HYDROMAGNETIC COUETTE FLOW BETWEEN TWO CONDUCTING POROUS WALLS

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The steady, incompressible hydromagnetic Couette flow between two conducting porous walls which are moving with different velocities has been discussed in detail. It is found that uniform suction or injection and the wall conductance considerably affect the flow parameters like magnetic and viscous drag and mass flow rate coefficients.

Notations

μ_e	magnetic permeability.
σ	electrical conductivity of the fluid.
$\vec{H}$	magnetic field vector.
H_y	applied magnetic field.
$\vec{J}$	current density.
$\vec{E}$	electric field.
$\vec{q}$	velocity of the fluid.
ρ	density of the fluid.
c	velocity of light.
η	electrical diffusivity $= \frac{1}{\sigma}\frac{c^2}{4\pi\mu_e}$.
ν	kinematic viscosity.
p_{rm}	magnetic Prandtl number $= \frac{\eta}{\nu}$.
a'	Alfven's wave velocity $= \left(\frac{\mu_e H_y^2}{4\pi\rho}\right)^{\frac{1}{2}}$.
R_m	magnetic Reynolds number $= \frac{a'L}{\nu}$.
M	Hartmann number $= a'L(\eta\nu)^{-\frac{1}{2}}$.

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σ_{w_1}, σ_{w_2}	conductivity of the lower and upper walls with fluid inside.
η_{w_1}, η_{w_2}	electrical diffusivity of the lower and upper walls with fluid inside.
h_1, h_2	thicknesses of the lower and upper walls.
ϕ_1, ϕ_2	electrical conductance parameters of the lower and upper walls.
C_{V_1}, C_{V_2}	viscous drag coefficients } at the lower and upper walls.
C_{M_1}, C_{M_2}	magnetic drag coefficients } at the lower and upper walls.
C_{T_1}, C_{T_2}	total drag coefficients } at the lower and upper walls.

1. Introduction

A basic problem on account of its varied application to problems of practical interest is that of flow between two surfaces fixed or moving relative to one another. The generalized Couette flow for incompressible and compressible fluids has been investigated by Schlichting (1953), Illingworth (1950) and Liepmann and Bleviss (1956). The effect of transverse magnetic field on a simple Couette flow has been studied by Lehnert (1952), Liepmann (1958) and Bleviss (1958). The analysis of hydromagnetic channel flow with porous walls is presented by Agarwal (1962).

In problems of heat-transfer, in addition to the part played by molecular conductivity in the laminar sub-layer near the walls, one has to take into account the effect of the migration of electrons also at high temperatures. The electrical conductivity of the walls can therefore be expected to play a significant part in the process because of the flow of electrons through the boundaries as a result of the electric current generated by the interaction of the magnetic field and the flow velocities. When the walls are porous the percolation of the fluid through the walls will render them electrically conducting even though the material of the walls may be non-conducting in the dry state. Hence in treating problems of flow between porous walls electrical conductivity of the walls has to be taken into account.

In this paper we have studied the steady, incompressible, hydromagnetic Couette flow between two conducting porous walls which are moving with different velocities. We find that the effect of suction or injection is characterized by a non-dimensional quantity v_0 and the effect of conductance of walls depends on the parameters ϕ_1 and ϕ_2. The flow parameters like the viscous drag, the magnetic drag and the mass flow rate coefficients are considerably affected by suction or injection and wall conductance parameters and are shown in Tables I to III and Figs. 2 to 8.

2. Basic Equations

The steady motion of an incompressible, viscous and electrically conducting fluid in the presence of a magnetic field is governed by the following equations: The Maxwell equations

$$\text{curl}\,\vec{H} = \frac{4\pi\vec{J}}{c}, \qquad (1)$$

$$\text{curl}\,\vec{E} = 0, \qquad (2)$$

$$\text{div}\,\vec{B} = 0, \qquad (3)$$

$$\vec{B} = \mu_e\vec{H}, \qquad (4)$$

Ohm's law

$$\vec{J} = \sigma\left[\vec{E}+\frac{1}{c}\left(\vec{q}\times\vec{B}\right)\right], \qquad (5)$$

the equation of continuity for an incompressible fluid

$$\text{div}\,\vec{q} = 0, \qquad (6)$$

and the momentum equation

$$\left[\vec{q}\,.\,\text{grad}\right]\vec{q} = -\frac{1}{\rho}\text{grad}\,p+\nu\nabla^2\vec{q}+\frac{\mu_e}{\rho c}\left(\vec{J}\times\vec{H}\right). \qquad (7)$$

The various symbols used here are shown in the notations. All the electromagnetic quantities are measured in Gaussian units.

