THE ACCEPTANCE FUNCTION OF A MAGNETIC COSMIC RAY SPECTROGRAPH

by P. S. BASU *and* B. C. NANDI, *Department of Physics, Regional Engineering College, Durgapur, India*

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An important parameter in any magnetic cosmic ray spectrograph is its acceptance function and its variation with magnetic deflection or momentum of the particle. A method has been described to determine this acceptance function which has been applied to find out its value in the arrangement of the Durgapur Cosmic Ray Spectrograph (DUCOSP).

INTRODUCTION

The coincidence trigger system of a cosmic ray spectrograph, which signals the arrival of a particle, mainly controls the acceptance of particles for different possible angles of arrival through the spectrograph. Particles of low momenta are deflected more than particles of high momenta in the magnetic field and hence the collecting power of the spectrograph decreases with the decrease of particle momentum. This fact is taken into account by calculating a function $A(p)$, called the acceptance function for particles of momentum p. A method has been described below to calculate this function which can be used for any arrangement of the detector system in a magnetic cosmic ray spectrograph. The particles by different processes while passing through the solid iron of the magnets will lose energy. The deflections of low energy particles are affected by the energy loss. In the present calculation this fact has also to be taken into account.

CALCULATION OF ACCEPTANCE FUNCTION

To find the coordinate of a particle trajectory at any detecting level it is necessary to calculate the linear displacement at that level due to magnetic field. Ashton *et al.* (1963) have calculated the linear magnetic displacement assuming that a particle of momentum p (Fig. 1) is incident on the magnet at a small angle θ with the normal to the surface and that the mean momentum loss per unit length in the iron is constant. The equation of the path of the particle can then be written as :

$$p-\alpha\int_0^x\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx=300\,B\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}\left(\frac{d^2y}{dx^2}\right)^{-1} \qquad ..(1)$$

where the directions of x and y are as indicated in Fig. 1.

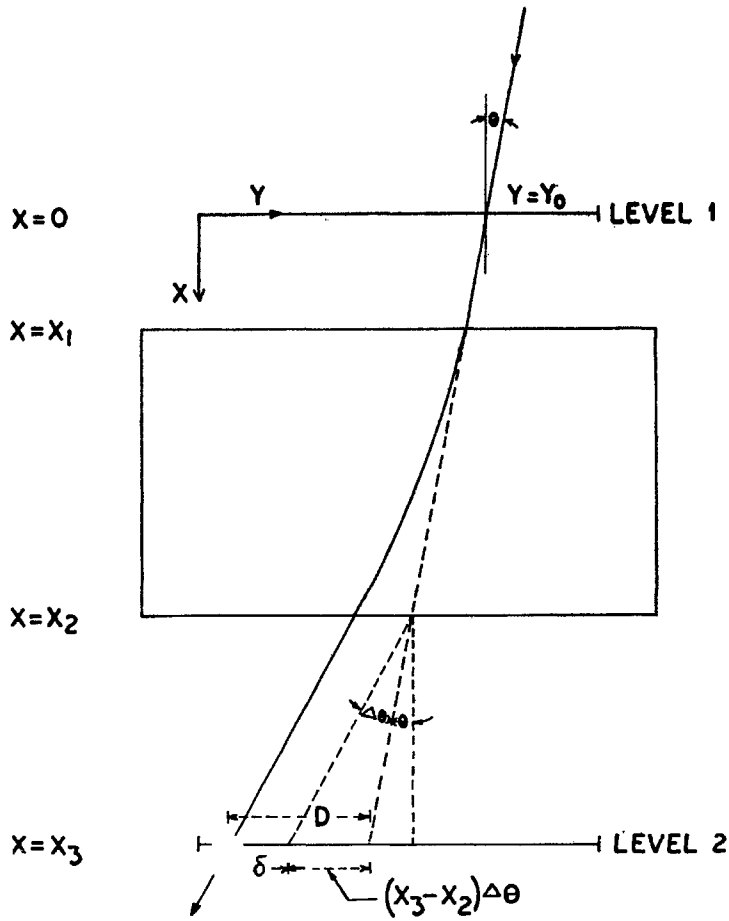

FIG. 1. The displacement due to magnetic field at a detecting level below the magnet

