DETERMINATION OF THE FREQUENCY OF A PERIODIC PULSE BY A MOVING COIL GALVANOMETER

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Starting from the fundamental galvanometer equation, it has been shown that the ratio of galvanometer deflection for a full wavelength sinusoidal pulse to that due to a half wavelength pulse is proportional to the ratio of the period of the pulse to that of the galvanometer. Mathematically, this may be expressed as:

$$\frac{\theta_2}{\theta_0} = \pi \cdot \frac{\tau}{T}$$

where θ_2 is the maximum throw experienced by the galvanometer coil due to the incidence of a full period pulse; θ_0 corresponding deflection due to a half period pulse;

τ the period of the pulse expressed as $E = E_0 \sin \frac{2\pi}{\tau} t$;

and T the free period of the galvanometer coil.

The result has been verified experimentally using a mechanically operated pulse generator.

INTRODUCTION

The torque acting on the coil of a d'Arsonval galvanometer is proportional to the direct current in the coil. For an alternating current of the type $I = I_{\max} \cos \omega t$, the galvanometer coil will be subject to a rapidly reversing torque, if $\omega \gg \omega_g$, where ω is the angular frequency of the coil and ω_g is the radian frequency of free motion of the coil.

However, the final deflection will be given by

$$\theta = (SI_m)(\omega_g/\omega)^2 \cos \omega t. \qquad \text{(1)}$$

The first factor in parentheses is the deflection which would be obtained if a steady current I_m is passed through the coil. The factor $(\omega_g/\omega)^2$ therefore measures the ratio of the maximum deflection with the alternating current to the steady deflection of the direct current. If the supply frequency is 60 cycles/sec ($\omega = 377$/s) and the period of the galvanometer is 10 seconds ($\omega_g = 0{\cdot}63$/s), the reduction in sensitivity is by a factor of $3{\cdot}6 \times 10^5$. Hence,

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to all intents and purposes, the galvanometer shows no deflection on alternating currents.

For alternating currents, therefore, the vibration galvanometer is sometimes employed at commercial frequencies. This is a modified d'Arsonval type instrument, which differs from the direct current galvanometer by having a moving system of very low moment of inertia and a very stiff suspension. The length of this suspension is adjustable allowing the torque per unit twist to be adjusted until the natural frequency of vibration is identical with the frequency of the applied alternating current.

The vibration galvanometer may also be made frequency selective by the series connection of a band pass or a low pass filter, either of which passes the fundamental frequency with little attenuation but discriminates strongly against harmonics.

Magnetic oscillographs, which employ d'Arsonval movements of short natural periods as their sensitive elements, were in use long before the cathode-ray oscilloscope was developed and still offer advantages over the newer instruments in some low frequency applications. A new method of determining the frequency of a L.F. sinusoidal alternating current with the help of an ordinary d'Arsonval galvanometer is described below.

Mathematical Formulation of the Problem

The fundamental galvanometer equation is

$$MK^2\frac{d^2\theta}{dt^2}+\left(\frac{A^2H^2}{R}+\alpha\right)\frac{d\theta}{dt}+c\theta = AH\cdot\frac{E}{R} \qquad \ldots \quad \ldots \quad (2)$$

where

t = time

MK^2 = moment of inertia of the coil including insulation and mirror

θ = angular distance of the coil from its position of equilibrium, measured in radians. θ is taken to be small so that $\sin\theta = \theta$

A = area of the coil, account being taken of the n number of turns

$AH\theta$ = total magnetic flux through the coil when displaced through a small angle θ

$AH\dfrac{d\theta}{dt}$ = back e.m.f. when the coil is rotating through an angular velocity $\dfrac{d\theta}{dt}$

R = resistance of the circuit

$\dfrac{A^2H^2}{R}\cdot\dfrac{d\theta}{dt}$ = retarding couple due to back e.m.f.

$\alpha\cdot\dfrac{d\theta}{dt}$ = retarding couple due to the air resistance (this could also be partly due to eddy currents induced in any metal parts attached to the moving system).

If the applied e.m.f. happens to be a periodic pulse of the form

$$E = E_0 \sin \frac{2\pi}{\tau} . t,$$

then the above equation (2) becomes

$$\frac{d^2\theta}{dt^2} + 2\mu\omega^2 . \frac{d\theta}{dt} + \omega^2 . \theta = \frac{AH}{MK^2} . i_0 . \sin \frac{2\pi}{\tau} t \qquad \ldots \quad \ldots \quad (3)$$

where

$$2\mu\omega^2 = \frac{1}{MK^2}\left(\frac{A^2H^2}{R} + \alpha\right);$$

$$\omega^2 = \frac{c}{MK^2} = \left(\frac{2\pi}{T}\right)^2;$$

$$i_0 = E_0/R.$$

T = free period of the galvanometer

and

τ = period of the pulse.

