A SINGLE FLUID MODEL APPROACH TO OBTAIN THE ION-ACOUSTIC WAVE DISPERSION AND DAMPING IN A WEAKLY IONISED GAS

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The macroscopic equations of motion for a weakly ionised gas where the collisions between the background neutral particles and the plasma particles cannot be ignored, have been considered. From these basic equations of motion the wave equation in "p" (the macroscopic pressure perturbation) and the corresponding dispersion relation has been obtained. The dispersion relation shows that the propagation constant has four roots of which two correspond to two distinct modes of propagation in the positive direction the other two corresponding to two modes of propagation in the negative direction. Considering the frequency region much below the ion plasma frequency it has been observed that one of the roots which is a solution of the particular mode of the general equation corresponds to a sonic speed which closely simulates the Tonks-Langmuir speed for ion-acoustic wave and shows considerable dispersion and damping. The solution indicates however that both the dispersion and damping can be reduced either by increasing the percentage of ionization or by lowering the background pressure. It is further pointed out that by measuring the phase velocity and attenuation constant it is possible to calculate the electron temperature and the plasma neutral collision frequency. The usefulness of the analysis for a sonic probe to obtain the plasma parameters has been discussed.

INTRODUCTION

That there exist various wave modes which can propagate through a plasma medium results from the fact that at least two distinct species viz., ions and electrons are present and there exist varieties of driving and restoring forces namely, electric and magnetic forces, pressure gradient, viscosity etc, in a plasma medium. Though theoretically all possible modes of wave motion can simultaneously exist in a plasma, the boundary conditions and the state of the plasma will essentially determine which of the restoring and inertial forces will predominate for the propagation of the corresponding wave mode. Of the different modes [broadly classified as: (*a*) electromagnetic; (*b*) electrostatic; (*c*) electro-acoustic; (*d*) magneto-acoustic] the electro-acoustic wave in many respects has the essential properties of an ordinary acoustic wave but part of the restoring force in this case is purely electrostatic in origin. When the frequencies of these waves are much larger than the electron plasma frequency, the wave is carried by electrons and is known as electronic-ultrasonic wave (Bhatnagar & Srivastava, 1971). At frequencies well below electron plasma frequency the waves are essentially carried by ions and are called ion-acoustic waves.

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These are low frequency plasma density (or plasma pressure) oscillations in which ions and electrons move in phase.

On the basis of fluid analysis it was first predicted in 1929 by Tonks and Langmuir (1929) that for isothermal plasma the phase velocity V_s is given by

$$V_s^2 = \frac{kT_e}{m_i}$$

The theory does not predict dispersion and damping of the waves. Considerable amount of work, both theoretical and experimental on the dispersion and damping of the waves has been published. There are two different approaches through which the dispersion relation for these waves can be obtained. The Boltzman-Transport equation approach predicts Landau damping (Landau 1945) and which approach is now limited to cases when the waves are externally excited by oscillatory electric field to grids (Gould 1964). The alternative hydrodynamic approach for a fully ionised plasma does not predict considerable dispersion and damping of ion-acoustic waves both for isothermal (Surdin 1962) and non isothermal plasma (Bhatnagar 1964 and Venkatarangan 1964). Even if the collisions between ions and electrons are included (Bhatnagar & Shrivastava 1971) the nature of dispersion relation does not differ much.

It has been suggested by several authors that the study of electro-acoustic wave propagation in a plasma (we use this as a general term to include both ion-acoustic and electronic-ultrasonic wave) may lead to a diagnostic method for plasma parameters. If, in particular, one utilizes the determination of phase velocity of ion-acoustic waves as a diagnostic method for a laboratory plasma it is necessary to consider the effect of collisions of plasma particles with a large background of neutral particles on the ion-acoustic waves.

Considering the effects of neutral particles, a theoretical calculation has been done by Hatta and Sato (1962) for ion-acoustic waves in an isothermal plasma where the fluid equations of electron and ion species have been considered separately and the differential wave equation for ion density has been obtained. The treatment is adequate so far as the ion-acoustic wave is only of concern; but in this treatment the macroscopic quantities viz., plasma velocity $\vec{U}$, current density $\vec{J}$, plasma density ρ and plasma pressure P are not involved; therefore the existence of other modes cannot be predicted. According to Spitzer (1962) the two intermingled fluid concept (Hatta & Sato 1962) is advantageous in those idealized cases where either the electrons or the positive ions remain at rest. These situations cannot be conceived at least in the case of ion oscillations.

