{curl } \mathbf{V}) = \text{curl } \{(\text{curl } \mathbf{h}) \times \mathbf{H}_0\}. \quad \ldots \quad (13)$$

Further since div $\mathbf{H}_0 = 0$ we find from (3)

$$\text{div } \mathbf{h} = 0. \quad \ldots \quad (14)$$

Then (6), (12), (13) and (14) are the fundamental equations to determine the small oscillations.

III. Solution of Problem

We solved these equations in cylindrical co-ordinate system and considered the radial and torsional oscillations only. Since div $\mathbf{V} = 0$ and div $\mathbf{h} = 0$, we may express $\mathbf{V}$ and $\mathbf{h}$ in terms of appropriate Stokes ' stream function', ψ and u respectively. At any time the family of curves ψ = constant and u = constant will then be the stream line of motion and the equations of the lines of force of the field of the currents induced by the motion of the fluid in permanent field respectively.

The resolutes of vectors $\mathbf{V}$ and $\mathbf{h}$ in the direction of r and θ increasing are given by

$$\left. \begin{aligned} v_r &= -\frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{\partial \psi}{\partial r} \\ h_r &= -\frac{1}{r}\frac{\partial u}{\partial \theta}, \quad h_\theta = \frac{\partial u}{\partial r}. \end{aligned} \right\} \quad \ldots \quad (15)$$

The permanent magnetic field due to an infinite axial line pole of strength m/unit length will be radial and its value will be given by

$$H_r = \frac{2m}{r}.$$

Let $\mu = \cos\theta$ and let Δ denote the operator

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial \mu^2} \quad \ldots \quad (16)$$

then (12) and (13) may be expressed in the form

$$\frac{\partial u}{\partial t} = \frac{2m}{r}\frac{\partial \psi}{\partial r} \qquad \text{.. .. .. ..} \quad (17)$$

$$4\pi\rho \frac{\partial}{\partial t}(\Delta\psi) = \frac{2m}{r}\frac{\partial}{\partial r}(\Delta u). \qquad \text{.. .. ..} \quad (18)$$

Assuming harmonic vibrations with the period time $2\pi/\lambda$, these equations become respectively

$$i\lambda u = \frac{2m}{r}\frac{\partial \psi}{\partial r} \qquad \text{.. .. .. ..} \quad (19)$$

$$4\pi\rho i\lambda \Delta\psi = \frac{2m}{r}\frac{\partial}{\partial r}(\Delta u). \qquad \text{.. .. ..} \quad (20)$$

These equations show that the phases of the functions u and ψ, hence, of the induced magnetic field $\mathbf{h}$ and velocity $\mathbf{V}$, differ by a quarter period. Eliminating u, between (19) and (20), we obtain finally differential equation for ψ.

$$-4\pi\rho\lambda^2 \Delta\psi = \frac{4m^2}{r}\frac{\partial}{\partial r}\left\{\Delta\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)\right\}. \qquad \text{.. ..} \quad (21)$$

To express this in non-dimensional form we put

$$r = \eta x$$

so that

$$\eta^4 = \frac{4m^2}{4\pi\rho\lambda^2}. \qquad \text{.. .. .. ..} \quad (22)$$

This reduces (21) to a form

$$\Delta_1\psi = -\frac{1}{x}\frac{\partial}{\partial x}\left\{\Delta_1\left(\frac{1}{x}\frac{\partial\psi}{\partial x}\right)\right\} \qquad \text{.. .. ..} \quad (23)$$

where

$$\Delta_1 = \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} + \frac{1-\mu^2}{x^2}\frac{\partial^2}{\partial\mu^2}. \qquad \text{.. .. ..} \quad (24)$$

Solution of (23) gives

$$\psi = \sum_n F_n(x)(1-\mu^2)^{\frac{1}{2}} P_n^1(\mu) \qquad \text{.. .. ..} \quad (25)$$

where P_n^1 is the associated Legendre function of degree n and order 1 and $F_n(x)$ is a function of x to be determined by the following fourth order differential equation

$$x^3 F_n^{\text{iv}} - 2x^2 F_n^{\text{iii}} + [(3-l)x + x^5]F_n^{\text{ii}} - [3(1-l) - x^4]F_n^{\text{i}} - lx^3 F_n = 0 \qquad \text{..} \quad (26)$$

where $l = n(n+1)$.

