ON UNSTEADY MHD FREE CONVECTIVE FLOW OF AN INCOMPRESSIBLE, VISCOUS, ELECTRICALLY CONDUCTING FLUID PAST AN INFINITE PLATE WITH TIME-DEPENDENT SUCTION

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(*Received 17 April 1976*)

An analysis of a two-dimensional unsteady free convective flow of an incompressible, viscous, electrically conducting fluid past an infinite porous plate with time-dependent suction is presented under transverse magnetic field. The induced magnetic field is neglected whereas Joule dissipation heat is not neglected. The problem is governed by the coupled non-linear equations and is solved by Fourier series method. Solutions are derived for the functions which effect the mean motions and are shown on graphs. During the course of discussion, the effects of ω (frequency), E (Eckert number), G (the Grashof number) and M (Magnetic field parameter) on the functions affecting the mean flow of mercury ($P=0.025$) are studied quantitatively.

INTRODUCTION

UNSTEADY free convective flow past an infinite or semi-infinite vertical plates has been studied by a number of researchers. The effect of the transverse magnetic field on the steady free convective flow has also been studied in quite a good number of papers. Usually in such types of flows, the viscous dissipative heat is neglected. Without neglecting the viscous dissipative heat, such problems were discussed by Gebhart and Mollendorf (1969), Gebhart (1962). The effect of suction on the unsteady free convective flow was discussed by Nanda and Sharma (1962), Pop (1968). In Pop (1968), variable suction was assumed. The hydromagnetic case of Pop (1968) was independently analysed by Pop (1969), Soundalgekar (1972*a*). The unsteady free convective flow with constant suction and without neglecting the viscous dissipative heat was presented by Soundalgekar (1972*b*) and the case of the variable suction in unsteady free convective flow past an infinite plate was discussed by Soundalgekar and Pop. In all these papers, the technique used for the solution of the problem was that suggested by Lighthill (1954). This technique is applicable for small amplitude flows, for in this case the mean flow is not affected by the frequency of the oscillating temperature. To understand the effects of the

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frequency of the oscillatory temperature of the plate about a constant non-zero mean, Kelly (1965) has suggested a method to analyse a non-linear phenomenon by Fourier Series method. Kelly (1965) discusses the analysis of the forced convection flow past an infinite porous plate with time-dependent suction with the help of a Fourier Series. The hydromagnetic case of Kelly's problem was presented by Soundalgekar (1970) in which the induced magnetic field was neglected.

The unsteady free convective flow past an infinite porous plate with time dependent suction and without viscous dissipative heat was presented by Govindrajulu (1971). His problem was governed by coupled linear equations. But the viscous dissipative heat being effective in fluids of large Prandtl number or in fluids at high gravity field, the problem of Govindrajulu (1971) was analysed by Soundalgekar and Birajdar (1974) without neglecting the viscous dissipative heat. The problem was then governed by coupled non-linear equations and the solutions were obtained by using Fourier series technique.

It is now the object of the present paper to study the effects of the transversely applied magnetic field on the unsteady free convective flow past an infinite plate with time-dependent suction. At low magnetic Reynolds number, the induced magnetic field can be neglected. But in the energy equation. Joule dissipation term is retained. Because of this term, the governing equations are coupled non-linear equations. These are solved by Fourier Series method and the approximate solutions are derived. The functions which affect the mean flow are shown on graphs. During the course of discussion, the effects of the different parameters are discussed in a quantitative manner.

Mathematical Analysis

Notation

B_0 — Magnetic field

c_p — specific heat at constant pressure

E — Eckert number

g — gravitational field

G — Grashof number

k — thermal conductivity of the fluid

P — Prandtl number

q' — rate of heat transfer

T' — temperature of fluid

T'_w — temperature of plate

T'_∞ — temperature of fluid in free stream

τ — time

u', v' — velocity components along x', y' directions

v_o — mean suction velocity

x', y' — co-ordinates along and normal to plate

μ — viscosity

ν — kinematic viscosity

ρ' — density of fluid

β — coefficient of volume expansion

ω' — frequency of the plate temperature

δ — amplitude of the suction velocity

σ — electrical conductivity of fluid

Here two-dimensional unsteady flow past an infinite vertical plate is assumed. The x'-axis is taken along the plate in the vertical direction and the y'-axis is taken normal to it. Let u', v' be the velocity components along and perpendicular to the plate. Then under these conditions, the physical variables are functions of y' and t' only. We also assume that the fluid properties are constant except that the influence of the density variation with temperature is considered only in the body force term and not affecting other terms in the momentum and the energy equations. With these assumptions, the flow is governed by the following equations :

$$\frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$\frac{\partial u}{\partial \tau} + v\frac{\partial u}{\partial y} g\beta = \left(T - T'_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2 u}{\rho}, \qquad \ldots(2)$$

and

$$\rho c_p\left(\frac{\partial T}{\partial \tau} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2. \qquad \ldots(3)$$

All the physical variables in equations (1)—(3) are defined in the notation. In addition to above assumptions, it is also assumed that the variation of expansion coefficient with temperature is very small and hence it can be neglected. Also the pressure term and the influence of the pressure on the density are negligible. In eqn. (3), the last term represents the heat due to Joule dissipation. Here polarisation effects are assumed to be negligible and hence $\overline{E}=0$, i.e., the electric field is also negligible.

From eqn. (1), we conclude that v is either constant or a function of time. We assume here the second case. Then following Kelly (1965), we have from (1), on integration,

$$v = -v_0[1 + \delta(e^{i\omega'\tau} + e^{-i\omega'\tau})], \qquad \ldots(4)$$

where v_0 is the mean steady suction velocity and the negative sign on the right hand side indicates that the suction velocity is directed towards the plate. Here δ is the non-dimensional amplitude of the time-dependent suction velocity.

