ON THE IONIZATION MEASUREMENTS AND THE PROCESS OF TRACK FORMATION IN NUCLEAR EMULSIONS

by A. P. SHARMA *and* P. S. GILL, F.N.I., *Department of Physics, Muslim University, Aligarh*

(*Received March 9, 1961*)

ABSTRACT

A process of track formation by an ionizing particle in nuclear emulsion is considered. Assuming a random distribution of grains in G-5 emulsions, a theoretical relation has been established for calculating the probability of development of a grain which depends on the characteristics of the emulsion and that of the charged particle producing the track. The experimental verification of the results has been done by measuring long gap lengths on identified tracks in G-5 emulsions exposed to cosmic radiation. The results give a convincing support to the mechanism of recombination and track formation and explain completely the experimental curves of grain density *vs.* specific energy loss.

1. INTRODUCTION

The analysis of tracks in nuclear emulsions is based on considerations of energy losses suffered by a charged particle traversing an emulsion in producing ionization by colliding with atomic electrons of the silver halide grains of the emulsion. The production of developable grains in an emulsion depends purely on this process, i.e. the ejection of electrons due to small energy transfers in the grains along the trajectory of the particle. Sufficiently ionized and sensitive grains are rendered developable during the process of development and a series of such developed grains makes the track visible under the high resolving power microscope. Measurement of grain density, i.e. number of developed grains per unit length along the track, is a very simple and standard method for the identification of a particle provided the grains are clearly resolved. With increasing grain density the grains overlap, which causes clogging. This occurs in unresolvable groups (grain density $\geqslant 100$ grains/100 microns) and hence direct grain counting becomes highly subjective and very difficult. The variation of grain density of a track in a particular emulsion, with the specific energy loss of a particle producing it, allows conclusions which are helpful in formulating the mechanism of track formation in emulsion.

An attempt has been made in the present paper to explain the process of ionization and latent image formation in the grains of an emulsion and to evaluate the effective number of electrons for various energy losses on the considerations of the recombination process. In fact, grain density or other track characteristics, which are measured, depend on the probability that a

silver halide crystal (grain) in the path of the particle has been impressioned and thus rendered developable. An exponential relation has been derived describing the dependence of probability of development (π) on the characteristics of the emulsion and the rate of energy loss (dE/dR) of the traversing particle. This probability has so far been assumed by many workers (Della Corte *et al.* 1953; Demers 1947; Fowler and Perkins 1955) to be of an exponential form but no conclusive reasoning has been put forward as the basis for such an assumption. Della Corte *et al.* (1953), Della Corte (1954) and Herz and Davis (1955) have given models of gap formation in nuclear emulsion and the probability of development involved in their relations depends on the measurable and assumed quantities along the track and does not give any relation involving the characteristics of the emulsions. Blatt (1955) has also discussed the models presented by the above-mentioned authors. Beiser (1952) and Blau (1949) explain in a qualitative way the saturation part of the specific energy loss *vs.* grain density curve given by Fowler (1950) and Fowler and Perkins (1951) by assuming space charge surrounding the sensitivity centres of the grain. Here it is shown that it is possible to give a quantitative explanation of the saturation part of the above-mentioned curve on the basis of recombination process alone. The verification has been done for the standard G-5 emulsions. The work has been continued for C-2 emulsions also and will be published separately.

2. Theory

(A) *Recombination process and effective number of electrons*

According to the theory of Gurney and Mott (1938) for the formation of latent image, any ionizing particle while passing through an emulsion grain (AgBr) loses energy and ejects electrons from its normal state in a Br^- ion to an energy level in the conduction band of AgBr according to the following process :

$$Br^- \rightleftharpoons Br + e^- \quad .. \quad .. \quad .. \quad .. \quad (1)$$

The liberated electron migrates through the crystal lattice leaving behind a neutral Br atom (positively charged potential hole) and is captured (trapped) at some sensitivity centre (in the grain) which is supposed to have a lower potential energy level. These sensitivity centres or active centres are generally specks of silver sulphide and silver or some other impurities and dislocations in the grain. The interstitial Ag^+ ions which are present due to Frenkel defects are always comparatively free to move through the crystal lattice and form a sort of ion gas within the crystal. This Ag^+ ion is attracted by the sensitivity centre where the electron has already been trapped in and forms a neutral Ag atom. This neutral atom is known as a latent image and the sensitivity centre becomes a development centre (nucleus) if it is

sufficiently enlarged with silver. The Br neutral liberated in the process is supposed to diffuse out of the grain and the gelatin may act as Br acceptor. Thus the formation of latent image* or development nuclei depends primarily on the number of sensitivity centres present in the AgBr grain and the number of electrons ejected in it by the passage of charged particle.

According to equation (1), the process of electron liberation is a reversible one and, therefore, the recombination of e^- with Br will always be possible at all concentrations, specially because the liberated Br takes a large time ($\sim 10^4$ sec)† to diffuse to the gelatin. This continuous process of recombination will reduce the number of electrons available for the formation of latent images. The rate of such non-useful recombinations increases with the large number of electrons liberated at higher values of specific ionization, dE/dR. The presence of recombination has been suggested by Demers (1947), Webb (1948) and Della Corte *et al.* (1953). Mitchell and Mott (1957) have also indicated that positive holes (Br atoms) surround the sensitivity centres, which trap the liberated electrons. Such a disposition of the positive holes and trapped electrons can very well be expected to result in recombination.

For calculating the number of recombined electrons in a grain at a particular rate of energy loss, dE/dR, we have to take into account the following simplifying assumptions:

(i) That an emulsion grain is spherical with a mean diameter of 0·27 micron in G-5 emulsion.‡

(ii) That the sensitivity centres are almost uniformly distributed in the grain and that their number varies between 1,500 and 20,000 (Mees 1948) for various types of grains. We have assumed the number between 1,500 and 2,000 (for G-5 emulsions).

(iii) That the whole energy loss in a grain is being utilized for the ejection of electrons from Br^- and that average value of the energy needed to liberate one electron from Br^- is 5·8 eV (Yamakawa 1951). Hence the maximum number of electrons produced in a grain will be (dE/dR) eV per micron multiplied by diameter of grain in microns and divided by 5·8.

(iv) That recombination will be a time dependent process.

The time needed for the latent image formation may be taken to be equal to that required by an Ag^+ ion to reach a negatively charged sensitivity centre to form a neutral Ag atom. The maximum time taken in such a process can be calculated by assuming that an Ag^+ ion is interstitial and free to

* Mitchell and Mott (1957) have reviewed the whole theory and have indicated some changes in the accepted theory of latent image formation. These changes, however, are not very significant so far as the present problem is concerned.

† Mitchell (1957) has given the value of diffusion as 10^{-5} cm^2 sec^{-1}. On the basis of Fick's law, we have calculated that the time taken by a layer of Br, liberated along the diameter of a G-5 grain (0·27 micron) to diffuse out of the grain will be $\sim 4{\cdot}7 \times 10^4$ sec.

‡ Private communication from Ilford Selo, London.

move within the crystal. It is subjected to the force of electrostatic attraction, and travels a distance equal to the grain diameter. The electrostatic force between two charges (each electronic), separated by a distance equal to the grain diameter situated in the medium of AgBr, of dielectric constant 12 will be $26{\cdot}34\times10^{-12}$ dyne in a grain of G-5 emulsion. This force will create an acceleration in Ag^+, and assuming that it starts from rest, the time required to cover a distance of 0·27 micron (the diameter of the G-5 grain) will be $1{\cdot}88\times10^{-8}$ sec.

That our assumptions are reasonable and that this magnitude of time is of the right order can be seen by comparison with Mitchell and Mott (1957) and Hamilton, Hamm and Brady (1956), who have indicated that lifetime of the electrons captured at a kink site is of the order of 10^{-8} sec. This may mean that if Ag^+ does not reach the electron within this time, the electron may be lost for latent image formation (the simplest way of being lost will be by recombination).