The purpose of the present paper is to derive the general wave equation and the dispersion relation using the macroscopic variables of plasma ($\vec{U}$, $\vec{J}$, ρ, P, etc.) and to make a unified treatment of the subject as a whole for a non-isothermal plasma taking the effect of collisions of the charged species with the neutral particles, and then to obtain from the general dispersion relation the phase velocity and attenuation constant of ion-acoustic waves at frequencies much below ion-plasma frequency.

Derivation of the Basic Equations in Terms of the Macroscopic Variables

The basic fluid equations for a fully ionized plasma, for a single fluid model for zero magnetic field and neglecting the effect of gravitational potential, can be written as

equation of motion :

$$\rho \frac{d\vec{U}}{dt} = -\nabla P \qquad \ldots \quad \text{(i)}$$

generalized Ohm's Law

$$\frac{1}{\epsilon_0 \omega_e^2} \frac{d\vec{J}}{dt} = E - \eta\vec{J} + \frac{1}{eN_e} \nabla P_e \qquad \ldots \quad \text{(ii)}$$

where rationalized units have been used.

Plasma momentum:

$$\rho\vec{U} = \left(N_i m_i \vec{U}_i + N_e m_e U_e \right) = \frac{m_i}{e} \left(\vec{J}_i - \vec{J}_e \right) \qquad \ldots \quad \text{(iii)}$$

Current density:

$$\vec{J} = e\,(N_i\vec{U}_i - N_e\vec{U}_e) = \vec{J}_i + \vec{J}_e \qquad \ldots \quad \text{(iv)}$$

Plasma pressure:

$$P = P_e + P_i$$

Mass density:

$$\rho = N_i m_i + N_e m_e \qquad \ldots \quad \text{(v)}$$

where, N_i, m_i, $\vec{U}_i$, P_i and $\vec{J}_i$ are respectively the particle density, particle mass, fluid velocity, fluid pressure and current density for ions and the quantities with suffix "e" correspond to those values for electrons,

ω_e = electron plasma frequency;
ϵ_o = free space permitivity;
Z_e = average ionic charge;
η = resistivity (for a fully ionized gas)

If the effect of momentum transfer due to collision between the charged particle species and the neutral particle species is considered the following quantities are to be added to the right hand side of the Eqs. (i) & (ii) respectively:

$$a = N_e \nu_{ea} \frac{m_e m_a}{m_e + m_a} \left(\vec{U}_a - \vec{U}_e \right) + N_i \nu_{ia} \frac{m_i m_a}{m_i + m_a} \left(\vec{U}_a - \vec{U}_i \right)$$

$$b = \frac{m_e}{\rho e^2}\, e\, N_i \nu_{ia} \frac{m_i m_a}{m_i + m_a} \left(\vec{U}_a - \vec{U}_i \right) - \frac{mi}{\rho e^2} N_e \nu_{ea} \frac{m_e m_a}{m_e + m_a} \left(\vec{U}_a - \vec{U}_e \right)$$

where U_a = macroscopic neutral atom velocity and ν_{ea} and ν_{ia} are, respectively, the effective electron-atom and ion-atom collision frequencies and m_a is the mass of a neutral atom.

Neglecting the terms of the order of m_e/m_i one can write

$$a = N_e \nu_{ea} m_i \left(\vec{U} - \vec{U_e}\right) + N_i \frac{\nu_{ia}}{2} m_i \left(\vec{U_a} - \vec{U_i}\right)$$

$$b = \frac{m_e}{\rho e^2} . e.N_i m_i \frac{\nu_{ia}}{2} \left(\vec{U_a} - \vec{U_i}\right) - \frac{m_i}{\rho e^2} . e.N_e m_i \nu_{ea} \left(\vec{U_a} - \vec{U_e}\right).$$

Considering the infinitesimal perturbation approximations (so that the equations are linearised) the momentum transfer equation and generalized Ohm's law equation become, considering the effect of collisions with neutral particles

