

HEAT TRANSFER BY LAMINAR MOTION OF AN ELASTICO-VISCOUS LIQUID BETWEEN TWO COAXIAL CIRCULAR CYLINDERS DUE TO LONGITUDINAL MOTION OF THE INNER CYLINDER

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The problem of heat transfer by the laminar flow of an elastico-viscous liquid through the annular space between two coaxial circular cylinders has been studied when the inner cylinder moves longitudinally with a uniform velocity and the outer cylinder is at rest. The flow phenomenon has been characterized by the non-dimensional numbers like elastic number R_e , radii ratio σ and Brinkman number B_r , and the effects of these numbers on velocity distribution, temperature field, shearing stresses at the walls, rate of heat transfer from the walls and the radial pressure variation have been studied.

1. INTRODUCTION

In recent years, the study of heat transfer by laminar flow of Newtonian and non-Newtonian liquids has gained considerable importance. Schlichting (1951) has discussed several cases of exact solutions of heat transfer in viscous liquids. Jain (1963) studied the problem of heat transfer by laminar flow of elastico-viscous liquids through parallel walls when both the walls are at uniform temperatures and Mishra (1965a) extended the problem to the case when the walls are at linearly varying temperature. Both Jain (1963) and Mishra (1965a) obtained an approximate solution of the differential equation governing the velocity field by considering the elasticity of the liquid to be small. Bhatnagar (1966) studied the same problem considered by Mishra (1965a) and obtained an exact solution of the differential equation governing the velocity field. Again, Mishra (1965b) studied the problem of heat transfer by the laminar flow of an elastico-viscous liquid through a circular tube when the wall is having linearly varying temperature and obtained an approximate solution of the differential equation for the velocity field. Šesták (1967) has made a few remarks on Mishra's (1965b) paper. Tikekar (1967) has resolved Mishra's (1965b) problem and following Bhatnagar's (1966) method of solution

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has given an exact solution of the differential equation for the velocity field. Recently, Mishra and Sinha Roy (1968) have studied the problem of heat transfer by the rotatory flow of an elastico-viscous liquid between two coaxial circular cylinders. The aim of this paper is to study the problem of heat transfer by the laminar flow of elastico-viscous liquid through the annular space between two coaxial circular cylinders when the inner cylinder moves longitudinally with a uniform velocity and the outer is at rest. The constitutive equations of elastico-viscous fluid model considered here are

$$p_{ij} + \lambda_1 \tilde{p}_{ij} = 2\mu d_{ij} + 4\mu_c d_{i\alpha} d_{\alpha j} \quad \dots \quad (1)$$

where p_{ij} is the stress tensor, λ_1 the elastic parameter having the dimension of time, μ and μ_c respectively the coefficients of viscosity and cross-viscosity, d_{ij} the rate-of-strain tensor given by

$$2d_{ij} = v_{i,j} + v_{j,i}$$

v^i being the velocity vector and the term $\tilde{p}_{ij}$ appearing in (1) is the rate of stress tensor and in contravariant form is given by

$$\tilde{p}^{ij} = \frac{\partial p^{ij}}{\partial t} + p^{ij}_{,k} v^k - p^{ik} v^j_{,k} - p^{kj} v^i_{,k} + p^{ij} v^k_{,k}.$$

The equations of motion and continuity are respectively

$$\rho \left[\frac{\partial v^i}{\partial t} + v^j v^i_{,j} \right] = -p_{,i} + p^{ij}_{,j} \quad \dots \quad (2)$$

and

$$v_{i,i} = 0 \quad \dots \quad (3)$$

where ρ is the density of the medium and p is an arbitrary isotropic pressure.

The heat energy equation is

$$\rho c \frac{DT}{Dt} = k \nabla^2 T + \phi \quad \dots \quad (4)$$

where T is the temperature, c and k are respectively the specific heat capacity and thermal conductivity with

$$\phi = p_{ij} d_{ij}$$

and

$$\begin{aligned} \frac{D}{Dt} &= \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} + \frac{v}{r} \frac{\partial}{\partial \theta} + w \frac{\partial}{\partial z} \\ \nabla^2 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \end{aligned}$$

are dissipation function, material derivative and Laplace operator respectively.