The number of AgBr molecules present in an average grain can be calculated from its crystal structure. AgBr is a cubic crystal of the NaCl type and has a cell size 5·755 Å. One cell contains four molecules. Hence a G-5 grain (0·27 micron in diameter) contains about $2{\cdot}16\times10^8$ molecules. This is also the total number of Ag^+ ions available in each grain for the latent image formation.

The rate of recombination of e^- with Br at any instant will be proportional to the number of e^- and Br present at that time. Let K_0 be the maximum number of electrons liberated at any instant by the passage of an ionizing particle through a grain, and let K_t be the number of electrons left behind at any time t. The corresponding number of Br atoms present will also obviously be K_0 and K_t respectively. Since the diffusion of Br out of the grain takes a large time ($\sim10^4$ sec), no Br is likely to diffuse in the short time ($\sim10^{-8}$ sec) required for the formation of a latent image. Thus $(K_0-K_t)=R$ is the number of electrons or Br atoms lost by the recombination in time t. The rate of loss of electrons will be proportional to the number of electrons and Br atoms (positively charged potential holes) present at that time, i.e. K_t^2 ; or

$$dR/dt\propto K_t^2. \quad .. \quad .. \quad .. \quad .. \quad (2)$$

We must bear in mind that the electrons could also be lost by the formation of a latent image. The loss of electrons in this process will depend on the number of sensitivity centres S, present. The greater the number S, the smaller will be the chance that electrons are left behind to recombine. Hence it would be reasonably assumed that the rate dR/dt at which electrons are lost by recombination also varies inversely as S. Hence,

$$dR/dt\propto K_t^2.\frac{1}{S}, \quad .. \quad .. \quad .. \quad .. \quad (3)$$

or

$$dR/dt=(A/S).(K_0-R)^2, \quad .. \quad .. \quad .. \quad (4)$$

where A is the constant of proportionality. An expression similar to (4) has been indicated by Gurney and Mott (1938) for the rate of recombination while dealing with the reciprocity law. It should be noted here that the time t is not the total time that has elapsed after the passage of an ionizing particle but the time needed for the latent image formation, i.e. the time needed for Ag^+ to reach e^-. This means that the present calculations will give the recombinations possible within the maximum time required for the latent-image formation. That is why we shall assume that no recombination is possible after the latent image is once formed.

From equation (4) we have

$$dR/(K_0-R)^2 = (A/S)dt. \quad .. \quad .. \quad .. \quad (5)$$

And on integrating (5) we have

$$1/(K_0-R) = (At/S)+P, \quad .. \quad .. \quad .. \quad (6)$$

where P is the constant of integration to be evaluated from the physical conditions of the experiment. When $t = 0$, we get $R = 0$ and hence

$$1/K_0 = P \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (7)$$

and

$$1/(K_0-R) = At/S+1/K_0, \quad .. \quad .. \quad .. \quad (8)$$

or

$$R = K_0/(1+S/AtK_0). \quad .. \quad .. \quad .. \quad .. \quad (9)$$

The above expression gives us the number of electrons which have recombined with Br.

For evaluating A (the constant of proportionality), we take the help of equation (4). Putting $dt = 1$, $S = 1$, and $(K_0-R)^2 = 1$, we get $dR = A$. This enables us to consider A as the number of electrons recombined in unit time in a grain having one sensitivity centre provided $(K_0-R)^2 = 1$. Let us consider the assumptions involved in this definition of A. The time allowed for recombination, $t = 1$ sec is very large as compared with the time required for the latent image formation, which is of the order of 10^{-8} sec, and may be considered to be infinitely large, hence this approaches a state where t tends to infinity (∞) and R tends to K_0 (equation 9). Also $S = 1$ is a very small value of sensitivity centres as compared with even the minimum value of sensitivity centres assumed to be present in a grain ($\sim$1,500). This value of S is such that it may be considered under the category when S tends to zero. Hence from equation (9), R should again tend to K_0. The third condition, $(K_0-R)^2 = 1$, will mean that $K_0-R = \pm 1$, i.e. 1 should be negligible as compared with both R and K_0. This is justifiable since K_0 is a very large number and hence $K_0 = R$, after a reasonable interval of time.

Thus, it can, now, be stated that A is the number of electrons recombined (i.e. R) when $R = K_0$. But K_0 by definition is the maximum number of

electrons liberated which is equal to the total number of AgBr molecules present in a grain. Thus A will be equal to $2{\cdot}16\times10^{8}$ (calculated earlier). This value of A, which is the constant of proportionality, will remain unchanged so long as the size and nature of the grain remain unchanged.

To calculate the maximum possible value of electrons lost by recombination with Br, we have to substitute in equation (9) the maximum value of the time t required for the formation of a latent image ($1{\cdot}88\times10^{-8}$ sec); the minimum value of S and the above value of A. Hence,

$$R_{\max} = \frac{K_0}{1+\frac{S}{4{\cdot}06K_0}} = \frac{K_0}{1+\frac{1500}{4{\cdot}06K_0}} \quad .. \quad .. \quad .. \quad .. \quad (10)$$

and the effective number of electrons (K_t) to be utilized for latent image formation can be calculated from the following relation:

$$K_t = K_0 - R_{\max} = \frac{K_0 S}{4{\cdot}06\,K_0+S} \quad .. \quad .. \quad .. \quad (11)$$

which is a function of specific energy loss, dE/dR.

(B) *Relation between probability of development, π, and the specific energy loss, dE/dR*

For deriving this relation we assume the following model of the grains distribution in the emulsion apart from the assumptions which we have made so far.

(i) That the grains have a random distribution in the gelatin of the emulsion and have varying sensitivities and sizes (Fig. 1).

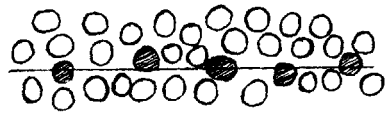

FIG. 1. Random distribution of grains along the trajectory of a charged particle.

(ii) We define the development nuclei as the number of sensitivity centres which will trap one or more electrons and to which interstitial Ag^{+} ions will be attracted to form Ag atom (latent image) and will initiate the reduction of that grain in the development process.

(iii) The average number of development nuclei per grain has been calculated by considering the total number of development nuclei in a certain track length and then dividing it by the number of grains in that length. For

the present calculations the passage of the charged particle has been assumed to be in a straight line.

(iv) The ionization in the gelatin has been neglected in comparison with the ionization in AgBr grains, because the emulsions are densely packed with AgBr molecules.

(v) Since the grains have a random distribution in the emulsion, the ionizing particle will have varying lengths of the passage through different grains. We have considered the average value of the passage through a grain to be 2/3 times the diameter of the grain (Brown 1953; Della Corte 1954).

(vi) In some cases the ejected electrons may be sufficiently energetic to eject another or more electrons from other Br^-. All such electrons are termed as secondary ionization electrons. The value on the basis of such secondary ionization is nearly twice the initial ionization (Price 1955). Hence the total number of electrons available for the latent image formation is considered to be twice the number given in the assumption (iii) of (2A).

(vii) The fading of the latent image has been neglected in the process.

The average number of development nuclei in a grain can be calculated in the light of the theory of de Langhe (1936, 1948) who had put forward a method of calculating the probability of development in thin emulsion plates in case of light exposures, when certain quanta of light are falling on them.

