

# OPACITY MEANS AND STELLAR ATMOSPHERES

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## ABSTRACT

The three kinds of means which are generally used in construction of a model of a 'nongrey atmosphere' of a star are compared for three effective temperatures, 41,700, 20,400 and 10,700° K, with  $\log g = 4.2$ . It is found that the kind of mean used does not show any wild influence on the physical parameters of the model. Thus, in general, for the study of 'nongrey stellar atmospheres' in mechanical equilibrium any kind of mean may be a good approximation.

## INTRODUCTION

The method of model atmospheres is a deductive method of analysing a problem theoretically. From the work of Strömngren (1940, 1944), the method of model atmosphere is found to predict successfully the observable properties of a stellar atmosphere. The most complicated, though a fundamental, quantity which one cannot evade in the construction of model atmosphere is the absorption coefficient due to various chemical elements in the stellar atmosphere. Earlier, models were constructed for 'grey atmosphere' which assumes that the absorption coefficient at every depth of the atmosphere is independent of frequency. But in fact the absorption coefficient not only varies appreciably with frequency but also it varies in an irregular manner. This departure from the 'grey atmosphere' is reflected in the observations in the form of flux discontinuity at series limits. Hence, to understand better the real state of the atmosphere, one is forced to consider the variation of absorption coefficient with frequency. This makes the problem complicated. Even at the present time the knowledge about 'nongrey stellar atmosphere' is inadequate because of the fact that the 'nongrey atmosphere' cannot be treated mathematically in a general manner. The only alternative to study 'nongrey atmosphere' is to reduce it to a 'grey atmosphere' by some method. If once this is done all the results derived for the 'grey atmosphere' can be applied to it also.

To reduce a 'nongrey atmosphere' to a 'grey atmosphere', a mean absorption coefficient  $\bar{k}$  is defined as follows:

$$\bar{k} = \frac{\int_0^{\infty} k_{\nu} w_{\nu} d\nu}{\int_0^{\infty} w_{\nu} d\nu} \quad \dots \quad \dots \quad \dots \quad (1)$$

Here  $w_\nu$  represents some sort of weight function. Hence the problem essentially reduces to the selection of a suitable weight function, such that the real 'nongrey atmosphere' is represented by this reduced 'grey atmosphere' to a best possible approximation.

### DIFFERENT KINDS OF WEIGHT FUNCTIONS

The weight functions required for the calculation of the mean value of  $\bar{k}$  have to be selected such that they have some bearing on the actual physical conditions existent in the stellar atmosphere. One can, therefore, connect these weight factors to radiation through the atmosphere or the processes of emission or absorption of energy in the outer layers of a star. Consequently, it is possible to define weight factor in three different ways, namely radiation pressure mean, true absorption mean and true emission mean. They are given below (Przybylski 1960) :

$$(i) \text{ Radiation pressure mean } \bar{k} = \frac{\int_0^\infty k_\nu F_\nu d\nu}{\int_0^\infty F_\nu d\nu} \quad \dots \quad (2)$$

$$(ii) \text{ True absorption mean } \bar{k} = \frac{\int_0^\infty k_\nu I_\nu d\nu}{\int_0^\infty I_\nu d\nu} \quad \dots \quad (3)$$

$$(iii) \text{ True emission mean } \bar{k} = \frac{\int_0^\infty k_\nu J_\nu d\nu}{\int_0^\infty J_\nu d\nu} \quad \dots \quad (4)$$

Here  $F_\nu$ ,  $I_\nu$  and  $J_\nu$  represent the monochromatic flux, intensity and source function of  $\nu$  radiation respectively. The main difficulty in the above weight functions is that they are not known rigorously until the actual model of the atmosphere is known. So it is necessary to make further simplifying approximations in the definition of weight functions. There are three such approximations which are given below :

(i) *Rosseland Mean*.—The Rosseland mean is an approximation to the radiation pressure mean defined by (2). It can be shown from the equation of radiative transfer that at any optical depth  $\tau_\nu$  the monochromatic flux is given by

$$F_\nu(\tau_\nu) = 2 \int_{\tau_\nu}^\infty B_\nu(t) E_2(t - \tau_\nu) dt - 2 \int_0^{\tau_\nu} B_\nu(t) E_2(\tau_\nu - t) dt \quad \dots \quad (5)$$

where  $B_\nu(t)$  is the source function at the optical depth  $t$  and  $E_2$  is the well-known second exponential integral. Expanding the source function in a Taylor's series and using only first two terms in the expansion, it can be shown

that for large values of  $\tau_\nu$  equation (5) reduces to

$$F_\nu \approx \frac{1}{3} \frac{dB_\nu}{dT} \frac{dT}{d\tau_{P_r}} \frac{\bar{k}_{P_r}}{k_\nu} \dots \dots \dots (6)$$

Here  $\tau_{P_r}$  represents the mean optical depth measured in terms of the radiation pressure mean  $\bar{k}_{P_r}$ . Thus from (6) the radiation pressure mean (2) reduces to

$$\frac{1}{\bar{k}_R} = \frac{\int_0^\infty \frac{1}{k_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu} \dots \dots \dots (7)$$

Substituting  $B_\nu(T)$ , the expression (7) simplifies to

$$\frac{1}{\bar{k}_R} = \frac{15}{4\pi^4} \int_0^\infty \frac{1}{k_\nu} \frac{u^4 e^{-u}}{(1-e^{-u})^2} du \dots \dots \dots (8)$$

where  $u = \frac{h\nu}{kT}$ . Thus once the chemical composition is known, Rosse-land mean can be calculated as a function of temperature and pressure. This mean is good for deep interior layers and less satisfactory for smaller values of optical depth since, in the latter case, the terms which were disregarded in the derivation of the above mean are not negligible.