2. FORMULATION OF THE PROBLEM

We shall consider the steady state flow of an elastico-viscous liquid contained between two concentric cylinders of radii a and b ($a > b$), when the

inner cylinder is moving with a uniform longitudinal velocity, the cylinders being assumed to be infinitely long. If we select cylindrical polar coordinates with z -axis along the axis of the tubes, the velocity field consistent with the continuity condition (3) can be taken as

$$v_r = 0; \quad v_\theta = 0; \quad v_z = w(r). \quad \dots \quad (5)$$

The only surviving rate-of-strain component is

$$d_{rz} = \frac{1}{2} \frac{dw}{dr} \quad \dots \quad (6)$$

and the non-zero stress components are

$$p_{rr} = \mu_c \left(\frac{dw}{dr} \right)^2 \quad \dots \quad (7)$$

$$p_{rz} = \mu \frac{dw}{dr} + \lambda_1 \mu_c \left(\frac{dw}{dr} \right)^3 \quad \dots \quad (8)$$

$$p_{zz} = (\mu_c + 2\lambda_1 \mu) \left(\frac{dw}{dr} \right)^2 + 2\lambda_1^2 \mu_c \left(\frac{dw}{dr} \right)^4. \quad \dots \quad (9)$$

The momentum equations under the present physical situation are

$$\frac{dp}{dr} = \frac{dp_{rr}}{dr} + \frac{1}{r} p_{rr} \quad \dots \quad (10)$$

$$\frac{\partial p}{\partial z} = \frac{dp_{rz}}{dr} + \frac{1}{r} p_{rz}. \quad \dots \quad (11)$$

Since the flow takes place due to shear only, that is due to the uniform motion of the inner cylinder, we have

$$\frac{\partial p}{\partial z} = 0$$

and, therefore, from (11) we get

$$\frac{d}{dr} (r p_{rz}) = 0$$

which on integration gives

$$p_{rz} = \frac{C_1}{r} \quad \dots \quad (12)$$

where C_1 is a constant of integration. From eqns. (7) and (10) we have

$$\frac{dp}{dr} = \frac{\mu_c}{r} \frac{d}{dr} \left\{ r \left(\frac{dw}{dr} \right)^2 \right\} \quad \dots \quad (13)$$

and from eqns. (8) and (12) we get

$$\mu \frac{dw}{dr} + \lambda_1 \mu_c \left(\frac{dw}{dr} \right)^3 = \frac{C_1}{r}. \quad \dots \quad (14)$$

The heat energy eqn. (4) is now reduced to

$$k \left[\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right] + \frac{C_1}{r} \frac{dw}{dr} = 0. \quad \dots \dots \dots (15)$$

If the inner cylinder ($r = b$) moves with a uniform velocity W and is at temperature T_b and the outer cylinder ($r = a$) is at rest and is at the temperature T_a , then the boundary conditions of the problem are

$$\left. \begin{aligned} r = b: w = W \text{ and } T = T_b \\ r = a: w = 0 \text{ and } T = T_a \end{aligned} \right\} \quad \dots \dots \dots (16)$$

We now introduce the non-dimensional parameters:

$$\begin{aligned} \bar{w} &= w/W, & \text{dimensionless velocity;} \\ \eta &= r/a, & \text{dimensionless radial distance;} \\ \sigma &= b/a \quad (0 < \sigma < 1), & \text{radii ratio;} \\ R_c &= \frac{\lambda_1 \mu_c W^2}{\mu a^2}, & \text{elastic number;} \\ \nu_c &= \frac{\mu_c}{\rho a^2}, & \text{cross-viscous number;} \\ P &= \frac{p}{\rho W^2}, & \text{dimensionless pressure;} \\ A &= \frac{C_1}{\mu W}, & \text{dimensionless constant of integration;} \\ \theta &= \frac{T - T_b}{T_a - T_b}, & \text{dimensionless temperature;} \\ Br &= P_r E = \frac{\mu W^2}{k(T_a - T_b)}, & \text{Brinkman number;} \\ P_r &= \frac{C \mu}{k}, & \text{Prandtl number;} \\ E &= \frac{W^2}{C(T_a - T_b)}. & \text{Eckert number;} \end{aligned}$$

the eqns. (13), (14) and (15) reduce to (omitting bars over $\bar{w}$ for convenience)

$$\frac{dP}{d\eta} = \frac{\nu_c}{\eta} \frac{d}{d\eta} \left\{ \eta \left(\frac{dw}{d\eta} \right)^2 \right\} \quad \dots \dots \dots (17)$$

$$\frac{dw}{d\eta} + R_c \left(\frac{dw}{d\eta} \right)^3 = \frac{A}{\eta} \quad \dots \dots \dots (18)$$

$$\eta^2 \frac{d^2 \theta}{d\eta^2} + \eta \frac{d\theta}{d\eta} + A Br \eta \frac{dw}{d\eta} = 0. \quad \dots \dots \dots (19)$$