Integrating Eqn. (1) over the whole length l of the magnet it can be shown that the angular deflection due to the magnetic field B existing through a length l is :

$$\triangle\theta = -\frac{300\,B}{\alpha}\int_0^l ln\left(1-\frac{\alpha x}{p}\right)dx$$

$$\text{or,}\quad \triangle\theta = \frac{300\,Bl}{p}\left[1+\tfrac{1}{2}\left(\frac{\alpha l}{p}\right)+\tfrac{1}{3}\left(\frac{\alpha l}{p}\right)^2+\tfrac{1}{4}\left(\frac{\alpha l}{p}\right)^3+\ldots.\right] \qquad ..(2)$$

The linear displacement at the emergent face of the magnet is :

$$\delta = -\frac{300\,B}{\alpha}\int_0^l ln\left(1-\frac{\alpha x}{p}\right)dx$$

$$\text{or,}\quad \delta = \frac{300\,Bl^2}{2p}\left[1+\tfrac{1}{3}\left(\frac{\alpha l}{p}\right)+\tfrac{1}{6}\left(\frac{\alpha l}{p}\right)^2+\tfrac{1}{10}\left(\frac{\alpha l}{p}\right)^3+\ldots.\right] \qquad ..(3)$$

Hence the linear magnetic deflection D at the detecting level at a distance (X_3-X_2) from the bottom surface of the magnet is :

$$D=\delta+(X_3-X_2)\triangle\theta \quad ..(4)$$

Taking the top counter as the reference level it is possible to determine the co-ordinates of the path of the particle at any other level below the magnet. For example the Y-coordinate at level $X=X_3$ (Fig. 1) of a particle of momentum p incident at a point Y_o on the top counter is :

$$Y_3=Y_0+\delta+(X_3-X_2)\triangle\theta-X_3\tan\theta \quad ..(5)$$

Due to the finite size of the bottom counter there is a finite range of zenith angles within which a particle of momentum p incident on the top level at $Y=Y_0$ will cross the bottom counter. If the Y-coordinates of the two extreme ends of the bottom counter are $Y=a$ and $Y=b$, then it is evident from geometrical consideration (Fig. 1) that the minimum and maximum zenith angles of this range are :

$$\theta_{min}=\tan^{-1}\left[\frac{Y_0+D-b}{X_3}\right] \quad ..(6)$$

$$\theta_{max}=\tan^{-1}\left[\frac{Y_0+D-a}{X_3}\right] \quad ..(7)$$

These minimum and maximum zenith angles can be calculated by shifting the point of incidence of the particle on the top counter from one end to the other. The area covered by the maximum and minimum zenith angles when plotted as a function of Y_0 is proportional to the acceptance of the particle of a particular momentum p. The corresponding area for particles of infinite momentum can be calculated by putting $\delta=\triangle\theta=0$. The ratio of these two areas is the relative acceptance function of particles of a particular momentum. The acceptance function at infinite momentum is nothing but the geometric acceptance of the arrangement. Therefore, the absolute value of the acceptance for momentum p can be obtained by multiplying this ratio with the geometric acceptance.

The above method can also be applied to arrangements having more than two detecting levels. In this case taking two detecting levels (of which one is the reference level, preferably the top level) at a time, the maximum and minimum zenith angles can be calculated for particles incident at different points on the top detecting level. Among the series of zenith angles θ_{max}^1, θ_{max}^2, θ_{max}^3 ..... and θ_{min}^1, θ_{min}^2, θ_{min}^3,..... so obtained for particles of a particular momentum incident on the reference level at a particular point, one has to select minimum value of θ_{max}s and maximum value of θ_{min}s to ensure that the particle should cross all the detecting levels and hence will be accepted by the instrument.

A similar procedure can be followed to calculate the acceptance function for a spectrograph having two or more magnets and several detecting levels. For spectrographs with two magnets the linear magnetic deflections D_1 and D_2 at distances X_1', and X_2' from the bottom surface of the first and second magnet respectively can easily be shown from the geometry as :

$$D_1=\delta_1+X_1'\triangle\theta_1 \quad ..(8)$$

$$D_2=\delta_1+\delta_2+X_1'\triangle\theta_1+X_2'\triangle\theta_2 \quad ..(9)$$

where δ_1, δ_2 are the linear displacements at the first and second magnets respectively and $\triangle\theta_1$, $\triangle\theta_2$ are the respective angular deflections.