In order to calculate the probability of development, consider the passage of an ionizing particle in an unprocessed emulsion of length l having n grains. According to our assumptions these n grains will have $n \times S (= N$, say) sensitivity centres in all. The value of π is to be calculated considering the variations in the sensitivity of different grains, which may be shown, firstly, by assuming the number of sensitivity centres to be varying in different grains and, secondly, by assuming the sensitivity centres to have different nature. Since we have assumed the average number of sensitivity centres per grain, the only mode of representing the variations in the sensitivity is through their nature, which means that different sensitivity centres will be capable of absorbing different number of electrons (and hence Ag^+ ions) to become developable. Since the variations in the size of the grain can also be represented in terms of the variations in the total number of sensitivity centres of each, the above formulation will represent both the variations. Hence let N, the total number of sensitivity centres of different types, be distributed as

$$N = N_0 + N_1 + N_2 + N_3 + \ldots \quad \ldots + N_i + \ldots \quad \ldots + N_\infty, \qquad \ldots \quad (12)$$

where N_0 is the number of sensitivity centres requiring zero electron, N_1 requiring one electron, N_2 requiring two electrons, and so on, to become developable. Thus, N_0 helps us in calculating the number of grains which will become developable even without the effect of ionization created by the

particle, which may be called the fog grains, if it is due to the basic nature of the emulsion. N_∞ will mean the sensitivity centres which will require infinite number of electrons to become developable, which means that these sensitivity centres will never become developable whatever be the number of electrons given to them. These sensitivity centres will be distributed in all the grains in a perfectly random fashion. Theoretically speaking, such a distribution is the most ideal one, but we assume that in practice the distribution will end at a finite number (say B). Then,

$$N = N_0+N_1+N_2+N_3+\ldots \ldots+N_i+\ldots \ldots+N_B \quad \ldots (13)$$

The value of B has been calculated by us and we derive it to be about 431 for $S = 1{,}750$. The value will change for the number of sensitivity centres per grain, e.g. $B = 370$ for $S = 1{,}500$, and $B = 493$ for $S = 2{,}000$. The details of these calculations are given in Appendix 1.

The number of effective electrons left after recombination with Br will also be distributed in all these sensitivity centres. Let us consider that the total number of effective electrons available for the latent image formation in the length l of emulsion are K, which are distributed over all the sensitivity centres of different types present in all the grains of this length of emulsion. Let the distribution be as follows :

$$K = K_1+K_2+K_3+\ldots \ldots+K_i+\ldots \ldots+K_B \quad \ldots (14)$$

where K_1 is the number of electrons distributed over N_1 sensitivity centres, K_2 over N_2 and K_i over N_i sensitivity centres requiring i electrons to become developable. This distribution will again stop at the same finite number where the number of sensitivity centres stops, i.e. K_B. These electrons produced by actual ionization will have nothing to do with N_0 sensitivity centres which are requiring zero number of electron to become developable, and if they become developable it is by their own virtue.

Thus, K_i electrons are distributed over N_i sensitivity centres requiring i electrons to become developable. The probability (p_x) that one of the N_i sensitivity centres gets x of these K_i electrons will be given by

$$p_x = \frac{{}^{K_i}C_x (N_i-1)^{K_i-x}}{N_i^{K_i}}, \quad \ldots (15)$$

where ${}^{K_i}C_x$ stands for combination, because this distribution will be perfectly random. As mentioned earlier, only those sensitivity centres of this group will become impressioned so as to be rendered developable which acquire i or more electrons. Hence the total probability of their becoming developable will be given by the term $\sum_{x=i}^{K_i} p_x$. Let us consider that N_i' is the number of sensitivity centres out of N_i which become actually developable, then the probability of their becoming developable can also be written as $\frac{N_i'}{N_i}$.

Hence, $$\frac{N_i'}{N_i} = \sum_{x=i}^{K_i} p_x. \quad .. \quad .. \quad .. \quad .. \quad (16)$$

In order to calculate the total number of sensitivity centres becoming developable (henceforth called development nuclei) in all the grains present in length l, we have to consider the total variation in them, i.e. between 0 and B.

Therefore,

$$N' = \sum_{i=0}^{B} N_i' = \sum_{i=0}^{B} N_i \sum_{x=i}^{K_i} p_x \quad .. \quad .. \quad .. \quad (17)$$

where N' is the total number of development nuclei in the length l of the emulsion. The average number of development nuclei per grain (N'', say) will be found from the following relation:

$$N'' = \frac{N'}{n} = \frac{1}{n} \sum_{i=0}^{B} N_i \sum_{x=i}^{K_i} {}^{K_i}C_x \frac{(N_i-1)^{K_i-x}}{N_i^{K_i}} \quad .. \quad .. \quad (18)$$

$$= \frac{1}{n} \sum_{i=0}^{B} N_i \left(\frac{N_i-1}{N_i}\right)^{K_i} \sum_{x=i}^{K_i} {}^{K_i}C_x (N_i-1)^{-x}. \quad .. \quad .. \quad (19)$$

The value of N'' can be calculated theoretically from the above relation provided we find a solution of this equation in terms of the known factors. Since it is rather impossible to find such a solution, we make certain assumptions to bring this equation to a useful form. One of the most fundamental assumptions which has simplified it is of uniform distribution, i.e. we assume that the sensitivity centres of different types are uniformly distributed in all the grains and all the effective electrons available are uniformly distributed over these sensitivity centres. The details of these calculations and assumptions have been included in Appendix 2. Finally, we arrive at the following form where N'' has been expressed in terms of the known factors:

$$N'' = \frac{1}{n}\left[N_0 + \frac{K}{B} e^{-K/N} \left(1-\frac{B}{N}\right)^{-1} \left\{1 + \left(\frac{K}{N}-\frac{B}{N}\right)\left(1-\frac{B}{N}\right)^{-1} + \frac{1}{2}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\right.\right.$$
$$\times \left(1-\frac{B}{N}\right)^{-2} + \frac{1}{6}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\left(\frac{K}{N}-\frac{3B}{N}\right)\left(1-\frac{B}{N}\right)^{-3}$$
$$+ \frac{1}{24}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\left(\frac{K}{N}-\frac{3B}{N}\right)\left(\frac{K}{N}-\frac{4B}{N}\right)$$
$$\left.\left. \times \left(1-\frac{B}{N}\right)^{-4} + \text{negligible terms} \right\}\right]. \quad .. \quad (20)$$

Equation (20) will show that if $K = 0$, i.e. when no ionization has been created in the length l of the emulsion, the average number of the development nuclei per grain (N'') will be equal to N_0/n which means that only fog grains will become developable (which should have been the case). If $K = \infty$, the expression is undeterminant. But we should remember that K is the effective

number of electrons available to the process of latent image formation and in the light of the table presented in Appendix 1, it can be noted that this number will never be infinite and rather becomes constant at higher ionizations and remains so till the end, when we consider the particle to be moving with a very low velocity.

N' represents the total number of development nuclei formed in a length l of the emulsion, containing n grains. These development nuclei will be distributed over all these grains in a random fashion. Hence the probability that one of these n grains acquires z of these N' development nuclei will be given by

$$p_z = {}^{N'}C_z \frac{(n-1)^{N'-z}}{n^{N'}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

If a grain acquires a number z of the development nuclei, we do not know whether it will be sufficiently impressioned so as to make the whole of it developable or not. Therefore, certain conditions are to be laid for fixing the minimum number of development nuclei to be obtained by a grain so that it becomes developable. The necessary condition of developability of a grain is that any grain which acquires at least one development nucleus will become developable. This condition helps us to calculate π, the probability that a grain will be impressioned as to be rendered developable.

Thus, $$\pi = 1-p_0, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

where p_0 represents the probability (from equation 21) that one of the n grains gets no development nuclei (i.e. zero development nuclei), which according to the above condition will be the probability of non-development.
From equation (21),

$$p_0 = {}^{N'}C_0 \frac{(n-1)^{N'}}{n^{N'}} = \left(\frac{n-1}{n}\right)^{n.N'} \simeq e^{-N'} \text{ (for large } n)$$

and, therefore, $$\pi = 1-e^{-N'}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

On combining equations (23) and (20), we get

$$\pi = 1-e^{-\frac{\alpha}{n\beta}e^{-\alpha(1-\beta)^{-1}[1+(\alpha-\beta)(1-\beta)^{-1}+\text{negligible terms}]}}, \quad \ldots \quad \ldots \quad (24)$$

where $\alpha = K/N$, K being the total number of effective electrons and N the total number of sensitivity centres in the grains of path length l.

$\beta = B/N$, B being the limiting number of effective electrons at maximum possible ionization.

n = total number of grains in length l of unprocessed emulsion.