The boundary conditions of the problem are

$$\left.\begin{array}{ll} u=0, \; T=T_w(\tau) & \text{at } y=0 \\ u=0, \; T\to T_\infty & \text{as } y\to\infty \end{array}\right\} \qquad \ldots(5)$$

Equations (2) and (3), in view of (4), reduce to following non-dimensional form

$$\frac{\partial\Phi}{\partial t}-\left[1+\delta\left(e^{i\omega t}+e^{-i\omega t}\right)\right]\frac{\partial\Phi}{\partial\eta}=G\theta+\frac{\partial^2\Phi}{\partial\eta^2}-M^2\Phi \qquad \ldots(6)$$

and

$$P\frac{\partial\theta}{\partial t}-P\left[1+\delta\left(e^{i\omega t}+e^{-i\omega t}\right)\right]\frac{\partial\theta}{\partial\eta}=\frac{\partial^2\theta}{\partial\eta^2}+M^2PE\Phi^2, \qquad \ldots(7)$$

where

$$\left.\begin{array}{l} t=v_0^2\,\tau/\nu, \quad \eta=v_0 y/\nu, \quad u=v_0\Phi \\ \theta=T-T_\infty/(T_w-T_\infty), \quad \omega=\nu\omega'/v_0^2, \\ P=\dfrac{\mu c_p}{K}, \quad \text{Prandtl number,} \\ M^2=\sigma B_0^2\,\nu/\rho' v_0^2, \quad \text{the magnetic field parameter} \\ E=\dfrac{v_0^2}{c_p\left(T'_w-T'_\infty\right)}, \quad \text{the Eckert number} \\ \text{and} \\ G=\dfrac{\nu g\beta\left(T'_w-T'_\infty\right)}{v_0^3}, \quad \text{the Grashof number.} \end{array}\right\} \qquad \ldots(8)$$

The corresponding boundary conditions are

$$\left.\begin{array}{lll} \Phi=0, & \theta=\theta_w(t) & \text{at } \eta=0 \\ \Phi=0, & \theta\to 0 & \text{as } \eta\to\infty. \end{array}\right\} \qquad \ldots(9)$$

We assume that the plate temperature fluctuates about a constant non-zero mean and hence it is represented by

$$\theta_w(t)=1+\delta\left(e^{i\omega t}+e^{-i\omega t}\right). \qquad \ldots(10)$$

The problem is now governed by the coupled non-linear equations (6) and (7) and they can be solved in an exact manner numerically. It is not possible for us to follow this method for want of adequate computer facilities. Hence we follow the approximate method of finding the solution. Following Kelly (1965), we now represent the velocity and the temperature by the following Fourier series :

$$\Phi(\eta) = \Phi_0(\eta) + \sum_{1}^{\infty} \Phi_n(\eta) e^{in\omega t} + \sum_{1}^{\infty} \widetilde{\Phi}_n(\eta)\, e^{-in\omega t} \qquad \text{...(11)}$$

and

$$\theta(\eta) = \theta_0(\eta) + \sum_{1}^{\infty} \theta_n(\eta)\, e^{in\omega t} + \sum_{1}^{\infty} \widetilde{\theta}_n(\eta)\, e^{-in\omega t} \;. \qquad \text{...(12)}$$

Here $\widetilde{\Phi}_n$ and $\widetilde{\theta}_n$ are the complex conjugates of Φ_n and θ_n respectively.

On substituting (11) and (12) in (6) and (7), equating the harmonic and the non-harmonic terms, we get the following set of coupled equations :
For $n = 0$,

$$\Phi_0'' + \Phi_0' - M^2\Phi_0 + \delta\left(\Phi_1' + \widetilde{\Phi}_1'\right) = -G\theta_0 \qquad \text{...(13)}$$

and

$$\theta_0'' + P\theta_0' + P\delta\left(\theta_1' + \widetilde{\theta}_1'\right) = -PM^2E\left(\Phi_0'^2 + 2\Phi_1'\widetilde{\Phi}_1'\right). \qquad \text{...(14)}$$

For $n = 1$,

$$\Phi_1'' + \Phi_1' - \left(M^2 + i\omega\right)\Phi_1 + \delta\left(\Phi_0' + \Phi_2'\right) = -G\theta_1. \qquad \text{...(15)}$$

$$\theta_1'' + P\theta_1' + P\delta\left(\theta_0' + \theta_2'\right) - Pi\omega\theta_1 = -2PM^2E\left(\Phi_0'\Phi_1' + \overline{\Phi}_1'\Phi_2'\right)$$

and for $n \geqslant 2$ $\quad \Phi_n'' + \Phi_n' + \delta\left(\Phi_{n-1}' + \Phi_{n+1}'\right) - \left(M^2 + in\omega\right)\Phi_n = -G\theta_n \qquad \text{...(16)}$

$$\theta_n'' + P\theta_n + P\delta\left(\theta_{n-1}' + \theta_{n+1}'\right) - Pi\omega n\,\theta_n$$

$$= -2M^2PE[\Phi_0'\Phi_n' + \widetilde{\Phi}_1'\Phi_{n+1}'] \qquad \text{...(17)}$$

and the boundary conditions are

$$\left.\begin{array}{lll} \Phi_n(0) = 0, & \Phi_n(\infty) = 0 & \text{for } n \geqslant 0, \\ \theta_0(0) = 1, & \theta_0(\infty) = 0, & \\ \theta_1(0) = \delta, & \widetilde{\theta}_1(0) = \delta, & \theta_1(\infty) = 0 \\ \text{and} & & \\ \theta_n(0) = 0, & \widetilde{\theta}_n(0) = 0 & \text{for } n \geqslant 2. \end{array}\right\} \qquad \text{...(18)}$$