The boundary conditions (16) now reduce to

$$\left. \begin{aligned} \eta = \sigma: w = 1 \text{ and } \theta = 0 \\ \eta = 1: w = 0 \text{ and } \theta = 1 \end{aligned} \right\} \quad \dots \dots \dots (20)$$

3. SOLUTION OF EQUATIONS

Equation (18) can be solved in a closed form in the following way. Writing

$$s = \frac{dw}{d\eta}$$

eqn. (18) can be written in the form

$$s + R_c s^3 = \frac{A}{\eta} \quad \dots \quad \dots \quad \dots \quad (21)$$

Differentiating (21) with respect to w , we get

$$\frac{ds}{dw} [1 + 3R_c s^2] = -\frac{s}{A} [1 + R_c s^2]^2 \quad \dots \quad \dots \quad (22)$$

which gives

$$dw = -A \left[\frac{1}{s} - \frac{R_c s}{1 + R_c s^2} \right] ds - \frac{2AR_c s ds}{(1 + R_c s^2)^2} \quad \dots \quad \dots \quad (23)$$

Integration of (23) gives

$$w = A \log \frac{\sqrt{1 + R_c s^2}}{s} + \frac{A}{1 + R_c s^2} + B \quad \dots \quad \dots \quad (24)$$

where B is another constant of integration. Eliminant of s between eqns. (21) and (24) gives the solution of (18). The constants A and B can be evaluated with the help of boundary conditions (20). It can be easily shown that for a Newtonian liquid

$$\text{and} \quad \left. \begin{aligned} R_c &= 0 \\ w &= \frac{\log \eta}{\log \sigma} \end{aligned} \right\} \quad \dots \quad \dots \quad \dots \quad (25)$$

But this form of solution is very difficult to be tackled for a physical problem. So we obtain an approximate solution of eqn. (18) by adopting a small perturbation method.

Perturbation Method

We consider a realistic assumption that the elastic number $R_c \ll 1$ and expand w and the constant A in eqn. (18) in an ascending power of R_c in the forms

$$w = w_0 + R_c w_1 + R_c^2 w_2 + \dots \quad \dots \quad \dots \quad (26)$$

$$A = A_0 + R_c A_1 + R_c^2 A_2 + \dots \quad \dots \quad \dots \quad (27)$$

Substituting the assumed expressions for w and A into eqn. (18) and equating the coefficients of like powers of R_c , we have

zeroth-order equation

$$\frac{dw_0}{d\eta} - \frac{A_0}{\eta} = 0 \quad \dots \quad \dots \quad \dots \quad (28)$$

first-order equation

$$\frac{dw_1}{d\eta} + \left(\frac{dw_0}{d\eta} \right)^3 - \frac{A_1}{\eta} = 0 \quad \dots \quad \dots \quad (29)$$

second-order equation

$$\frac{dw_2}{d\eta} + 3 \left(\frac{dw_0}{d\eta} \right)^2 \frac{dw_1}{d\eta} - \frac{A_2}{\eta} = 0 \quad \dots \quad (30)$$

and so on. The boundary conditions on w_0 , w_1 and w_2 , etc., are

$$\left. \begin{aligned} \eta = \sigma: w_0 = 1, w_1 = w_2 = \dots = 0 \\ \eta = 1: w_0 = w_1 = w_2 = \dots = 0 \end{aligned} \right\} \quad \dots \quad (31)$$