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla p + \frac{m_i}{e} \nu_{ea} \vec{j_e} - \frac{m_e}{e} \nu'_{ia} \vec{j_i} \qquad \ldots \text{(vi)}$$

where $\nu'_{ia} = \nu_{ia}/2$

and

$$\frac{1}{\epsilon_0 \omega_e^2} \frac{\partial \vec{j}}{\partial t} = \vec{E} + \frac{1}{eN_{e0}} \nabla p_e - \frac{m_i m_e}{\rho e^2} \nu'_{ia} \vec{j_i} - \frac{m_i}{\rho e^2} \nu_{ea} \vec{j_e} \qquad \ldots \text{(vii)}$$

where we have assumed $\eta = 0$.

In the above equations the variables $\vec{E}$, p, $\vec{u}$ and $\vec{j}$ represent the perturbed values of electric field, pressure, velocity and current density. The suffix 'e', 'i' and 'a' denote the values corresponding to the electron, ion and neutral species respectively. The terms with suffix zero correspond to the corresponding quiescent values.

In arriving at the above equation, a driftless plasma in absence of external electric field has been considered and all the quiescent values of $\vec{E}$, $\vec{U}$ and $\vec{J}$ have been neglected. Again, since ionization has been assumed to be weak, the density of neutral particles is much greater than that of the charged particles and $\vec{U_a} = 0$, is also a plausible assumption.

Expressing $\vec{j_i}$ and $\vec{j_a}$ in terms of $\vec{u}$ & $\vec{j}$ from equations (vi) & (vii) one can write after certain approximation,

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla p - \rho \nu_{pa} \vec{u} + \beta \vec{j} \qquad \ldots (1)$$

$$\frac{1}{\epsilon_0 \omega_e^2} \frac{\partial \vec{j}}{\partial t} = \vec{E} + \frac{1}{eN_{e0}} \nabla p_e - \frac{\vec{j}}{\sigma_e} + \beta \vec{u} \qquad \ldots (2)$$

which are respectively the linearised plasma momentum transfer equation and generalised Ohm's law equation in presence of neutral particles,

where $1/\sigma_e = \frac{m_i^2}{\rho e^2} \nu_{ea}$, $\beta = \frac{m_i}{e} \nu_{ea}$ and $\nu_{pa} = \nu'_{ia} + \nu_{ea}$

The other fluid equations are the continuity equations of mass and charge,

$$\frac{\partial n_i}{\partial t} = -\frac{\rho}{m_i}\nabla.\vec{u} \qquad \ldots (3)$$

$$\frac{\partial n_e}{\partial t} = -\frac{\rho}{m_i}\nabla.\vec{u} + \frac{1}{e}\nabla.\vec{j} \qquad \ldots (4)$$

where n_i and n_e are the perturbed values of ion & electron density respectively. Poisson's equation:

$$\nabla.\vec{E} = \frac{e}{\epsilon_0}\left(n_i - n_e\right) \qquad \ldots (5)$$

Equation of state (for non-isothermal plasma) :—

$$p = \gamma_i n_i k T_i + \gamma_e n_e k T_e \qquad \ldots (6)$$

where T_e and T_i are electron temperature and ion temperature respectively. We shall assume for adiabatic gas constants $\gamma_i = \gamma_e = \gamma$

Wave Equation

Taking divergence on both sides of Eqs. (1) and (2) and using Eqs. (3), (4), (5) & (6), one can eliminate the variables $\vec{u}$, $\vec{E}$ and $\vec{j}$ so that only the pressure variables are retained

$$\frac{1}{V_s^2}\left[A\frac{\partial^4 p}{\partial t^4} + B\frac{\partial^3 p}{\partial t^3} + C\frac{\partial^2 p}{\partial t^2} + D\frac{\partial p}{\partial t}\right]$$

$$= F\nabla^2 p + G\frac{\partial}{\partial t}\nabla^2 p + H\frac{\partial}{\partial t}\nabla^2 p_e + A\frac{\partial^2}{\partial t^2}\nabla^2 p + S\frac{\partial^2}{\partial t^2}\nabla^2 p_e \qquad \ldots (7)$$