Here and henceforth, the primes denote differentiation with respect to η only. eqns. (13)—(17) are still coupled non-linear equations and hence their exact solutions are difficult. So we employ the series solutions by expanding Φ's and θ's in powers of δ where δ is assumed to be $<<1$.

$$\left.\begin{aligned} \Phi_n(\eta) &= \sum_{n=0}^{\infty} \Phi_{nj}\delta^j \\ \theta_n(\eta) &= \sum_{n=0}^{\infty} \theta_{nj}\delta^j \end{aligned}\right\} \qquad \ldots(19)$$

Substituting (19) in (13)—(18) and equating the coefficients of like powers of δ, we have again a set of coupled non-linear equations :

$$\Phi_{00}'' + \Phi_{00}' - M^2\Phi_{00} = -G\theta_{00}, \qquad \ldots(20)$$

$$\Phi_{01}'' + \Phi_{01}' - M^2\Phi_{01} + \Phi_{10}' + \tilde{\Phi}_{10}' = -G\theta_{01}, \qquad \ldots(21)$$

$$\Phi_{02}'' + \Phi_{02}' - M^2\Phi_{02} + \left(\Phi_{11}' + \tilde{\Phi}_{11}'\right) = -G\theta_{02}, \qquad \ldots(22)$$

$$\Phi_{10}'' + \Phi_{10}' - \left(M^2 + i\omega\right)\Phi_{10} = -G\theta_{11}, \qquad \ldots(23)$$

$$\Phi_{11}'' + \Phi_{11}' + \Phi_{10}' + \Phi_{20}' - \left(M^2 + i\omega\right)\Phi_{11} = -G\theta_{11}, \qquad \ldots(24)$$

$$\Phi_{20}'' + \Phi_{20}' - \left(M^2 + 2i\omega\right)\Phi_{20} = -G\theta_{20} \qquad \ldots(25)$$

and

$$\Phi_{12}'' + \Phi_{12}' + \Phi_{01}' + \Phi_{03}' - \left(M^2 + 2i\omega\right)\Phi_{12} = -G\theta_{12}. \qquad \ldots(26)$$

with boundary conditions

$$\left.\begin{aligned} &\Phi_{n0}(0) = 0,\ \Phi_{n0}(\infty) = 0 \text{ for } n = 0, 1, 2\ldots, \\ &\Phi_{0n}(0) = 0,\ \Phi_{0n}(\infty) = 0 \text{ for } n = 0, 1, 2\ldots, \end{aligned}\right\} \qquad \ldots(27)$$

and

$$\theta_{00}'' + P\theta_{00}' = -PM^2E\left(\Phi_{00}^2 + 2\Phi_{10}\tilde{\Phi}_{10}\right), \qquad \ldots(28)$$

$$\theta_{01}'' + P\theta_{01}' + P\left(\theta_{10}' + \tilde{\theta}_{10}'\right) =$$

$$= -2PM^2E\left(\Phi_{00}\Phi_{01} + \tilde{\Phi}_{10}\Phi_{11} + \Phi_{10}\tilde{\Phi}_{11}\right). \qquad \ldots(29)$$

Then

$$\theta_{02}'' + P\theta_{02}' + P\left(\theta_{11}' + \tilde{\theta}_{11}'\right) = -PM^2E\Big[\Phi_{01}'^2 + 2\Phi_{00}\,\Phi_{02} + 2\Phi_{11}\,\tilde{\Phi}_{11} + 2\Phi_{10}\,\tilde{\Phi}_{12} + 2\Phi_{12}\,\tilde{\Phi}_{10}\Big] \quad ...(30)$$

$$\theta_{10}'' + P\theta_{10}' - i\omega P\,\theta_{10} = -\,2PM^2E\left(\Phi_{00}\,\Phi_{10} + \tilde{\Phi}_{10}\,\Phi_{20}\right) \quad ...(31)$$

$$\theta_{11}'' + P\,\theta_{11}' + P\theta_{00} + P\theta_{20}' - i\,\omega\,P\,\theta_{11} = -\,2PM^2E\,(\Phi_{01}\Phi_{10} + \Phi_{00}\Phi_{11} + \tilde{\Phi}_{11}\Phi_{20} + \tilde{\Phi}_{10}\Phi_{21} \quad ...(32)$$

$$\theta_{20}'' + P\,\theta_{20}' - 2i\,\omega\,P\theta_{20} = -\,2PM^2E\,(\Phi_{00}\Phi_{20} + \tilde{\Phi}_{10}\Phi_{30}) \quad ...(33)$$

$$\theta_{21}'' + P\,\theta_{21}' + P\,\theta_{10}' + P\,\theta_{30}' - 2i\,\omega\,P\,\theta_{21} = -\,2PM^2E\,(\Phi_{01}\Phi_{20} + \Phi_{00}\Phi_{21} + \tilde{\Phi}_{11}\Phi_{31} + \tilde{\Phi}_{10}\Phi_{31}) \quad ...(34)$$

and the boundary conditions are

$$\left.\begin{array}{ll}\theta_{00}\,(0) = 1, & \theta_{00}\,(\infty) = 0 \\ \theta_{n0}\,(0) = 0, & \theta_{n0}\,(\infty) = 0,\ n \geqslant 1\end{array}\right\} \quad ...(35)$$

