SCATTERING OF PULSES OF RADIATION BY BOUND ELECTRONS INTO THEIR NATURAL FREQUENCY

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Scattering of pulses of radiation by bound electrons into their natural frequency is studied. If the natural frequency of the electrons is close to the mean frequency of incident radiation, the effect should be observable with the laser beams available these days.

It is well known that when electromagnetic waves are scattered by electrons harmonics (Blaton 1931, Sen Gupta 1952, Fried 1961, Vachaspati 1962, Brown and Kibble 1964, Fried and Eberly 1964, Bali and Dutt 1965, Dutt and Bali 1966, Prakash 1967, 1969, Prakash and Vachaspati 1967*a*, *b*) and combination frequencies (Fried and Frank 1964, Prakash and Vachaspati 1968*a*, *b*) are also produced. It has also been shown (Vachaspati 1964, Goldman 1964, Prakash 1967, Prakash and Vachaspati 1967*c*, 1968*c*, Prakash and Chandra, 1968*a*) that when a pulse of radiation falls on a free electron the electron experiences an impulse during the growth and decay of the pulse and gains, in effect, an average momentum proportional to the instantaneous intensity. This results in an intensity dependent frequency shift. Recently, the authors (Prakash and Chandra 1968*b*) have shown that if a pulse of radiation is incident on a bound electron, oscillations of the electron with its natural frequency are produced during the period when the amplitude of the pulse is changing. This gives rise to generation of natural frequency in scattering.

In these calculations, the authors considered an instantly growing pulse of radiation. Further investigations show that this mathematical idealization works well only for those physical pulses which grow in a time much less than the mean periodic time of the incident radiation. The pulses available at present do not have such a small growth time. This necessitates investigations for the case of slowly growing pulses of radiation. In this paper, we have studied this case. We have shown that this effect is observable with the laser beams available these days if the natural frequency of the electrons is close to the mean frequency of the incident radiation.

The generation of natural frequency can easily be understood by considering the distribution of frequencies in the incident pulse of radiation. If $I(\omega)$ $d\omega$ is the intensity in the frequency interval $d\omega$ at the frequency ω, the energy scattered per unit volume per second in this interval is $\sigma(\omega)\ I(\omega)\ d\omega$, where* $\sigma(\omega) = (8\pi/3)\ (e^2/m)^2\omega^4[(\omega^2-\Omega^2)^2+(2e^2/3m)^2\omega^6]^{-1}$ is the Rayleigh scattering—cross-section, Ω being the natural frequency of the bound electron. Obviously, if the incident pulses of radiation have the mean frequency $\bar{\omega}$, i.e. if $I(\omega)$ is peaked at $\omega = \bar{\omega}$, $\sigma(\omega)I(\omega)\ d\omega$ is peaked at both $\omega = \Omega$ and at $\omega = \bar{\omega}$. This means that if a pulse of radiation is incident on a bound electron, the natural frequency of the bound electron is also present in the scattered radiation. At $\omega \simeq \bar{\omega}$, $\sigma(\omega)\ I(\omega) \simeq \sigma(\bar{\omega})I(\omega)$. The intensity distribution in the scattered radiation, near $\omega = \bar{\omega}$, is therefore the same as in the incident radiation. At $\omega \simeq \Omega$ $\sigma(\omega)\ I(\omega) \simeq \sigma(\omega)I(\Omega)$. This shows that the intensity of the natural frequency in the scattered radiation is proportional to $I(\Omega)$. The emission of natural frequency will therefore be enhanced if $I(\Omega)$ is appreciable, i.e. if $\bar{\omega}$ is near Ω. Rayleigh did not get the natural frequency in scattering as he considered only the monochromatic waves (*see* Jackson 1966), for which $I(\Omega) = 0$.

Consider an electron bound to a harmonic oscillator potential of frequency ω_0 in the electromagnetic field represented by the vector potential, $A(x,\ t) = e_1\ A_0\ F(T)\ \cos(\bar{\omega}T+\chi)$, where $e_1 = (1,\ 0,\ 0)$, $T = t-x_3$, and $F(T)$ is the function representing the growth of the pulse ($F(T) = 0$ for $T \leqslant 0$, $0 < F(T) < 1$ for $0 < T < T_G$, $F(T) = 1$ for $T \geqslant T_G$). The equation of motion of the electron in linear approximation, viz. $\ddot{\xi}-b\dddot{\xi}+\omega_0{}^2\xi = (e/m)\ \dot{A}$, where dots denote partial differentiation with respect to t and $\xi \equiv \xi\ (x,\ t)$ is the displacement at time t of the electron whose centre of attraction is at x and $b = 2e^2/3m$, can be solved; it gives