(ii) *Chandrasekhar Mean.*—To a first approximation Chandrasekhar replaced the actual flux by a grey body flux. Thus the Chandrasekhar's mean is given by

$$\bar{k}_{Ch} = \frac{\int_0^\infty k_\nu F_\nu^{(1)} d\nu}{\int_0^\infty F_\nu^{(1)} d\nu} \dots \dots \dots (9)$$

Chandrasekhar (1945) has given the values of the weight factors  $\frac{F_\alpha^{(1)}}{F^{(1)}}$  in the

form of table, where  $\alpha = \frac{h\nu}{kT_e}$  and  $T_e$  being the effective temperature of the star.  $F^{(1)}$  is integrated flux over whole frequency. Chandrasekhar mean is a good approximation if the variation of the absorption coefficient with frequency is not large. But the actual variation of  $F$  with  $\nu$  is not known.

(iii) *Planck Mean.*—For a local thermodynamic equilibrium the source function is given by the Planck function, i.e.  $J_\nu = B_\nu$ . Hence, for this case, the true emission mean defined by (4) simplifies to

$$\bar{k}_{Pl} = \frac{\int_0^\infty k_\nu B_\nu d\nu}{\int_0^\infty B_\nu d\nu} \dots \dots \dots (10)$$

Since the weight functions given above come from different considerations, the structure of the atmosphere may reflect the essential differences in these different means. In the present paper, therefore, an attempt is made to make a comparative study of the model atmospheres for stars with three effective temperatures, namely  $T_e = 41,700$ ,  $20,400$  and  $10,700^\circ$  K with  $\log g = 4.2$  calculated with the above three means. These temperatures cover a range from 05 stars to A0 stars. Recently, a similar study has also been made by Przybylski (1960) for a solar type star. Some models used in this study were already available in previous literature, and others had to be constructed. Here two models have been computed using Planck's mean for the stars of effective temperature  $41,700^\circ$  K and  $10,700^\circ$  K. Surface gravity is taken to be  $\log g = 4.2$  and composition  $H : He = 85 : 15$  by number. The opacity in the atmosphere due to continuous absorption from hydrogen, neutral helium, ionized helium, negative hydrogen ions and Thomson scattering by free electrons is taken into account. The choice of these above parameters was mainly due to the fact that with these values above mentioned other models were already constructed by previous workers (Underhill 1951, 1957a; Saito 1954; Saito and Uesugi 1959).

#### MODEL ATMOSPHERE

The fundamental equation of mechanical equilibrium of a stellar atmosphere is

$$\frac{dP_g}{d\tau} = \frac{g}{\bar{K} + \sigma} - \frac{\sigma_R}{c} T_e^4 \quad \dots \quad (11)$$

where  $\sigma_R$  is the Stefan-Boltzmann Constant and  $\tau$  is the optical depth of the atmosphere. Other symbols have their usual meanings. Second term accounts for the radiation pressure. Here Planck mean absorption coefficient has been used. Following grey body  $T-\tau$  relation is used

$$T^4 = T_0^4 (1 + \frac{2}{3}\tau) \quad \dots \quad (12)$$

where  $T_0$  is the surface temperature.

The total continuous absorption coefficient due to hydrogen, negative hydrogen ions, neutral and ionized helium for the assumed composition is given by

$$\begin{aligned} \bar{K} = & 0.585(1-x_H)\{\bar{k}(H) + \bar{k}(H^-)\} + 0.415(1-x_{He^+} - x_{He^{++}}) \times \\ & \bar{k}(He^0) + 0.415x_{He^+}\bar{k}(He^+) \quad \dots \quad (13) \end{aligned}$$

where  $\bar{k}$ 's are the mass absorption coefficients per gram of the corresponding element averaged over all frequencies and  $x_H$ ,  $x_{He^+}$  and  $x_{He^{++}}$  are the fractions of ionized hydrogen, singly ionized helium and doubly ionized helium,

respectively, in terms of total of the corresponding element. These fractions are obtained from Saha's equation in the following form :

$$\text{Log } \frac{x_H}{1-x_H} = -13.53\theta + 2.5 \log T - 0.477 - \log P_e \quad \dots \quad (14)$$

$$\text{Log } \frac{N_{He^+}}{N_{He^0}} = -24.46\theta + 2.5 \log T + 0.125 - \log P_e \quad \dots \quad (15)$$

$$\text{Log } \frac{N_{He^{++}}}{N_{He^+}} = -54.14\theta + 2.5 \log T - 0.477 - \log P_e \quad \dots \quad (16)$$