To solve these coupled non-linear equations (20)—(35), we again assume that the Joule dissipation heat is superimposed on the flow. Mathematically, this is given by

$$\left.\begin{array}{l}\Phi_{001} = \Phi_{001} + E\,\Phi_{002} \\ \theta_{00} = \theta_{001} + E\,\theta_{002} \\ \Phi_{01} = \Phi_{011} + E\Phi_{012} \\ \theta_{01} = \theta_{011} + E\,\theta_{012} \\ \Phi_{10} = \Phi_{101} + E\,\Phi_{102} \\ \theta_{10} = \theta_{101} + E\,\theta_{102} \\ \Phi_{11} = \Phi_{111} + E\,\Phi_{112} \\ \theta_{11} = \theta_{111} + E\,\theta_{112} \\ \Phi_{20} = \Phi_{201} + E\,\Phi_{202} \\ \theta_{20} = \theta_{201} + E\,\theta_{202}\end{array}\right\} \quad ...(36)$$

Substituting (36) into (20) — (35), equating the coefficients of different powers of E, we have

$$\Phi_{001}'' + \Phi_{001}' - M^2\,\Phi_{001} = -\,G\,\theta_{001}\,, \quad ...(37)$$

$$\Phi''_{002}+\Phi'_{002}-M^2\,\Phi_{002}=-G\,\theta_{002}\,, \qquad \ldots(38)$$

$$\left.\begin{array}{ll}\Phi_{001}(0)=0, & \Phi_{001}(\infty)=0\\ \Phi_{002}(0)=0, & \Phi_{002}(\infty)=0\end{array}\right\}, \qquad \ldots(39)$$

$$\Phi''_{011}+\Phi'_{011}-M^2\,\Phi_{011}+\Phi'_{101}+\tilde{\Phi}'_{101}=-G\,\theta_{011}\,, \qquad \ldots(40)$$

$$\Phi''_{012}+\Phi'_{012}-M^2\,\Phi_{012}+\Phi'_{102}+\tilde{\Phi}'_{102}=-G\,\theta_{012}\,. \qquad \ldots(41)$$

$$\left.\begin{array}{ll}\Phi_{011}(0)=0, & \Phi_{011}(\infty)=0\\ \Phi_{012}(0)=0, & \Phi_{012}(\infty)=0\end{array}\right\}, \qquad \ldots(42)$$

$$\Phi''_{101}+\Phi'_{101}-(M^2+i\,\omega)\Phi_{101}=-G\,\theta_{101}, \qquad \ldots(43)$$

$$\Phi''_{102}+\Phi'_{102}-(M^2+i\,\omega)\,\Phi_{102}=-G\,\theta_{102}, \qquad \ldots(44)$$

$$\left.\begin{array}{ll}\Phi_{101}(0)=0, & \Phi_{101}(\infty)=0,\\ \Phi_{102}(0)=0, & \Phi_{102}(\infty)=0.\end{array}\right\} \qquad \ldots(45)$$

$$\Phi''_{111}+\Phi'_{111}+\Phi'_{001}+\Phi'_{201}-(M^2+i\,\omega)\,\Phi_{111}=-G\theta_{111}, \qquad \ldots(46)$$

$$\Phi''_{112}+\Phi'_{112}+\Phi'_{002}+\Phi'_{202}-(M^2+i\,\omega)\,\Phi_{112}=-G\theta_{112}, \qquad \ldots(47)$$

$$\left.\begin{array}{ll}\Phi_{111}(0)=0, & \Phi_{111}(\infty)=0,\\ \Phi_{112}(0)=0, & \Phi_{112}(\infty)=0.\end{array}\right\} \qquad \ldots(48)$$

$$\Phi''_{201}+\Phi'_{201}-(M^2+2\,i\omega)\,\Phi_{201}=-G\theta_{201}, \qquad \ldots(49)$$

$$\Phi''_{202}+\Phi'_{202}-(M^2+2\,i\,\omega)\,\Phi_{202}=-G\theta_{202}, \qquad \ldots(50)$$

$$\left.\begin{array}{ll}\Phi_{201}(0)=0, & \Phi_{201}(\infty)=0,\\ \Phi_{202}(0)=0, & \Phi_{202}(\infty)=0,\end{array}\right\} \qquad \ldots(51)$$

$$\Phi''_{211}+\Phi'_{211}+\Phi'_{101}+\Phi'_{301}-(M^2+2i\,\omega)\Phi_{211}=-G\theta_{211}, \qquad \ldots(52)$$

$$\Phi''_{212}+\Phi'_{212}+\Phi'_{102}+\Phi'_{302}-(M^2+2i\,\omega)\,\Phi_{212}=-G\,\theta_{212}, \qquad \ldots(53)$$

$$\left.\begin{array}{ll}\Phi_{211}(0)=0, & \Phi_{211}(\infty)=0,\\ \Phi_{211}(0)=0, & \Phi_{212}(\infty)=0.\end{array}\right\} \qquad \ldots(54)$$

$$\Phi''_{021}+\Phi'_{021}-M^2\Phi_{021}+\left(\Phi'_{111}+\tilde{\Phi}'_{111}\right)=-G\theta_{021}, \qquad \ldots(55)$$

$$\Phi_{022}'' + \Phi_{022}' - M^2 \Phi_{022}' + \left(\Phi_{112}' + \widetilde{\Phi}_{112}'\right) = -G\theta_{022}, \qquad \ldots(56)$$

$$\left.\begin{array}{ll} \Phi_{021}(0) = 0, & \Phi_{021}(\infty) = 0, \\ \Phi_{022}(0) = 0, & \Phi_{022}(\infty) = 0. \end{array}\right\} \qquad \ldots(57)$$

$$\theta_{001}'' + P\theta_{001}' = 0, \qquad \ldots(57a)$$

$$\theta_{002}'' + P\theta_{002}' = -PM^2\left(\Phi_{001}'^{2} + 2\Phi_{101}\widetilde{\Phi}_{101}\right), \qquad \ldots(58)$$

