ON SINGULAR SURFACES OF ORDER ONE IN MAGNETOGASDYNAMICS

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In this paper, we have studied the effect of magnetic field on singular surfaces of order one which, in ideal non-conducting gases, are of three types. It has been shown that for the first type, the first derivative of velocity is continuous and that the singular surfaces of order one exist in one dimensional unsteady flow. For the third type, the velocity of sonic wave, for a particular medium, is directly proportional to the effective velocity of sound. This constant of proportionality is one in the case of a weak magnetic field. Further, it has been deduced that the curvature vector is continuous when the magnetic field is orthogonal to the direction of flow.

1. INTRODUCTION

Thomas (1957 *a, b*) discussed the growth and decay of sonic discontinuities in ideal gases by taking the pressure p and the density ρ as the thermodynamical variables for the gas. Kaul (1961) took the entropy s and the local velocity of sound c in place of p and ρ as the thermodynamical variables and discussed, with certain advantages, the singular surfaces of order one in ideal gases. He arrived at the conclusion that the singular surfaces of order one were of three types. Across two of these the divergence of velocity was continuous and that they were similar in the sense that there was no gas flow across them. Across the third type of singular surfaces of order one, the first space derivatives of entropy were continuous but those of velocity and local velocity of sound were discontinuous. We have in this paper, studied the effect of an aligned magnetic field on the flow and have deduced that while there is hardly any change in the second type, remarkably significant results are obtained for the first and third types which are mentioned in the last section of the paper. The case of weak magnetic field is also discussed.

2. BASIC EQUATIONS

Assuming the fluid to be devoid of viscosity and thermal conductivity, the equations governing magnetogasdynamic flow are (Kanwal 1960)

$$\frac{\partial \rho}{\partial t} + v_i \rho_{,i} + \rho v_{i,i} = 0, \qquad \text{...(2.1)}$$

$$\frac{\partial v_i}{\partial t} + v_k v_{i,k} + \frac{1}{\rho} p_i + (H_k H_{k,i} - H_k H_{i,k}) / 4\pi \rho = 0, \qquad \text{...(2.2)}$$

$$\frac{\partial H_i}{\partial t} + H_{i,j} v_j + H_i v_{k,k} - v_{i,j} H_j = 0, \qquad \text{...(2.3)}$$

$$\frac{\partial s}{\partial t} + v_i\, s_{,i} = 0, \qquad \ldots(2.4)$$

and
$$s = \frac{R}{\gamma - 1} \log \theta + \text{constant}, \qquad \ldots(2.5)$$

where
$$\theta = p^*/\rho^\gamma \text{ ; and } p^* = p + \frac{H^2}{8\pi},$$

where x_i are the Cartesian orthogonal coordinates for space so that there is no difference between the covariant and contravariant components of the vectors. v_i denotes the velocity vector, H_i the magnetic field vector, ρ the density, p the pressure, s the entropy and γ, as usual, is the gas constant. A comma (,) denotes partial differentiation with respect to the coordinates x_i and Einstein's summation convention has been followed.

Now as given by Verma (1967), if we put the thermodynamic pressure,

$$p = \beta p^* \quad , \quad 0 < \beta < 1, \qquad \ldots(2.6)$$

the effective sound velocity c is given by,

$$c^2 = \frac{\gamma^* p^*}{\rho}, \qquad \ldots(2.7)$$

where

$$\gamma^* = \gamma\beta + 2(1-\beta).$$

Written in terms of the effective sound velocity, Eqns. (2.1) and (2.2) assume the following alternative forms,

$$\frac{2}{\gamma-1}\frac{\partial c}{\partial t} + \frac{2}{\gamma-1} v_k\, c_{,k} + c\, v_{k,k} = 0 \qquad \ldots(2.8)$$

and

$$\frac{\partial v_i}{\partial t} + v_k\, v_{i,k} + \frac{2\gamma c}{\gamma^*(\gamma-1)}\, c_{,1} - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H_k H_{i,k} = \frac{c^2}{\gamma^*(\gamma-1)\, c_v} s_{,i}. \ldots(2.9)$$

