THE SIMULATION OF THIRD ORDER SYSTEMS WITH A LEADING TIME-CONSTANT BY A SINGLE OPERATIONAL AMPLIFIER—III

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In the previous papers (Wadhwa 1964*a*, *b*) on the same topic six basic circuits each employing three capacitors and six resistors and capable of simulating, under certain conditions, third order systems with a leading time-constant were presented. In this paper two more basic circuits for simulating the same type of third order systems are presented. The circuits are analysed, the design formulae obtained and the conditions of their physical realizability discussed.

Introduction

In previous communications (Wadhwa 1964*a*, *b*) on this subject six basic circuits each capable of simulating, under certain conditions, a particular type of the general third order linear systems, that is systems with a leading time-constant, were presented.

The purpose of this paper is to present two more basic circuits, discuss the design formulae and the conditions of their physical realizability.

Third Order System Simulation

Two basic circuits, each employing three capacitors and six resistors (with a certain arbitrary choice of resistor values) and capable of simulating, under certain conditions, third order systems with a leading time-constant, that is

$$F(S) = -\frac{b_0(b_1S+1)}{a_3S^3+a_2S^2+a_1S+1}, \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

are shown in Fig. 1.

Transfer function for the circuit of Fig. 1(*a*) can be shown to be

$$\frac{E_0}{E_1} = -\frac{\frac{\beta^2}{(3\beta+1)}(RC_5S+1)}{\frac{\beta^2}{(3\beta+1)}R^3C_2C_5C_6S^3+\left[\frac{\beta}{(3\beta+1)}R^2C_2C_5+\frac{\beta(\alpha+2\alpha\beta+\beta)}{\alpha(3\beta+1)}R^2C_2C_6+\frac{\beta(2\beta+1)}{(3\beta+1)}R^2C_5C_6\right]S^2+\left[\frac{\beta}{(3\beta+1)}RC_2+RC_5+\frac{\beta(2\alpha+3\alpha\beta+2\beta)+(\alpha+2\alpha\beta+\beta)}{\alpha(3\beta+1)}RC_6\right]S+1} \quad \ldots \quad (2)$$

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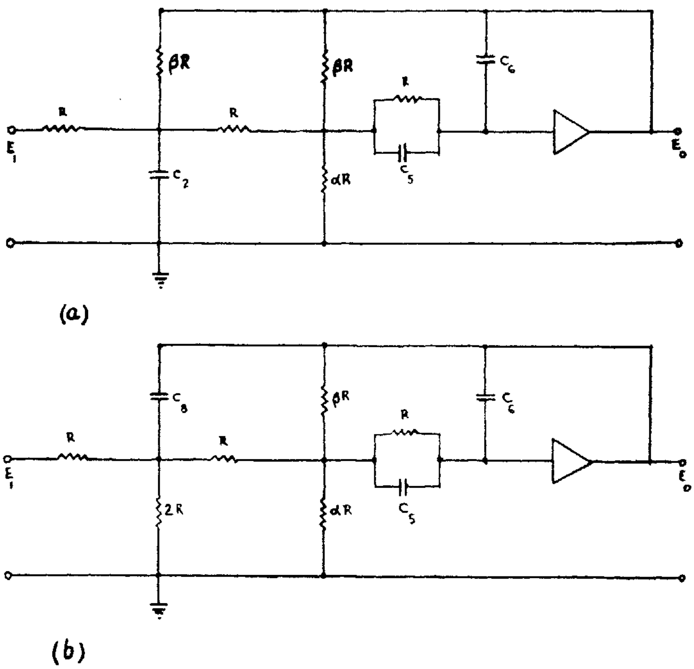

FIG. 1. Networks for the simulation of $\frac{E_0}{E_1} = -\frac{b_0(b_1S+1)}{a_3S^3+a_2S^2+a_1S+1}$.

(1) and (2) will be identical if

$$b_0 = \frac{\beta^2}{(3\beta+1)} \quad \ldots \quad (3)$$

$$b_1 = T_5 \quad \ldots \quad (4)$$

$$a_1 = \frac{\beta}{(3\beta+1)}T_2+T_5+\frac{\beta(2\alpha+3\alpha\beta+2\beta)+(\alpha+2\alpha\beta+\beta)}{\alpha(3\beta+1)}T_6 \quad \ldots \quad (5)$$

$$a_2 = \frac{\beta}{(3\beta+1)}T_2T_5+\frac{\beta(\alpha+2\alpha\beta+\beta)}{\alpha(3\beta+1)}T_2T_6+\frac{\beta(2\beta+1)}{(3\beta+1)}T_5T_6 \quad \ldots \quad (6)$$

$$a_3 = \frac{\beta^2}{(3\beta+1)}T_2T_5T_6 \quad \ldots \quad (7)$$

where

$$T_n = RC_n. \quad \ldots \quad (8)$$

The solution of (3) gives

$$\beta = \frac{3b_0+\sqrt{(9b_0^2+4b_0)}}{2} \quad \ldots \quad (9)$$

as the negative root is inadmissible.