3. Experiment

To find the values of probability of development π, gaplength measurements were made on some proton tracks in 600 μ thick G-5 emulsion plates

(Sharma and Gill 1960) exposed at Gulmarg. The identity of the proton tracks was made practically certain by scattering measurements using constant sagitta method with the scattering scheme of Fay, Gottstein and Hain (1954). As these tracks could not give the minimum ionization region (Sharma and Gill 1960), measurements were carried out on a G-5 stack (600 μ thick pellicles) on well identified, long and flat tracks from π—μ decays. All these selected tracks had an angle of dip $\leqslant 10^\circ$ and were ending in the emulsion.

The values of dE/dR *vs.* R for G-5 emulsions were derived from Barkas range-energy tables, UCRL-3769, and were found with respect to various ranges of a particular track. Each track was subdivided into various segments of ranges such that the increase of energy loss was 1 meV/gm cm^{-2} in each one in the low ionization region and 2 meV/gm cm^{-2} in the high ionization region, i.e. towards the end of the track. For each segment, the lengths of gaps greater than a certain cutoff value were measured and their number were counted.

Some of the measurements were done on a microscope with an objective 95$\times$ and eyepiece 15$\times$. The micrometer scale had one small division equal to 0·4166 μ. The measurements on π—μ tracks were done on a microscope having a filar micrometer eyepiece attached with a drum or rotating head. Each division of the drum recorded 0·0415 micron. This allowed an easy alignment of the gap edges with the hair of the drum micrometer.

The values of probability of development for various segments were calculated by means of the following relation due to Della Corte (1954) :

$$\pi = \frac{d_0}{(L_g/N_g)_{\geqslant \xi} - \xi + d_0}, \qquad \ldots \quad (25)$$

where $d_0 = 0{\cdot}2\ \mu$, ξ = cutoff value and $(L_g/N_g) \geqslant \xi$ means the mean gaplength with respect to a cutoff value ξ.

From the above relation (25), the experimental values of π have been calculated for various flat sections of each track corresponding to various ranges of the specific ionization dE/dR and are shown in column (3) of Table 2.

4. Calculations and Results

In Table 1, the values of K_0 (column 2) for various values of dE/dR (column 1) have been calculated on the basis that the average value of energy needed to liberate one electron from Br^- is 5·8 eV (Yamakawa 1951) as shown in assumption (iii) of (2A). The values of dE/dR in meV/g cm^{-2} have been converted to keV/micron by considering the density of emulsion to be 3·8 g/c.c. The corresponding values of K_0 as calculated by Della Corte *et al.* (1953) have also been shown in the same column (2). Slight difference in our values and those of Della Corte *et al.* are probably due to the difference

TABLE 1

Effective number of electrons K_t, and efficiency of the process η

(1) dE/dR		(2) K_0		(3) R		(4) K_t		(5) η calculated		(6)
				For		For		For		η experimental values of Della Corte *et al.* *
meV/g cm^{-2}	keV/μ	Our values	Values of Della Corte *et al.* *	$S_1 = 1{,}500$	$S_2 = 2{,}000$	$S_1 = 1{,}500$	$S_2 = 2{,}000$	$S_1 = 1{,}500$	$S_2 = 2{,}000$	
2·35	0·89	41·4	40·1	4·2	3·2	37·2	38·2	0·90	0·92	0·93
4·50	1·71	79·5	76·8	14·1	11·0	65·4	68·5	0·82	0·86	0·87
6·50	2·47	114·9	111·0	27·3	21·8	87·6	93·1	0·76	0·81	0·83
8·42	3·19	148·3	143·7	42·5	34·3	105·8	114·0	0·71	0·77	0·80
10·40	3·95	183·7	177·7	61·0	49·9	122·7	133·8	0·67	0·73	0·76
12·25	4·65	216·2	209·1	79·8	65·9	136·4	150·3	0·63	0·70	0·73
14·50	5·51	256·2	247·5	105·0	87·7	151·2	168·5	0·59	0·66	0·70
17·65	6·71	312·0	301·3	142·5	120·9	169·5	191·1	0·54	0·61	0·67
21·40	8·13	378·0	356·3	190·9	164·3	187·1	213·7	0·50	0·57	0·63
25·47	9·68	450·1	436·5	245·9	214·3	204·2	235·8	0·45	0·52	0·59

* Della Corte *et al.* (1953).

in the conversion factors used. The values of K_t (column 4) have been calculated from the relation (11). Columns (5) and (6) give the values of η (efficiency of the process of electron utilization) which is the ratio of K_t and K_0 values calculated by us and experimentally determined by Della Corte *et al.* (1953). The values of η have been calculated for two different values of sensitivity centres $S_1 = 1{,}500$ and $S_2 = 2{,}000$.

Table 2 gives the theoretically calculated values of probability of development π according to the relation (24) for various ranges of specific ionization dE/dR. The experimental values of π determined by us and also due to Della Corte *et al.* (1953) are given in column (3). In calculating N'', the exponent of the relation (23) from relation (20), we have not taken into account the factor N_0, the reasons for which have been given in Appendix 2. We have started from the value of $dE/dR = 2{\cdot}00$ meV/g cm^{-2} ($\sim 0{\cdot}76$ keV/μ) which is in the relativistic region and have gradually gone to a value of 27·00 meV/g cm^{-2} ($\sim 10{\cdot}26$ keV/μ) at which a singly-charged particle will have relative velocity (v/c) equal to 0·16, i.e. the particle is sufficiently slow. The values of K (the number of effective electrons) have been obtained on the basis of the primary and secondary ionizations [assumption (vi) of 2(B)]. The series of equation (20) for the value of N'' is a rapidly converging one and hence the contributions of factors having square terms are exceedingly small, and those of factors having powers -3 or -4 are almost negligible. In order to simplify the process of calculations, we have neglected these higher power terms. But we have thoroughly checked that this does not affect the accuracy of our results. The values of π have been calculated for three different values of sensitivity centres of a grain, viz. $S_1 = 1{,}500$, $S_2 = 2{,}000$ and $S_3 = 1{,}750$ (average) and the limiting values of π between sensitivity centres 1,500 and 2,000 are shown in the same column (2).

5. Discussion of Results and Conclusion

Table 1 gives complete information regarding the recombination process taking place in the grains. The values of R given in the table are the maximum values of recombined electrons. The actual values will always be less than the given values because the time required for completing the process of latent image formation will always be less than the maximum value considered by us. Moreover, the value of S (number of sensitivity centres) will be more than the minimum number considered. Thus, the values of η calculated by us will be less than the actual values (because our calculations give minimum value of K_t). This fact is clear from the values of η for $S = 1{,}500$ and the corresponding experimental values of Della Corte *et al.* (1953). Some part of the difference in the two values can be attributed to the difference in the factor (k/a) used by Della Corte *et al.* (1953) and corresponding factor

