SELF-ENERGY DIVERGENCE LOOPS IN SPINOR FIELDS

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In freely propagating spinor fields, divergence loops are considered to occur and a renormalization is shown to change the mass of the particle into a matrix of integrals. Canonical form for dilation current is obtained without improvements on the stress tensor. This is seen necessary for divergence loops to occur and the improvement terms of Callan-Coleman-Jackiew are seen to vanish in a generalized formulation of improved Belinfanté Stress Tensor. Its shown that one-loop divergences are not equivalent to gauge transformation of fields, but are cancelled by altering the mass of the field.

INTRODUCTION

Occurrence of divergence loops has been considered by several authors (Callen *et al.* 1970) as a ϕ^4 term in free Lagrangian for scalar fields. There is nothing to forbid such loops in spinor fields, and I want to consider their renormalization by method of addition of counter terms to the Lagrangian. For scalar fields, the renormalization gives an imaginary scalar addition to the mass and clearly pushes the mass pole in the complex plane towards positive imaginary axis. In case of spinors, the self-energy divergence loops could be either contact loops or the spin-interaction loops. Spin interaction introduces γ^{μ}-matrices and their renormalization counter terms must have coefficients containing integrals of scalar matrices. I have shown finally for 'One-Loop-Renormalisation' that this has an effect of adding a matrix of scalar integrals towards the positive imaginary axis in the mass pole. This imaginary part of the mass is linear in the strengths of interacting terms and is directly proportional to the mass of the spinor under consideration. I begin with the local field theory and the energy-momentum tensor has been attempted by adding Higgin's type of term and no such improvements are found to have any softer trace than the canonical energy momentum tensor. Then I list a set of fifteen possible terms exhausting all single-vertex four-point self-interaction loops. 'Stress Tensor' exhibits the non-compatibity of divergence loops with the improvement terms for spinors. Section 'Improvements' gives the equality of canonical stress tensor with the Callan-Coleman-Jackiew stress tensor and shows that improvements are not needed for spin-$\frac{1}{2}$ field. One-loop-divergences are thus admissible. The section titled 'Theorem' proves Ward-Identities for the matrix elements of energy momentum tensor and shows that these are finite and renormalizable. It also explicitly displays the self-energy divergence counter terms and the net effect of renormalization.

HIGGIN'S TYPE IMPROVEMENTS

Spinor fields are described by the free Lagrangian

$$L = \bar{\Psi}\left(\frac{i\overleftrightarrow{\partial}_{\mu}\gamma^{\mu}}{2} - m\right)\Psi \quad \quad .. \quad .. \quad .. \quad (1)$$

where m is the mass of the particle. The symmetric stress-tensor obtained from Lagrangian (1) has the form

$$T_S{}^{\mu\nu} = \bar{\Psi}\, \frac{i\overleftrightarrow{\partial}}{2}{}^{(\mu}\gamma^{\nu)}\, \Psi \qquad \ldots \qquad \ldots \qquad \ldots \quad (2)$$

and if we add a Higgin's type (Callen *et. al.* 1970) term

$$T_H{}^{\mu\nu} = \frac{1}{6m}\, (\partial^\mu \partial^\nu - g^{\mu\nu} \partial^2)\, \bar{\Psi}\, \Psi \qquad \ldots \qquad \ldots \quad (3)$$

which satisfies the conservation law

$$\partial_\nu\, T_H{}^{\mu\nu} = 0$$

we find that total stress tensor

$$\theta^{\mu\nu} = T^{\mu\nu} = T_H{}^{\mu\nu}$$

satisfies the conservation law

$$\partial_\nu\, \theta^{\mu\nu} = 0$$

and has a trace

$$g_{\mu\nu}\, \theta^{\mu\nu} = \bar{\Psi}\frac{i\gamma\overleftrightarrow{\partial}}{2}\,\Psi + \frac{\partial^2}{2m}\,\bar{\Psi}\,\Psi$$

$$= \frac{1}{m}\, \bar{\Psi}{}^{,\alpha}\Psi_{\alpha} \qquad \ldots \qquad \ldots \qquad \ldots \quad (4)$$

where we use equations of motion to simplify the trace. The unimproved stress tensor trace