Solving the eqns. (28)–(30) with the boundary conditions (31) we obtain zeroth-order solution

$$w^{(0)} = \frac{\log \eta}{\log \sigma} \quad \dots \quad (32)$$

$$A^{(0)} = \frac{1}{\log \sigma} \quad \dots \quad (33)$$

first-order solution

$$w^{(1)} = \frac{\log \eta}{\log \sigma} + \frac{R_c}{2(\log \sigma)^3} \left[\frac{1-\eta^2}{\eta^2} + \frac{(\sigma^2-1)}{\sigma^2} \frac{\log \eta}{\log \sigma} \right] \quad \dots \quad (34)$$

$$A^{(1)} = \frac{1}{\log \sigma} - \frac{R_c(1-\sigma^2)}{2\sigma^2(\log \sigma)^4} \quad \dots \quad (35)$$

second-order solution

$$\begin{aligned} w^{(2)} = & \frac{\log \eta}{\log \sigma} + \frac{R_c}{2(\log \sigma)^3} \left[\frac{1-\eta^2}{\eta^2} + \frac{(\sigma^2-1)}{\sigma^2} \frac{\log \eta}{\log \sigma} \right] \\ & + \frac{3R_c^2}{4(\log \sigma)^5} \left[\frac{\eta^4-1}{\eta^4} + \frac{(1-\sigma^2)(\eta^2-1)}{\eta^2\sigma^2 \log \sigma} \right. \\ & \left. + \left\{ \frac{(1-\sigma^2)^2}{\sigma^4 \log \sigma} + \frac{(1-\sigma^4)}{\sigma^4} \right\} \frac{\log \eta}{\log \sigma} \right] \quad \dots \quad (36) \end{aligned}$$

$$\begin{aligned} A^{(2)} = & \frac{1}{\log \sigma} - \frac{R_c(1-\sigma^2)}{2\sigma^2(\log \sigma)^4} \\ & + \frac{3R_c^2}{4(\log \sigma)^6} \left[\frac{(1-\sigma^2)^2}{\sigma^4 \log \sigma} + \frac{1-\sigma^4}{\sigma^4} \right]. \quad \dots \quad (37) \end{aligned}$$

In a similar manner the higher order perturbation solutions can be obtained, but the algebra involved will naturally be very complicated. But, since we have restricted our discussions to very short relaxation time (small elastic number), we retain only up to the terms containing squares of R_c . Hence from (26) and (27)

$$w^{(2)} = w_0 + R_c w_1 + R_c^2 w_2 + O(R_c^3) \quad \dots \quad (38)$$

$$A^{(2)} = A_0 + R_c A_1 + R_c^2 A_2 + O(R_c^3). \quad \dots \quad (39)$$

Substituting the second order perturbation solutions for w ($= w^{(2)}$) and A ($= A^{(2)}$) into the energy equation (19) we have

$$\begin{aligned} \eta^2 \frac{d^2 \theta}{d\eta^2} + \eta \frac{d\theta}{d\eta} = & -A^{(2)} B_r \left[\frac{1}{\log \sigma} - \frac{R_c}{2(\log \sigma)^3} \left\{ \frac{2}{\eta^2} + \frac{(1-\sigma^2)}{\sigma^2 \log \sigma} \right\} \right. \\ & + \frac{3R_c^2}{4(\log \sigma)^5} \left\{ \frac{4}{\eta^4} + \frac{2(1-\sigma^2)}{\eta^2 \sigma^2 \log \sigma} \right. \\ & \left. \left. + \frac{1-\sigma^4}{\sigma^4 \log \sigma} + \frac{(1-\sigma^2)^2}{\sigma^4 (\log \sigma)^2} \right\} \right]. \quad \dots \dots \dots (40) \end{aligned}$$

The solution of (40) subject to the boundary conditions (20) on temperature distribution is given by

$$\begin{aligned} \theta = & D + C \log \eta - \frac{1}{2} A^{(2)} B_r \left[\frac{(\log \eta)^2}{\log \sigma} - \frac{R_c}{2(\log \sigma)^3} \left\{ \frac{1}{\eta^2} + \frac{1-\sigma^2}{\sigma^2} \frac{(\log \eta)^2}{\log \sigma} \right\} \right. \\ & + \frac{3R_c^2}{4(\log \sigma)^5} \left\{ \frac{1}{2\eta^4} + \frac{1-\sigma^2}{\eta^2 \sigma^2 \log \sigma} + \frac{(1-\sigma^4)(\log \eta)^2}{\sigma^4 \log \sigma} \right. \\ & \left. \left. + \frac{(1-\sigma^2)^2 (\log \eta)^2}{\sigma^4 (\log \sigma)^2} \right\} \right] \quad \dots \dots \dots (41) \end{aligned}$$