$$\left.\begin{array}{ll} \theta_{001}(0) = 1, & \theta_{001}(\infty) = 0, \\ \theta_{002}(0) = 0, & \theta_{002}(\infty) = 0. \end{array}\right\} \qquad \ldots(59)$$

$$\theta_{011}'' + P\theta_{011}' + P\theta_{101}' + P\widetilde{\theta}_{101}' = 0, \qquad \ldots(60)$$

$$\theta_{012}'' + P\theta_{012}' + P\left(\theta_{102}' + \widetilde{\theta}_{102}'\right) =$$

$$-2PM^2(\Phi_{001}\Phi_{011} + \widetilde{\Phi}_{101}\Phi_{111} + \Phi_{101}\widetilde{\Phi}_{111}), \qquad \ldots(61)$$

$$\left.\begin{array}{ll} \theta_{011}(0) = 0, & \theta_{011}(\infty) = 0, \\ \theta_{012}(0) = 0, & \theta_{012}(\infty) = 0. \end{array}\right\} \qquad \ldots(62)$$

$$\theta_{101}'' + P\theta_{101}' - i\omega P\theta_{101} = 0, \qquad \ldots(63)$$

$$\theta_{102}'' + P\theta_{102}' - i\omega P\theta_{102} = -2PM^2(\Phi_{001}\Phi_{101} + \Phi_{101}^{2}\Phi_{201}), \qquad \ldots(64)$$

$$\left.\begin{array}{ll} \theta_{101}(0) = 0, & \theta_{101}(\infty) = 0, \\ \theta_{102}(0) = 0, & \theta_{102}(\infty) = 0. \end{array}\right\} \qquad \ldots(65)$$

$$\theta_{111}'' + P\theta_{111}' + P\theta_{001}' + P\theta_{201}' - i\omega P\theta_{111} = 0, \qquad \ldots(66)$$

$$\theta_{112}'' + P\theta_{112}' + P\theta_{002}' + P\theta_{202}' - i\omega P\theta_{112} =$$

$$-2PM^2(\Phi_{011}\Phi_{101} + \Phi_{001}\Phi_{111} + \widetilde{\Phi}_{111}\Phi_{201} + \widetilde{\Phi}_{201}\Phi_{211} \qquad \ldots(67)$$

$$\left.\begin{array}{ll} \theta_{111}(0) = 1, & \theta_{111}(\infty) = 0, \\ \theta_{112}(0) = 0, & \theta_{112}(\infty) = 0. \end{array}\right\} \qquad \ldots(68)$$

$$\theta_{201}'' + P\theta_{201}' - 2i\omega P\theta_{201} = 0, \qquad \ldots(69)$$

$$\theta_{202}'' + P\theta_{201}' - 2i\omega P\theta_{202} = -2PM^2(\Phi_{001}\Phi_{201} + \tilde{\Phi}_{101}\Phi_{301}), \quad \ldots(70)$$

$$\left.\begin{array}{ll}\theta_{201}(0) = 0, & \theta_{201}(\infty) = 0,\\ \theta_{202}(0) = 0, & \theta_{201}(\infty) = 0.\end{array}\right\} \quad \ldots(71)$$

$$\theta_{211}'' + P\theta_{211}' + P\theta_{101}' + P\theta_{301}' - 2i\omega P\theta_{211} = 0, \quad \ldots(72)$$

$$\theta_{212}'' + P\theta_{212}' + P\theta_{102}' + P\theta_{302}' - 2i\omega P\theta_{212} =$$

$$-2PM^2(\Phi_{011}\Phi_{201} + \Phi_{001}\Phi_{211} + \tilde{\Phi}_{111}\Phi_{301} + \tilde{\Phi}_{101}\Phi_{311}) \quad \ldots(73)$$

$$\left.\begin{array}{ll}\theta_{211}(0) = 0, & \theta_{211}(\infty) = 0,\\ \theta_{212}(0) = 0, & \theta_{212}(\infty) = 0.\end{array}\right\} \quad \ldots(74)$$

$$\theta_{021}'' + P\theta_{021}' + P\left(\theta_{111}' + \theta_{111}'\right) = 0, \quad \ldots(75)$$

$$\theta_{022}'' + P\theta_{022}' + P\left(\theta_{112}' + \tilde{\theta}_{112}'\right) =$$

$$-PM^2\left(\Phi_{011}^2 + 2.\Phi_{001}\Phi_{021} + 2\Phi_{111}\Phi_{111}' + 2\Phi_{101}\tilde{\Phi}_{121} + 2\Phi_{121}\tilde{\Phi}_{101}\right), \quad \ldots(76)$$

$$\left.\begin{array}{ll}\theta_{021}(0) = 0, & \theta_{021}(\infty) = 0,\\ \theta_{022}(0) = 0, & \theta_{022}(\infty) = 0.\end{array}\right\} \quad \ldots(77)$$