$$g_{\mu\nu}\, T^{\mu\nu} = m\, \bar{\Psi}\, \Psi \qquad \ldots \qquad \ldots \qquad \ldots \quad (5)$$

is seen softer (Callen *et al*, 1970) than the improved one. This is an observation. We shall show later that improvements are not required for spin 1/2 fields. First, we consider what happens if divergence loops occur in free propagation of the spinor fields. The Lagrangian describing presence of such loops is

$$L = \bar{\Psi}\left(\frac{i\,\overleftrightarrow{\partial_\mu}}{2}\,\gamma^\mu - m\right)\Psi - \lambda_0\, \bar{\Psi}\Psi\bar{\Psi}\Psi - \lambda_1 \bar{\Psi}\,\gamma^\mu\,\Psi\,\bar{\Psi}\,\gamma_\mu\,\Psi \qquad \ldots \quad (6)$$

where the first addition is a contact loop and the second addition is a current-current interaction loop. The field equation alters :

$$[i\,\gamma\,\partial - m - 2\,\lambda_0\,\bar{\Psi}\,\Psi - 2\,\lambda_1\,\bar{\Psi}\,\gamma^\mu\,\Psi\,\gamma_\mu]\,\Psi = 0 \qquad \ldots \qquad \ldots \quad (7)$$

Closed Set

Upto first order, divergence loops given by only one four-point vertex, the following terms are closed under renormalization :

$[g^{\mu\nu}\bar{\Psi}\Psi,\ g^{\mu\nu}\bar{\Psi}\,\gamma\,\partial\Psi,\ g^{\mu\nu}\,\partial^\alpha\,\bar{\Psi}\,\partial_\alpha\,\Psi,\ g^{\mu\nu}\,\partial^2\,\bar{\Psi}\Psi,\ g^{\mu\nu}\,\bar{\Psi}\,\partial^2\,\Psi,$

$\bar{\Psi}\gamma^{(\mu}\gamma^{\nu)}\Psi$, $\partial^{(\mu}\bar{\Psi}\gamma^{\nu)}\Psi$, $\partial^{\mu}\partial^{\nu}\bar{\Psi}\Psi$, $\bar{\Psi}\partial^{(\mu}\partial^{\nu)}\Psi$, $\partial^{\alpha}\bar{\Psi}\gamma_{\alpha}\gamma^{(\mu}\partial^{\nu)}\Psi$,

$\bar{\Psi}\gamma^{(\alpha}\partial^{\nu)}\gamma_{\alpha}\partial^{\alpha}\Psi$, $\partial^{\alpha}\partial^{(\alpha}\bar{\Psi}\gamma_{\alpha}\gamma^{\nu)}\Psi$, $\bar{\Psi}\gamma_{\alpha}\gamma^{(\mu}\partial^{\alpha}\partial^{\nu)}\Psi$,

$g^{\mu\nu}\bar{\Psi}\Psi\bar{\Psi}\Psi$, $\bar{\Psi}\gamma^{(\mu}\Psi\bar{\Psi}\gamma^{\nu)}\Psi]$

We take into account the following facts in selecting terms :

(1)–*Symmetry in the indices* μ *and* ν *because the stress tensor is symmetric in* μ *and* ν.

(2)–$\gamma^{(\mu}\gamma^{\nu)}$ *factors are omitted since they reduce to* $g^{\mu\nu}$.

(3)–*Only non-derivative couplings are included in the loop diagrams. Terms like* $\bar{\Psi}\frac{i\overleftrightarrow{\partial}^{(\mu}}{2}\Psi\bar{\Psi}\frac{i\overleftrightarrow{\partial}^{\nu)}}{2}\Psi$ and $\bar{\Psi}\gamma^{(\mu}\Psi\bar{\Psi}\frac{i\overleftrightarrow{\partial}^{\nu)}}{2}\Psi^{\mu}$ *will carry out energy and momenta and forbid a freely propagating wave from free propagation, i.e., we omit self-decaying interactions and Lagrangian does not possess any contracted tensors in divergent loops.*