where

$$D = 1 + \epsilon_1$$

$$C = -\frac{1}{\log \sigma} + \epsilon_2$$

$$\epsilon_1 = \frac{1}{2} A^{(2)} B_r \left[-\frac{R_c}{2(\log \sigma)^3} + \frac{3R_c^2}{4(\log \sigma)^5} \left\{ \frac{1}{2} + \frac{1-\sigma^2}{\sigma^2 \log \sigma} \right\} \right]$$

$$\begin{aligned} \epsilon_2 = & \frac{1}{2} A^{(2)} B_r \left[1 - \frac{R_c(1-\sigma^2) \log(e\sigma)}{2\sigma^2 (\log \sigma)^4} + \frac{3R_c^2(1-\sigma^2)}{4\sigma^4 (\log \sigma)^5} \left\{ \frac{1}{2}(1+\sigma^2) \log(e\sigma^2) \right. \right. \\ & \left. \left. + \frac{(1-\sigma^2)}{\log \sigma} \log(e\sigma) \right\} \right]. \end{aligned}$$

If the walls are at equal temperature $T_a = T_b$, we define

$$\theta = \frac{T - T_a}{T_a} \quad \text{and} \quad B_r = \frac{\mu W^2}{k T_a},$$

and solve eqn. (40) with the boundary conditions

$$\theta = 0 \quad \text{at} \quad \eta = \sigma \quad \text{and} \quad \eta = 1.$$

We get the same solution (41), but the constants C and D are given by

$$C = \epsilon_2 \quad \text{and} \quad D = \epsilon_1.$$

The shearing stress at any cross-section of the annulus is obtained from eqns. (8), (38) and (39) and can be put in the form

$$\frac{a p_{rz}}{\mu W} = \frac{1}{\eta \log \sigma} \left[1 - \frac{(1-\sigma^2) R_c}{2\sigma^2 (\log \sigma)^3} + \frac{3(1-\sigma^2) R_c^2}{4\sigma^4 (\log \sigma)^5} \left\{ \frac{1-\sigma^2}{\log \sigma} + 1 + \sigma^2 \right\} \right]. \quad \dots \quad (42)$$

The skin-frictions at the walls τ_{out} and τ_{in} are obtained from (42) by putting $\eta = 1$ and $\eta = \sigma$ respectively. The radial pressure variation at any cross-section is obtained by integrating eqn. (17) and it is reduced to the form

$$P = P(\eta) - P(1) = \nu_c \left[\frac{1-\eta^2}{2\eta^2(\log \sigma)^2} - \frac{R_c}{2(\log \sigma)^4} \left\{ \frac{3(1-\eta^4)}{\eta^4} + \frac{(1-\sigma^2)(1-\eta^2)}{\eta^2\sigma^2 \log \sigma} \right\} \right. \\ \left. + \frac{R_c^2}{4(\log \sigma)^6} \left\{ \frac{70(1-\eta^6)}{3\eta^6} + \frac{12(1-\sigma^2)(1-\eta^4)}{\eta^4\sigma^2 \log \sigma} \right. \right. \\ \left. \left. + \frac{7(1-\eta^2)}{2\eta^2} \left(\frac{1-\sigma^2}{\sigma^2 \log \sigma} \right)^2 + \frac{3(1-\sigma^4)(1-\eta^2)}{\eta^2\sigma^4 \log \sigma} \right\} \right]. \quad \dots (43)$$

The rate of heat transfer from the outer cylinder denoted by q_0 is given by

$$q_0 = \left(\frac{d\theta}{d\eta} \right)_{\eta=1} = C - \frac{1}{2} A^{(2)} B_r \epsilon_3 \quad \dots \quad (44)$$

and the rate of heat transfer from the inner cylinder q_i is given by

$$q_i = \left(\frac{d\theta}{d\eta} \right)_{\eta=\sigma} = \frac{C}{\sigma} - \frac{1}{2\sigma} A^{(2)} B_r [2 + \epsilon_3] \quad \dots \quad (45)$$

where

$$\epsilon_3 = \frac{R_c}{(\log \sigma)^3} - \frac{3R_c^2}{2(\log \sigma)^5} \left\{ 1 + \frac{(1-\sigma^2)}{\sigma^2 \log \sigma} \right\}.$$

4. DISCUSSION OF RESULTS

In this paper the problem of heat transfer by the laminar flow of an elastico-viscous liquid through the annular space between two coaxial circular cylinders has been studied when the inner cylinder moves longitudinally with a uniform velocity and the outer cylinder is at rest. The flow phenomenon has been characterized by the non-dimensional numbers like elastic number R_c , radii ratio σ , and Brinkman number B_r and the effects of these numbers on velocity distribution, temperature field, shearing stresses at the walls, rate of heat transfer from the walls and the radial pressure variation have been studied. For drawing the graphs we have used a normalized distance variable

$$\lambda = \frac{\eta - \sigma}{1 - \sigma}$$

such that as η varies from σ to 1; the variable λ varies from zero to one.