Equations (37)—(78) are solved and the expressions for the non-trivial solutions are given below :

$$\theta_{001}(\eta) = e^{-P\eta}, \quad \ldots(78)$$

$$\theta_{002}(\eta) = \frac{PM^2G^2}{(P^2 - P - M^2)^2(4h^2 - 2hP)}\left(e^{-P\eta} - e^{-2h\eta}\right)$$

$$- \frac{2PM^2G^2}{(P^2 - P - M^2)^2(h^2 + hP)}\left(e^{-P\eta} - e^{-(h+P)\eta}\right)$$

$$+ \frac{M^2G^2}{2P(P^2 - P - M^2)^2}\left(e^{-P\eta} - e^{-2P\eta}\right), \quad \ldots(79)$$

$$h = \frac{1 + \sqrt{1 + 4M^2}}{2},$$

$$\Phi_{001}(\eta) = \frac{G}{P^2 - P - M^2}\left(e^{-h\eta} - e^{-P\eta}\right), \quad \ldots(80)$$

$$\Phi_{002}(\eta) = \frac{M^2G^3}{(P^2-P-M^2)^3}\left[\frac{P}{4h^2-2hP} - \frac{2P}{h^2+hP} + \frac{1}{2P(P^2-P-M^2)}\right]$$

$$(e^{-h\eta} - e^{-P\eta})$$

$$-\frac{PM^2G^3}{(P^2-P-M^2)^2(4h^2-2h-M^2)(4h^2-2hP)}\left(e^{-h\eta} - e^{-2h\eta}\right) +$$

$$+\frac{2PM^2G^3}{(P^2-P-M^2)^2(h^2+hP)\{(h+P)^2-(h+P)-M^2\}} \times$$

$$\left(e^{-h\eta} - e^{-(h+P)\eta}\right)$$

$$-\frac{M^2G^3}{2P(P^2-P-M^2)^2(4P^2-2P-M^2)}\left(e^{-h\eta} - e^{-2P\eta}\right), \qquad \ldots(81)$$

$$\theta_{111}(\eta) = \left(1-\frac{iP}{\omega}\right)e^{-k\eta} + \frac{iP}{\omega}e^{-P\eta}, \qquad \ldots(82)$$

$$k = \frac{P+\sqrt{P^2+4iP\omega}}{2},$$

$$\theta_{111}(\eta) = \frac{G}{P^2-P-i\omega-M^2}\left(\frac{P}{P^2-P-M^2}+\frac{iP}{\omega}\right)\left(e^{-l\eta}-e^{-P\eta}\right)$$

$$+\frac{G\left(1-\frac{iP}{\omega}\right)}{k^2-k-i\omega-M^2}\left(e^{-l\eta}-e^{-k\eta}\right),$$

$$-\frac{Gh}{(P^2-P-M^2)(h^2-h-i\omega-M^2)}\left(e^{-l\eta}-e^{-h\eta}\right) \qquad \ldots(83)$$

and

$$l = \frac{1+\sqrt{(1+4M^2)+4i\omega}}{2}.$$

The expressions for θ_{112}, Φ_{112}, θ_{021}, Φ_{021}, θ_{022}, Φ_{022} being very lengthy, they are not mentioned here to save space.

Thus the functions which affect the mean flow field are now given by

$$\left.\begin{aligned} \Phi_{00} &= \Phi_{001} + E\Phi_{002} \\ \Phi_{02} &= \Phi_{021} + E\Phi_{022} \\ \theta_{00} &= \theta_{001} + E\theta_{002} \\ \text{and} \quad \theta_{02} &= \theta_{021} + E\theta_{022}. \end{aligned}\right\} \qquad \ldots(84)$$

Hence the mean flow is

$$\left.\begin{aligned} \Phi &= \Phi_{00} + \delta^2\Phi_{02} \\ \text{and} \quad \theta &= \theta_{00} + \delta^2\theta_{02}. \end{aligned}\right\} \qquad \ldots(85)$$

We observe from (82), (83) that Φ_{11}, θ_{11} are the nontrivial solutions. Hence from (22) and (30), we see that Φ_{02} and θ_{02} are affected by Φ_{11} and θ_{11} and through these quantities Φ_{02} and θ_{02} are affected by ω, the frequency. Hence, the mean flow is affected by the frequency of the oscillating temperature of the plate and also the applied magnetic field. The functions which influence the velocity and the temperature field of mercury ($P = 0.025$) viz., Φ_{00}, Φ_{02}; θ_{00}, θ_{02} are shown on figures 1—4.