TABLE 2

Theoretical and experimental values of probability of development π

(1) (dE/dR) interval		(2) Theoretical values of π				(3) Experimental values of π	
meV/g cm^{-2}	keV/μ	For $S_1 = 1{,}500$	For $S_3 = 1{,}750$	For $S_2 = 2{,}000$	For sensitivity centres between 1,500 and 2,000	Our values	Values of Della Corte *et al.* *
2– 3	0·76– 1·14	0·1063–0·1477	0·0934–0·1309	0·0832–0·1174	0·0832–0·1477	0·085	0·087
3– 4	1·14– 1·52	0·1477–0·1832	0·1309–0·1640	0·1174–0·1479	0·1174–0·1832	0·133	0·133
4– 5	1·52– 1·90	0·1832–0·2141	0·1640–0·1926	0·1479–0·1749	0·1479–0·2141	0·147	0·148
5– 6	1·90– 2·28	0·2141–0·2408	0·1926–0·2182	0·1749–0·1990	0·1749–0·2408	0·190	0·189
6– 7	2·28– 2·66	0·2408–0·2643	0·2182–0·2413	0·1990–0·2210	0·1990–0·2643	0·221	0·220
7– 8	2·66– 3·04	0·2643–0·2855	0·2413–0·2615	0·2210–0·2412	0·2210–0·2855	0·250	0·253
8– 9	3·04– 3·42	0·2855–0·3044	0·2615–0·2801	0·2412–0·2594	0·2412–0·3044	0·268	0·264
9–10	3·42– 3·80	0·3044–0·3214	0·2801–0·2970	0·2594–0·2755	0·2595–0·3214	0·301	0·302
10–11	3·80– 4·18	0·3214–0·3367	0·2970–0·3123	0·2755–0·2907	0·2755–0·3367	0·305	0·308
11–12	4·18– 4·56	0·3367–0·3505	0·3123–0·3265	0·2907–0·3046	0·2907–0·3505	0·318	0·320
12–13	4·56– 4·94	0·3505–0·3631	0·3265–0·3392	0·3046–0·3176	0·3046–0·3631	0·340	0·346
13–14	4·94– 5·32	0·3631–0·3748	0·3392–0·3511	0·3176–0·3296	0·3176–0·3748	0·359	0·367
14–15	5·32– 5·70	0·3748–0·3857	0·3511–0·3619	0·3296–0·3403	0·3296–0·3857	0·388	0·401
15–17	5·70– 6·46	0·3857–0·4042	0·3619–0·3816	0·3403–0·3605	0·3403–0·4042	0·399	0·406
17–19	6·46– 7·22	0·4042–0·4208	0·3816–0·3987	0·3605–0·3782	0·3605–0·4208	0·420	0·423
19–21	7·22– 7·98	0·4208–0·4347	0·3987–0·4122	0·3782–0·3934	0·3782–0·4347	0·425	0·430
21–23	7·98– 8·74	0·4347–0·4474	0·4122–0·4266	0·3934–0·4072	0·3934–0·4474	0·439	0·449
23–25	8·74– 9·50	0·4474–0·4576	0·4266–0·4382	0·4072–0·4190	0·4072–0·4576	0·459	0·479
25–27	9·50–10·26	0·4576–0·4679	0·4382–0·4486	0·4190–0·4300	0·4190–0·4679	0·468	0·481

* Della Corte *et al.* (1953).

in our calculations. The value of Della Corte is arrived at to explain the experimental data, while our value, which is not different from this value, has been reached by theoretical considerations. As explained above, if we increase the value of S from the minimum, the number of recombined electrons will decrease and we should approach the actual state of affairs inside the grain. It is clear from Table I that the values of Della Corte *et al.* come in better agreement with our values if the value of S is taken as 2,000. We do not have any experimental evidence for the number of sensitivity centres present in the grain but assuming the values of Della Corte *et al.* to be reasonably correct, our theory could be used to predict the number of sensitivity centres per grain of a G-5 emulsion as about 2,000.

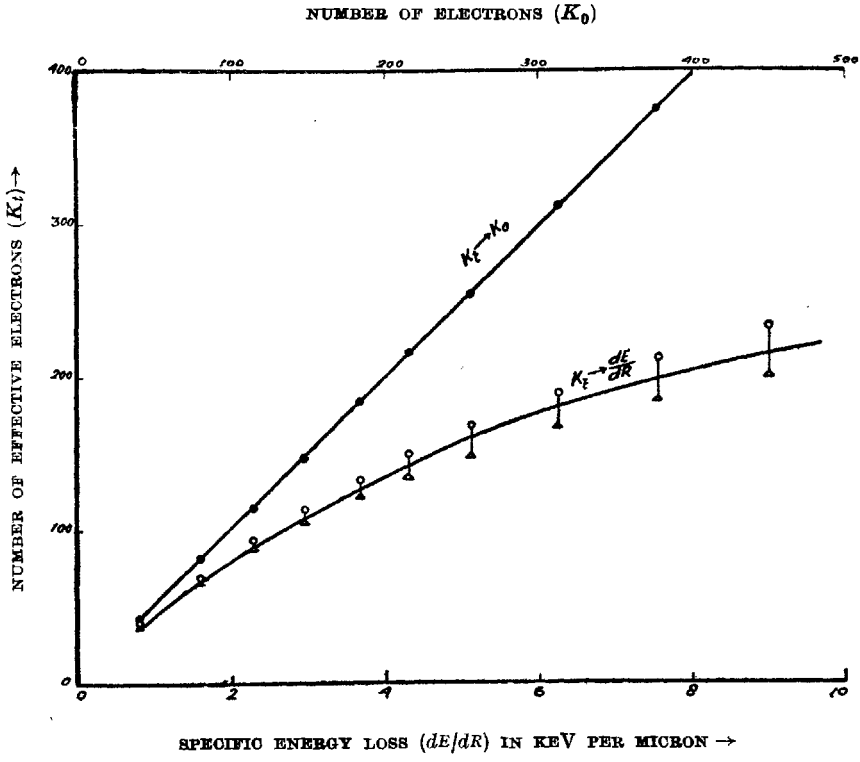

FIG. 2. Variation of the number of electrons (K_t), effective in the formation of latent image with (i) the specific energy loss (dE/dR) in keV/micron, and (ii) the maximum number of electrons (K_0), produced in a G-5 emulsion grain. Values of K_t have been plotted for S = 1,500 (△ points) and S = 2,000 (0 points). ● points indicate the absence of recombination process.

The curve in Fig. 2 indicates that the values of K_t, the number of effective electrons, reach a saturation point as the number of electrons (K_0) increases. Thus, the number of electrons available for the latent image

formation becomes almost constant and independent of ionization (dE/dR). This value will be about 200 electrons per grain.

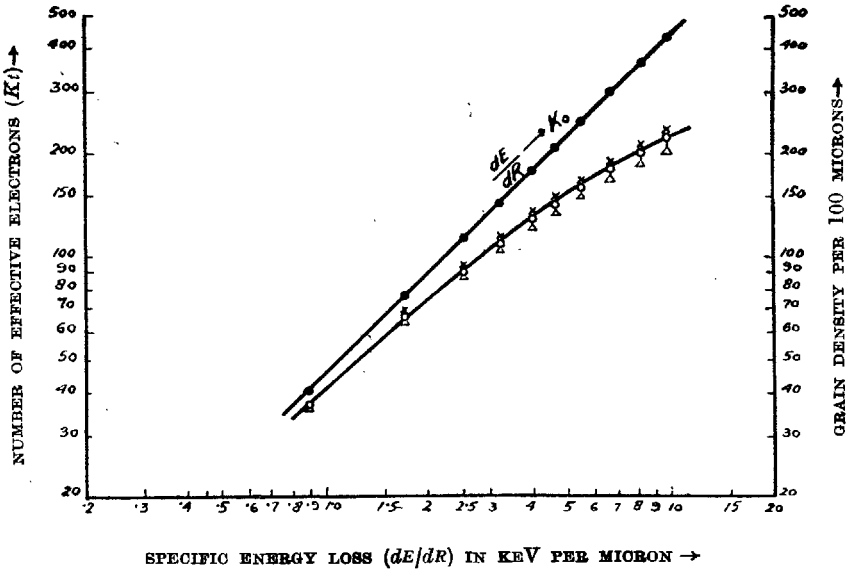

FIG. 3. Variation of specific energy loss (dE/dR) in keV/micron, with the number of effective electrons (K_t) plotted for $S = 1{,}500$ (Δ points) and $S = 2{,}000$ ($\times$ points) on a double log scale. 0 points on the curve represent the variation of grain density with specific energy loss as shown by Fowler and Perkins (1951). The upper curve indicates the variation of dE/dR with maximum number of electrons (K_0) on the same scale.