(i) *Cross-viscous coefficient and the elastic parameter*—From eqn. (18) it is quite clear that the velocity field is affected by the elasticity of the liquid only when the liquid possesses cross-viscosity and vice versa. But eqn. (17) shows that cross-viscosity modifies the radial pressure field even in the absence of elasticity of the liquid. Equations (38), (42), (44) and (45) show that the temperature distribution, shearing stress at the walls, and the rate of heat

transfer from the walls are affected by the elasticity of the liquid only if the liquid possesses cross-viscosity.

(ii) *Velocity field*—A small perturbation technique has been used to obtain an approximate solution for the velocity field for small values of the elastic number. Fig. 1 represents the velocity distribution for different

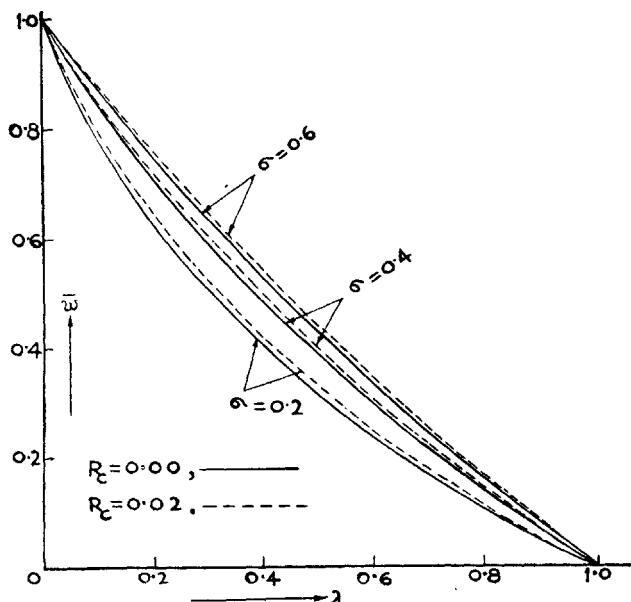


FIG. 1. Velocity distribution for different values of radii ratio and the elastic number.

values of the radii ratio and the elastic number. Fig. 1 shows that the velocity at any point in the annulus increases with the increase in the value of σ , the radii ratio and the elastic elements in the viscous liquid increase it further. As the gap between the cylinders becomes small, that is σ is nearer to 1, the velocity profile approaches to be a linear distribution.

(iii) *Temperature field*—Fig. 2 represents the temperature distribution when the walls are at unequal temperature. It shows that when the Brinkman number is zero, the elasticity of the liquid does not affect the temperature field. This is also clear from eqn. (19) where the Brinkman number is coupled with the expression containing the elastic part. The temperature at any point of the annulus increases with the increase in the values of the Brinkman number and for non-zero Brinkman number, the elasticity of the liquid increases the temperature rise at any point. The temperature profile gradually increases in the annulus till the Brinkman number is $0 \leq Br < 4$, but for $Br \geq 4$ the peak appears nearly at the middle of the annulus. The elasticity of the liquid does not affect the position where the peak appears.

In Fig. 3 are drawn the curves for temperature distribution when the walls are at equal temperature. Fig. 3 shows that when $B_r = 0$, the entire annulus is at the same temperature and the elasticity of the liquid does not affect the temperature. When $B_r > 0$, the temperature at any point increases with B and the elasticity of the liquid increases it further. When the walls