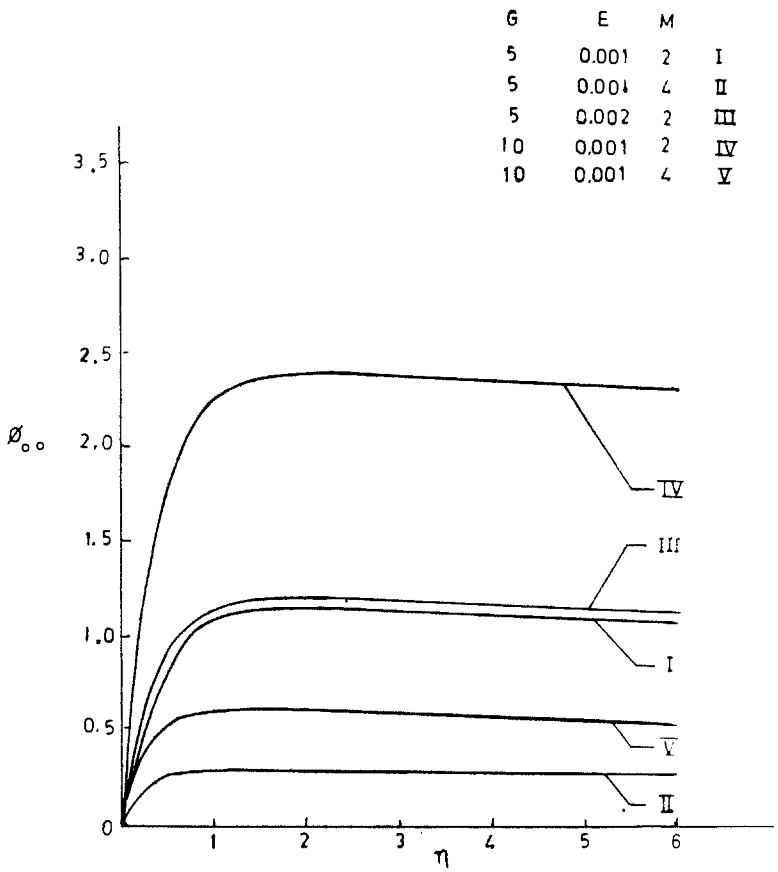

Fig. 1

The function Φ_{00} is plotted on Fig. 1. It is observed from this figure that an increase in M leads to a decrease in the value of the function. Quantitatively, we see here that for $G=5$, $E=0.001$, $\gamma=2$, there is a 68 per cent drop in the value of Φ_{00}, when M is increased from 2 to 4. But an increase in G leads to an increase in Φ_{00}. Thus, for $E=0.001$, $\gamma=2$, $M=2$, an increase in G from 5 to 10 leads to 92 per cent rise in the value of Φ_{00}. The function Φ_{02} is shown on Fig. 2. An increase in ω or M leads to an increase in Φ_{02}, when G and E are constant. But an increase in G leads to a decrease in Φ_{02}. The function θ_{00} affecting the mean temperature is shown on Fig. 3. The function θ_{00} is not

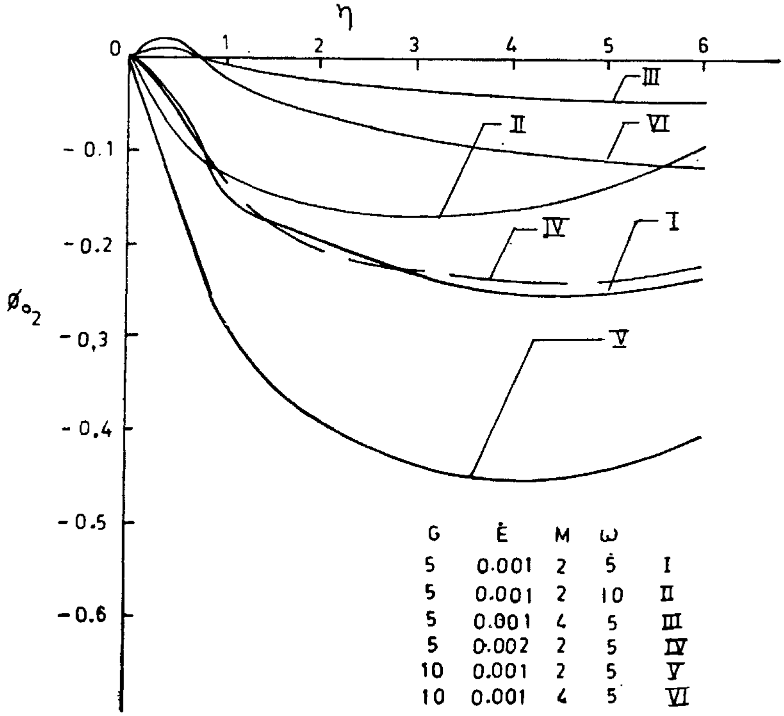
η
0
1
2
3
4
5
6
- 0.1
- 0.2
- 0,3
- 0.4
- 0.5
- 0.6
ϕ_{o_2}
III
II
VI
IV
I
V
G Ė M ω
5 0.001 2 5 I
5 0.001 2 10 II
5 0.001 4 5 III
5 0.002 2 5 IV
10 0.001 2 5 V
10 0.001 4 5 VI

Fig. 2

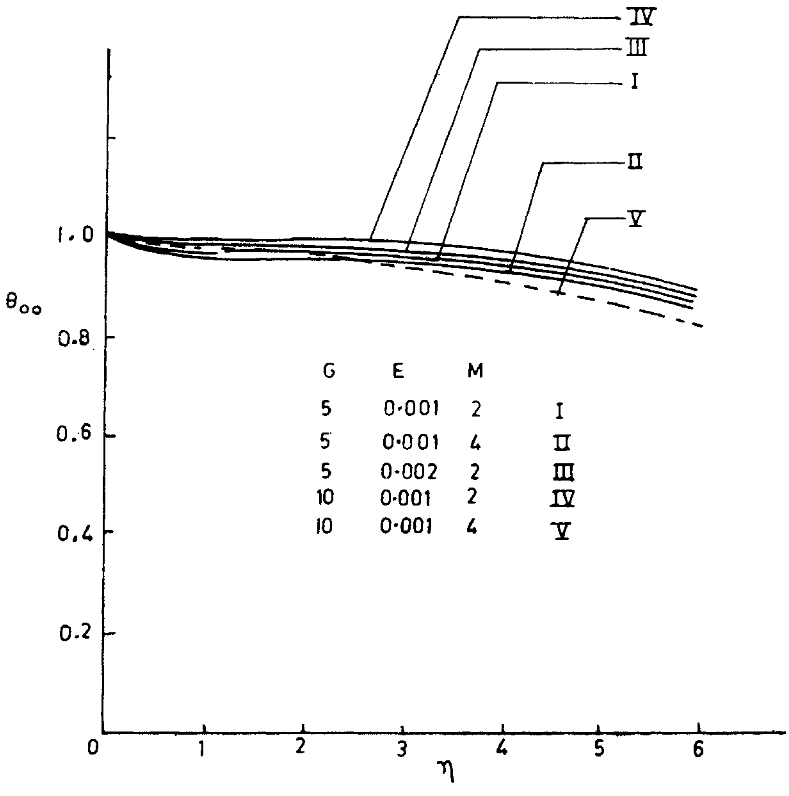
IV
III
I
II
V
1.0
θ_{oo}
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
η
G E M
5 0·001 2 I
5 0·001 4 II
5 0·002 2 III
10 0·001 2 IV
10 0·001 4 V