(*a*) *Solution of the equation for* F_n *in series*—

Equation (26) has been solved in a power series. Substituting

$$F_n = \sum_0^\infty a_k x^{\alpha+k} \qquad \text{.. .. .. ..} \quad (27)$$

in (26), the indicial equation is found to be

$$\alpha^4-8\alpha^3+(20-l)\alpha^2-4(4-l)\alpha = 0$$

the roots of which are

$$0,\ 2-\sqrt{l},\ 2+\sqrt{l},\ \text{and } 4. \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

The series is found to proceed by the powers of x^4, so that (27) may be written

$$F_n = \sum_0^\infty A_k x^{\alpha+4k},$$

where A's are determined from the recurrence relation

$$\frac{A_k}{A_{k-1}} = -\frac{(\alpha+4k-4-\sqrt{l})(\alpha+4k-4+\sqrt{l})}{(\alpha+4k)(\alpha+4k-4)(\alpha+4k-2+\sqrt{l})(\alpha+4k-2-\sqrt{l})}.$$

(*b*) *Discussion of the four solutions of* F_n—

Case (i) $\alpha = 0$. Here the series (29) is finite at the axis ($r = 0$) and

$$\frac{A_k}{A_{k-1}} = -\frac{(4k-4-\sqrt{l})(4k-4+\sqrt{l})}{4k(4k-4)(4k-2+\sqrt{l})(4k-2-\sqrt{l})}. \quad \ldots \quad \ldots \quad (29)$$

If $l = (4k-4)^2$, the series terminates; if l is not of the form $(4k-2)^2$, the series converges absolutely and uniformly for all values of x. If $l = (4k-2)^2$, some of the coefficients of the series become infinite. It is found that the velocity and magnetic resolutes become infinite at the axis. Hence we neglect this solution.

Case (ii) $\alpha = 2-\sqrt{l}$. Here

$$\frac{A_k}{A_{k-1}} = -\frac{(4k-2-2\sqrt{l})(4k-2)}{(4k+2-\sqrt{l})(4k-2-\sqrt{l})(4k-2\sqrt{l})4k}.$$

If $l = (2k-1)^2$, the series terminates. The series is absolutely and uniformly convergent for all values of $|x| > 0$ provided some of the factors in the denominator does not vanish. The solution is not finite at the axis for $l > 4$. When $l < 4$ the velocity and magnetic field is again infinite at the axis. Hence this solution must also be neglected.

Case (iii) $\alpha = 2+\sqrt{l}$. Here

$$\frac{A_k}{A_{k-1}} = -\frac{(4k-2)(4k-2+2\sqrt{l})}{(4k+2+\sqrt{l})(4k-2+\sqrt{l})(4k+2\sqrt{l})4k} \quad \ldots \quad (30)$$

and the series is uniformly and absolutely convergent for all values of x. Moreover, all variables are finite or zero at the axis and so this gives a possible solution $F_n(x)$. We shall denote this solution by $F_n^{(1)}(x)$.

Case (iv) $\alpha = 4$. Here

$$\frac{A_k}{A_{k-1}} = -\frac{(4k-\sqrt{l})(4k+\sqrt{l})}{(4k+4)(4k)(4k+2+\sqrt{l})(4k+2-\sqrt{l})} \quad \ldots \quad \ldots \quad (31)$$

and the series is uniformly and absolutely convergent for all values of x except when $l = (4k+2)^2$. If $l = (4k+2)^2$ some of the coefficients become infinite and a second solution must be sought. As we have restricted ourselves to the lower harmonics, hence this restriction over l will not interfere. To this solution we shall denote by $F^{(2)}_n(x)$.

The required solution of (26) is thus of the form

$$F_n(x) = AF^{(1)}_n(x) + BF^{(2)}_n(x), \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

where A and B are arbitrary constants.