Our results also yield a modification in the explanation of the specific energy loss *vs.* grain density curve due to Fowler (1950) and Fowler and Perkins (1951). In their curve (Fig. 3), it has been shown that the relation between the two does not remain linear after the value of the specific energy loss becomes about 2·6–3·0 keV per micron. Beiser (1952) and Blau (1949) have introduced the idea of a space charge around the sensitivity centres to explain their results. According to them, the deviations from linearity occur due to the inefficiency in the process of electron utilization in the latent image formation. Our results indicate that the results of Fowler and Perkins can be fully explained by the recombination of the electrons with Br and that there is no need for the assumption of the formation of space charge. The results of Fig. 2 indicate that for lower values of K_0 the number of recombined electrons is very small, and values of K_t lie close to the values of K_0. An appreciable deviation in the values of K from K_0 occurs only after a value corresponding to value of dE/dR of the order of 3 keV per micron. It means that up to 3 keV per micron the relation between dE/dR and the

number of effective electrons (K_t) can be assumed to be the same as the relation between dE/dR and the maximum number of electrons (K_0). The relation will not remain the same after this value of dE/dR, because the number of recombined electrons increases.

In Fig. 3, we have plotted the values of dE/dR in keV per micron and the number of effective electrons on a double log scale. The curve of Fowler and Perkins (between dE/dR and grain density) has also been shown on the same scale in the same figure. The results indicate the following :

(i) The nature of the variations in the two curves is almost similar.

(ii) The relation remains linear up to a value of about 3 keV per micron in both the curves and after that both indicate saturation of similar nature.

The process of the development of a grain is dependent entirely on the size of the latent image formed within the grain and hence will be dependent on the number of electrons available for its formation. Thus grain density should be directly connected with the number of effective electrons K_t. Since log K_t is linear with log (dE/dR) up to a value of 3 keV per micron energy loss, the relation of log of grain density and log of dE/dR should also be linear up to this point. This is exhibited by the curve of Fowler and Perkins. Fig. 3 also shows that the variations beyond this point are also similar in the two cases. Thus, the entire curve of Fowler and Perkins can be explained in terms of the variations in the number of effective electrons K_t, with specific energy loss. The deviation from the linearity can also be explained in terms of an increase in the number of electrons recombining with Br at higher values of ionization. Thus, there does not seem to be any necessity of considering the formation of a space charge (Beiser 1952; Blau 1949).

Relation (24) presents a method of calculating the probability of development π, provided we know the specific ionization created by the particle and the basic characteristics of the emulsion. Table 2 shows that the values of π (column 2) calculated theoretically are in perfect agreement with the experimental values of Della Corte *et al.* and also our experimental values (column 3). Only a slight discrepancy arises at higher values of specific ionization. However, the disagreement is so nominal that it can easily be considered to be within the experimental error (Della Corte 1958). The slight variation may be due to the difficulty in measurement of such a high specific ionization and also the effect of fog grains which we have not considered in the calculations. The difference in the two experimental values (column 3) may be due to the fact that for finding dE/dR values Della Corte *et al.* have used the range *vs.* energy-loss curves given by Bradt *et al.* (1950) while we have used the range energy table due to Barkas.

Dodd and Waller (1951) have shown that for G-5 emulsion the probability that a grain traversed by a particle will be made developable for relativistic tracks (i.e. minimum ionization), as calculated from the relation of Demers for normal development, will be 0·095, which is in perfect agreement with our value of π for minimum ionization region ($\sim 0{\cdot}7$ or $0{\cdot}8$ keV/μ).

Thus the similar theoretical and experimental values of π give a convincing support to the work presented in this paper, i.e. the processes of recombination; distribution of effective number of electrons in a random fashion in different sensitivity centres of the grains; formation of development nuclei in a grain and, finally, the process of track formation in nuclear emulsions.

The probability of development π is a much more reliable parameter than grain density. Unlike grain density, π is independent of degree of development up to values of dE/dR as high as nearly 15 times the minimum value (Della Corte *et al.* 1953). Hence, instead of scattering *vs.* grain density measurements for finding the mass of a particle, scattering *vs.* probability of development measurements can be done. The latter method is much more reliable especially for high grain density regions.

Table 2 shows that the theoretical values of π decrease by considering an increased number of sensitivity centres (and hence large grains) and it explains why the exposures for weakly ionizing particles should be made in emulsions whose grain size is large while the exposures for highly-ionizing particles, e.g. alpha particles, fission fragments, etc., should be made in emulsions having fine grains.

ACKNOWLEDGEMENT

One of the authors (A. P. S.) is thankful to Prof. M. G. K. Menon and Dr. R. R. Daniel, of the Tata Institute of Fundamental Research, Bombay, for permitting him to carry out measurements on their emulsion stack.

6. APPENDIX 1

Evaluation of 'B' (equation 13)

We have shown that the number of effective electrons left after recombination with Br in a grain of AgBr can be represented by the expression $K_t = SK_0/(4{\cdot}06\ K_0+S)$. The validity of this expression has already been discussed.

As the velocity of the particle decreases, i.e. specific energy loss increases, the number of electrons produced in a grain also increases with the result that the recombination process will be more predominant. In the following Table 3 we have given the values of effective number of electrons per grain of G-5 emulsions for different values of relative velocities ($\beta_1 = v/c$), v being

TABLE 3

Values of effective number of electrons per grain

Charge (z)	β_1 ($= v/c$)	Energy loss (dE/dR) keV/μ	Number of electrons produced per grain	Number of effective electrons			
				For $S = 1{,}750$		For $S = 1{,}500$	For $S = 2{,}000$
				Considering primary ionization alone	Considering secondary ionization also		
1	0·1489	9·68	450·10	220·2	291·5	202·9	235·2
	0·1200	13·26	616·59	253·7	319·4	231·0	273·8
	0·1000	17·31	804·91	280·7	340·0	253·2	305·6
	0·0800	23·97	1114·61	310·8	361·2	277·5	341·6
	0·0600	36·48	1696·32	343·7	382·4	303·4	381·8
	0·0400	65·95	3066·21	377·9	402·7	329·7	424·4
	0·0200	181·40	8435·10	410·1	420·2	354·0	465·4
	0·0100	499·10	23208·15	423·2	427·0	363·6	482·4
	0·0010	14400·00	669600·00	430·7	430·8	369·3	492·2
	0·0001	415200·00	19306800·00	431·0	431·0	369·5	492·6
	0·00008	575000·00	26737500·00	431·1	431·0	369·5	492·6
	0·00001	$11{\cdot}98\times10^6$	$55{\cdot}707\times10^7$	431·1	431·0	369·5	492·6
2	0·10	69·21	3218·26	380·1	403·9	331·4	427·3
	0·01	1996·00	92814·00	429·0	430·1	368·0	489·9
	0·001	$57{\cdot}57\times10^3$	$26{\cdot}77\times10^5$	430·9	430·9	369·4	492·5
	0·00001	$47{\cdot}88\times10^6$	$222{\cdot}64\times10^7$	431·0	431·0	369·5	492·6

the velocity of the particle and c, the velocity of light. The dE/dR values in keV/μ have been calculated by the relation of Fowler and Perkins (1951), viz. $dE/dR = 0{\cdot}6\ Z^2/\beta_1^{1{\cdot}46}$.

Table 3 indicates that the number of the effective electrons reaches a constant value at very low β_1, and remains almost constant for all further values of β_1. This number is about 431, if we consider the number of sensitivity centres to be 1,750. However, if the value of the sensitivity centres is taken to be 2,000 or 1,500 (limits on two sides), the value of effective number of electrons changes to 493 and 370 respectively. Moreover, this constant value is independent of the charge and the secondary ionization. The value of β_1 of the particle has been taken as low as 0·00001 when it creates a very heavy ionization, but the number of effective electrons remains constant. However, there does not seem to be any necessity of considering the values of β_1 beyond 0·001.

Further, we note that at such a high-energy loss the grain will attain a sufficiently strong latent image and is likely to be reduced definitely in the development process. In the light of the above statement, nearly 431 number of effective electrons will definitely make a grain developable (for a grain with $S = 1{,}500$).

Considering the distribution of these maximum number of effective electrons in the entire grain, it can be noted from what we have discussed that different number of these electrons will be captured by different types of sensitivity centres having different capacities (or sensitivities). In this process of distribution some of these sensitivity centres may acquire sufficient number of electrons so as to be rendered developable, while others may not have enough electrons. We have already considered that a grain will become developable even if it has acquired one development nucleus.