Stress Tensor

The total stress tensor can be written as

$$\theta^{\mu\nu} = \left(\bar{\Psi}\frac{i\overleftrightarrow{\partial}^{(\mu}\gamma^{\nu)}}{2}\Psi - g^{\mu\nu}L\right) + \frac{1}{6m}(\partial^{\mu}\partial^{\nu} - g^{\mu\nu}\partial^{2})\bar{\Psi}\Psi$$

and the trace is altered from

$$T_{\mu}{}^{\mu} = \bar{\Psi}(m - 2\lambda_0\bar{\Psi}\Psi - 2\lambda_1\bar{\Psi}\gamma^{\mu}\Psi\gamma_{\mu})\Psi$$

to

$$\theta_{\mu}{}^{\mu} = \frac{1}{m}\bar{\Psi}^{,\mu}\Psi_{,\mu} + \bar{\Psi}(-4\lambda_0\bar{\Psi}\Psi + 4\lambda_0\bar{\Psi}\Psi\,\bar{\Psi}\Psi$$
$$-4\bar{\lambda}_1\bar{\Psi}\gamma^{\mu}\Psi\gamma_{\mu} + 4\bar{\lambda}_1\bar{\Psi}\gamma^{\mu}\bar{\Psi}\Psi\gamma_{\mu}\Psi)\Psi.$$

$T_{\mu}{}^{\mu}$ has nothing but a current -current interaction piece plus a contact piece. $\theta_{\mu}{}^{\mu}$ carried the hexagonal piece $4\lambda_0\bar{\Psi}(\bar{\Psi}\Psi\,\bar{\Psi}\Psi)\Psi$ and $4\lambda_1\bar{\Psi}(\bar{\Psi}\gamma^{\mu}\Psi\bar{\Psi}\gamma_{\mu}\Psi)\Psi$ which are outside the closed set and the trace are not renormalizable. Therefore either loop-diagram or improvements are to be discarded.

Improvements

If we calculate the improvements by a standard method, the dilation current is given by

$$D^{\mu} = X_{\nu}T^{\mu\nu} + \partial_{\nu}\sigma_{+}{}^{\mu\nu} \quad .. \quad .. \ (8)$$

where

$$\partial_{\nu}\sigma^{\mu\nu} \equiv \pi^{\mu}.D\Psi + \pi_{\lambda}\Sigma^{\mu\lambda}\Psi \quad .. \quad .. \ (9)$$

π^{μ} are given by free field Lagrangian

$$\pi^{\mu} = \frac{\partial L}{\partial\partial_{\mu}\Psi}$$

and D is the dimension of spinor field Ψ and is equal to 3/2. $\Sigma^{\mu\lambda}$ is the spin matrix. Explicit calculation of (9) shows that

$$\begin{aligned}\partial_\nu \sigma^{\mu\nu} &= \frac{i}{2}\overline{\Psi}\gamma^\mu \frac{3}{2}\Psi + \frac{i}{2}\overline{\Psi}\gamma_\lambda \frac{1}{4}(\gamma^\mu\gamma^\lambda - \gamma^\lambda\gamma^\mu)\Psi \\ &= \frac{i}{2}\overline{\Psi}\gamma^\mu \frac{3}{2}\Psi + \frac{i}{2}\Psi\left(-\frac{3}{2}\right)\gamma^\mu \Psi \\ &= 0\end{aligned}$$

and we can choose $\sigma^{\mu\nu} \equiv 0$

so, the improvements vanish :

$$T^{\mu\nu} R = 0$$

and the dilation current has the form

$$D^\mu = X_\nu T^{\mu\nu}$$

and $\theta^{\mu\nu} = T^{\mu\nu}$

i.e., improvements are not needed for spin 1/2 fields. *Loop diagrams can exist and should be renormalizable.*