IV. The Magnetic Field associated with the Oscillations

For points within the cylinder the stream function of the magnetic field is given by (19) and (25)

$$u = -i(4\pi\rho)^{\frac{1}{2}} x^{-1} \sum F^{\mathrm{i}}_n(x)(1-\mu^2)^{\frac{1}{2}} P^1_n(\mu), \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

where F^{i}_n is the derivative of F_n with respect to x.

To find the magnetic field outside the cylinder we have to find solutions of the equations

$$\mathrm{curl}\, \mathbf{H} = 0, \ \mathrm{div}\, \mathbf{H} = 0. \qquad \ldots \quad \ldots \quad \ldots \quad (34)$$

Since the field outside will only differ slightly from the field of the axial magnetic line pole, we write as before

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}'. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

Using u' for the stream function of the induced field h', we have from (34) that

$$\mathrm{curl}\, \mathbf{h}' = 0 \ \text{or} \ \Delta_1 u' = 0, \qquad \ldots \quad \ldots \quad \ldots \quad (36)$$

where Δ_1 is given by (24).

Boundary condition.—Let the value of x at the surface of the cylinder be denoted by x_1, so that by (22)

$$a = \eta x_1. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

To satisfy the continuity of the radial resolutes of the magnetic field at the surface, the solution of (36) must involve the Legendre function P^1_n and must vanish at infinity. Such a solution is

$$u' = \sum_n B_n x^{-\sqrt{l}} (1-\mu^2)^{\frac{1}{2}} P'_n, \qquad \ldots \quad \ldots \quad \ldots \quad (38)$$

where Bn's are constants to be determined. Continuity of magnetic resolutes at the surface $x = x_1$ then gives two conditions:

$$x_1^{\sqrt{l}-1} F^{\mathrm{i}}_n(x_1) = iB_n (4\pi\rho)^{-\frac{1}{2}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

$$\sqrt{l} B_n x_1^{1-\sqrt{l}} = i(4\pi\rho)^{\frac{1}{2}} \left[x_1 F^{\mathrm{ii}}_n(x_1) - F^{\mathrm{i}}_n(x_1)\right].$$

Elimination of B_n gives

$$(1-\sqrt{l}) F^{\mathrm{i}}_n(x_1) = x_1 F^{\mathrm{ii}}_n(x_1) \qquad \ldots \quad \ldots \quad \ldots \quad (40)$$

and

$$u' = -i(4\pi\rho)^{\frac{1}{2}} \sum x_1^{\sqrt{l}-1} x^{-\sqrt{l}} F^{\mathrm{i}}_n(x_1) (1-\mu^2)^{\frac{1}{2}} P'_n(\mu). \quad \ldots \quad \ldots \quad (41)$$

V. The Electric Field associated with the Oscillations

The electric field within the cylinder is given by (7) or since **V** is small, approximately by

$$\mathbf{E} = -\mathbf{V}\times\mathbf{H}_0.$$

Since **E** is wholly axial, its intensity E is given by

$$E = (4\pi\rho)^{\frac{1}{2}}\lambda\sum F^1_n(x)\,x^{-1}(1-\mu^2)^{\frac{1}{2}}P^1_n\,(\mu). \qquad \ldots \quad \ldots \quad (42)$$

Outside the cylinder the electric field satisfies the condition

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} \quad \text{or} \quad \text{curl } \mathbf{E} = -\frac{\partial h'}{\partial t}$$

by (35). Using the stream function u' for the field h' given by (41), and the fact that E is continuous at the surface of the cylinder, we find

$$E = i\lambda u', \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

where u' is given by (41).