where, $A = \frac{e}{m_e}\frac{T_i + T_e}{T_e}$, $\quad B = e.\frac{T_i + T_e}{T_e}\left(\frac{\nu_{ea}}{m_e} + \frac{\nu^1_{ia}}{m_i}\right)$

$$C = \frac{e}{m_i}\frac{T_i + T_e}{T_e}\omega_e^2 + \frac{e}{m_e}\nu'_{ia}\nu_{ea}\frac{T_i + T_e}{T_e} + \frac{e}{m_e}\nu_{ea}^2\left(\frac{T_i}{T_e} + \frac{T_i}{T_i + T_e}\right)$$

$$D = \frac{e}{m_i}\frac{T_i + T_e}{T_e}\omega_e^2\,\nu_{pa}, \qquad F = \frac{e}{m_i}\frac{T_i + T_e}{T_e}\omega_e^2$$

$$G = \frac{e}{m_e}\nu_{ea}\frac{T_i}{T_e}, \qquad H = \frac{e}{m_e}\left(\nu'_{ia} + \nu_{ea}\frac{T_i}{T_i + T_e}\right) \text{ and } S = e/m_e$$

To eliminate $\nabla^2 p_e$ term from eq. (7), one can assume to a first approximation that

$$\rho\frac{\partial\vec{u}}{\partial t} \simeq -\nabla p \qquad \ldots (8)$$

with this approximation $\nabla^2 p_e$ term can easily be eliminated and Eq. (7) becomes

$$\frac{1}{V_s^2}\left[A\frac{\partial^5 p}{\partial t^5} + B\frac{\partial^4 p}{\partial t^4} + C\frac{\partial^3 p}{\partial t^3} + D\frac{\partial^2 p}{\partial t^2}\right]$$

$$= F\frac{\partial}{\partial t}\nabla^2 p + G\frac{\partial^2}{\partial t^2}\nabla^2 p + A_1\frac{\partial^3}{\partial t^3}\nabla^2 p - H_1\nabla^4 p - S_1\frac{\partial}{\partial t}\nabla^4 p \ldots (9)$$

where $\quad (A + S) = A_1\,, \quad H\dfrac{\gamma k T_i}{m_i} = H_1 \quad \text{and} \quad S\dfrac{\gamma k T_i}{m_i} = S_1.$

The above equation is the required wave equation for a weakly ionized plasma medium. The equation is a partial differential equation, fifth order in time derivative and fourth order in space derivative of the pressure perturbation 'p'.

Dispersion Relation

Considering one dimensional case, if one assumes that $p = p_0\, e^{i(\chi x - \omega t)}$ be a solution of Eq. (9) the required dispersion relation between the frequency ω and propagation constant χ can be obtained as:

$$\frac{1}{V_s^2}\Big[-i\omega^5 A - B\omega^4 + iC\omega^3 + D\omega^2\Big]$$

$$= (iF\omega - G\omega^2 - iA_1\omega^3)\,\chi^2 + (iS_1\omega - H_1)\,\chi^4 \qquad \dots (10)$$

or, inserting the values of the constants,

$$\frac{1}{V_s^2}\Bigg[i\Bigg\{\underline{-\frac{e}{m_i}\cdot\frac{T_i+T_e}{T_e}}\cdot\,\omega^2 + \frac{e}{m_i}\,\frac{T_i+T_e}{T_e}\,\omega_e^2 + \frac{e}{m_e}\,\nu'_{ia}\,\nu_{ea}\,\frac{T_i+T_e}{T_e}$$

$$\underline{+\frac{e}{m_e}\,\nu_{ea}\left(\frac{T_i}{T_e} + \frac{T_i}{T_i+T_e}\right)}\Bigg\}\,\omega^3 - \Bigg\{e.\,\frac{T_i+T_e}{T_e}.\left(\frac{\nu_{ea}}{m_e} + \frac{\nu'_{ie}}{m_i}\right).\,\omega^2 + \frac{e}{m_i}\,\frac{T_i+T_e}{T_e}\,\nu_{pa}\omega_e^2\Bigg\}\omega^2$$