Fractions of helium in different stages of ionization are given by

$$x_{He^+} = \frac{N_{He^+}}{N_{He^0} + N_{He^+} + N_{He^{++}}} = \frac{\left(\frac{N_{He^+}}{N_{He^0}}\right)}{1 + \frac{N_{He^+}}{N_{He^0}} \left[1 + \frac{N_{He^{++}}}{N_{He^+}}\right]} \equiv C \quad \dots \quad (17)$$

$$x_{He^{++}} = \frac{N_{He^{++}}}{N_{He^0} + N_{He^+} + N_{He^{++}}} = C \left(\frac{N_{He^{++}}}{N_{He^+}}\right) \quad \dots \quad (18)$$

and

$$(1 - x_{He^+} - x_{He^{++}}) = \frac{1}{1 + \frac{N_{He^+}}{N_{He^0}} \left[1 + \frac{N_{He^{++}}}{N_{He^+}}\right]} \quad \dots \quad (19)$$

METHOD OF CALCULATION OF MEAN ABSORPTION COEFFICIENTS

(i) *Hydrogen.*—The monochromatic continuous absorption coefficient corrected for stimulated emission per gram of neutral hydrogen neglecting the Gaunt factor is given by

$$k_\nu(H) = \frac{F(H)D(H)}{u^3} (1 - e^{-u}) \quad \dots \quad (20)$$

where

$$F(H) = \frac{64\pi^4 m e^{10}}{3\sqrt{3} \, ch^3 m_H} \frac{e^{-u_1(H)}}{(kT)^3}$$

and

$$D(H) = \sum_{1 < n}^{n=9} \frac{e^{u_n(H)}}{n^3} + \frac{e^{u_{10}(H)}}{2u_1(H)}$$

Here  $u = \frac{h\nu}{kT}$  and  $u_n(H)$  is the value of  $u$  at the series limit  $n$ ,

i.e. 
$$u_n(H) = \frac{h\nu_n}{kT} = \frac{1}{n^2} \frac{R_H hc}{kT}$$

Other symbols have their usual meanings. Averaging (20) over all frequencies according to the definition (10) we get

$$\bar{k}(H) = \frac{15}{\pi^4} \Sigma F(H) D(H) [e^{-u_a} - e^{-u_b}] \dots \dots \dots (21)$$

where  $u_a$  and  $u_b$  are the values of  $u$  at the long and short wavelength ends of the range of  $F(H) D(H)$  respectively.

(ii) *Ionized Helium*.—Using 'hydrogen-like' approximation we get

$$\bar{k}(He^+) = \frac{15}{\pi^4} \Sigma F(He^+) D(He^+) [e^{-u_a} - e^{-u_b}] \dots \dots (22)$$

where

$$F(He^+) = \frac{64\pi^4 m e^{10}}{3\sqrt{3} c h^3 m_{He}} \frac{(2)^4}{(kT)^3} e^{-u_1(He^+)}$$

and

$$D(He^+) = \sum_{1 < n}^{n=9} \frac{e^{u_n(He^+)}}{n^3} + \frac{e^{u_{10}(He^+)}}{2u_1(He^+)}$$

The values of  $F(H) D(H)$  and  $F(He^+) D(He^+)$  have been tabulated by Underhill (1950) for the temperature range 25,200° K to 80,000° K. These tables were extended to the lower temperature also.

(iii) *Neutral Helium*.—Absorption coefficient arising from different levels was calculated and then added together to get the total absorption coefficient due to neutral helium for  $n \leq 2$  as follows:

Assuming the population of atoms to be given according to Boltzmann's distribution law, the monochromatic continuous absorption coefficient per gram of neutral helium, corrected for stimulated emission, from the levels  $n \leq 2$  is given by

$$k_\nu(He^0, n \leq 2) = \Sigma \frac{g_{1S}}{m_{He}} e^{-\frac{\psi_{1S}}{kT}} a_\nu(S) (1 - e^{-h\nu/kT}) \dots \dots (23)$$

where  $a_\nu(S)$  is the atomic absorption coefficient arising from the level  $S$  for the frequency  $\nu$ ,  $g_{1S}$  and  $\psi_{1S}$  are the statistical weight and the excitation potential of the level  $S$ . A table giving all the relevant information like the excitation potentials, statistical weights, etc., which have been used in the present calculations, are given by Ueno (1954) and Underhill (1950). The atomic absorption coefficient  $a_\nu(S)$  for the continua arising from the ground level  $1^1S$  and from the excited states  $2^1S$  and  $2^3S$  levels are computed from the tables of Huang (1948) using the values derived from the momentum interaction. The values of the atomic absorption coefficient for the series  $2^1P - n^1D$  and  $2^3P - n^3D$  have been interpolated from the curves given by

Goldberg (1939). For the series  $2^1P-n^1S$  and  $2^3P-n^3S$  the following approximate formulae have been used :

$$a_\nu(He^0) \simeq \frac{1}{2R} \frac{\pi e^2}{mc} 10^{-15.80} \nu_n^{4.3} \nu^{-3.3} \dots \dots \dots (24)$$

for  $2^3P-n^3S$  series and

$$a_\nu(He^0) \simeq \frac{1}{2R} \frac{\pi e^2}{mc} 10^{-15.71} \nu_n^{4.6} \nu^{-3.6} \dots \dots \dots (25)$$

for  $2^1P-n^1S$  series. Here  $\nu$  denotes the frequency of the series limit and  $R$  the Rydberg constant.