3. Flow between Porous Conducting Walls

We consider the case of a fully developed flow between parallel walls in the presence of transverse applied magnetic field. The problem under consideration is shown schematically in Fig. 1.

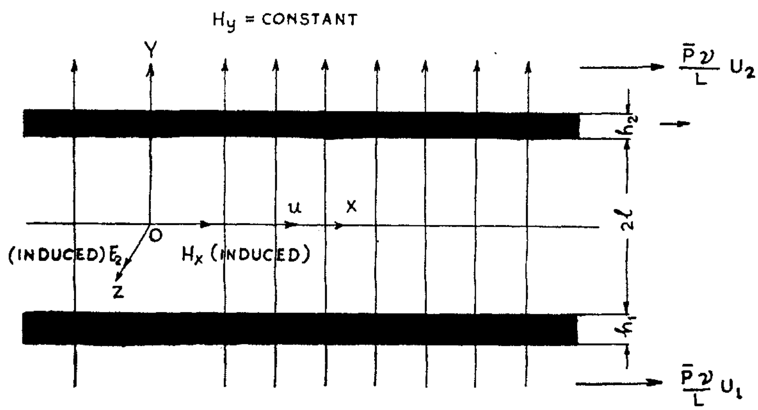

Fig. 1. Channel configuration and coordinate system.

We assume all the physical quantities except the pressure p to be functions of y alone.

We set

$$\vec{q} = u\vec{i}+v\vec{j}, \quad \vec{H} = H_x\vec{i}+H_y\vec{j}, \quad p = p(x, y). \qquad (8)$$

Substituting (8) into (3), (4), (6) and (1) we get

$$H_y = \text{constant} = H_0,$$

$$v = V_0$$

and

$$\vec{J} = J_z \vec{K} = -\frac{c}{4\pi}\frac{dH_x}{dy}\vec{K}. \quad \ldots \quad (9)$$

Taking curl of (1) and using (2) and (5) we obtain

$$\frac{c^2}{4\pi\mu_e\sigma}\,\text{curl curl}\,\vec{H} = \text{curl}\left(\vec{q}\times\vec{H}\right). \quad \ldots \quad (10)$$

The momentum equation (7) in view of (8) gives

$$V_0\frac{du}{dy} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2u}{dy^2} + \frac{\mu_e}{4\pi\rho}H_y\frac{dH_x}{dy}, \quad \ldots \quad (11)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\mu_e}{4\pi\rho}H_x\frac{dH_x}{dy} = 0. \quad \ldots \quad (12)$$

Integrating equation (12), we have

$$\left(p + \frac{\mu_e}{8\pi}H_x^2\right) = \text{function of } x \text{ alone}$$

$$= P(x). \quad \ldots \quad (13)$$

Substituting (13) into (11), we obtain

$$\nu\frac{d^2u}{dy^2} + \frac{\mu_e}{4\pi\rho}H_y\frac{dH_x}{dy} - V_0\frac{du}{dy} = \frac{1}{\rho}\frac{dP}{dx}. \quad \ldots \quad (14)$$

Equation (10) can be written as

$$\eta\frac{d^2H_x}{dy^2} + H_y\frac{du}{dy} - V_0\frac{dH_x}{dy} = 0. \quad \ldots \quad (15)$$

The boundary conditions are

$$u = a'U_1\frac{\overline{P}}{R_m} \quad \text{at} \quad y = -L,$$

$$u = a'U_2\frac{\overline{P}}{R_m} \quad \text{at} \quad y = +L. \quad \ldots \quad (16)$$

The physical statement of the electromagnetic boundary conditions (*see* Chang and Yen 1962; Shercliff 1956) is that the tangential components of $\vec{E}$ and $\vec{H}$ must be continuous across the intersurface. The mathematical consequence of the above statement is as follows:

For the lower wall equation (15) can be written as

$$\frac{d^2H_{xw_1}}{dy^2} - \frac{V_0}{\eta_{w_1}}\frac{dH_{xw_1}}{dy} = 0. \quad \ldots \quad (17)$$