$$= \Bigg[i\,\frac{e}{m_i}\,\frac{T_i+T_e}{T_e}\omega_e^2\,\omega - \underline{\frac{e}{m_e}\,\nu_{ea}\,\frac{T_i}{T_e}}\,.\,\omega^2 - i\left(\underline{\frac{e}{m_i}\,\frac{T_i+T_e}{T_e} + \frac{e}{m_e}}\right)\omega^3\Bigg]\chi^2$$

$$+\Bigg[i\,\frac{\gamma k T_i}{m_i}.\,\frac{e}{m_i}\,.\,\omega - \frac{\gamma k T_i}{m_i}\,\frac{e}{m_e}\left(\nu'_{ia} + \nu_{ea}\,\frac{T_i}{T_i+T_e}\right)\Bigg]\chi^4 \qquad \dots (11)$$

From Eq. (10) or Eq. (11) it is evident that χ has four roots (for any particular excitation frequency ω) of which two correspond to two distinct modes propagating in the positive direction and the other two correspond to the same propagating in the negative direction.

Phase Velocity and Attenuation of the Ion-Acoustic Mode

Though in principle, it is possible to obtain exact solution of Eq. (11) for χ, only the situation here will be considered where the following conditions are satisfied:

$$V_s^2\ |\chi^2|\ \ll \omega_i^2 \qquad \dots (x)$$

$$\omega\nu_{pa} \ll \omega_1^2 \qquad \dots (y)$$

$$\nu'_{ia}\ \nu_{ea} \ll \omega_i^2 \qquad \dots (z)$$

and $\quad \omega \ll \omega_i$

where $\quad \omega_i\ (= e^2 N_{e0}\,/\,m_i\epsilon_0)$ is the ion plasma frequency.

In fact the above conditions are satisfied in most recent experimental conditions concerning ion-acoustic waves, if ω is kept in the audio range.

With the above assumptions one can neglect the underlined terms in Eq. (11) compared to other terms (*vide* Appendix) and the Eq. (11) is simplified to

$$V^2\ V_i^2\left\{\omega - i\left(\nu'_{ia} + \frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}\chi^4 - V_s^2\,\omega_e^2\,\omega\chi^2$$

$$+\left(\omega_e^2\,\omega - i\nu_{pa}\,\omega_e^2\right)\omega^2 = 0 \qquad \dots (12)$$

where, $V_i^2 = \gamma k T_i m_i$

From Eq. (12) two roots of χ^2 can be determined:

$$\chi^2 = \frac{V_s^2\omega_e^2\omega \pm \sqrt{\left(V_s^\circ\omega_e^2\omega\right)^2 - 4V_e^2V_i^2\left\{\omega - i\left(\nu'_{ia} + \frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}\omega_e^2\left(\omega - i\nu_{ea}\right)\omega^2}}{2V_e^2V_i^2\left\{\omega - i\left(\nu_{ia}' + \frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}} \qquad \dots (13)$$

In order to make the quantity under the radical a perfect square the term

$$\left[\frac{2V_e^2V_i^2\omega}{V_s^2}\left\{\omega - i\left(\nu_{ia}^1 + \frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}\left(\omega - i\nu_{pa}\right)\right]^2$$

is added which is very small in comparison with the second term under the radical (*vide* appendix). Thus one gets one of the roots of

$$\chi^2 = \frac{\omega}{V_s^2}\left(\omega - i\nu_{pa}\right) \qquad \dots (14)$$

In the other root of χ^2 the electron density (in the form of ω_e^2) occurs explicitly and this corresponds to the electron wave mode and at frequencies much below electron plasma frequency, this root corresponds to large damping (this has been shown by Bhatnagar & Shrivastava 1971) for a fully ionized plasma.