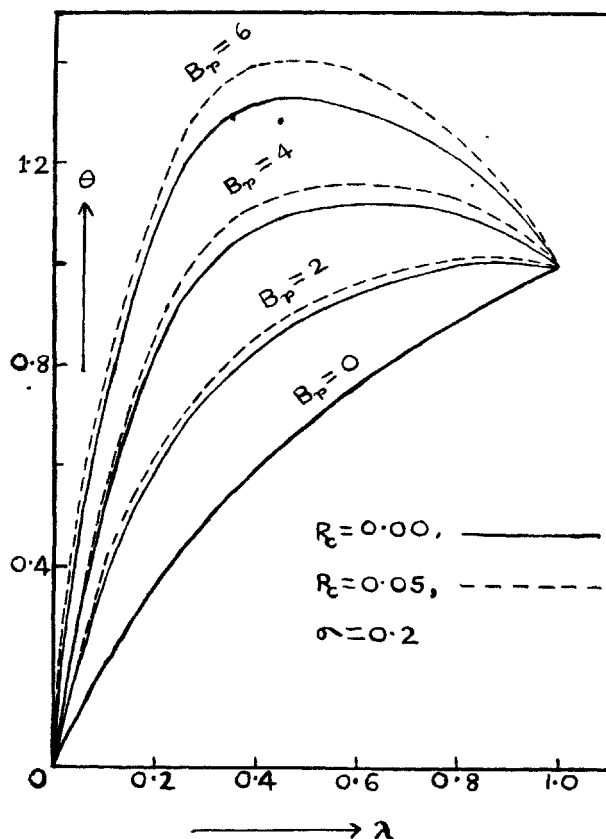


FIG. 2. Temperature distribution for different values of Brinkman number and elastic number (walls are at unequal temperature).

are at equal temperature, from Fig. 3 it is clear that the point of maximum temperature is nearer to the inner wall and for all values of B_r , this point remains the same.

(iv) *Shearing stresses at the walls*—Fig. 4 represents the shearing stresses at the outer and inner (τ_0 and τ_1 respectively) cylinders. For small values of the elastic number the shearing stresses at both the cylinders increase with the elastic number and then decrease.

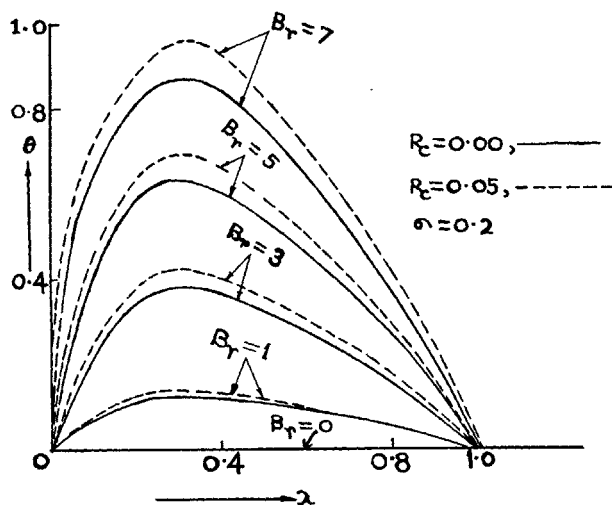


FIG. 3. Temperature distribution for different values of Brinkman number and elastic number (walls are at equal temperature).

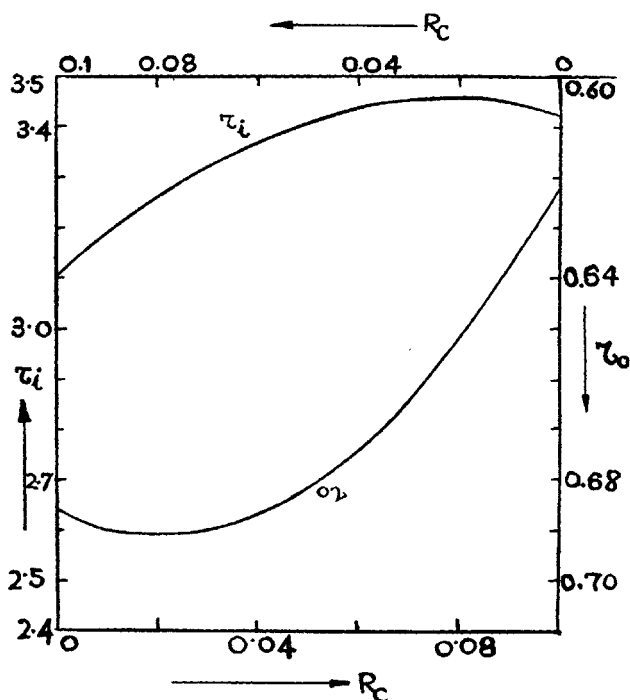


FIG. 4. Shearing stress for different values of the elastic number (radii ratio is 0.2).