In order to have an estimate of the maximum capacity of sensitivity centres to acquire electrons to become developable (i.e. value of B), let us consider an ideal distribution in the light of the above discussion. Let us assume a case when all the electrons are going to one sensitivity centre. If the maximum number of electrons (431) are available and are going only to one sensitivity centre, then under the condition that this grain should become developable, we can have the following arguments. If the number of sensitivity centres present in the grain require more than 431 electrons, and if all the electrons happen to go to such a sensitivity centre, it will not become developable even with this number of maximum available electrons. This will be against the condition that the grain must become developable at such an extremely low velocity. The case of all the electrons going to one sensitivity centre alone is a very rare process, but it gives a probability that the grain may not become developable specially if we assume the grain to have sensitivity centres requiring more than 431 electrons. In order to avoid all

such probabilities, however small they are, we can reasonably assume that the sensitivity centres present in a grain do not extend up to infinity, as given in equation (12) but will have an ending limit at a finite number N_B, i.e. sensitivity centres requiring B electrons to become developable and B will have a value of 431 for 1,750 sensitivity centres (Table 3).

7. Appendix 2

Evaluation of N″ (equation 19)

$$N'' = \frac{1}{n}\sum_{i=0}^{B} N_i \left(\frac{N_i-1}{N_i}\right)^{K_i} \sum_{x=i}^{K_i} {}^{K_i}C_x (N_i-1)^{-x} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1')$$

$$= \frac{1}{n}\sum_{i=0}^{B} N_i \left(\frac{N_i-1}{N_i}\right)^{K_i} \left[\frac{\lfloor K_i}{\lfloor i \lfloor K_i-i}(N_i-1)^{-i} + \frac{\lfloor K_i}{\lfloor i+1 \lfloor K_i-(i+1)}(N_i-1)^{-(i+1)} + \ldots \right.$$

$$\left. + \frac{\lfloor K_i}{\lfloor K_i \lfloor 0}(N_i-1)^{-K_i}\right]$$

or

$$N'' = \frac{1}{n}\left[N_0\left(\frac{N_0-1}{N_0}\right)^{K_0'}\left\{1 + K_0'(N_0-1)^{-1} + \ldots\ldots\ldots + (N_0-1)^{-K_0'}\right\}\right.$$

$$+ N_1\left(\frac{N_1-1}{N_1}\right)^{K_1}\left\{K_1(N_1-1)^{-1} + \frac{1}{2}K_1(K_1-1)(N_1-1)^{-2}\right.$$

$$\left. + \frac{1}{6}K_1(K_1-1)(K_1-2)(N_1-1)^{-3} + \ldots\ldots\ldots + (N_1-1)^{-K_1}\right\}$$

$$+ N_2\left(\frac{N_2-1}{N_2}\right)^{K_2}\left\{\frac{1}{2}K_2(K_2-1)(N_2-1)^{-2} + \frac{1}{6}K_2(K_2-1)(K_2-2)\right.$$

$$\left. \times (N_2-1)^{-3} + \ldots. + (N_2-1)^{-K_2}\right\}$$

$$+ \ldots\ldots\ldots\ldots\ldots$$

$$+ N_B\left(\frac{N_B-1}{N_B}\right)^{K_B}\left\{\frac{K_B(K_B-1)(K_B-2)\ldots.[K_B-(B+1)]}{1, 2, 3, \ldots. B}\right.$$

$$\left.\left. \times (N_B-1)^{-B} + \ldots. + (N_B-1)^{-K_B}\right\}\right] \quad .. \quad .. \quad .. \quad .. \quad (2')$$

We consider that K_0' is not a part of the effective electrons, hence it may be put equal to zero. If K_0' plays some part, it is due to some other factors, but not due to the ionization under consideration. Hence, the terms containing K_0' can be ignored.

Thus,

$$N'' = \frac{1}{n}\left[N_0 + N_1\left(1-\frac{1}{N_1}\right)^{K_1}\left\{K_1(N_1-1)^{-1} + \frac{1}{2}K_1(K_1-1)(N_1-1)^{-2}\right.\right.$$

$$\left.\left. + \frac{1}{6}K_1(K_1-1)(K_1-2)(N_1-1)^{-3} + \ldots\ldots\ldots + (N_1-1)^{-K_1}\right\}\right.$$

$$+N_2\left(1-\frac{1}{N_2}\right)^{K_2}\left\{\frac{1}{2}K_2(K_2-1)(N_2-1)^{-2}+\frac{1}{6}K_2(K_2-1)(K_2-2)\right.$$
$$\left.\times(N_2-1)^{-3}+\ldots.+(N_2-1)^{-K_2}\right\}$$
$$+N_3\left(1-\frac{1}{N_3}\right)^{K_3}\left\{\frac{1}{6}K_3(K_3-1)(K_3-2)(N_3-1)^{-3}+\frac{1}{24}K_3(K_3-1)\right.$$
$$\left.\times(K_3-2)(K_3-3)(N_3-1)^{-4}+\ldots.+(N_3-1)^{-K_3}\right\}$$
$$+\ldots\ldots\ldots\ldots$$
$$+N_B\left(1-\frac{1}{N_B}\right)^{K_B}\left\{\frac{K_B(K_B-1)(K_B-2)\ldots.[K_B-(B+1)]}{1,\,2,\,3,\ldots.B}\right.$$
$$\left.\times(N_B-1)^{-B}+\ldots.+(N_B-1)^{-K_B}\right\}\Big]$$
$$\ldots\ldots\ldots\ldots\ldots\ldots(3')$$

Equation (3′) is the most general form of the distribution and its value can be found if K_1, K_2, K_3 ..... and N_1, N_2, N_3 ....., etc., are known.

As it is impossible to get the exact magnitude of K_1, K_2, K_3 .... and N_1, N_2, N_3 ....., etc., we use the relations $K_i/N_i = K/N$, where i has values 1, 2, 3, ... B and $\bar{N} = N-N_0$ which means that the sensitivity centres of different sensitivities are uniformly distributed and the distribution of electrons over these different sensitivity centres is also uniform. This, of course, will be a special case of uniform distribution, but will not affect the ratio of the distribution as shown by the equation $K_i/N_i = \frac{K/B}{N/B} = \frac{K}{N}$. We can justify the above uniform distribution in the following way.

Let us consider a single grain. The passage of the charged particle and the liberation of electrons in the grain is a phenomenon which does not take a long time, while the formation of latent image takes a comparatively longer time. The velocity of electrons is much faster as compared to that of Ag^+ ions, therefore in a short time these liberated electrons get spread over all the sensitivity centres of the grain, and the distribution becomes uniform and not localized in any part of the grain. Hence, it will not be wrong to consider that in all the grains the electrons distributed over all different sensitivity centres are mostly uniform. Further, since the sensitivity of the grain may be considered to be uniform, it will mean that different grains may have a perfect uniformity as regards the distribution of the sensitivity centres of different types. Expressing these assumptions mathematically, we have

$$K_i = K/B \text{ and } N_i = N/B. \quad .. \quad .. \quad .. \quad (4')$$

However, we still hold that equation (3′) will represent the most general distribution whose probable case is referred in the relation (4′). This has

also been considered to be the only and the best method of representing equation (3′) in terms of known factors so that it may be useful for further calculations.