Solution of (17) takes the form

$$H_{xw_1} = A + B \exp \frac{V_0 y}{\eta_{w_1}} \quad \ldots \quad (18)$$

where A and B are constants determined from the conditions of vanishing of H_x outside the boundary and continuity of H_x at $y = -L$,

i.e.

$$H_{xw_1} = 0 \quad \text{at} \quad y = -(L+h_1)$$

$$H_{xf})_{y=-L} = H_{xw_1})_{y=-L} \quad \ldots \quad (19)$$

where f designates the value in the fluid. Then the induced magnetic field in the lower wall is given by

$$H_{xw_1} = \frac{H_{xf})_{y=-L}}{\exp\left(-\frac{V_0 L}{\eta_{w_1}}\right)\left[1-\exp\left(-\frac{V_0 h_1}{\eta_{w_1}}\right)\right]}\left[\exp\frac{V_0 y}{\eta_{w_1}} - \exp\left\{-\frac{V_0}{\eta_{w_1}}(L+h_1)\right\}\right]. \quad (20)$$

Using (1) and (5), we get

$$-\frac{c}{4\pi}\frac{dH_x}{dy} = \sigma\left[E_z + \frac{1}{c}(uB_y - V_0 B_x)\right]. \quad \ldots \quad (21)$$

From (21), we obtain

$$-E_{zf})_{y=-L} = \frac{c}{4\pi\sigma}\frac{dH_{xf}}{dy}\Big)_{y=-L} + \frac{\mu_e a' U_1 \frac{P}{R_m}}{c} H_y - \frac{\mu_e V_0}{c} H_{xf})_{y=-L}, \quad \ldots \quad (22)$$

and

$$-E_{zw_1})_{y=-L} = \frac{c}{4\pi\sigma_{w_1}}\frac{dH_{xw_1}}{dy}\Big)_{y=-L} + \frac{\mu_e a' U_1 \frac{P}{R_m}}{c} H_y - \frac{\mu_e V_0}{c} H_{xw_1})_{y=-L}, \quad (23)$$

Continuity of E_z between the wall and fluid at $y = -L$ requires

$$E_{zf})_{y=-L} = E_{zw_1})_{y=-L}. \quad \ldots \quad (24)$$

The condition (24) together with (19) gives

$$\frac{1}{\sigma}\frac{dH_{xf}}{dy}\Big)_{y=-L} = \frac{1}{\sigma_{w_1}}\frac{dH_{xw_1}}{dy}\Big)_{y=-L}. \quad \ldots \quad (25)$$

Using (20) the equation (25) takes the form

$$\frac{dH_{xf}}{dy}\Big)_{y=-L} - \frac{\sigma}{\sigma_{w_1}}\frac{V_0}{\eta_{w_1}}\frac{1}{\left[1-\exp\left(-\frac{V_0 h_1}{\eta_{w_1}}\right)\right]} H_{xf})_{y=-L} = 0. \quad \ldots \quad (26)$$

Equation (26) can be written as

$$\frac{dH_x}{dy} - \frac{\sigma}{\sigma_{w_1}}\frac{V_0}{\eta_{w_1}}\frac{1}{\left[1-\exp\left(-\frac{V_0 h_1}{\eta_{w_1}}\right)\right]} H_x = 0$$

$$\text{at} \quad y = -L. \quad \ldots \quad (27)$$

For the upper wall the corresponding boundary condition can be derived as

$$\frac{dH_x}{dy}+\frac{\sigma}{\sigma_{w2}}\frac{V_0}{\eta_{w2}}\frac{1}{\left(\exp\frac{V_0 h_2}{\eta_{w2}}-1\right)}H_x=0 \quad \text{at} \quad y=L. \qquad (28)$$

Using the transformation

$$\bar{u}=\frac{u}{a'},\ \bar{H}_x=\frac{H_x}{H_y},\ \xi=\frac{y}{L},\ v_0=\frac{V_0}{a'}$$

and

$$\bar{P}=-\frac{1}{\rho\nu^2L^{-3}}\frac{dP}{dx} \qquad (29)$$

into the equations (14) to (16), (27) and (28) we get

$$\frac{d^2\bar{u}}{d\xi^2}+R_m\frac{d\bar{H}_x}{d\xi}-R_m v_0\frac{d\bar{u}}{d\xi}=-\frac{\bar{P}}{R_m}, \qquad (30)$$

$$p_{rm}\frac{d^2\bar{H}_x}{d\xi^2}+R_m\frac{d\bar{u}}{d\xi}-R_m v_0\frac{d\bar{H}_x}{d\xi}=0, \qquad (31)$$

with the boundary conditions

$$\left.\begin{aligned}\bar{u}&=U_1\frac{\bar{P}}{R_m} \quad \text{at} \quad \xi=-1,\\ \bar{u}&=U_2\frac{\bar{P}}{R_m} \quad \text{at} \quad \xi=+1,\end{aligned}\right\} \qquad (32)$$