The solution of the above equation (3) under the initial conditions $\theta = 0$ and $\frac{d\theta}{dt} = 0$ is

$$\theta = B . e^{-\mu\left(\frac{2\pi}{T}\right)^2 . t} . \cos\left(\frac{2\pi}{T} . \sqrt{1-\mu^2\omega^2} . t + \epsilon\right)$$

$$+ \frac{AH . i_0}{c} . \frac{\sin\left[\frac{2\pi}{\tau} . t - \tan^{-1}\left\{2\mu\left(\frac{2\pi}{\tau}\right)\Big/\left(1-\frac{T^2}{\tau^2}\right)\right\}\right]}{\left[\left(1-\frac{T^2}{\tau^2}\right)^2 + 4\mu^2\left(\frac{2\pi}{\tau}\right)^2\right]^{\frac{1}{2}}} \qquad \ldots \quad (4)$$

where

$$\epsilon = \tan^{-1}\frac{\left(1-\frac{T^2}{\tau^2}\right) - 2\mu^2\left(\frac{2\pi}{T}\right)^2}{2 . \left(\frac{2\pi}{T}\right) . \mu\left[1-\mu^2\left(\frac{2\pi}{T}\right)^2\right]^{\frac{1}{2}}}$$

and

$$B = \frac{AHi_0}{c} . \frac{2\mu\left(\frac{2\pi}{\tau}\right)}{\left(1-\frac{T^2}{\tau^2}\right)^2 + 4\mu^2\left(\frac{2\pi}{\tau}\right)^2} \sec \epsilon.$$

This solution holds for the case $\mu\omega < 1$. If T be large enough such that any term involving $\left(\frac{2\pi}{T}\right)^2$ is negligible in comparison to the other terms, then

$$\epsilon = \tan^{-1}\frac{1-\frac{T^2}{\tau^2}}{2\mu\left(\frac{2\pi}{T}\right)}$$

and

$$B = \frac{AHi_0}{c} \cdot \frac{\frac{T}{\tau}}{1-\frac{T^2}{\tau^2}}.$$

Hence the solution (4) reduces to

$$\theta = \frac{AHi_0}{c\left(1-\frac{T^2}{\tau^2}\right)} \cdot \left[-\frac{T}{\tau} . \sin \left(\frac{2\pi}{T} t - \tan^{-1} \frac{2\mu\left(\frac{2\pi}{T}\right)}{1-\frac{T^2}{\tau^2}} \right) \right.$$

$$\left. + \sin \left(\frac{2\pi}{\tau} t - \tan^{-1} \frac{2\mu\left(\frac{2\pi}{\tau}\right)}{1-\frac{T^2}{\tau^2}} \right) \right].$$

Now

$$\tan^{-1} \frac{2\mu\tau^2\left(\frac{2\pi}{T}\right)}{\tau^2-T^2} \approx \tan^{-1}\left(-\frac{4\mu\pi\tau^2}{T^3}\right),$$

which is negligible in comparison to $\frac{2\pi}{T}t$;

and

$$\tan^{-1} \frac{2\mu\left(\frac{2\pi}{\tau}\right)}{1-\frac{T^2}{\tau^2}} \approx \tan^{-1}\left[-\frac{2\mu(2\pi\tau)}{T^2}\right],$$

which is negligible in comparison to $\frac{2\pi}{\tau}t$.

Thus

$$\theta = \frac{AHi_0}{c\left(1-\frac{T^2}{\tau^2}\right)} \left[\sin \frac{2\pi}{\tau} t - \frac{T}{\tau} . \sin \frac{2\pi}{T} t\right] \qquad \ldots \quad \ldots \quad (5)$$

and

$$\frac{d\theta}{dt} = \frac{2\pi . AHi_0}{\tau . c . \left(1-\frac{T^2}{\tau^2}\right)} \left[\cos \frac{2\pi}{\tau} t - \cos \frac{2\pi}{T} t\right]. \qquad \ldots \quad \ldots \quad (6)$$

If the galvanometer experienced a single half pulse, $\dot{\theta}$ would become $\dot{\theta}_1$ at the end of the time, where

$$\dot{\theta}_1 = \frac{2\pi . AHi_0}{\tau c\left(\frac{T^2}{\tau^2}-1\right)} \left[1+\cos \pi . \frac{\tau}{T}\right] \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

and its deflection θ_1 would be

$$\theta_1 = \frac{AHi_0}{c\left(\frac{T^2}{\tau^2}-1\right)}\left[\frac{T}{\tau}\,.\sin\left(\pi\,.\frac{\tau}{T}\right)\right]. \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