If $\Sigma\,(t)$ denotes a moving singular surface of order one, relative to at least one of the first derivatives of s, c or v_i and a square bracket [ ] denotes the discontinuity or jump in a quantity across $\Sigma\,(t)$, the compatibility conditions of first order are (Thomas 1957 *b*)

$$\left.\begin{aligned}
[\,v_{i,k}\,] &= \lambda_i \nu_k\,, & \left[\frac{\partial v_i}{\partial t}\right] &= \lambda_i\, G,\\
[\,c_{,i}\,] &= \xi \nu_i\,, & \left[\frac{\partial c}{\partial t}\right] &= -\,\xi\, G,\\
[\,s_{,i}\,] &= \zeta \nu_i\,, & \left[\frac{\partial s}{\partial t}\right] &= -\,\zeta\, G,\\
[\,H_{i,k}\,] &= \eta_i \nu_k\,, \text{ and} & \left[\frac{\partial H_i}{\partial t}\right] &= \eta_i\, G,
\end{aligned}\right\} \qquad \ldots(2.10)$$

where ν_i are the components of the unit normal vector ν to $\Sigma\ (t)$; G denotes the velocity of $\Sigma\ (t)$ in the direction of the unit normal and λ_i, ξ, ζ and η_i are suitable functions defined over the surface $\Sigma\ (t)$.

From eqns. (2.3), (2.4), (2.8) and (2.9), with the help of the above compatibility conditions, we get,

$$\frac{2}{\gamma-1}(G-v_n)\,\xi = c\lambda_i \nu_i, \qquad \ldots(2.11)$$

$$(v_n-G)\,\lambda_i + \frac{2\gamma c}{\gamma^*(\gamma-1)}\xi\nu_i - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi\ c^{2/(\gamma-1)}}\, H_k\, \nu_k \eta_i = \frac{c^2}{\gamma^*(\gamma-1)c_\nu}\ \zeta\nu_i, \qquad \ldots(2.12)$$

$$(v_n-G)\ \eta_i + H_i\, \lambda_k\, \nu_k - H_j\, \lambda_i\, \nu_j = 0 \qquad \ldots(2.13)$$

and

$$(v_n-G)\,\zeta = 0, \qquad \ldots(2.14)$$

where

$$v_n = v_i\, \nu_i.$$

Eqn. (2.14) implies that either,

(*i*) $(v_n - G) = 0,\ \zeta = 0$

or

(*ii*) $(v_n - G) = 0,\ \zeta \neq 0$

or

(*iii*) $(v_n - G) \neq 0,\ \zeta = 0.$

Case (*i*) — If $v_n - G = 0$, $\zeta = 0$; then from (2.11),

$$\lambda_i\, \nu_i = 0. \qquad \ldots(2.15)$$

Here it is assumed that c is not zero anywhere on $\Sigma(t)$. Eqn. (2.15) implies that divergence of velocity is continuous. Also, since $H_i\, \nu_i = H_n$, from (2.12) and (2.13), we have

$$\frac{2\gamma c}{\gamma^*\,(\gamma-1)}\ \xi\ \nu_i = \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi\ c^{2/(\gamma-1)}}\ H_n\ \eta_i \qquad \ldots(2.16)$$

and

$$H_i \lambda_k \nu_k = H_j \nu_j \lambda_i$$

or

$$\lambda_i H_n = 0 \qquad \ldots(2.17)$$

by virtue of (2.15).

Thus the surface $\Sigma\,(t)$ is such that,

$$v_n - G = 0,\ \zeta = 0,\ \lambda_i \nu_i = 0,\ \lambda_i H_n = 0 \text{ and } \frac{2\gamma c}{\gamma^* (\gamma - 1)} \xi \nu_i = \frac{\gamma^* \theta^{1/(\gamma-1)}}{4\pi\, c^{2/(\gamma-1)}} H_n \eta_i.$$

Thus the divergence of velocity, first order derivative of entropy and first order derivative of velocity are continuous across $\Sigma(t)$. In the case when all λ_i vanish, $\Sigma\,(t)$ remains a singular surface of order one. The vorticity vector ω^k must be continuous across $\Sigma\,(t)$, so that,

$$[\,\omega^k\,] = \nu \epsilon^{ijk}\,[\,v_{j,i}\,] = \epsilon^{ijk} \lambda_j \nu_i = 0,$$

where ϵ^{ijk} is the usual skew symmetric permutation tensor. Differentiating (2.4) with respect to x_j, and remembering that first order derivatives of entropy are continuous across $\Sigma\,(t)$ for the case under consideration, we get,

$$\left[\frac{\partial^2 s}{\partial x_j \partial t}\right] + v_i\,[s_{,ij}] + s_{,i}\,[v_{i,j}] = 0. \qquad \ldots(2.18)$$