Following Underhill (1950), for levels  $n \geq 3$  and for the free-free transitions, the continuous absorption coefficient per gram of neutral helium taking into account stimulated emission is given by

$$k_\nu(He^0, n \geq 3) = \frac{F(He^0, n \geq 3)D(He^0, n \geq 3)}{u^3} (1 - e^{-u}) \dots \dots (26)$$

where

$$F(He^0, n \geq 3) = \frac{64\pi^4 m e^{10} e^{-u_1(He^0)}}{3\sqrt{3} c h^3 m_{He} (kT)^3}$$

and

$$D(He^0, n \geq 3) = Z_{eff}^4 \left[ \sum_{3 \leq n}^5 \frac{e^{u_n}}{n^3} + \frac{(e^{u_5} - 1)}{2u_1} \right] + \frac{2}{u_1}$$

Since the effective charge,  $Z_{eff}$ , does not change appreciably with the principal quantum number  $n$ , the mean value of  $Z_{eff} = 1.0226$  for the levels  $n = 3, 4$  and  $5$  has been used. We, therefore, get

$$k_\nu(He^0) = k_\nu(He^0, n \leq 2) + k_\nu(He^0, n \geq 3) \dots \dots (27)$$

Thus knowing  $k_\nu(He^0)$  as a function of  $\nu$ ,  $\bar{k}(He^0)$  is calculated from the following equation :

$$\bar{k}(He^0) = \frac{\int_0^\infty k_\nu(He^0) B_\nu d\nu}{\int_0^\infty B_\nu d\nu} \dots \dots \dots (28)$$

The integration over frequency is performed using Cote's five-point formula by calculating  $k_\nu(He^0)$ 's at equidistant frequency intervals, in the wavelength regions

$$\lambda\lambda 228-504.3, \lambda\lambda 504.3-911.6, \lambda\lambda 911.6-1458,$$

$$\lambda\lambda 1458-3646, \lambda\lambda 3646-8204 \text{ and } \lambda\lambda 8204-14590$$

and then adding the contribution from each wavelength range.

(iv) *Negative Hydrogen Ions*.—The monochromatic continuous absorption coefficient of negative hydrogen ions per gram of neutral hydrogen taking into account stimulated emission is given by

$$k_\nu(H^-) = \frac{a_\nu(H^-)}{m_H} P_e \quad \dots \quad (29)$$

where  $a_\nu(H^-)$  is the atomic absorption coefficient per neutral hydrogen atom per unit electron pressure and  $P_e$  is the electron pressure. The atomic absorption coefficients  $a_\nu(H^-)$  have been tabulated by Chandrasekhar and Breen (1946) for  $\theta = 0.5$  to  $2.0$  and for wavelengths  $\geq 4050$  A.U.

Averaging (29) over all frequencies we get

$$\bar{k}(H^-) = \frac{P_e}{m_H} \frac{\int_0^\infty a_\nu(H^-) B_\nu d\nu}{\int_0^\infty B_\nu d\nu} \quad \dots \quad (30)$$

These integrations were evaluated graphically.

(v) *Thomson Scattering by Free Electrons*.—For our chosen composition of the atmosphere,  $\sigma$  at any depth is given by

$$\sigma = 0.397 [0.585 x_H + 0.104 (x_{H_e^-} + 2x_{H_e^{++}})] \quad \dots \quad (31)$$

It has been shown by Rudkjøbing (1947) that the coefficient of electron scattering should not be corrected for stimulated emission.

From these  $\bar{k}$ 's tables of  $\bar{K}$  of equation (13) and  $\sigma$  are prepared as a function of temperature and electron pressure. In the actual calculation, the values of  $\bar{K}$  and  $\sigma$  at different optical depths are interpolated from these tables.

#### INTEGRATION OF THE MODEL ATMOSPHERE

(i) *For the High Temperature Star*.—( $T_e = 41,700^\circ$  K.)

To integrate the equation of mechanical equilibrium  $\tau$  is taken as independent variable,  $P_e$  as dependent variable and the following relation between gas pressure and electron pressure is explicitly assumed:

$$P_g = 2P_e \dots \quad (32)$$

For numerical integration the equation (11) then is simplified to

$$\Delta P_e = \frac{1}{2} \left\{ \frac{g}{\bar{K} + \sigma} - \frac{\sigma_R}{c} T_e^4 \right\} \Delta \tau \quad \dots \quad (33)$$

At every step  $\bar{K}$  and  $\sigma$  were interpolated and from (33)  $\Delta P_e$  was computed. The final value of  $P_e$  was checked by the following relation:

$$\frac{1}{2} \{ \Delta P_{e_\tau} + \Delta P_{e_{\tau - \Delta \tau}} \} = P_{e_\tau} - P_{e_{\tau - \Delta \tau}} \quad \dots \quad (34)$$



The model atmosphere in mechanical equilibrium thus constructed is given in Table I.