Theorem

If we define the matrix elements of the stress tensor and the trace of the stress tensor by vaccuum expectation of a T^* product :

$$\Gamma^{(n)}_{\mu\nu}(X; X_1 .. X_n) \equiv \langle 0 \mid T^* \theta_{\mu\nu}(x)\, \Psi'(X_1) .. \Psi'(X_n) \mid 0\rangle$$

$$\Gamma^{(n)} \quad (X; X_1 .. X_n) \equiv \langle \mid T^* \theta(X)\, \Psi'(X_1) .. \Psi'(X_n) \mid 0\rangle$$

and of the Green's function by vaccuum expectation of a T-product,

$$G^{(n)}(X_1 .. X_n) \equiv \langle 0 \mid T\, \Psi'(X_1) .. \Psi'(X_n) \mid 0\rangle$$

and use the anti-commutator

$$\{\Psi(X), \overline{\Psi}(y)\}\, \delta(X^\circ - Y^\circ) = \gamma^\circ \delta^4(X-Y)$$

to calculate the matrix elements, we obtain for Fourier transformed components, the relations

$$k^\mu \Gamma^{(n)}_{\mu\nu}(k; p_1 .. p_n) = (-)^{n-1} \sum_p (p_1+k)\, G^{(n)}(p_1+k, p_2 .. p_n)$$

$$\Gamma^{(n)}_{\mu\nu}(0; 0000) = (-)^{n-1} g_{\mu\nu}(n-1)\, G^{(n)}(0; 0000)$$

$$\begin{aligned}g^{\mu\nu}\Gamma^{(n)}_{\mu\nu}(k; p_1 .. p_n) &= \Gamma^{(n)}(k; p_1 .. p_n) \\ &\quad + (-)^{n-1}\sum_p G^{(n)}(k+p_1\, p_2 .. p_n)\end{aligned}$$

where superscript n denotes number of Fermion field $\Psi'(X)$ and prime indicates that we begin with normalized fields. The result is identical to the case of scalar fields except for the power of (—1) arising from Pauli's exclusion principle. 'k' is the momenta associated with interpolating fields.

Closure of trace $\theta_\mu{}^\mu$ and tensor $\theta_{\mu\nu}$ implies that $\Gamma^{(n)}(k; p_1 .. p_n)$ and $\Gamma_{\mu\nu}{}^{(n)}(k; p_1 .. p_n)$ are finite and therefore a linear epansion for $\Gamma_{\mu\nu}{}^{(n)}$ within the closed set has the form

$$\Gamma^{(2)}_{\mu\nu}(k; p_1 p_2) = g_{\mu\nu}(C_1 + C_2 \gamma p + C_3 \gamma k + C_4 p^2 + C_5 k^2)$$
$$+ C_6(\gamma_\mu p_\nu + p_\mu \gamma_\nu) + C_7(-g_{\mu\nu}\gamma k + \gamma_\mu k_\nu + k_\mu \gamma_\nu)$$
$$+ C_8 p_\mu p_\nu + C_9(k_\mu k_\nu - g_{\mu\nu} k^2)$$
$$+ (C_{10}\gamma k + C_{11}\gamma p)(\gamma_\mu p_\nu + p_\mu \gamma_\nu)$$
$$+ C_{12}\gamma k(-g_{\mu\nu}\gamma k + \gamma_\mu k_\nu + k_\mu \gamma_\nu)$$
$$+ C_{13}\gamma p(\gamma_\mu k_\nu + k_\mu \gamma_\nu)$$

upto 2nd order in momenta.

The Ward identities show that $k^\mu \Gamma_{\mu\nu}{}^{(2)}$ is finite, and we have

$$k^\mu \Gamma^{(2)}_{\mu\nu} = k_\nu(C_1 + C_2\gamma p + C_3\gamma k + C_4 p^2 + C_5 k^2) + C_6(\gamma k p_\nu + kp\,\gamma_\nu)$$
$$+ C_7(k^2\gamma_\nu) + C_8(k\,.\,p)p_\nu + C_{10}(k^2 p_\nu + \gamma k \gamma_\nu\, k\,.\,p)$$
$$+ C_{11}(\gamma p\,\gamma k p_\nu + k\,.\,p\,\gamma p\,\gamma_\nu) + C_{12}(\gamma k k^2 \gamma_\nu)$$
$$+ C_{13}(\gamma p\,\gamma k\, k_\nu + k^2 \gamma p\,\gamma_\nu)$$

which is a linear combination of independent vectors; therefore the coefficients C_1 to C_8 and C_{10} to C_{13} are finite. The finiteness of the trace