Elimination of α, T_2 and T_5 from (4) to (7) gives a cubic

$$A_1T_6^3-A_2T_6^2+A_3T_6-A_4 = 0 \quad \ldots \quad (10)$$

which will have either one or three real roots depending on whether its discriminant Δ is positive or negative, where

$$\left.\begin{aligned} A_1 &= b_0^2 b_1^3(2\beta+1)^2 \\ A_2 &= b_0 b_1(2\beta+1)\{a_2 b_1\beta - a_3(2\beta+1)\} + a_3 b_1\beta^2(\beta+1) \\ A_3 &= a_3 b_0 b_1^2(2\beta+1) + a_3 b_1 \beta^2(a_1 - b_1) \\ A_4 &= a_3^2\beta \end{aligned}\right\} \quad \ldots \quad (11)$$

Now, it can be shown by a process of reasoning similar to that already discussed in detail elsewhere (Wadhwa 1963) that at least one positive real set of α, β, T_2, T_5 and T_6 exists, provided

$$\left.\begin{aligned} &(a_1 - b_0) > 0 \\ &b_1\beta(a_1-b_1)^2 > 4a_3(\beta+1) \\ &\{a_2 b_1\beta - a_3(2\beta+1)\} > 0 \\ &\{a_2 b_1\beta - a_3(2\beta+1)\}^2 > 4a_3 b_0 b_1^3(2\beta+1) \end{aligned}\right\} \quad \ldots \quad (12)$$

and either

$$OQ > OB > OP > OA \quad \ldots \quad (13)$$

or

$$\Delta = 4p^3 + 27q^2 < 0 \quad \ldots \quad (14)$$

where

$$\left.\begin{aligned} OP &= \frac{b_1\beta(a_1-b_1) - \sqrt{\left\{b_1^2\beta^2(a_1-b_1)^2 - 4a_3 b_1\beta(\beta+1)\right\}}}{2b_1\beta(\beta+1)} \\ OQ &= \frac{b_1\beta(a_1-b_1) + \sqrt{\left\{b_1^2\beta^2(a_1-b_1)^2 - 4a_3 b_1\beta(\beta+1)\right\}}}{2b_1\beta(\beta+1)} \\ OA &= \frac{\{a_2 b_1\beta - a_3(2\beta+1)\} - \sqrt{[\{a_2 b_1\beta - a_3(2\beta+1)\}^2 - 4a_3 b_0 b_1^3(2\beta+1)]}}{2b_0 b_1^2(2\beta+1)} \\ OB &= \frac{\{a_2 b_1\beta - a_3(2\beta+1)\} + \sqrt{[\{a_2 b_1\beta - a_3(2\beta+1)\}^2 - 4a_3 b_0 b_1^3(2\beta+1)]}}{2b_0 b_1^2(2\beta+1)} \\ p &= \frac{A_3}{A_1} - \frac{1}{3}\left(\frac{A_2}{A_1}\right)^2 \\ q &= -\frac{A_4}{A_1} + \frac{1}{3}\left(\frac{A_2 A_3}{A_1^2}\right) - \frac{2}{27}\left(\frac{A_2}{A_1}\right)^3 \end{aligned}\right\} \quad \ldots \quad (15)$$

To summarize, if the inequalities of expressions (12) and either (13) or (14) are satisfied then at least one and possibly two or three sets of positive real α, β, T_2, T_5 and T_6 exist for which the circuit of Fig. 1(a) is physically realizable.

The circuit component values may be determined by first solving (9) and (10) for β and T_6 respectively. Since T_5 is known directly from (4), therefore T_2 can then be easily determined from (7), and α from either (5) or (6).