TABLE I  
 Model atmosphere for  $T_e = 41,700^\circ \text{K}$  and  
 $\log g = 4.2$

$\tau$	$T^\circ \text{K}$	$\theta$	Log $P_g$ bars	Log $(\bar{K} + \sigma)$ $\text{cm}^2/\text{gm}$
0.01	35,131	0.144	2.56	1.82
0.02	35,259	0.143	2.72	1.90
0.03	35,387	0.142	2.81	1.96
0.04	35,513	0.142	2.88	1.99
0.05	35,638	0.141	2.93	0.02
0.06	35,762	0.141	2.98	0.05
0.07	35,884	0.141	3.01	0.07
0.08	36,006	0.140	3.04	0.08
0.09	36,126	0.140	3.07	0.10
0.10	36,245	0.139	3.09	0.11
0.20	37,373	0.135	3.19	0.15
0.30	38,407	0.131	3.31	0.21
0.40	39,364	0.128	3.39	0.23
0.50	40,256	0.125	3.45	0.23
0.60	41,092	0.123	3.50	0.24
0.70	42,073	0.120	3.54	0.24
0.80	42,626	0.118	3.58	0.24
0.90	43,335	0.116	3.62	0.24
1.00	44,010	0.115	3.65	0.24
1.20	45,275	0.111	3.70	0.23
1.40	46,442	0.109	3.76	0.24
1.60	47,527	0.106	3.81	0.25
1.80	48,542	0.104	3.85	0.26
2.00	49,498	0.102	3.88	0.26
2.20	50,401	0.100	3.92	0.27
2.40	51,257	0.0983	3.95	0.25
2.60	52,073	0.0968	3.98	0.24
2.80	52,853	0.0954	4.01	0.23
3.00	53,599	0.0940	4.04	0.22

(ii) For Low Temperature Star.—( $T_e = 10,700^\circ \text{K}$ .)

In the integration of the equation of mechanical equilibrium the exact relation between the gas pressure and the electron pressure given below has been used :

$$\frac{P_g}{P_e} = \frac{N_H(1+x_H) + N_{He}(1+x_{He^+} + 2x_{He^{++}})}{N_H x_H + N_{He}(x_{He^+} + 2x_{He^{++}})} \dots \dots \dots (35)$$

From the point where the model became convectively unstable, onwards the adiabatic temperature gradient was used. After Underhill (1949) this adiabatic gradient was calculated as follows :

$$\Delta_{Ad} = \left( \frac{d \log T}{d \log P_g} \right)_{Ad} = \frac{4(B+1)P_r/P_g + 1 + A}{4(A+4)P_r/P_g + 2.5 + A \left( 2.5 + \frac{\psi i}{kT} \right)} \dots (36)$$

where

$$P_r = \frac{1}{3}aT^4.$$

Here 'a' is the Stefan-Boltzmann constant.

Also

$$B = N_i \left\{ \frac{1 + \bar{x}}{x_i(1-x_i)} + \frac{N_i}{\bar{x}} \right\}^{-1},$$