Since χ is complex, $\chi = \chi' + i\chi''$

and $\chi^2 = (\chi'^2 - \chi''^2) + i2\chi'\chi''$ ... (15)

Using Eqs. (14) and (7) and equating real and imaginary parts one can solve χ' and χ'' hence we obtain the phase velocity V_p and attenuation χ'' given by

$$V_p^2 = \frac{\omega^2}{\chi'^2} = \frac{2.\ \dfrac{\gamma k\,(T_i+T_e)}{m_i}}{1+\sqrt{1+\nu_{pa}^2/\omega^2}} \qquad \dots (16)$$

$$\chi''^2 = \frac{(\sqrt{1+\nu_{pa}^2/\omega^2} - 1)}{2.\ \dfrac{\gamma k\,(T_i+T_e)}{m_e}}.\ \omega^2 \qquad \dots (17)$$

Discussion

(*a*) *Effect of plasma-neutral collision frequency*—The above expression (Eq. 16) for phase velocity of ion-acoustic wave can be compared to that obtained by Bhatnagar & Shrivastava (1971) who considered fully ionized plasma and obtained the expression for phase velocity as

$$\nu_p^2 = \gamma k \, (T_i + T_e)/m_i$$

when collision with neutral molecules is taken into consideration as in the case of partially ionised plasma, both the phase velocity and attenuation are functions of the excitation frequency and the parameter ν_{pa} (one may call it as affective plasma neutral collision frequency) as is evident from Eqs. (16) and (17).

From the above relations it is easily seen that when $\nu_{pa} \rightarrow 0$ $V_p^2 \simeq \dfrac{\gamma k \, (T_i + T_e)}{m_i}$ and $\chi'' = 0$ as obtained by Bhatnagar and Srivastava (1971), previously.

Further it is noted that keeping the percentage of ionization constant if the pressure is reduced, the assumption made previously namely $\nu'_{ia} \, \nu_{ea} \ll \omega_i^2$ becomes more valid and if the value of ω, which usually lies in the sonic range be such that $\nu_{pa}^2 \ll \omega^2$, the phase velocity is found to be given by

$$V_p^2 \simeq \frac{\gamma k \, (T_e + T_i)}{m_i}$$

the wave shows no attenuation.

Thus it is evident that both dispersion and damping of ion-acoustic waves can be reduced in two ways; either by increasing the percentage of ionization or by lowering the gas pressure.

(*b*) *Dependence of ion-acoustic phase velocity and damping with back-ground pressure*—Dependence of ion-acoustic phase velocity and damping with background gas pressure can be predicted from the relations (16) and (17). Since both ν_{ia} and ν_{ea} are proportional to the neutral particle density i.e., background neutral gas pressure P_n one can write ν_{pa} as $\nu_{pa} = \alpha \, P_n$ where α is a suitable constant of proportionality related to the effective cross-sections of collision of charged particles with the neutrals. Thus variation of phase velocity and damping becomes the function of gas pressure through the relation

$$V_p^2 = 2V_s^2 \Big/ \left(1 + \sqrt{1 + \alpha^2 P_n^2 \big/ \omega^2} \right)$$

and

$$\chi''^2 = \left(\sqrt{1 + \alpha^2 P_n^2 \big/ \omega^2} - 1 \right) \omega^2 / 2V_s^2$$

(*c*) *Measurement of Electron Temperature*—In many cases, particularly in the discharge plasma $T_e \gg T_i$ so neglecting T_i it becomes apparent from Eqs. (16) and (17) that both T_e and ν_{pa} can be determined by measuring phase velocity and damping of ion-acoustic waves.

It is argued that, in a weakly ionized plasma, it is a reasonable approximation (Uman 1964) to use actual collision frequencies in place of the effective collision frequencies which have been used in the hydrodynamic equations. Thus determination of ν_{pa} by sonic probe method will be a further verification of the above assumption. It is also clear that the difference between ν_{pa}-effective (measured with sonic-probe technique) and ν_{pa}-elastic (determined from the measured elastic scattering cross-sections of electrons and ion with neutral atoms) would be a measure of the inelasticity of collisions if wall effects are ignored. Thus for a three component plasma ν_{pa} becomes a useful parameter to be studied.

(*d*) *Use of Acoustic Transducers as Sonic Probe*—If the waves are excited electrostatically or with oscillatory magnetic field (Little 1962) $u_a = 0$ is a plausible assumption. But if instead transducers are used to excite ion-acoustic waves the assumption, $u_a = 0$ will no longer be true. In those situations treatment should be more general than that given above and separate fluid equations for the neutral particles should also have to be considered and also more general definitions Ψ, P and $\vec{U}$ are to be used. Consequently it would predict another purely acoustic wave mode which will be carried by neutral particles alone.