$$g^{\mu\nu}\Gamma^{(n)}_{\mu\nu}(k; p_1 p_2) = 4(C_1 + C_2\gamma p + C_3\gamma k + C_4 p^2 + C_5 k^2) + 2\,C_6\gamma p$$
$$- 2\,C_7\gamma k + C_8 p^2 - 3\,k^2 C_9 + 2\,C_{10}\gamma k\,\gamma p$$
$$+ 2\,C_{11} p^2 + C_{12}k(-2\,\gamma k) + C_{13}\gamma p\,\gamma k$$
$$= 4\,C_1 + (4\,C_2 + 2\,C_6)\gamma p + (4\,C_3 - 2\,C_7)\gamma k$$
$$+ (4\,C_4 + C_8 + 2\,C_{11})p^2 + (C_5 - 3\,C_9 - 2\,C_{12})k^2$$
$$+ 2\,C_{10}\gamma k\,\gamma p + C_{13}\gamma p\,\gamma k$$

shows that C_9 is also finite.

Matrix elements of stress tensor with the quadratic pieces of Lagrangian

$$\Gamma^{(4)}_{\mu\nu}(0; 0000) = C_{14}\, g^{\mu\nu} + C_{15}\, F^{abcd}\,[(\gamma_\mu\, ab\,(\gamma_\mu)\, cd]$$

when contracted with the momenta variable k_μ.

$$k^\mu\,\Gamma^{(4)}_{\mu\nu}(0; 0000) = C_{14}\, k_\nu + \tfrac{1}{2}\, C_{15}\, F^{abcd}\,[(\gamma k)_{ab}\,\gamma_{\nu\, cd} + \gamma_{\nu\, ab}\,(\gamma k)_{cd}]$$

shows that C_{14} and C_{15} are also finite. F^{abcd} is a function of Kronecker δ's contracting the spinor indices. Thus the stress tensor $\theta_{\mu\nu}$ has finite matrix elements and is renormalizable.

One-Loop-Renormalization

To the Lagrangian

$$L = \overline{\Psi}\left(\frac{i\gamma \overleftrightarrow{\partial}}{2} - m\right)\Psi - \lambda_0\,\overline{\Psi}\Psi\,\overline{\Psi}\Psi - \lambda_1\,\overline{\Psi}\,\gamma^\mu\,\Psi\overline{\Psi}\,\gamma_\mu\,\Psi$$

If we add the so called counter terms

$$A\,\overline{\Psi}\,\frac{i\gamma\overleftrightarrow{\partial}}{2}\,\Psi + B\,\overline{\Psi}\Psi + C\,\overline{\Psi}\Psi\,\overline{\Psi}\Psi + D\,\Psi\,\gamma^\mu\,\Psi\overline{\Psi}\,\gamma_\mu\,\Psi$$

to negate the effect of divergent terms we note that normalization counter terms change the field's only by a gauge transformation

$$\Psi \rightarrow \Psi' = (1 + i\sqrt{A})\,\Psi$$

$$L \rightarrow \overline{\Psi}\ \frac{i\gamma \overleftrightarrow{\partial}}{2}\Psi' + \overline{\Psi}'\Psi'\left(\frac{-m+B}{1+A}\right) + \frac{-\lambda_0+C}{(1+A)^2}\ \overline{\Psi}'\Psi'\ \overline{\Psi}'\Psi'$$
$$+ \frac{-\lambda_1+D}{(1+A)^2}\ \overline{\Psi}'\gamma^{\mu}\Psi'\overline{\Psi}'\gamma_{\mu}\Psi'$$

provided $\dfrac{-m+B}{1+A} = -m$ or $B = -A$

$$\frac{-\lambda_0+C}{(1+A)^2} = -\lambda_0 \quad \text{or} \quad C = -\lambda_0\, A(A+2)$$

and, $\dfrac{-\lambda_1+D}{(1+A)^2} = -\lambda_1$ or $D = -\lambda_1\, A(A+2)$

The loop-diagrams are however still present. We will show that a single loop can be cancelled by altering the mass of the field. This is accomplished by retaining B as non-zero and altering A, C and D to zero. The mass changes from m to $(m+B)$ provided

$$B\Psi_a(X) = 2\lambda_0\,\Psi_a(X)\overline{\Psi}_b(X)\overline{\Psi}_b(X) + 2\lambda_{1\,ab}\,\Psi_b(X)\,\overline{\Psi}_c(X)\,\gamma_{\mu cd}\,\Psi_d(X)$$