$$A = B(2.5 + \psi i/kT)$$

and

$$\bar{x} = \Sigma N_i x_i.$$

TABLE II

Model atmosphere for  $T_e = 10,700^\circ K$  and  $\log g = 4.2$

$\tau$	$T^\circ K$	$\theta$	Log $P_g$ bars	Log $P_e$ bars	Log $(\bar{\kappa} + \sigma)$ cm <sup>2</sup> /gm	Log $\Delta_{ad}$	Log $\Delta_{rad}$
0.01	9,033	0.558	1.47	1.10	0.84		
0.02	9,067	0.556	1.69	1.29	0.97	$\bar{1}.10$	$\bar{2}.04$
0.03	9,100	0.554	1.81	1.40	1.05		
0.04	9,132	0.552	1.89	1.47	1.10		
0.05	9,164	0.550	1.95	1.52	1.14		
0.06	9,196	0.548	2.00	1.57	1.17	$\bar{1}.06$	$\bar{2}.52$
0.07	9,228	0.546	2.04	1.61	1.20		
0.08	9,258	0.544	2.08	1.64	1.23		
0.09	9,290	0.543	2.11	1.68	1.25		
0.10	9,320	0.541	2.14	1.70	1.28	$\bar{1}.05$	$\bar{2}.75$
0.20	9,610	0.525	2.23	1.83	1.45		
0.30	9,876	0.510	2.35	1.97	1.57	$\bar{1}.03$	$\bar{1}.18$
0.40	10,122	0.498	2.45	2.08	1.65		
0.50	10,352	0.487	2.55	2.18	1.71		
0.60	10,566	0.477	2.64	2.27	1.76	$\bar{1}.02$	$\bar{1}.58$
0.70	10,769	0.468	2.71	2.35	1.81		
0.80	10,961	0.460	2.79	2.43	1.85		
0.90	11,143	0.452	2.86	2.51	1.89	$\bar{1}.03$	$\bar{1}.84$
1.00	11,317	0.445	2.92	2.57	1.92		
1.20	11,642	0.433	3.03	2.68	1.97	$\bar{1}.04$	0.02
1.40	11,942	0.422	3.13	2.78	2.02		
1.60	12,221	0.412	3.21	2.87	2.06		
1.80	12,482	0.404	3.29	2.95	2.09	$\bar{1}.05$	0.32
2.00	12,728	0.396	3.37	3.03	2.12		
2.20	12,960	0.389	3.43	3.10	2.15		
2.40	13,181	0.382	3.48	3.15	2.17	$\bar{1}.08$	0.51
2.60	13,390	0.376	3.54	3.20	2.20		
2.80	13,591	0.371	3.59	3.25	2.22		
3.00	13,783	0.366	3.64	3.29	2.24	$\bar{1}.10$	0.67
3.50	14,230	0.354	3.74	3.40	2.28		
4.00	14,639	0.344	3.83	3.48	2.32		
4.50	15,016	0.336	3.90	3.56	2.34		
5.00	15,367	0.328	3.97	3.64	2.37	$\bar{1}.20$	1.05
6.00	16,004	0.315	4.07	3.75	2.40		
7.00	16,574	0.304	4.16	3.84	2.43	$\bar{1}.25$	1.19
8.00	17,090	0.295	4.23	3.91	2.44		
9.00	17,562	0.287	4.30	3.95	2.44		
10.00	18,000	0.280	4.35	4.00	2.44	$\bar{1}.31$	1.26
20.00	21,236	0.237	4.63	4.32	2.22	$\bar{1}.39$	0.96
30.00	23,439	0.215	4.79	4.49	2.15		
40.00	25,152	0.200	4.90	4.60	2.12	$\bar{1}.47$	0.75
50.00	26,573	0.190	4.97	4.67	2.09		
60.00	27,798	0.181	5.04	4.74	2.06	$\bar{1}.51$	0.63

$\bar{x}$  denotes the mean degree of ionization,  $N_i$  the abundance of elements by number such that  $\sum N_i = 1$  and  $x_i$  the degree of ionization of elements. The summation is carried over all the elements considered.

It is found that for the present model atmosphere convection starts at about  $\tau \cong 0.2$ . The structure of the atmosphere in the convective zone is obtained by integrating equation (36). A table of adiabatic gradient as a function of temperature and electron pressure in the relevant ranges was first prepared and interpolated values were used in the actual model construction.

The structure of the atmosphere thus obtained is given in Table II.

### COMPARISON OF THE MODEL ATMOSPHERES

For the convenience of comparison, the curves of log opacity per unit electron pressure *vs.* log  $\tau$  and log  $P_e$  *vs.* log  $\tau$  for these models, with the corresponding curves for the models with different means from the literature, are given below. For the sake of completeness similar curves for  $T_e = 20,400^\circ$  K are also reproduced from the study of Saito (1954) and Underhill (1957*a*). These are illustrated in Figs. 1 to 6.

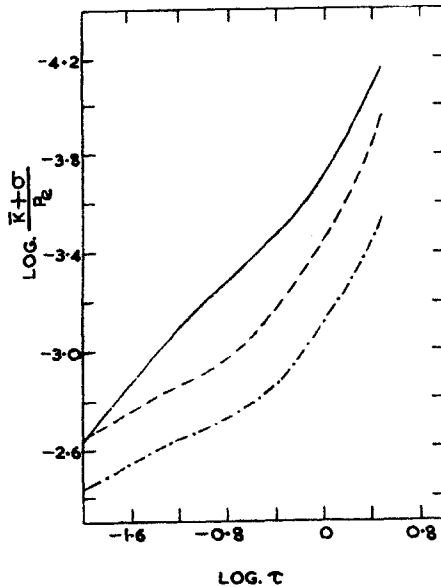


FIG. 1. Curves of  $\log \frac{\bar{\kappa} + \sigma}{P_e}$  *vs.*  $\log \tau$  for  $T_e = 41,700^\circ$  K,  $\log g = 4.2$ . Continuous, broken and dot-dash lines represent Rosseland model, Chandrasekhar model and Planck model respectively.

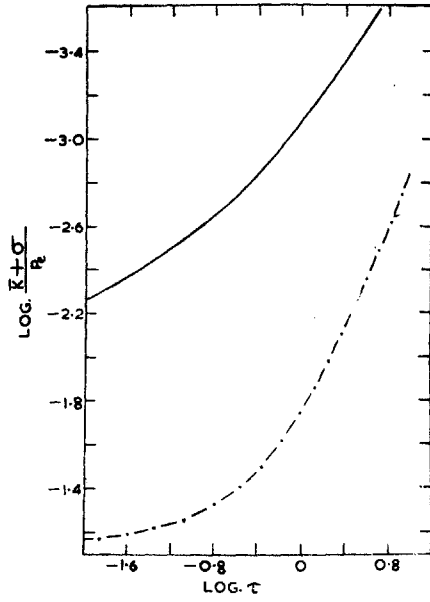


FIG. 2. Curves of  $\log \frac{\bar{K} + \sigma}{P_e}$  vs.  $\log \tau$  for  $T_e = 20,400^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dot-dash lines represent Rosseland model and Planck model respectively.