Further the second order compatibility conditions (Thomas 1957*b*) for s become

$$[\,s_{,ij}\,] = \bar{\zeta} \nu_i \nu_j, \left[\frac{\partial^2 s}{\partial x_j\, \partial t}\right] = -G\, \bar{\zeta} \nu_j, \left[\frac{\partial^2 s}{\partial t^2}\right] = G^2 \bar{\zeta}, \qquad \ldots(2.19)$$

where $\bar{\zeta}$ is some suitable scalar defined over $\Sigma\,(t)$. The equations (2.18) and (2.19) give

$$(G - v_n)\, \bar{\zeta} \nu_j = \lambda_i\, s_{,i}\, \nu_j.$$

Since $v_n - G = 0$ for the case under consideration,

$$\nu_j \lambda_i s_{,i} = 0$$

or,

$$\lambda_i s_{,i} = 0 \quad (\text{since all } \nu_j \neq 0). \qquad \ldots(2.20)$$

Equation (2.20) imposes a further restriction on λ_i; the other restriction being the continuity of the divergence of velocity across $\Sigma\,(t)$, that is,

$$\lambda_i \nu_i = 0.$$

From this relation it follows that the type of singular surface given by case (*i*) is possible in one dimensional unsteady gas flows.

Case (ii) — If $v_n - G = 0$ and $\zeta \neq 0$, then from (2.11), $\lambda_i \nu_i = 0$, if c is nowhere zero on Σ (t). Also from (2.12) and (2.13) we obtain,

$$\frac{2\gamma c}{\gamma^* (\gamma-1)} \xi \nu_i - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H_n \eta_i = \frac{c^2}{\gamma^* (\gamma-1) c_v} \zeta \nu_i$$

and

$$H_i \lambda_k \nu_k = H_j \lambda_i \nu_j$$

or

$$\lambda_i H_n = 0.$$

The divergence of velocity and first order derivative of velocity are continuous across Σ (t). Hence the vorticity vector ω^k also must be continuous across $\Sigma(t)$, that is,

$$[\omega^k] = \epsilon^{ijk} [v_{j.i}] = 0.$$

In both the cases discussed above, divergence of velocity, first order derivative of velocity and vorticity vector are continuous across Σ (t). They differ only in the sense that first order derivative of entropy is continuous in the first case whereas it is discontinuous in the second case. Also, $v_n - G = 0$ in both the cases. This condition can be expressed by saying that the velocity G of surface Σ (t) is equal to the normal component of the velocity of the gas. The first and second type of singular surfaces propagate in the gas entirely by the motion of the particles on these surfaces.

Case (iii) — If $v_n - G \neq 0$ and $\zeta = 0$, from (2.12) we get,

$$(v_n - G) \lambda_i + \frac{2\gamma c}{\gamma^* (\gamma-1)} \xi \nu_i - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H_n \eta_i = 0. \qquad \ldots(2.21)$$

Multiplying both sides by ν_i,

$$(v_n - G) \lambda + \frac{2\gamma c}{\gamma^* (\gamma-1)} \xi = \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H_n \eta, \qquad \ldots(2.22)$$

where

$$\eta_i \nu_i = \eta, \quad \lambda_i \nu_i = \lambda, \quad \nu_i \nu_i = 1.$$

Putting the values of λ and η from (2.11) and (2.13), we obtain,

$$\frac{2}{\gamma-1}\left\{ (G - v_n)^2 - \frac{\gamma}{\gamma^*} c^2 \right\} \xi = 0$$

or

$$G - v_n = \sqrt{\frac{\gamma}{\gamma^*}}\, c. \qquad \ldots(2.23)$$

Multiplying both sides of (2.13) by ν_i, we get

$$(v_n - G) \eta_i \nu_i = 0.$$

But, since $v_n - G \neq 0$, we have

$$\eta_i \, v_i = 0, \qquad \ldots(2.24)$$

which implies that divergence of magnetic field vector is continuous across the surface Σ (t). The equations (2.11) and (2.13) give

$$\lambda = \sqrt{\frac{\gamma}{\gamma^*}} \; \frac{2}{\gamma-1} \; \xi. \qquad \ldots(2.25)$$

Also from (2.21) and (2.25), we obtain

$$\sqrt{\frac{\gamma}{\gamma^*}} \, (\lambda \, v_i - \lambda_i) \, c = \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi \; c^{2/(\gamma-1)}} \, H_n \, \eta_i,$$

which, as a consequence of (2.13) yields,

$$\lambda_i = \lambda \left[\frac{\dfrac{\gamma}{\gamma^*} \, c^2 v_i - \dfrac{(\gamma^*\theta)^{1/(\gamma-1)}}{(4\pi c^{2/(\gamma-1)}} \, H_i \, \eta_i}{\dfrac{\gamma}{\gamma^*} c^2 - \dfrac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi \; c^{2/(\gamma-1)}} \, H_n^2} \right] \qquad \ldots(2.26)$$