These quantities are given by :

$$\delta_1 = \frac{300\,Bl_1^2}{2p_1}\left[1+\tfrac{1}{3}\left(\frac{\alpha_1 l_1}{p_1}\right)+\tfrac{1}{6}\left(\frac{\alpha_1 l_1}{p_1}\right)^2+\tfrac{1}{10}\left(\frac{\alpha_1 l_1}{p_1}\right)^3+\ldots\ldots\right]$$

$$\delta_2 = \frac{300\,Bl_2^2}{2p_2}\left[1+\tfrac{1}{3}\left(\frac{\alpha_2 l_2}{p_2}\right)+\tfrac{1}{6}\left(\frac{\alpha_2 l_2}{p_2}\right)^2+\tfrac{1}{10}\left(\frac{\alpha_2 l_2}{p_2}\right)^3+\ldots\ldots\right]$$

$$\triangle\theta_1 = \frac{300\,Bl_1}{p_1}\left[1+\tfrac{1}{2}\left(\frac{\alpha_1 l_1}{p_1}\right)+\tfrac{1}{3}\left(\frac{\alpha_1 l_1}{p_1}\right)^2+\tfrac{1}{4}\left(\frac{\alpha_1 l_1}{p_1}\right)^3+\ldots\ldots\right]$$

$$\triangle\theta_2 = \frac{300\,Bl_2}{p_2}\left[1+\tfrac{1}{2}\left(\frac{\alpha_2 l_2}{p_2}\right)+\tfrac{1}{3}\left(\frac{\alpha_2 l_2}{p_2}\right)^2+\tfrac{1}{4}\left(\frac{\alpha_2 l_2}{p_2}\right)^3+\ldots\ldots\right]$$

where p_1 is the momentum of the particle incident on the first magnet of length l_1, and $p_2 = p_1 - \alpha_1 l_1$ is the momentum of the particle incident on the second magnet of length l_2; α_1 and α_2 are the mean energy losses per unit length in the first and second magnets respectively.

Using eqns. 8 and 9 it is possible to calculate the coordinates at any detecting level below the first and second magnets and use the same procedure to determine the acceptance as a function of momentum.

Acceptance Function of DUCOSP

The characteristics of the Durgapur Cosmic Ray Spectrograph (DUCOSP) have been described by Nandi and Sinha (1970). The previous arrangement of the spectrograph was used to measure the momentum spectrum (Nandi and Sinha 1972*a*) and the charge ratio (Nandi and Sinha 1972*b*) of cosmic ray muons. The detecting system of the earlier spectrograph has now been modified for measuring the specific ionisation of muons in a cloud chamber placed at the bottom of the spectrograph. A schematic diagram of the spectrograph in the new arrangement is shown in Fig. 2. The acceptance of the spectrograph is defined by the flash tube trays T_1, T_3 and G. M. counter trays G_1, G_3 and G_4. The maximum and minimum zenith angles for different levels with respect to the reference level G_1 are as follows :