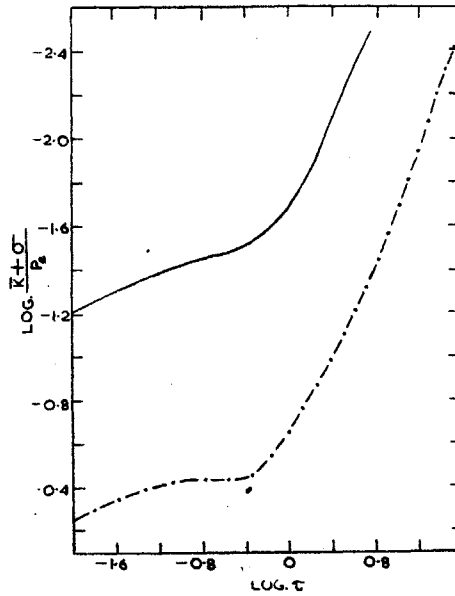


FIG. 3. Curves of  $\log \frac{\bar{K} + \sigma}{P_e}$  vs.  $\log \tau$  for  $T_e = 10,700^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dot-dash lines represent Rosseland model and Planck model respectively.

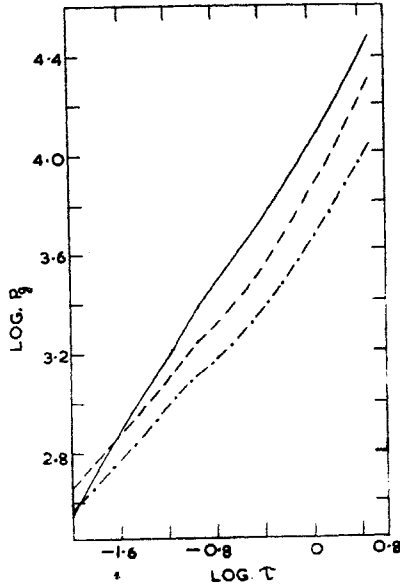


FIG. 4. Curves of  $\log P_g$  vs.  $\log \tau$  for  $T_s = 41,700^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous, broken and dot-dash lines represent Rosseland model, Chandrasekhar model and Planck model respectively.

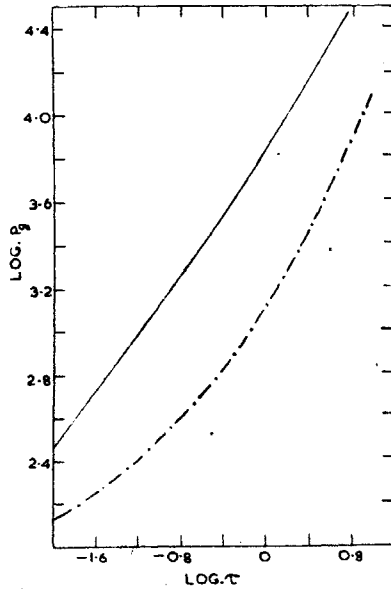


FIG. 5. Curves of  $\log P_g$  vs.  $\log \tau$  for  $T_s = 20,400^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dot-dash lines represent Rosseland model and Planck model respectively.

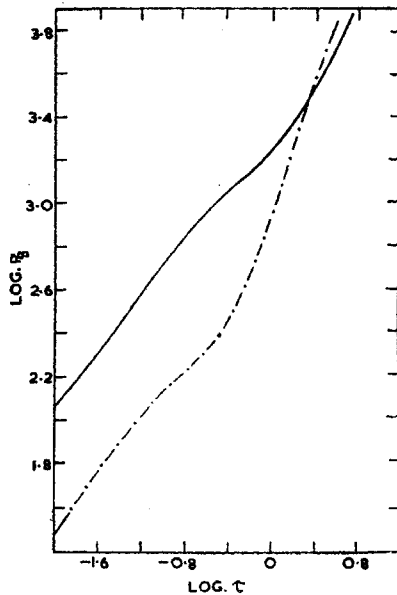


FIG. 6. Curves of  $\log P_g$  vs.  $\log \tau$  for  $T_e = 10,700^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dot-dash lines represent Rosseland model and Planck model respectively.

The trend of the pressure curves and mean absorption per unit electron pressure curves for all the model atmospheres are similar. It is seen that the model constructed with the Planck mean always gives a larger value of the opacity than the models with Rosseland mean at the same optical depth of the atmosphere. The curves also show that the mean absorption per unit electron pressure almost goes parallel ultimately, while the distance between these parallels increases as the temperature goes down, meaning that the corresponding factor becomes larger. It may be remarked at this stage that the Planck model for the middle temperature has different composition (i.e. pure hydrogen) which will have corresponding effect on  $\bar{K}$ . Actually the opacity in the present case is more than what it would be if the composition was  $H : He = 85 : 15$ , since hydrogen is the main contributor to opacity in this temperature range. Table III gives the values of Planck mean in terms of Rosseland mean at few values of the optical depth.