$$
\begin{aligned}
\xi = e_1 g \int_{-\infty}^{\infty} dT' F(T')\ \cos\ (\bar{\omega}T'+\chi)[\Omega^{-1}\theta(T-T')e^{-\alpha(T-T')} \\
\times \mathrm{Re}\ \{(1+\alpha\beta-i\beta\Omega)^{-1}(\Omega+i\alpha)e^{i\Omega(T-T')}\} \\
+\{(1+\alpha\beta)^2+\beta^2\Omega^2\}^{-1}\theta(T'-T)e^{(1/\beta)(T-T')}] \qquad \ldots \quad \ldots \quad \ldots \quad (1)
\end{aligned}
$$

where $g = eA_0/m$, θ is the unit step function, Re denotes the real part, and $\Omega = \omega_0-b^2\omega_0{}^3+\ldots$, $\alpha = \frac{1}{2}b\omega_0{}^2-b^3\omega_0{}^4+\ldots$, $\beta^{-1} = b^{-1}+b\omega_0{}^2+\ldots$ (*see* Prakash and Vachaspati 1968*d*). This solution shows that if $T_G \ll \bar{\omega}^{-1}$, $F(T')$ can be replaced by $\theta(T)$; this case has been studied by the authors (Prakash and Chandra 1968*b*). Equation (1) also shows that if $T_G \gg \alpha^{-1}$, $F(T')$ in the integrand can be replaced by $F(T)$ and the solution therefore involves the terms of frequency $\bar{\omega}$ only.

* We take $c = 1$ and work in the Gaussian system of units.

Let us consider the case for which* $F(T) = 0$ for $T \geqslant 0, = T/T_G$ for $0 < T < T_G$ and $= 1$ for $T \geqslant T_G$. Equation (1) then leads to

$$\xi = \frac{g}{2\Omega T_G} \operatorname{Re} \left[\frac{(\Omega+i\alpha)e^{i\Omega T}}{1+\alpha\beta-i\beta\Omega} \left\{ \theta(T-T_G)e^{-\alpha(T-T_G)} \right.\right.$$

$$\times \left[\frac{\exp\left[-i(\Omega-\bar{\omega})T_G+i\chi\right]}{(\Omega-\bar{\omega}+i\alpha)^2} + \frac{\exp\left[-i(\Omega+\bar{\omega})T_G-i\chi\right]}{(\Omega+\bar{\omega}+i\alpha)^2} \right]$$

$$\left.\left. -\theta(T)e^{-\alpha T} \left[\frac{e^{i\chi}}{(\Omega-\bar{\omega}+i\alpha)^2} + \frac{e^{-i\chi}}{(\Omega+\bar{\omega}+i\alpha)^2} \right] \right\} \right] \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

+ terms of frequency $\bar{\omega}$.

Let us take $\alpha \ll \Omega$, $\bar{\omega}$, $\Omega \pm \bar{\omega} \ll \beta^{-1}$. If n pulses of radiation are incident per second, the cross-section for generation of the frequency Ω is then found to be†

$$\sigma(\Omega) = \sigma_T(n/2\alpha)(\Omega T_G)^{-2}\{[\Omega/(\Omega-\bar{\omega})]^4+[\Omega/(\Omega+\bar{\omega})]^4\} \quad \ldots \quad \ldots \quad (3)$$

where $\sigma_T = (8\pi/3)\ (e^2/m)^2$ is the Thomson scattering cross-section.

The above expression for $\sigma(\Omega)$ involves a resonance‡ at $\bar{\omega} \simeq \Omega$. If $\bar{\omega} = \Omega(1+\epsilon)$, where§ $|\epsilon| \ll 1$, $\sigma(\Omega)/\sigma_T \simeq (n/2\alpha)(\Omega T_G)^{-2}|\epsilon|^{-4}$. For Ω in the optical region, $\alpha \sim 10^{-8}$ sec^{-1} and $\Omega^2 \sim 10^{-31}$sec^{-2}. If we take $n \sim 10^3$ sec^{-1} and $T_G \sim 10^{-9}$ sec, for $|\epsilon| \sim 10^{-2}$, $\sigma(\Omega)/\sigma_T \sim 10^{-9}$. This effect should, therefore, be detectable with the intense pulses of radiation available in the laser beams.

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* We take $T_G \gg \bar{\omega}^{-1}$ but not $\gg \alpha^{-1}$.

† We assume that for different pulses statistical variations $> |\Omega-\omega|^{-1}$ occur in T_G.

‡ At $\bar{\omega} \simeq \Omega$, $(\Omega-\bar{\omega})^4$ in eqn. (3) should be replaced by $[(\Omega-\bar{\omega}♀)^2+\alpha^2]^2$.

§ We take $\alpha/\Omega \ll |\epsilon| \ll 1$.

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