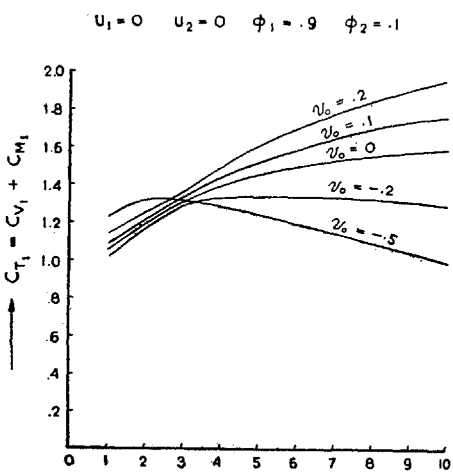

FIG. 2. Coefficient of total drag C_{T1} versus Hartmann number M for different values of suction parameter v_0.

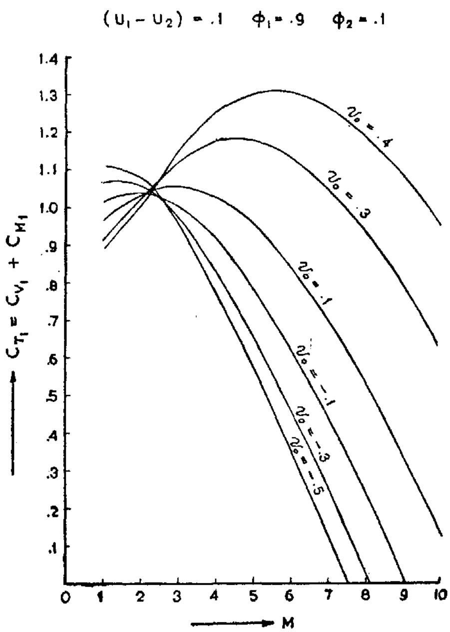

FIG. 3. Coefficient of total drag C_{T1} versus Hartmann number M for different values of suction parameter v_0.

and

$$\frac{d\overline{H}_x}{d\xi} - \frac{1}{\phi_1}\overline{H}_x = 0 \quad \text{at} \quad \xi = -1, \qquad \text{(33)}$$

$$\frac{d\overline{H}_x}{d\xi} + \frac{1}{\phi_2}\overline{H}_x = 0 \quad \text{at} \quad \xi = +1. \qquad \text{(34)}$$

The electrical conductance parameters ϕ_1 and ϕ_2 are given by

$$\phi_1 = \left(\frac{\sigma_{w1}}{\sigma}\right)\left[\frac{p_1\left\{1-\exp\left(-\frac{R_m v_0}{p_1}\frac{h_1}{L}\right)\right\}}{R_m v_0}\right], \qquad \text{(35)}$$

$$\phi_2 = \left(\frac{\sigma_{w2}}{\sigma}\right)\left[\frac{p_2\left(\exp\frac{R_m v_0}{p_2}\frac{h_2}{L}-1\right)}{R_m v_0}\right] \qquad \text{(36)}$$

where

$$p_1 = \frac{\eta_{w1}}{\nu} \quad \text{and} \quad p_2 = \frac{\eta_{w2}}{\nu}.$$

Solving the boundary value problem (30) to (34) we get

$$\left(\frac{u}{\nu L^{-1}}\right)\frac{1}{\overline{P}} = \frac{\frac{a}{\alpha}e^{\alpha\xi}+\frac{b}{\beta}e^{\beta\xi}-R_m v_0\xi+g}{R_m^2(1-v_0^2)} \qquad \text{(37)}$$

and

$$\left(\frac{H_x}{H_y}\right)\frac{R_m^2}{\overline{P}} = \frac{\frac{a}{\alpha}(R_m v_0-\alpha)e^{\alpha\xi}+\frac{b}{\beta}(R_m v_0-\beta)e^{\beta\xi}-R_m^2\xi+f}{R_m^2(1-v_0^2)} \qquad \text{(38)}$$