The throw would continue beyond θ_1, to the extent θ_0 given by the energy equation

$$\tfrac{1}{2}MK^2\,.\,\dot{\theta}_1^{\,2} = \tfrac{1}{2}c(\theta_0^{\,2}-\theta_1^{\,2}) \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

or

$$\left(\frac{T}{2\pi}\right)^2.\,\dot{\theta}_1^{\,2}+\theta_1^{\,2} = \theta_0^{\,2}. \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Therefore,

$$\theta_0^{\,2} = \left[\frac{AHi_0}{\frac{T^2}{\tau^2}-1}\right]^2.\frac{T^2}{c^2\tau^2}\left[1+\cos\pi\frac{\tau}{T}\right]^2$$

$$+\left[\frac{AHi_0}{\frac{T^2}{\tau^2}-1}\right]^2.\frac{T^2}{c^2\tau^2}.\sin^2\pi\,.\frac{\tau}{T} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

or

$$\theta_0 = 2^{\frac{1}{2}}\,.\left[\frac{AHi_0}{\frac{T^2}{\tau^2}-1}\right].\frac{T}{c\tau}\left[1+\cos\pi\,.\frac{\tau}{T}\right]^{\frac{1}{2}}. \quad \ldots \quad \ldots \quad (12)$$

If the pulse had continued for one complete period and no more, we should have as follows:

we seek a maximum for eqn. (5) by putting $\frac{d\theta}{dt} = 0$

i.e.

$$\cos\frac{2\pi t}{\tau} - \cos\frac{2\pi t}{T} = 0.$$

Hence, either

$$\frac{2\pi t}{\tau} - \frac{2\pi t}{T} = 2\pi n_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

or

$$\frac{2\pi t}{\tau} = 2\pi n_0 - \frac{2\pi t}{T} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where n_0 is any positive integer.
From eqn. (13), we get

$$t_{\max} = \left(\frac{T\tau}{T-\tau}\right).\,n_0$$

while from eqn. (14), we get

$$t_{\max} = \frac{n_0 T\tau}{T+\tau}.$$

The maximum which occurs for $t < \tau$ is that for

$$t_{\max} = \frac{T\tau}{T+\tau} \quad \text{(taking } n_0 = 1\text{)},$$

then from eqn. (5)

$$\theta_2 = \frac{AHi_0}{c\left(\frac{T^2}{\tau^2} - 1\right)}\left[-\sin\frac{2\pi T}{T+\tau} + \frac{T}{\tau}\sin\frac{2\pi\tau}{T+\tau}\right].$$

Therefore,

$$\frac{\theta_2}{\theta_0} = \frac{\sin\frac{2\pi\tau}{T+\tau} - \frac{\tau}{T}\cdot\sin\frac{2\pi T}{T+\tau}}{2^{\frac{1}{2}}\left[1+\cos\pi\frac{\tau}{T}\right]^{\frac{1}{2}}}.$$

$$= \frac{2\pi\cdot\frac{\tau}{T}}{2}$$

$$= \pi\cdot\frac{\tau}{T} \qquad \ldots \quad (15)$$

since $\left(\frac{\tau}{T}\right)^2$ is negligibly small.

Experimental Arrangement

The experimental arrangement is simple and diagrammatically represented in Fig. 1.

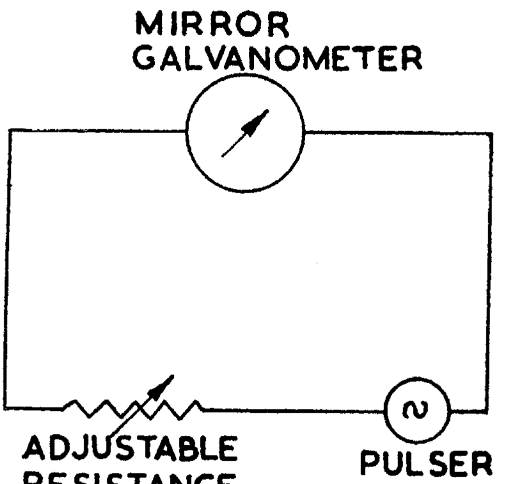

Fig. 1. Galvanometer circuit

It consists essentially of a ballistic galvanometer with known constants, a series resistor and a mechanical pulser. A single full pulse or a half pulse can be injected into the circuit and the corresponding throws of the galvanometer spot of light is recorded.