(v) *Rate of heat transfer from the cylinders*—Fig. 5 represents the rate of heat transfer from the cylinders for different values of Brinkman number

and elastic number. The rate of heat transfer from the inner cylinder increases linearly with the increase in the value of the Brinkman number and the elasticity of the liquid increases it. The rate of heat transfer from the outer cylinder decreases linearly with the increase in the value of the Brinkman number and the elasticity of the liquid decreases it further. For the viscous liquid cooling of the outer cylinder occurs when $Br \geq 2$, but the elasticity of the liquid reduces this value.

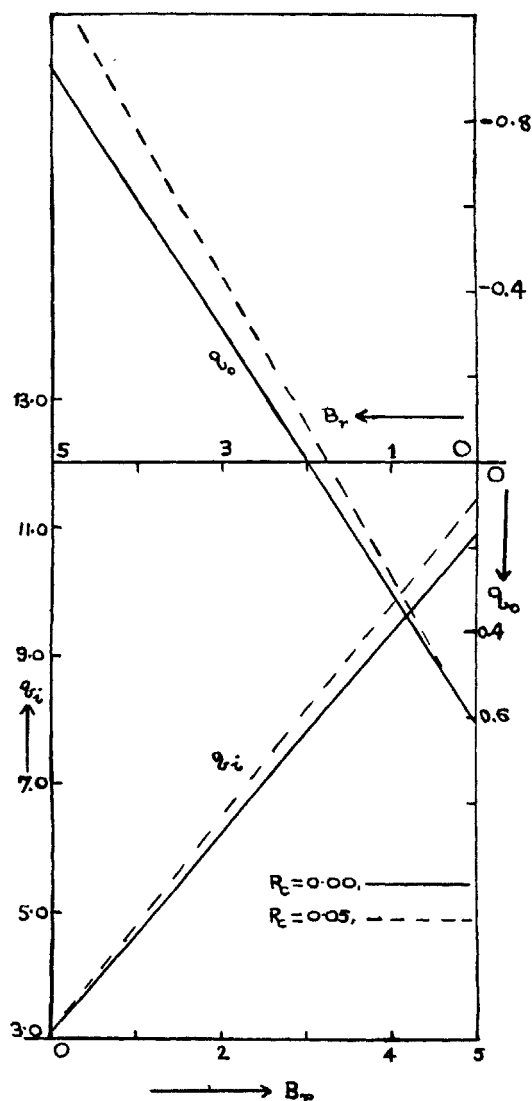


FIG. 5. Rate of heat transfer for different values of Brinkman number and elastic number (radii ratio is 0.2).

(vi) *Radial pressure variation*—The radial pressure variation is plotted in Fig. 6 which shows that this decreases gradually as the outer cylinder is approached. For a viscous liquid, at the inner cylinder $\bar{P} = P(\sigma) - P(1) = 2.3147$. The effect of elasticity of the liquid is to decrease the pressure in a liquid layer near the inner cylinder and to increase it to a small extent as the outer cylinder is approached.

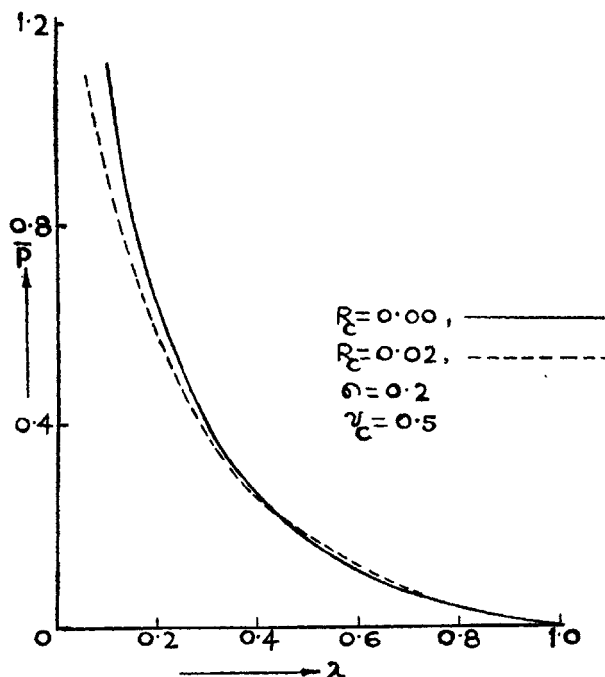


FIG. 6. Radial pressure variation for different values of elastic number (radii ratio is 0.2).

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