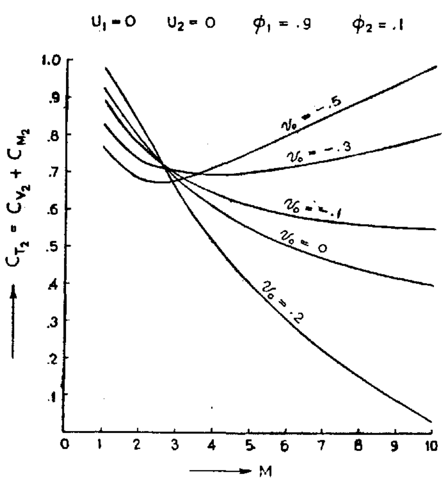

FIG. 4. Coefficient of total drag C_{T2} versus Hartmann number M for different values of suction parameter v_0.

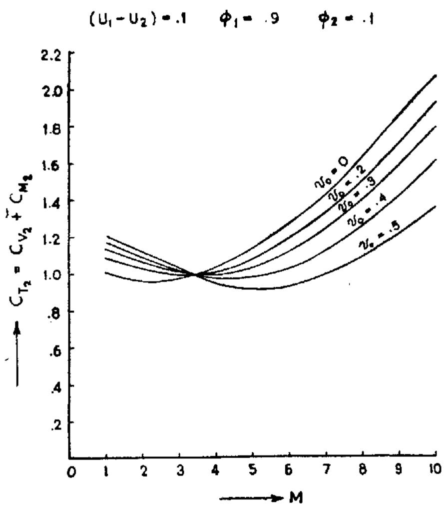

FIG. 5. Coefficient of total drag C_{T2} versus Hartmann number M for different values of suction parameter v_0.

where a, b, f, g are given by

$$\frac{a}{\alpha}\sinh\alpha = R_m v_0 - \frac{b}{\beta}\sinh\beta + \frac{(U_2-U_1)}{2}R_m^2(1-v_0^2) \quad \ldots \quad (39)$$

$$g\sinh\alpha = -R_m v_0\cosh\alpha - \frac{b}{\beta}\sinh(\alpha-\beta) + \tfrac{1}{2}(U_1 e^{\alpha} - U_2 e^{-\alpha})R_m^2(1-v_0^2), \quad (40)$$

$$\begin{aligned} f\sinh\alpha = \frac{b}{\beta}\big[&(\alpha-R_m v_0)(\alpha\phi_1-1)e^{-\alpha}\sinh\beta \\ &+(R_m v_0-\beta)(\beta\phi_1-1)e^{-\beta}\sinh\alpha\big] \\ &-\Big[\Big\{R_m v_0 + \frac{(U_2-U_1)}{2}R_m^2(1-v_0^2)\Big\}(\alpha-R_m v_0)(\alpha\phi_1-1)e^{-\alpha} \\ &+R_m^2(\phi_1+1)\sinh\alpha\Big] \quad \ldots \quad (41) \end{aligned}$$

$$\frac{b}{\beta} = \frac{\Big[\Big\{R_m v_0 + \frac{(U_2-U_1)R_m^2(1-v_0^2)}{2}\Big\}(\alpha-R_m v_0)\{\alpha(\phi_1 e^{-\alpha}+\phi_2 e^{\alpha})+2\sinh\alpha\} + R_m^2(\phi_1+\phi_2+2)\sinh\alpha\Big]}{\big[(\alpha-R_m v_0)\sinh\beta\{\alpha(\phi_1 e^{-\alpha}+\phi_2 e^{\alpha})+2\sinh\alpha\} + (R_m v_0-\beta)\sinh\alpha \times\{\beta(\phi_1 e^{-\beta}+\phi_2 e^{\beta})+2\sinh\beta\}\big]} \quad (42)$$

and α, β are the roots (evidently real) of the quadratic equation

$$\lambda^2 - R_m v_0\left(1+\frac{M^2}{R_m^2}\right)\lambda - M^2(1-v_0^2) = 0 . \quad \ldots \quad (43)$$

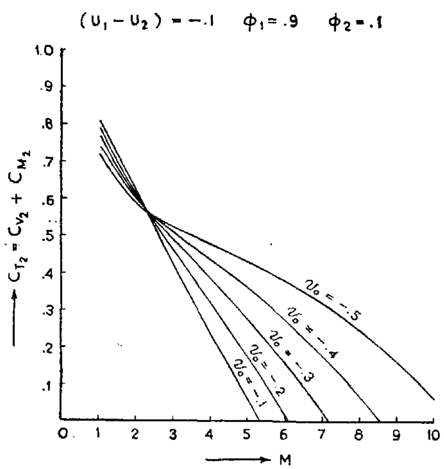

FIG. 6. Coefficient of total drag C_{T_2} versus Hartmann number M for different values of suction parameter v_0.