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APPENDIX

(*a*) *The approximations made in arriving at Eq.* (12) *from Eq.* (11):

It has been pointed earlier that the underlined terms in Eq. (12) have been neglected in comparison to the other terms. This statement will be verified by considering each term separately.

1st term:—

Since $\omega^2 \ll \omega_e^2$ the 1st term can be neglected in comparison to the 2nd term.

3rd term:—

The third term is $\frac{e}{m_e} \nu'_{ia} \nu_{ea} \frac{T_i+T_e}{T_e} = \frac{m}{m_e} \frac{e}{m_i} \nu'_{ia} \nu_{ea} \frac{T_i+T_e}{T_e}$

Since the 2nd term can be written as $\frac{m_i}{m_e} \cdot \frac{e}{m_i} \frac{T_i+T_e}{T_e} \cdot \omega_i^2$

Thus from the condition (z) it is evident that the third term may be neglected in comparison with the second term.

4th term:—

From a similar argument as above this term can be shown very small as compared to the second term.

5th term:—

$$e\,\frac{T_i+T_e}{T_e}\left(\frac{\nu_{ea}}{m_e}+\frac{\nu_{ia}'}{m_i}\right)\omega^2 = e\,\frac{T_i+T_e}{T_e}\cdot\frac{\nu_{ea}}{m_e}\cdot\omega^2 + e\,\frac{T_i+T_e\nu_{ia}'}{T_e\quad m_i}\cdot\omega^2$$

The second portion of this term can obviously be neglected in comparison with the sixth term i.e., $\frac{e}{m_i}\,\frac{T_i+T_e}{T_e}\,\nu_{pa}\,\omega_e^2$ since $\nu_{ia}' < \nu_{pa}$ and $\omega^2 \ll \omega_e^2$. Again expressing the 6th term in terms of ω_i^2 it will become evident that the 1st portion of the fifth term may also be neglected in comparison with the sixth term.

From similar arguments it can be shown without ambiguity that the eighth term may be neglected in comparison with the 6th term [condition (x) is to be used] and the ninth term may be neglected in comparison to the seventh term.

(b) *On the approximation made in arriving at Eq.* (14) *from Eq.* (13):

To prove under the assumptions (x), (y) and (z)

$$\left[\frac{2V_e^2\,V_i^2\omega}{V_s^2}\left\{\omega - i\left(\nu_{ia}'+\frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}\left(\omega - iV_{pa}\right)\right]^2$$

$$\ll 4\,V_e^2\,V_i^2\left\{\omega - i\left(\nu_{ia}'+\frac{V_i^2}{V_s^2}\nu_{ea}\right)\right\}\left(\omega - i\nu_{pa}\right)\omega_e^2\,\omega^2$$

It will be sufficient to show that

$$\left|\left[\frac{2V_e^2V_i^2\omega}{V_s^2}\left(\omega - i\nu_{pa}^1\right)\left(\omega - i\nu_{pa}\right)\right]^2\right|$$

$$\ll \left|\,4V_e^2\,V_i^2\left(\omega - i\nu_{pa}'\right)\left(\omega - i\nu_{pa}\right)\omega_s^2\,\omega^2\right.$$

where $\qquad \nu_{pa}' = \omega - i\left(\nu_{ia}' + \frac{V_i^2}{V_s^2}\nu_{ea}\right)$

Let $\left|\left[\left(\omega - i\nu_{pa}'\right)\left(\omega - i\nu_{pa}^1\right)\right]^2\right| = \psi^4$ and obviously the value of ψ^2 is of the order of ω^2 or ν_{pa}^2.

Thus we have to show that

$$\frac{4V_e^4V_i^4\omega^2}{V_s^4}\cdot\psi^4 \ll 4V_e^2\,V_i^2\,\psi^2\,\omega_e^2\,\omega^2$$

or to show that

$$V_e^2\,V_i^2\,\omega^2\,\psi^2\,\psi^2 \ll V_s^4\,.\,\omega^2.\,\psi^2.\,\omega_e$$

which is apparent since,

$$\omega_e^2 = \frac{m_i}{m_e}\,\omega_i^2 \quad \text{and } T_i$$

is in general less than T_e.

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