Fig. 3

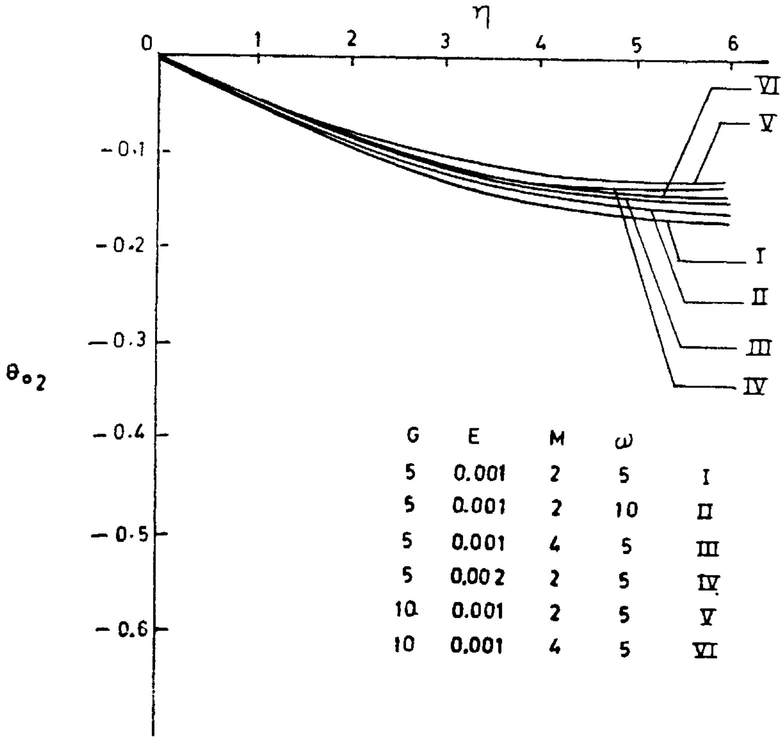

Fig. 4

affected significantly by G, E or M. But the function θ_{02} is slightly affected. It increases with increasing ω. θ_{02} increases with increasing M at small values of G whereas it decreases with increasing M at large values of G. An increase in G also leads to an increase in θ_{02}.

From the velocity field and the temperature field given by (86), we now derive the expressions for the mean skin-friction and the mean rate of heat transfer. They are given by

$$\tau = \tau'/\rho v_0^2 = \frac{d\Phi_0}{d\eta}\bigg|_{\eta=0} = \frac{d\Phi_{00}}{d\eta}\bigg|_{\eta=0} + \delta^2 \frac{d\Phi_{02}}{d\eta}\bigg|_{\eta=0} \qquad \ldots(86)$$

and

$$q = \frac{q_0' \nu}{k\left(T_\omega' - T_\infty'\right) v_0} = -\frac{d\theta_0}{d\eta}\bigg|_{\eta=0} = -\left(\frac{d\theta_{00}}{d\eta}\right)_{\eta=0} + \delta^2 \frac{d\theta_{02}}{d\eta}\bigg|_{\eta=0} \qquad \ldots(87)$$

We have substituted the expressions for Φ_{00}, Φ_{02}, θ_{00}, θ_{02} in (86) and (87) and the numerical values of τ and q are entered in Table I. We observe from this table that an increase in ω leads to a decrease in the mean skin-friction. Thus, for $G=5$, $E=0.001$, $M=2$, there is 0.31 per cent decrease in the mean skin-friction when ω is increased from 5 to 10. An increase in M also leads to a decrease in the mean skin-friction. For possible experimental verification, we observe here that for $G=5$, $E=0.001$, $\omega=5$, there is 124 per cent decrease in the mean skin-friction when M is increased from 2 to 4. Also, greater viscous dissipative heat leads to an increase in the mean skin-friction. An increase in G also leads to an increase in τ. Quantitatively, we see here that for $E=0.001$, $M=2$, there is 101 per cent rise in the value of τ when G is increased from 5 to 10.

TABLE I

Values of τ. $\delta=0.2$

G	E	$M \backslash \omega$	5	10	15	20
5	0.001	2	3.1775	3.1619	3.1585	3.1580
5	0.001	4	1.4154	1.4178	1.4163	1.4144
5	0.002	2	3.1835	3.1678	3.1644	3.1639
10	0.001	2	6.3910	6.3592	6.3523	6.3514

Values q. $\delta=0.2$

G	E	$M \backslash \omega$	5	10	15	20
5	0.001	2	0.02393	0.023992	0.024000	0.024002
5	0.001	4	0.026197	0.026218	0.026227	0.026230
5	0.002	2	0.020802	0.020985	0.021000	0.021004
10	0.001	4	0.014690	0.014902	0.014998	0.015006

Now a close study of the values of the rate of heat transfer shows that it is not affected significantly by large values of the frequency ω. An increase in M leads to an increase in q whereas an increase in E or G leads to a decrease in the value of q.

CONCLUSIONS

The conclusions are as follows :—

1. An increase in M leads to a decrease in Φ_{00} and an increase in G leads to an increase in Φ_{00}.
2. An increase in ω or M leads to an increase in Φ_{02} whereas with rising G, there is a decrease in Φ_{02}.
3. The functions θ_{00}, θ_{02} are not significantly affected by the frequency ω.
4. The mean skin-friction decreases with increasing ω or M. Greater viscous dissipative heat or an increase in G leads to an increase in the mean skin-friction.

5. The rate of heat transfer is not significantly affected by large ω. An increase in M leads to an increase in q whereas it decreases with increasing E or G.

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