(*i*) for level T_1

$$\theta_{min1} = \tan^{-1}\left(\frac{Y_{ta}-Y_0}{X_{t1}}\right)$$

$$\theta_{max1} = \tan^{-1}\left(\frac{Y_{tb}-Y_0}{X_{t1}}\right)$$

(*ii*) for level T_3

$$\theta_{min2} = \tan^{-1}\left(\frac{Y_0+D_{t3}-Y_{tb}}{X_{t3}}\right)$$

$$\theta_{max2} = \tan^{-1}\left(\frac{Y_0+D_{t3}-Y_{ta}}{X_{t3}}\right)$$

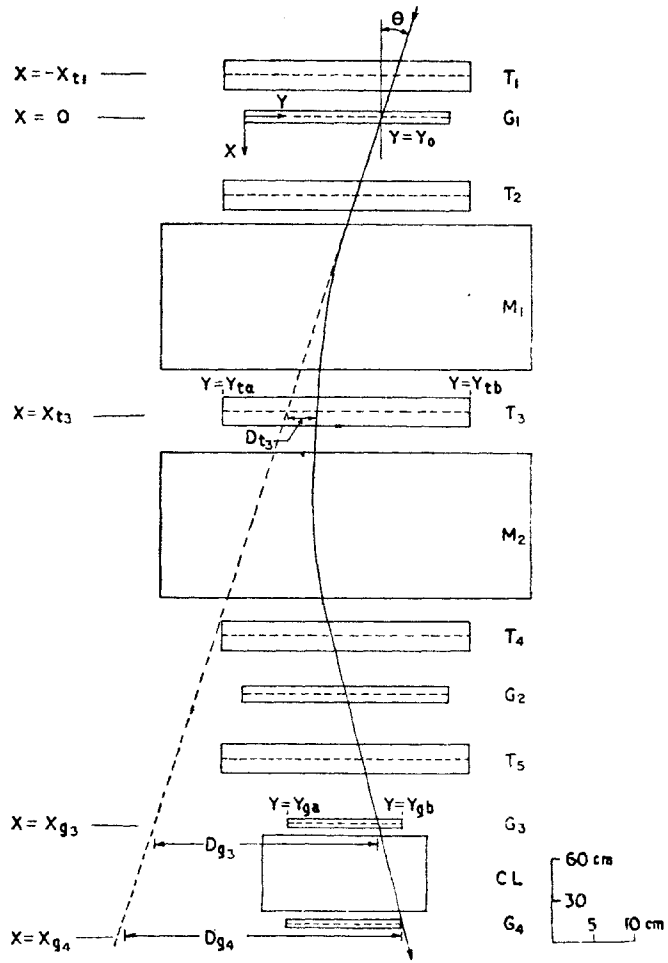

FIG. 2. Schematic diagram of the Durgapur Cosmic Ray Spectrograph : T_1, T_2, T_3, T_4, T_5, Neon flash tube trays ; M_1, M_2, Magnets ; G_1, G_2, G_3, G_4, G. M. counter trays ; C_1, Cloud chamber