VI. The Variation of Pressure in the Cylinder

This is most readily derived from the equation of motion (11), viz.

$$4\pi\rho\frac{\partial \mathbf{V}}{\partial t} = (\text{curl } \mathbf{h})\times\mathbf{H}_0 - 4\pi \text{ grad } p'.$$

Considering the azimuthal resolutes of this equation, we find that p' is of the form

$$p' = -(i\rho\lambda)\sum G_n(x)\,P_n(\mu), \qquad \ldots \quad \ldots \quad \ldots \quad (44)$$

where $G_n(x)$ is a function of x given by

$$G_n(x) = \left[x^{-1}\frac{\partial^3}{\partial x^3} - x^{-2}\frac{\partial^2}{\partial x^2} + x^{-3}\left\{(1-l)+x^4\right\}\frac{\partial}{\partial x}\right]F_n(x). \qquad \ldots \quad (45)$$

We shall denote by $G^{(1)}_n$ and $G^{(2)}_n$ the G functions corresponding to $F^{(1)}_n$ and $F^{(2)}_n$ respectively. Let

$$F^{(1)}_n = x^{2+\sqrt{l}}\sum_0^\infty A_k x^{4k}\,;\quad F^{(2)}_n = x^4\sum_0^\infty A'_k x^{4k}, \qquad \ldots \quad \ldots \quad (46)$$

where $A_0 = A'_0 = 1$ and A_k and A'_k are given by (30) and (31).

Then we find that

$$G^{(1)}_n(x) = \sum_0^\infty \frac{l}{4k+2+\sqrt{l}}A_k x^{4k+2+\sqrt{l}} \qquad \ldots \quad \ldots \quad \ldots \quad (47)$$

$$G^{(2)}_n(x) = 4(4-l) + \sum_0^\infty \frac{l}{4k+4}A'_k x^{4k+4}, \qquad \ldots \quad \ldots \quad \ldots \quad (48)$$

where $l = n(n+1)$. The series are absolutely and uniformly convergent for all values of x.

VII. The Period of Normal Modes of Vibrations

At the boundary of the cylinder $\frac{dp}{dt} = 0$ and since $p' \alpha e^{i\lambda t}$ and $p = P + p'$, therefore

$$p' = 0. \quad \dots \quad (49)$$

From (32) and (44) we have

$$G_n(x) = AG_n^{(1)}(x) + BG_n^{(2)}(x),$$

where A and B are the same arbitrary constants as used in (34). The condition (49) then requires by (44) that

$$G_n(x_1) = 0$$

or

$$AG_n^{(1)}(x_1) + BG_n^{(2)}(x_1) = 0. \quad \dots \quad (50)$$

Again (40) gives

$$(1-\sqrt{l})\left[A\frac{\partial F_n^{(1)}}{\partial x} + B\frac{\partial F_n^{(2)}}{\partial x}\right]_{x=x_1} = x_1\left[A\frac{\partial^2 F_n^{(1)}}{\partial x^2} + B\frac{\partial^2 F_n^{(2)}}{\partial x^2}\right]_{x=x_1} \quad \dots \quad (51)$$

Eliminating the ratio B/A between (50) and (51) gives the period equation for the normal modes of vibrations

$$G_n^{(1)} H_n^{(2)} = G_n^{(2)} H_n^{(1)}, \quad \dots \quad (52)$$

where

$$H_n^{(1)} = (1-\sqrt{l})\left(\frac{\partial F_n^{(1)}}{\partial x}\right)_{x=x_1} - x_1\left(\frac{\partial^2 F_n^{(1)}}{\partial x^2}\right)_{x=x_1}$$

$$H_n^{(2)} = (1-\sqrt{l})\left(\frac{\partial F_n^{(2)}}{\partial x}\right)_{x=x_1} - x_1\left(\frac{\partial^2 F_n^{(2)}}{\partial x^2}\right)_{x=x_1} \quad \dots \quad (53)$$

Equation (52) determines the value of x_1 and hence by (22) and (37) the period $T\left(=\frac{2\pi}{\lambda}\right)$ of the normal modes. We find

$$T = \frac{\pi a^2}{m}\left(\frac{4\pi\rho}{x_1^4}\right)^{\frac{1}{2}} \quad \dots \quad (54)$$

which may be expressed in terms of the surface value of permanent magnetic field $H_s\left(=\frac{2m}{a}\right)$ in the form

$$T = \frac{2\pi a}{H_s}\left(\frac{4\pi\rho}{x_1^4}\right)^{\frac{1}{2}}. \quad \dots \quad (55)$$