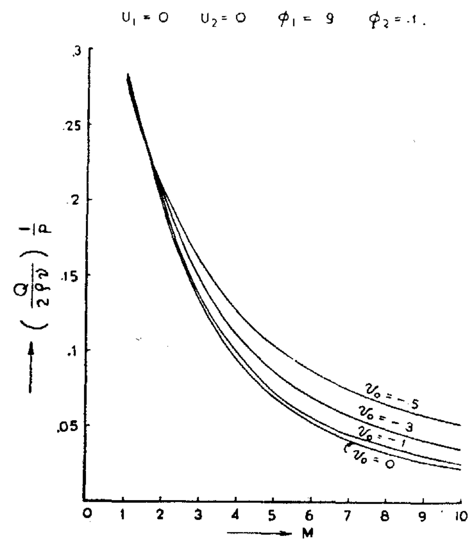

FIG. 7. Mass flow rate Q versus Hartmann number M for different values of suction parameter v_0.

The coefficients of viscous drag and magnetic drag defined by

$$C_V = -\left[\rho_\nu\left(\frac{\partial u}{\partial y}\right)_w \frac{1}{\rho\nu^2 L^{-2}}\right]\frac{1}{\bar{P}}, \qquad (44)$$

$$C_M = -\left[\frac{1}{4\pi}(\mu_e H_x H_y)_w \frac{1}{\rho\nu^2 L^{-2}}\right]\frac{1}{\bar{P}}, \qquad (45)$$

lead to

$$C_{V_1} = \frac{(ae^{-\alpha}+be^{-\beta}-R_m v_0)}{R_m^2(1-v_0^2)}, \qquad (46)$$

$$C_{V_2} = -\frac{ae^{\alpha}+be^{\beta}-R_m v_0}{R_m^2(1-v_0^2)}, \qquad (47)$$

$$C_{M_1} = \frac{\frac{a}{\alpha}(R_m v_0-\alpha)e^{-\alpha}+\frac{b}{\beta}(R_m v_0-\beta)e^{-\beta}+R_m^2+f}{R_m^2(1-v_0^2)}, \qquad (48)$$

$$C_{M_2} = -\frac{\frac{a}{\alpha}(R_m v_0-\alpha)e^{\alpha}+\frac{b}{\beta}(R_m v_0-\beta)e^{\beta}-R_m^2+f}{R_m^2(1-v_0^2)}. \qquad (49)$$

The mass flow rate is found to be

$$Q = \int_{-L}^{L} \rho u\, dy = \frac{2\rho\bar{P}\left[\frac{a}{\alpha^2}\sinh\alpha+\frac{b}{\beta^2}\sinh\beta+g\right]}{R_m^2(1-v_0^2)}. \qquad (50)$$

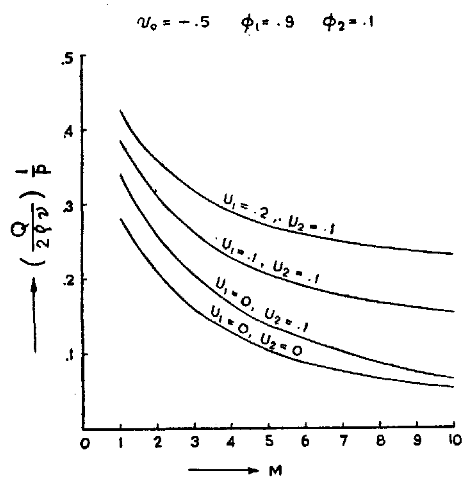

FIG. 8. Mass flow rate Q versus Hartmann number M for different wall velocities.

The physical parameters have been calculated for various values of M, v_0, U_1, U_2, ϕ_1 and ϕ_2 and are shown in Tables I to III and Figs. 2 to 8, which have been prepared on the Electronic Digital Computer installed at Hindustan Aircraft Limited, Bangalore.