(*iii*) for level G_3

$$\theta_{min3} = \tan^{-1}\left(\frac{Y_0+D_{g3}-Y_{gb}}{X_{g3}}\right)$$

$$\theta_{max3} = \tan^{-1}\left(\frac{Y_0+D_{g3}-Y_{ga}}{X_{g3}}\right)$$

(*iv*) for level G_4

$$\theta_{min4} = \tan^{-1}\left(\frac{Y_0+D_{g4}-Y_{gb}}{X_{g4}}\right)$$

$$\theta_{max4} = \tan^{-1}\left(\frac{Y_0+D_{g4}-Y_{ga}}{X_{g4}}\right)$$

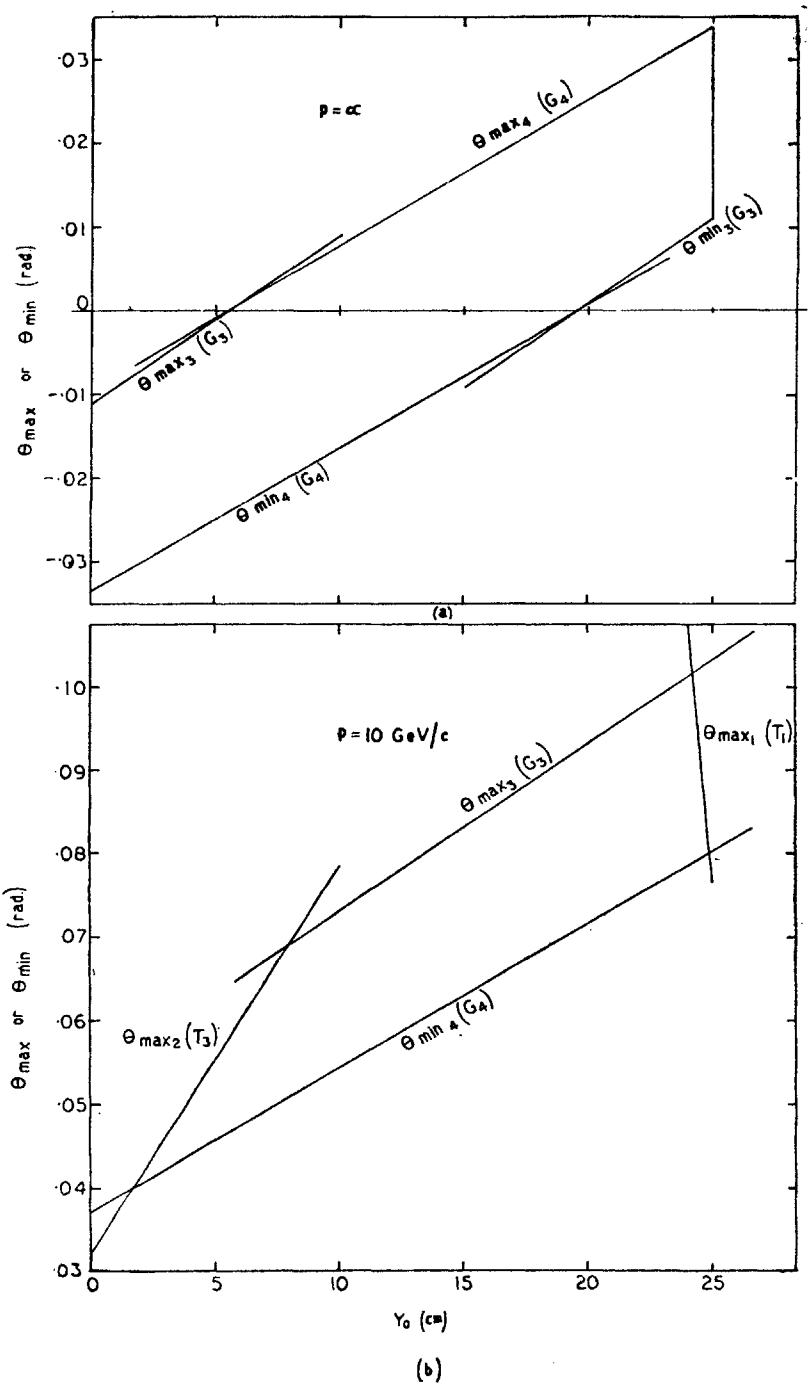

FIG. 3. Plot of maximum and minimum zenith angles as a function of position on G_1 level : (a) for particles of infinite momentum ; (b) for particles of momentum 10 GeV/c