Hence,

$$N'' = \frac{1}{n}\Big[N_0 + \frac{\bar{N}}{\bar{B}}\Big(1-\frac{B}{\bar{N}}\Big)^{K/B}\Big\{\frac{K}{\bar{B}}\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-1} + \frac{1}{2}\frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-2} + \ldots\ldots\ldots + \Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-K/B}\Big\}$$

$$+\frac{\bar{N}}{\bar{B}}\Big(1-\frac{B}{\bar{N}}\Big)^{K/B}\Big\{\frac{K}{2\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-2} + \frac{1}{6}\frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{K}{\bar{B}}-2\Big) \times\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-3} + \ldots. + \Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-K/B}\Big\}$$

$$+\frac{\bar{N}}{\bar{B}}\Big(1-\frac{B}{\bar{N}}\Big)^{K/B}\Big\{\frac{1}{6}\frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{K}{\bar{B}}-2\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-3} + \ldots. + \Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-K/B}\Big\}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\frac{\bar{N}}{\bar{B}}\Big(1-\frac{B}{\bar{N}}\Big)^{K/B}\Big\{\frac{K/B(K/B-1)(K/B-2)\ldots.[K/B-(B+1)]}{1.\ 2.\ 3.\ \ldots.\ B}\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-B} + \ldots. + \Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-K/B}\Big\}\Big].$$

But

$$\Big(1-\frac{B}{\bar{N}}\Big)^{K/B} = \Big(1-\frac{B}{\bar{N}}\Big)^{\frac{K}{\bar{N}}\cdot\frac{\bar{N}}{B}} \simeq e^{-K/\bar{N}} \text{ (for large values of } \bar{N}).$$

Thus,

$$N'' = \frac{1}{n}\Big[N_0 + \frac{\bar{N}}{\bar{B}}e^{-K/\bar{N}}\Big\{\frac{K}{\bar{B}}\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-1} + \frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-2} + \frac{1}{2}\frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big) \times\Big(\frac{K}{\bar{B}}-2\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-3} + \frac{1}{6}\frac{K}{\bar{B}}\Big(\frac{K}{\bar{B}}-1\Big)\Big(\frac{K}{\bar{B}}-2\Big)\Big(\frac{K}{\bar{B}}-3\Big)\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-4} + \ldots. + B \times\Big(\frac{\bar{N}}{\bar{B}}-1\Big)^{-K/B}\Big\}\Big]$$

$$= \frac{1}{n}\Big[N_0 + \frac{K}{\bar{B}}e^{-K/\bar{N}}\Big(1-\frac{B}{\bar{N}}\Big)^{-1}\Big\{1 + \Big(\frac{K}{\bar{N}}-\frac{B}{\bar{N}}\Big)\Big(1-\frac{B}{\bar{N}}\Big)^{-1} + \frac{1}{2}\Big(\frac{K}{\bar{N}}-\frac{B}{\bar{N}}\Big) \times\Big(\frac{K}{\bar{N}}-\frac{2B}{\bar{N}}\Big)\Big(1-\frac{B}{\bar{N}}\Big)^{-2} + \frac{1}{6}\Big(\frac{K}{\bar{N}}-\frac{B}{\bar{N}}\Big)\Big(\frac{K}{\bar{N}}-\frac{2B}{\bar{N}}\Big)\Big(\frac{K}{\bar{N}}-\frac{3B}{\bar{N}}\Big)\Big(1-\frac{B}{\bar{N}}\Big)^{-3} + \frac{1}{24}\Big(\frac{K}{\bar{N}}-\frac{B}{\bar{N}}\Big)\Big(\frac{K}{\bar{N}}-\frac{2B}{\bar{N}}\Big)\Big(\frac{K}{\bar{N}}-\frac{3B}{\bar{N}}\Big)\Big(\frac{K}{\bar{N}}-\frac{4B}{\bar{N}}\Big)\Big(1-\frac{B}{\bar{N}}\Big)^{-4} + \text{negligible terms}\Big\}\Big]. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5')$$

As a matter of fact N_0 in the above relation (5′) will be a very small factor, because considering the basic theories of development of nuclear emulsions, we find that no grain should have the potentiality (sensitivity) to become developable by its own virtue under normal development conditions. If the fog grains are present they are due to various factors, viz. radioactive contaminations, impurities, etc. Curves of Dodd and Waller (1951) give the value of fog grains to be about 10 per 100 microns of emulsion tracks which means that only about 10 sensitivity centres in a total of about $275 \times 1{,}750$ belong to category N_0, i.e. they require no electron to become developable. This small number (~ 10) indicates the maximum limit of fog grains to be included in the category N_0, because it includes the fog grains created by the reasons mentioned earlier. Thus, this number can easily be considered to be negligible for all further derivations.

Thus, finally, we have

$$N'' = \frac{K}{nB} e^{-K/N} \left(1-\frac{B}{N}\right)^{-1} \left[1+\left(\frac{K}{N}-\frac{B}{N}\right)\left(1-\frac{B}{N}\right)^{-1}+\frac{1}{2}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\left(1-\frac{B}{N}\right)^{-2}\right.$$
$$+\frac{1}{6}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\left(\frac{K}{N}-\frac{3B}{N}\right)\left(1-\frac{B}{N}\right)^{-3}$$
$$+\frac{1}{24}\left(\frac{K}{N}-\frac{B}{N}\right)\left(\frac{K}{N}-\frac{2B}{N}\right)\left(\frac{K}{N}-\frac{3B}{N}\right)\left(\frac{K}{N}-\frac{4B}{N}\right)\left(1-\frac{B}{N}\right)^{-4}$$
$$\left.+ \text{ negligible terms}\right]. \quad .. \quad .. \quad .. \quad .. \quad (6')$$

References

Beiser, A. (1952). *Rev. mod. Phys.*, **24**, 273–311.

Blatt, J. M. (1955). *Aust. J. Phys.*, **8**, 248–272.

Blau, M. (1949). *Phys. Rev.*, **75**, 279–282.

Bradt, H. L., Kaplon, M. F., and Peters, B. (1950). *Helv. phys. acta.*, **23**, 24–63.

Brown, L. M. (1953). *Phys. Rev.*, **90**, 95–97.

Della Corte, M. (1954). *Nuovo Cim.*, **12**, 28–36.

——— (1958). Private communication.

Della Corte, M., Ramat, M., and Ronchi, L. (1953). *Nuovo Cim.*, **10**, 958–970.

de Langhe, J. E. (1936). *Physica*, **3**, 904.

——— (1948). Theory of Photographic Process by C. E. K. Mees. The Macmillan Co., New York, 221.

Demers, P. (1947). *Canad. J. Res.*, A **25**, 223–251.

Dodd, E. C., and Waller, C. (1951). Effects of Diluting Nuclear Emulsion Type G-5 with Gelatin. Fundamental Mechanism of Photographic Sensitivity. Butterworths Scientific Publs., London, 266–271.

Fay, H., Gottstein, K., and Hain, K. (1954). *Nuovo Cim.* (Suppl.), **11**, 234–263.

Fowler, P. H. (1950). *Phil. Mag.*, **41**, 168–184.

Fowler, P. H., and Perkins, D. H. (1951). Application of the Photographic Plate to the Investigation of Problems in Nuclear Physics and Cosmic Rays. Fundamental Mechanism of Photographic Sensitivity. Butterworths Scientific Publs., London, 340–345.

——— (1955). *Phil. Mag.*, **46**, 587–610.

Gurney, R. W., and Mott, N. F. (1938). *Proc. roy. Soc.*, A **164**, 151–167.
Hamilton, J. F., Hamm, F. A., and Brady, L. E. (1956). *J. appl. Phys.*, **27**, 874–885.
Herz, A. J., and Davis, G. (1955). *Aust. J. Phys.*, **8**, 129–135.
Mees, C. E. K. (1948). Theory of Photographic Process. The Macmillan Co., New York, 162.
Mitchell, J. W. (1957). *Rep. Progr. Phys.*, **20**, 433.
Mitchell, J. W., and Mott, N. F. (1957). *Phil. Mag.*, **2**, 1149–1170.
Price, B. T. (1955). *Rep. Progr. Phys.*, **18**, 52–82.
Sharma, A. P., and Gill, P. S. (1960). Ionization measurements in nuclear emulsions. Proceedings Cosmic Ray Symposium, Ahmedabad, 353–359.
Webb, J. H. (1948). *Phys. Rev.*, **74**, 511–532.
Yamakawa, K. A. (1951). *Ibid.*, **82**, 522–526.