TABLE I

Mass flow rate coefficient $= \dfrac{Q}{2\rho\nu\bar{P}}$

(a) For $U_1 = 0, \quad U_2 = 0, \quad v_0 = -0{\cdot}2$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	0·3118	0·2671	0·2236	0·1885	0·1615	0·1407
0·9	0·1	0·2837	0·2020	0·1419	0·1033	0·0785	0·0618
9	1	0·2526	0·1551	0·1001	0·0701	0·0523	0·0409

(b) For $U_1 = 0{\cdot}2, \quad U_2 = 0{\cdot}1, \quad v_0 = 0{\cdot}3$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	0·4664	0·4235	0·3815	0·3475	0·3210	0·3002
0·9	0·1	0·4393	0·3587	0·2987	0·2607	0·2366	0·2209
9	1	0·4031	0·2979	0·2424	0·2150	0·2005	0·1924

TABLE II

Viscous drag coefficient C_{V_1} *at the lower wall*

(a) For $(U_1 - U_2) = 0, \quad v_0 = -0{\cdot}2$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	1·0804	1·1114	1·1122	1·1058	1·0997	1·0950
0·9	0·1	0·9785	0·8194	0·6643	0·5476	0·4625	0·3988
9	1	0·8659	0·6086	0·4356	0·3296	0·2614	0·2147

(b) For $(U_1 - U_2) = 0{\cdot}1, \quad v_0 = 0{\cdot}3$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	0·8332	0·7671	0·7373	0·7164	0·6949	0·6714
0·9	0·1	0·7615	0·5860	0·4699	0·3870	0·3226	0·2702
9	1	0·6659	0·4162	0·2880	0·2137	0·1637	0·1262

TABLE III

Viscous drag coefficient C_{V_2} *at the upper wall*

(*a*) For $(U_1 - U_2) = 0$, $v_0 = -0\cdot2$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	0·9196	0·8886	0·8878	0·8942	0·9003	0·9050
0·9	0·1	0·8403	0·6882	0·5926	0·5300	0·4855	0·4520
9	1	0·7526	0·5436	0·4419	0·3878	0·3546	0·3321

(*b*) For $(U_1 - U_2) = 0\cdot1$, $v_0 = 0\cdot3$

ϕ_1	ϕ_2 \ M	1	2	3	4	5	6
0	0	1·2092	1·3178	1·3900	1·4533	1·5172	1·5832
0·9	0·1	1·1049	0·9983	0·8872	0·8229	0·8013	0·8109
9	1	0·9658	0·6989	0·5451	0·4912	0·4957	0·5338

4. Conclusions and Discussion

Chang and Yen (1962) have discussed the effect of wall conductance on the flow parameters for the case of fully developed Poiseuille flow. Here in this paper for the magnetic Prandtl number $p_{rm} = 2$ we have examined the effect of (i) wall conductance, (ii) the motion of the walls and (iii) suction or injection on the physical parameters in the generalized hydromagnetic Couette flow.

The following conclusions can be obtained:

(1) From the equations (37) to (50) it is seen that the viscous drag, the magnetic drag and the mass flow rate coefficients depend upon the individual values of the electrical conductance parameters ϕ_1 and ϕ_2 of the walls. The drag coefficients vary linearly with the relative velocity of the walls whereas the mass flow rate depends upon the individual values of U_1 and U_2.

(2) The numerical results in Tables I to III show that the effect of increase in wall conductance parameters ϕ_1 and ϕ_2 is to decrease the viscous drag and the mass flow rate.

(3) The total drag, when the walls are in relative motion ($U_1 > U_2$), is decreased at the lower wall and increases at the upper wall as compared with the case when the walls are at relative rest. However, when the upper wall moves with a velocity greater than that of the lower wall ($U_2 > U_1$) the total drag at the upper wall is decreased (Figs. 2 to 6).

(4) Increase in suction increases the total drag at the lower wall for small values of M and decreases it for large values of M. The value of M at which this change takes place appears to be 3 roughly. The reverse is true, as could be expected, for the total drag at the upper wall. The results for injection are just opposite to that of suction (Figs. 2 to 6).

(5) The mass flow rate increases with the increase in suction or with increase in U_1 and U_2 (Figs. 7 and 8).

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