We take the anticommutator with $\overline{\Psi}_e(y)$

$$B\{\Psi_a(X)\,\overline{\Psi}_c(Y)\} = 2\lambda_0\{\Psi_a(X)\,\overline{\Psi}_b(X)\,\Psi_b(X),\ \overline{\Psi}(Y)\}$$
$$+ 2\lambda_1\{\gamma_{ab}{}^{\mu}\,\Psi_b(X)\,\overline{\Psi}_c(X)\,\gamma_{\mu\,cd}\,\Psi_d(X),\ \overline{\Psi}_e(Y)\}$$

and use the relations

$$\{abc,\ d\} = ab\{c,\ d\} - a\{b,\ d\}\ c + \{a,\ d\}\ bc$$

$$\{\Psi_a(X),\ \overline{\Psi}_c(Y)\} = i(i\gamma\,\partial_X + m)\ \triangle\ (X\_Y;\ m)$$

and, $\{\overline{\Psi}_a(X),\ \overline{\Psi}_c(Y)\} = 0$

then we get,

$$B(i\gamma\,\partial_x + m)_{ae}\ i\triangle\ (x-y:m) = 2\lambda_0\, i\,([i\gamma\,\partial_x+m]_{ae}\ \triangle\ (x-y;\ m\,\overline{\Psi}_b(x)\,\Psi_b(x)$$
$$+ \Psi_a(x)\,\overline{\Psi}_b(x)\,[i\gamma\,\partial_x+m]_{be}\,(x-y;\ m))$$
$$+ 2\lambda_1\,\gamma_{ab}{}^{\mu}\ i([i\gamma\,\partial_x+m)]_{be}\ \triangle\ (x-y;\ m)\ \overline{\Psi}_c(x)$$
$$\gamma_{\mu\,cd}\,\Psi_d(x) + \Psi_b(x)\,\Psi_c(x)\,\gamma_{\mu\,cd}\,[i\gamma\,\partial_x+m]_{de}$$
$$\triangle\ (x-y;\ m)$$

and taking the vacuum expectation on both sides, we find that

$$B\,\delta_{ae}\, m\,\delta^4(x-y) = -2\lambda_0\, i\left(\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{\gamma k-m}\right)_{ae}\ \delta^4(x-y)\right.$$
$$\left.+\int\frac{d^4k}{(2\pi)^2}\left(\frac{1}{\gamma k-m}\right)_{be}\ \delta_{ab}\,\delta^4(x-y)\right)$$
$$-2\lambda_1\, i\,\delta^4(x-y)\,\gamma_{ab}{}^{\mu}\left(\int\frac{d^4k}{(2\pi)^4}\left[\left(\frac{1}{\gamma k-m}\right)\gamma_{\mu\,cd}\ \delta_{cd}\right.\right.$$

$$+ \gamma_{\mu\, bd} \left(\frac{1}{\gamma k - m}\right)_{de} \Big]\Big)$$

$$= - 4\, \lambda_0\, i \int \frac{d^4 k}{(2\pi)^2} \left(\frac{1}{\gamma k - m}\right)_{ae} \delta^4 (x-y)$$

$$- 8\, \lambda_1\, i \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{\gamma k - m}\right)_{ae}$$

Thus the counter terms give a mass renormalization such that

$$B\, \delta_{ae} = - \frac{i}{m}\, 4(\lambda_0 + 2\, \lambda_1) \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{\gamma k - m}\right)_{ae}$$

and the field equation then reads

$$\left[i\gamma\partial - m - \frac{i}{m}\, 4(\lambda_0 + 2\, \lambda_1) \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{\gamma k - m}\right)\right] \Psi = 0$$

The self-energy divergence loops are considered as contact loops and the spin-interaction loops. The spin interaction introduces γ^{μ}—matrices and their re normalization counter terms contain coefficients containing integrals of scalar matrices. This has an effect of adding a matrix of scalar integrals towards the positive imaginary axis in the mass pole. This imaginary part of the mass is linear in the strengths of interaction and is proportional to the mass of the spinor under consideration. This clearly indicates the existence of a new generation of perticles.

REFERENCE

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