Thus $\Sigma(t)$ is such that,

$$G_n - v_n = \sqrt{\frac{\gamma}{\gamma^*}} \, c \; ; \bar{\zeta} = 0 \; , \; \eta_i \, v_i = 0, \; \lambda = \sqrt{\frac{\gamma}{\gamma^*}} \; \frac{2}{\gamma-1} \, \xi$$

$$\lambda_i = \lambda \left[\frac{\dfrac{\gamma}{\gamma^*} c^2 \, v_i - \dfrac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi \; c^{2/(\gamma-1)}} \, H_n \, H_i}{\dfrac{\gamma}{\gamma^*} \; c^2 - \dfrac{(\gamma^*\theta)^{(1/\gamma-1)}}{4\pi c^{2/(\gamma-1)}} \, H^2 n} \right]$$

This singular surface is the so-called sonic wave with speed $\sqrt{\frac{\gamma}{\gamma^*}} \, C$. The first order derivative of entropy is continuous. The ratio of the two scalars λ and ξ which give the jumps in the first derivatives of velocity and effective velocity of sound is a function of γ and β and hence is constant for a particular gas.

From (2.25), we have

$$\frac{\delta\lambda}{\delta t} = \sqrt{\frac{\gamma}{\gamma^*}} \; \frac{2}{\gamma-1} \; \frac{\delta \, \xi}{\delta t}, \qquad \ldots(2.27)$$

where the operator $\frac{\delta}{\delta t}$ is the same as defined by Thomas (1957*b*). This direct proportionality between λ and ξ and that between $\frac{\partial\lambda}{\delta t}$ and $\frac{\delta\xi}{\delta t}$ account for considerable implication in discussing the growth and decay of a sonic wave propagating in a gas under a magnetic field.

SECOND DERIVATIVES OF ENTROPY

The second derivatives of entropy across the sonic wave will be shown to be discontinuous in general and jump in them can be expressed in terms of the jumps in the first order derivatives of velocity. Since the first order derivatives of entropy are continuous across a sonic wave, the second order compatibility conditions (Thomas 1957*b*) can be written as,

$$\left[\ s_{,ij}\ \right] = \bar{\zeta}\, \nu_i\, \nu_j,\ \left[\frac{\partial^2 s}{\partial x_i \partial t}\right] = -\,G\,\bar{\zeta}\,\nu_j,\ \left[\frac{\partial^2 s}{\partial t^2}\right] = G^2\bar{\zeta}. \qquad \ldots(3.1)$$

Differentiating (2.4) with respect to x_j and using the fact that the velocity and the first order derivatives of entropy are continuous across $\Sigma\ (t)$, we get,

$$\left[\frac{\partial^2 s}{\partial x_j \partial t}\right] + v_i\left[\ s_{,ij}\ \right] + s_{,i}\left[\ v_{i,j}\ \right] = 0.$$

From (2.10) and (3.1), we have

$$\bar{\zeta} = \frac{\lambda_i s_{,i}}{G - v_n}\,. \qquad \ldots(3.2)$$

In case the sonic wave propagates in a gas with constant entropy, (3.2) gives $\bar{\zeta} = 0$, implying that the second derivatives of entropy are also continuous across $\Sigma\ (t)$.

THE VORTICITY COMPONENTS

Although the first derivative of velocity is discontinuous across a sonic wave, vorticity vector will suffer no jump across $\Sigma\ (t)$. We have

$$[\ \omega^k\ ] = \epsilon^{ijk}\,[\ v_{j,i}\ ] = \lambda_j\, \nu_i\, \epsilon^{ijk}$$

It is obvious to see that after substituting for λ_j from (2.26), this gives

$$[\omega^k] = 0.$$

This shows that, vorticity vector is continuous across the sonic wave.

CURVATURE OF STREAM LINE

We shall show in this section that the curvature vector of a stream line is discontinuous in general across a sonic wave.

If t_i, n_i, b_i are the components of unit tangent, unit normal, and unit binormal vectors to a stream line at a point, then curvature vector R_i is defined as

$$R_i = \frac{dt_i}{dl} = r n_i, \qquad \ldots(5.1)$$

where r is the curvature of a stream line and l denotes the arc length measured along the stream line in the direction of the flow.