Thus one may conclude that the Planck opacity curve approaches the other two curves as the effective temperature increases.

From the gas pressure curves also it can be seen that there is a tendency of these two curves to approach together as we go towards larger optical depths. For the low temperature it can be easily seen from Table III as it also contains the gas pressure of Rosseland model in terms of Planck model. It can be seen from the table that there is no appreciable effect of the effective temperature variation on this. Only one can probably conclude that

TABLE III  
Comparison of  $\bar{K}$  and  $P_g$  in several model atmospheres

$\tau$	$T_e = 41,700^\circ \text{K}$		$T_e = 10,700^\circ \text{K}$	
	Planck mean in terms of Rosseland mean	Gas pressure of Rosseland model in terms of Planck model	Planck mean in terms of Rosseland mean	Gas pressure of Rosseland model in terms of Planck model
0.02	1.6	1.2	3.8	1.7
0.6	1.6	1.6	4.0	1.7
1.0	1.6	1.6	5.3	1.5
2.0	1.7	1.6	9.3	1.2
3.0	1.6	1.6	13.8	-1.1

for cooler stars the approach of the two pressure curves is faster than for the hotter stars.

The curves of radiative temperature gradient and adiabatic temperature gradient for the stars of  $T_e = 10,700^\circ \text{K}$  are shown in Figs. 7 and 8.

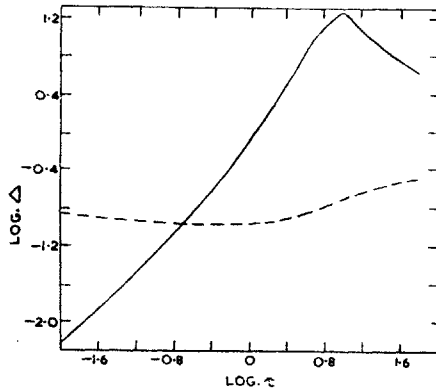


FIG. 7. Curves of log of adiabatic and radiative gradients vs. log  $\tau$  for Planck model  $T_e = 10,700^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dash lines represent radiative and adiabatic gradients respectively.

It can be seen that for Planck model convection sets in at about  $\tau \approx 0.2$  and for Rosseland model at about  $\tau \approx 0.1$ . So the optical depth at which the convection starts is not much different in the two cases. It seems that in the beginning as  $\tau$  increases the gap between the adiabatic temperature gradient and radiative temperature gradient also increases. This difference is larger in the case of Planck mean than the corresponding difference for Rosseland mean. But these  $\tau$ 's still represent only the outer layers of the star. As one goes into the interior layers, i.e. towards larger values of  $\tau$ , the two gradients converge rapidly in both cases, so that in the convective zone

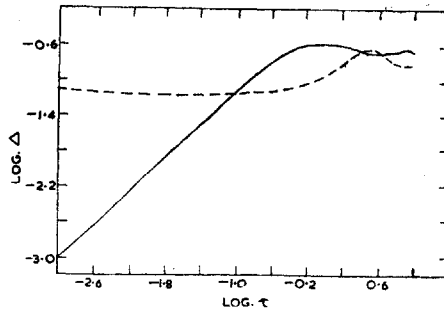


Fig. 8. Curves of log of adiabatic and radiative gradients *vs.* log  $\tau$  for Rosseland model  $T_0 = 10,700^\circ \text{K}$ ,  $\log g = 4.2$ . Continuous and dash lines represent radiative and adiabatic gradients respectively.

the use of adiabatic temperature gradient may affect the results for the very outer layers only but on the whole the physical structure of the whole atmosphere will not be affected very much.

#### CONCLUSION

Thus from the above study it may be concluded that none of the physical parameters show any wild influence of the kind of mean used. In fact they do show a little discordance in these outer layers but mostly due to the large value of the opacity in the Planck mean. The larger the opacity the larger the discordance. The other test for the best choice of a kind of mean is the constancy of the flux with optical depth. This point is not worked out in detail in the present note. From the model atmospheres constructed by a large number of workers so far using one or the other of the above three means, it is known that the net flux remains fairly constant with depth in all the models. Underhill (1957*b*) by comparing the model atmospheres for  $T_0 = 16,800^\circ \text{K}$  and  $\log g = 3.8$ , derived with the three kinds of mean, has concluded that the best constancy of the flux is obtained in the case of Planck mean, again probably due to largeness of the opacity of this mean. A semi-empirical proof for this fact has also been given by Saito (1959). Thus combining all these one can easily conclude that for the study of stellar atmospheres in mechanical equilibrium any kind of mean can be good enough in general. They differ only in details. But for detailed study even mechanical equilibrium breaks down. In these very outer layers even use of adiabatic temperature gradient has also to be modified and actual temperature gradient has to be taken into account if any study in greater details has to be made.

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