The roots of equation (52) have been calculated numerically for the fundamental mode of the first and second harmonics. They are approximately $x_1 = 1{\cdot}90$ and $x_1 = 2{\cdot}02$ for the two harmonics respectively. The periods of oscillation for the fundamental mode of the first and second harmonics are 1,070 and 950 days respectively. The values used for radius $a = 1{\cdot}09 \times 10^{11}$ cm., density $\rho = 0{\cdot}93$ and surface magnetic field $H_s = 7{,}000$ gauss are same as were used by Ferraro and Memory (1952). The periods of the oscillation of star in their case were 894 and 890 days for the first two harmonics respectively. Thus there is a very little difference in time period for the two harmonics. In the present case the period of oscillations of the cylinder is rapidly decreasing as we go from first harmonic to second harmonic.

VIII. The Velocity and Velocity Stream Function

We have calculated the velocity and velocity stream function for the fundamental mode of the first two harmonics. The surface of the cylinder for two cases corresponds to $x = x_1 = 1{\cdot}90$ and $2{\cdot}02$ respectively. The stream functions for the first and second harmonics are given by (56) and (57). (Omitting time factor.)

$$\psi = cx^{3\cdot 414}(1-k_1x^{0\cdot 586})\sin^2\theta \qquad (56)$$

and

$$\psi = 3cx^4(x^{0\cdot 449}-k_2)\sin^2\theta\cos\theta, \qquad (57)$$

where

$$k_1 = 0{\cdot}3096 \qquad (58)$$

$$k_2 = 1{\cdot}3646. \qquad (59)$$

The stream lines of motion, given by $\psi =$ constant, are drawn for the first two harmonics in the first quadrant only (see Figs. 1(a) and 1(b)). The radial and azimuthal resolutes of velocity are (omitting time factor)

$$\left.\begin{aligned} v_r &= 2c\eta x^{2\cdot 414}(1-k_1x^{0\cdot 586})\sin\theta\cos\theta \\ v_\theta &= c\eta x^{2\cdot 414}(3{\cdot}414-4k_1x^{0\cdot 586})\sin^2\theta \end{aligned}\right\} \qquad (60)$$

for the first harmonic, and

$$\left.\begin{aligned} v_r &= 3c\eta x^3(x^{0\cdot 449}-k_2)(3\cos^2\theta-1)\sin\theta \\ v_\theta &= 3c\eta x^3(4{\cdot}449x^{0\cdot 449}-4k_2)\sin^2\theta\cos\theta \end{aligned}\right\} \qquad (61)$$

for the second harmonic. Here k_1, k_2 and η are given by (58), (59) and (22) respectively.

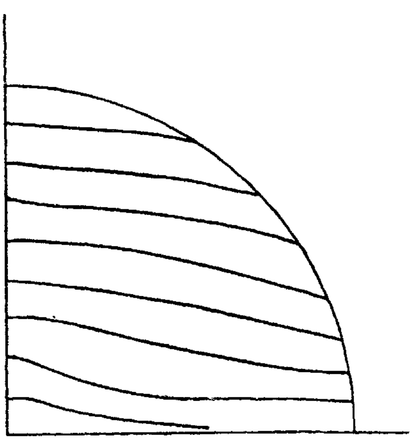

Fig. 1(a)

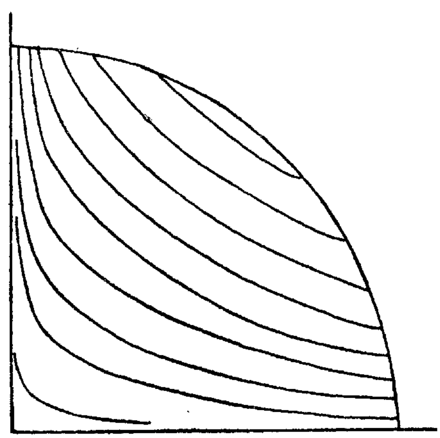

Fig. 1(b)

Fig. 1(a). Stream lines for the fundamental mode of the first harmonic vibration.
Fig. 1(b). Stream lines for the fundamental mode of the second harmonic vibration.