If μ_i denotes the discontinuity in the components of curvature vector across $\Sigma(t)$ then,

$$\mu_i = \left[\frac{dt_i}{dl}\right] = \left[\frac{d}{dl}\left(\frac{v_i}{q}\right)\right] \quad , \quad q = \sqrt{v_i v_i} \; .$$

But as given by Thomas (1957*a*)

$$\frac{dt_i}{dl} = \frac{1}{q}\frac{dv_i}{dl} - \frac{v_i}{q^3}\frac{d}{dl}(v_i v_i) = \frac{t_j v_{i,j}}{q} - \frac{t_k v_j v_j v_{j,k}}{q^3} .$$

Since q, v_i, t_i are continuous across the sonic wave, we get by using compatibility condition,

$$\mu_i = \left(\frac{\lambda_i}{q} - \frac{v_i v_j \lambda_j}{q^3}\right) t_k \nu_k$$

If θ is the angle between unit normal to the sonic wave and the tangent to the stream line at the point where it crosses $\Sigma(t)$,

$$t_j \nu_j = \cos\theta .$$

Then

$$\mu_i = \left(\frac{\lambda_i}{q} - \frac{v_i v_j \lambda_j}{q^3}\right)\cos\theta \qquad \ldots(5.2)$$

or

$$\mu_i = \frac{\lambda\cos\theta}{q}\left[\frac{\frac{\gamma}{\gamma^*}c^2\nu_i - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H_n H_i}{\frac{\gamma}{\gamma^*}c^2 - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}\cdot H^2{}_n} - \frac{v_i v_j}{q^2}\,\frac{\frac{\gamma}{\gamma^*}c^2\nu_j - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H_n H_j}{\frac{\gamma}{\gamma^*}c^2 - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H^2{}_n}\right]$$

Multiplying both sides by ν_i and writing $\mu = \mu_i \nu_i$, we get the following relation between the two scalars λ and μ

$$\mu = \frac{\lambda\cos\theta}{q}\left[1 - \frac{v_n}{q^2}\,\frac{\frac{\gamma}{\gamma^*}c^2 u_n - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H_n H_j v_j}{\frac{\gamma}{\gamma^*}c^2 - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{1/(\gamma-1)}}H_n{}^2}\right] \qquad \ldots(5.3)$$

Equation (5.2) is the required relation giving the jumps in the curvature vector of a stream line across a sonic wave.

If the stream line crosses the sonic wave orthogonally then by (5.2), $\mu_i = 0$. Again, since λ cannot be zero for a sonic wave, μ_i would be zero only if either $\theta = \frac{\pi}{2}$, or

$$\frac{\gamma}{\gamma^*}c^2 v_i - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H_n H_i = \frac{v_i v_i}{q^2}\left(\frac{\gamma}{\gamma^*}c^2 v_j - \frac{(\gamma^*\theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}}H_n H_j\right)$$

$\theta = \frac{\pi}{2}$ gives the trivial case, when the streamline does not cross the wave. The other condition gives,

$$v_i = q^2 \frac{\frac{\gamma}{\gamma^*} c^2 v_i - \frac{(\gamma^* \theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H_n H_i}{\frac{\gamma}{\gamma^*} c^2 v_n - \frac{(\gamma^* \theta)^{1/\gamma-1)}}{4\pi c^2/(\gamma-1)} H_n H_j v_j} . \qquad ...(5.4)$$

In the case when the magnetic field is normal to the velocity,

$$q^2 = \frac{\frac{\gamma}{\gamma^*} c^2}{\frac{\gamma}{\gamma^*} c^2 - \frac{(\gamma^* \theta)^{1/(\gamma-1)}}{4\pi c^{2/(\gamma-1)}} H^2{}_n} v^2{}_n, \qquad ...(5.5)$$

which shows that velocity is in the direction of the unit normal. Hence, we conclude that whenever magnetic field is normal to the velocity, curvature vector of a stream line crossing a sonic wave, is continuous if and only if it crosses the wave orthogonally.

Case of Weak Magnetic Field

In case of a weak magnetic field, p^* will tend to p, consequently, $\beta \to 1$ and $\gamma^* \to \gamma$. In this case, the singular surface moves with velocity c and there is no change in the results obtained in the earlier sections.

Conclusion

For the case (*i*) the effects of the magnetic field are ;—

(*a*) The first order derivative of velocity is continuous.

(*b*) When all λ_i vanish, Σ (*t*) remains a singular surface of order one.

(*c*) The singular surface of order one exists in one dimensional unsteady flow.

For the case (*iii*), the effect of the magnetic field are :

(a) The velocity of sonic wave is given by, $G - v_n = \sqrt{\frac{\gamma}{\gamma^*}} \; C$

(*b*) λ_i which gives the jump in the first derivative of the velocity is not derivable from a single scalar.

(*c*) Whenever the magnetic field is normal to the velocity, curvature vector of stream line crossing the sonic wave is continuous.

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