Referring to the circuit of Fig. 1(b), its transfer function can be shown to be

$$\frac{E_0}{E_1} = -\frac{\frac{2}{5}\beta(RC_5S+1)}{\frac{2}{5}\beta R^3C_5C_6C_8S^3+\left[\beta R^2C_5C_6+\frac{2(\beta+1)}{5}R^2C_5C_8+\frac{2(\alpha+2\alpha\beta+\beta)}{5\alpha}R^2C_6C_8\right]S^2+\left[RC_5+\frac{(5\alpha+8\alpha\beta+5\beta)}{5\alpha}RC_6+\frac{2(\beta+1)}{5}RC_8\right]S+1} \quad .. \quad (16)$$

(1) and (16) will be identical if

$$b_0 = \tfrac{2}{5}\beta \quad .. \quad (17)$$

$$b_1 = T_5 \quad .. \quad (18)$$

$$a_1 = T_5+\frac{(5\alpha+8\alpha\beta+5\beta)}{5\alpha}T_6+\frac{2(\beta+1)}{5}T_8 \quad .. \quad (19)$$

$$a_2 = \beta T_5T_6+\frac{2(\beta+1)}{5}T_5T_8+\frac{2(\alpha+2\alpha\beta+\beta)}{5\alpha}T_6T_8 \quad .. \quad (20)$$

$$a_3 = \tfrac{2}{5}\beta T_5T_6T_8 \quad .. \quad (21)$$

where

$$T_n = RC_n. \quad .. \quad (22)$$

Elimination of α, β, T_5 and T_8 from (17) to (21) gives a cubic

$$T_6^3-B_1T_6^2+B_2T_6-B_3 = 0 \quad .. \quad (23)$$

which will have either one or three real roots depending on whether its discriminant Δ is positive or negative, where

$$\left.\begin{aligned} B_1 &= \frac{2(5a_2b_1-2a_3)}{25b_0b_1^2} \\ B_2 &= \frac{2a_3(2a_1+5b_0b_1)}{25b_0^2b_1^2} \\ B_3 &= \frac{4a_3^2(2+5b_0)}{125b_0^3b_1^3} \end{aligned}\right\}. \quad .. \quad (24)$$

As already mentioned, it can be shown (Wadhwa 1963) that at least one set of positive real α, β, T_5, T_6 and T_8 exists, provided

$$\left.\begin{aligned} &(a_1-b_1) > 0 \\ &5b_0b_1(a_1-b_1)^2 > 4a_3(1+4b_0)(2+5b_0) \\ &\{5a_2b_0b_1-2a_3(1+5b_0)\} > 0 \\ &\{5a_2b_0b_1-2a_3(1+5b_0)\}^2 > 50a_3b_0^2b_1^3(2+5b_0) \end{aligned}\right\} \quad .. \quad (25)$$

and either

$$OQ' > OB' > OP' > OA' \quad .. \quad (26)$$

or

$$\left.\begin{aligned}(5a_2b_1-2a_3) > 0 \\ \Delta' = 4p_1^3+27q_1^2 < 0\end{aligned}\right\} \quad \ldots \quad (27)$$

where

$$\left.\begin{aligned}
OP' &= \frac{5b_0b_1(a_1-b_1)-\sqrt{\left\{25b_0^2b_1^2(a_1-b_1)^2-20a_3b_0b_1(1+4b_0)(2+5b_0)\right\}}}{10b_0b_1(1+4b_0)} \\
OQ' &= \frac{5b_0b_1(a_1-b_1)+\sqrt{\left\{25b_0^2b_1^2(a_1-b_1)^2-20a_3b_0b_1(1+4b_0)(2+5b_0)\right\}}}{10b_0b_1(1+4b_0)} \\
OA' &= \frac{\{5a_2b_0b_1-2a_3(1+5b_0)\}-\sqrt{[\{5a_2b_0b_1-2a_3(1+5b_0)\}^2-50a_3b_0^2b_1^3(2+5b_0)]}}{25b_0^2b_1^2} \\
OB' &= \frac{\{5a_2b_0b_1-2a_3(1+5b_0)\}+\sqrt{[\{5a_2b_0b_1-2a_3(1+5b_0)\}^2-50a_3b_0^2b_1^3(2+5b_0)]}}{25b_0^2b_1^2} \\
p_1 &= B_2-\tfrac{1}{3}B_1^2 \\
q_1 &= -B_3+\frac{B_1B_2}{3}-\frac{2}{27}B_1^3
\end{aligned}\right\} \quad (28)$$

To summarize, if the inequalities of (25) and either (26) or (27) are satisfied then at least one and possibly two or three sets of positive real α, β, T_5, T_6 and T_8 exist for which the circuit of Fig. 1(*b*) is physically realizable.

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