IX. The Magnetic Field of the Induced Current inside the Cylinder

The corresponding stream function for the magnetic field of the induced current inside the cylinder is given by (62) and (63) for the case of the fundamental mode of the first two harmonics (omitting time factor)

$$u = -i(4\pi\rho)^{\frac{1}{2}}\,(3{\cdot}414)\,c(x^{1\cdot414}-k_3x^2)\sin^2\theta \qquad \ldots \quad \ldots \quad (62)$$

$$u = -i(4\pi\rho)^{\frac{1}{2}}\,(13{\cdot}347)\,c(x^{2\cdot449}-k_4x^2)\sin^2\theta\cos\theta, \qquad \ldots \quad (63)$$

where

$$k_3 = 0{\cdot}3627 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (64)$$

$$k_4 = 1{\cdot}2102. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (65)$$

The magnetic lines of forces are given by u = const. for the additional magnetic field in the fundamental mode of the first two harmonics are drawn in the first quadrant only (see Figs. 2(*a*) and 2(*b*)). The radial and azimuthal resolutes of the magnetic field of the induced current inside the cylinder are (omitting time factor)

$$h_r = -i(4\pi\rho)^{\frac{1}{2}}\,\eta(6{\cdot}828)\,c(x^{0\cdot414}-k_3x)\sin\theta\cos\theta$$

$$h_\theta = -i(4\pi\rho)^{\frac{1}{2}}\,\eta(4{\cdot}827)\,c(x^{0\cdot414}-1{\cdot}414k_3x)\sin^2\theta \qquad \ldots \quad \ldots \quad (66)$$

for the first harmonic and

$$h_r = -i(4\pi\rho)^{\frac{1}{2}}\,\eta(4{\cdot}449)\,3c(x^{2\cdot449}-k_4x^2)\,(3\cos^2\theta-1)\sin\theta$$

$$h_\theta = -i(4\pi\rho)^{\frac{1}{2}}\,\eta(4{\cdot}449)\,3c(2{\cdot}449x^{1\cdot449}-2k_4x)\sin^2\theta\cos\theta \qquad \ldots \quad (67)$$

for the second harmonic. The values of k_3, k_4 and η are given by (64), (65) and (22) respectively.

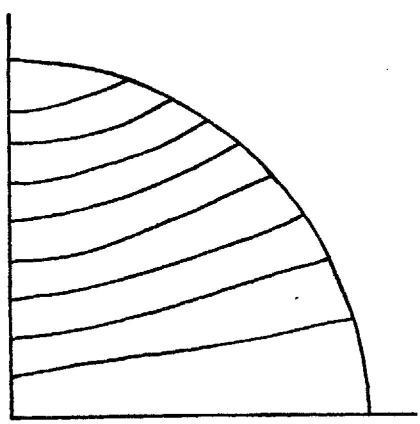

Fig. 2(*a*)

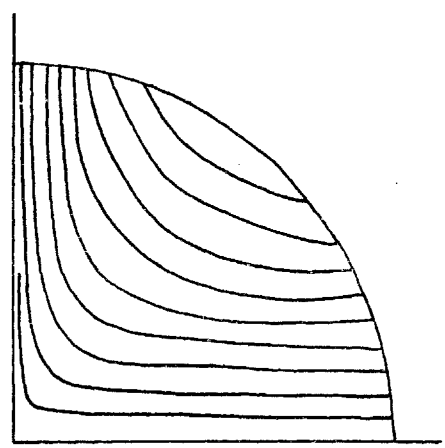

Fig. 2(*b*)

Fig. 2(*a*). Magnetic lines of force for the additional field in the fundamental mode of the first harmonic vibration.

Fig. 2(*b*). Magnetic lines of force for the additional field in the fundamental mode of the second harmonic vibration.

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SUMMARY

In the present paper, the discussion of the oscillation of a cylindrical fluid in the presence of permanent magnetic field has been considered. The magnetic field has been assumed due to infinite axial line magnetic pole.

REFERENCE

Ferraro, V. C. A., and Memory, D. F. (1952). Oscillations of a star in its own magnetic field. An illustrated problem, *M.N.*, **112**, 361.

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