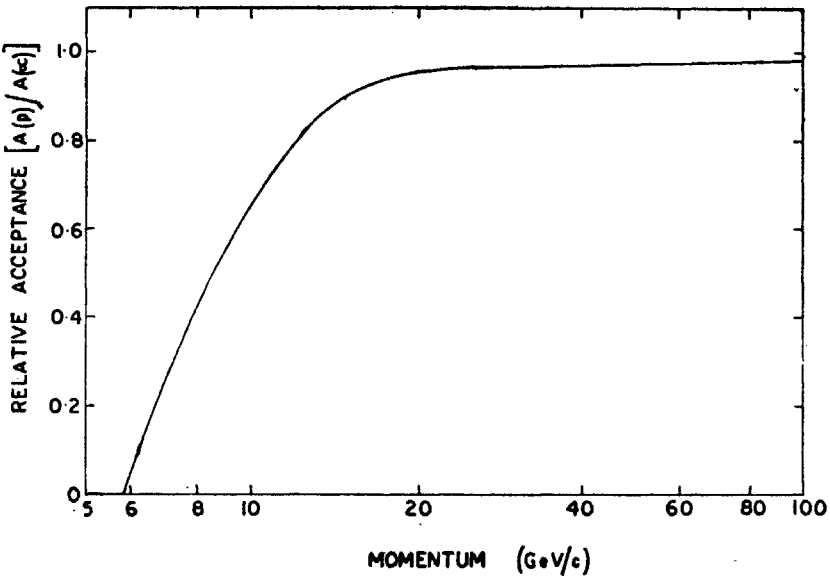

FIG. 4. The variation of acceptance function of the spectrograph with momentum

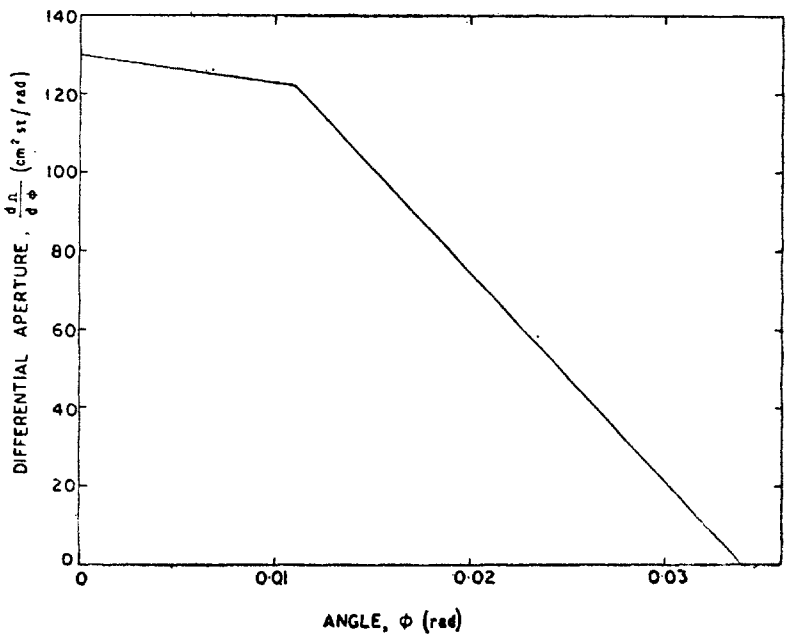

FIG. 5. The variation of the differential aperture as a function of the projected angle in the front plane

where all the dimensions are indicated in Fig. 2. From the above equations $\theta_{min}{}^{s}$ and $\theta_{max}{}^{s}$ have been calculated, and using the selection criterion discussed in section 2, θ_{min} and θ_{max} are plotted as a function of Y_0. Fig. 3 shows two such plots, one for particles of momentum 10 GeV/c and the other for particles of infinite momentum. The ratio of the two areas is the relative acceptance function for the particles of momentum 10 GeV/c with respect to the infinite momentum particle. These ratios are calculated for different momenta from 5.8 GeV/c to 100 GeV/c and plotted in Fig. 4. The plot of Fig. 4, therefore, represents the variation of the relative acceptance function of DUCOSP with momentum. The relative acceptance calculated from Fig. 3 for particles of momentum 10 GeV/c is 0.66.

Geometrical Acceptance of DUCOSP

The geometrical acceptance of the spectrograph is the acceptance function at zero magnetic field or the acceptance function for particles of infinite momentum with the magnetic field on. The plot in Fig. 3 for infinite momentum particle shows that the geometrical acceptance of DUCOSP is determined by the G. M. counter trays G_1, G_3, and G_4. In order to calculate this acceptance (collecting power), the formula for the differential aperture given by Lovati *et al.* (1954) for counter trays of equal dimension has been modified for unequal counter trays and applied to the present arrangement of the spectrograph. Fig. 5 shows the plot of the differential aperture as a function of the projected angle in the front plane. The collecting power which is the area under this curve has been calculated to be 2.79 ± 0.05 cm^2 st.

Discussion

The advantage of the method described in this paper is that it can be applied to any arrangement of detector systems and magnets of a spectrograph. The effect of scattering on the acceptance of the particle has not, however, been taken into account.

The small-angle-approximation limits the validity of the method for particles incident at large angle on the top detecting level. This is the case for low momentum particles and for spectrographs having large detecting areas. For particles incident at an angle of about 20°, error in the acceptance has been found to be about five per cent. The following procedure can be adopted to correct the acceptance function when the angle involved is large. The approximate value of θ_{min} and θ_{max} are calculated by using the value of D obtained from Eqn. 4. The value of D is then again calculated by using the exact relation $D=\delta+(x_3-x_2)[\tan(\theta+\triangle\theta)-\tan\theta]$ (*see* Fig. 1) and putting $\theta=\theta_{max}$ or θ_{min} calculated from the approximate relation. θ_{max} and θ_{min} are then again calculated putting this new value of D in Eqns 6 and 7. This procedure is repeated until there is practically no change of θ_{max} or θ_{min}. The final value of θ_{min} and θ_{max} are used to calculate the acceptance function.

The maximum angle in the present spectrograph is 9° so that it was not necessary to apply successive approximation to calculate θ_{min} and θ_{max}.

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