Pulse generator

A laboratory demonstration type A.C. dynamo was initially used to produce the required pulses. It consisted of a coil rotated by some external mechanical power, between the two poles of a horse-shoe magnet. The induced voltage generated in the coil can be led outside by slip rings and behaves as an alternating source of current. Unfortunately due to malfunctioning of the slip-ring contacts, pure sinusoidal waveform was difficult to obtain. And so, a commercial-type cycle dynamo was used instead, which consisted of a fixed coil and a rotating system of permanent magnets. The dynamo unit is shown in Fig. 2.

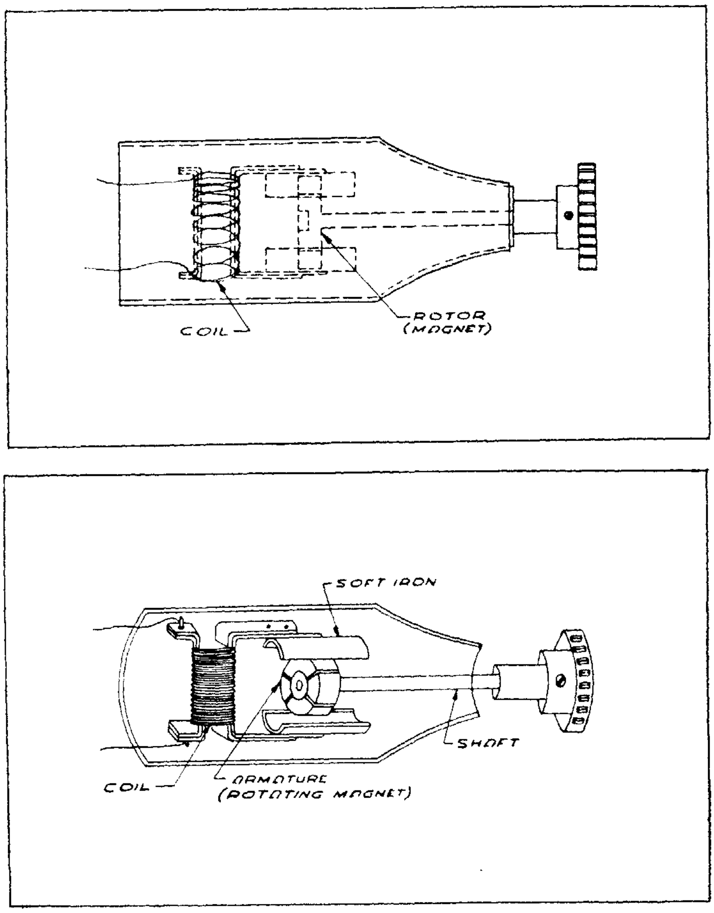

FIG. 2. Pulse generator

The period of the sinusoidal pulse depending upon the speed of rotation of the magnets could be conveniently adjusted by an array of gears having predetermined ratios. The time-duration for a half pulse or a full pulse could also be accurately controlled by means of suitable stops in the driving mechanism. The time-period of the pulses are checked and calibrated with the help of a cathode-ray oscillograph.

Experimental Results

It may be seen from equation (15) that

$$\tau = \frac{T}{\pi} \cdot \frac{\theta_2}{\theta_0}$$

where

τ = period of the injected sinusoidal pulse
T = period of the galvanometer
θ_2 = maximum deflection due to a full period pulse
θ_0 = corresponding deflection due to a half period pulse.

Table I shows a typical set of data for a particular setting of the pulse generator.

Table I

No. of measurement	Galvanometer deflection in cm		Mean deflection for half pulse θ_0 (cm)	Mean deflection for full pulse θ_2 (cm)
	Full pulse	Half pulse		
1	1·3	18·7		
2	1·4	18·4		
3	1·3	18·4	18·47	1·33
4	1·3	18·4		

Free period of galvanometer used, T = 7·14 sec.

Therefore, frequency of the pulse

$$= \frac{1}{\tau} = \frac{\pi}{T} \cdot \frac{\theta_0}{\theta_2} = \frac{\pi}{7{\cdot}14} \times \frac{18{\cdot}47}{1{\cdot}33}$$

$$= 6 \text{ cycles/sec.}$$

Actual frequency of the sinusoidal pulse generated by the pulse generator and checked by C.R.O. = 6 cycles per second.

Acknowledgements

We wish to pay homage to the memory of the late Dr. W. F. G. Swann, Director, Bartol Research Foundation, Swarthmore, Pa, U.S.A., who initially suggested the galvanometer problem.

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