PHOTONS IN A LEWIS FIELD

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A test particle in the gravitational field of a rotating cylinder acquires angular velocity about the same axis of rotation as that of the body itself. In the present paper, we study the restrictions imposed upon the motion of photons by this angular velocity in such a field.

INTRODUCTION

The emission of photons from spherical bodies has been studied by Synge (1966). Motions of particles in a Kerr field have been investigated in detail by F. De Felice and M: Calvani (1972). Bannerjee (1968) and Krori and Chaudhury (1978) have studied the escape of photons from cylindrical bodies. But so far no attention appears to have been paid to the behaviour of photons in the gravitational field due to a rotating cylinder. Here the effect of the angular velocity of photons, caused by the rotation of a cylinder, on their motion in the gravitational field of the cylinder has been investigated and some interesting results obtained.

CALCULATIONS

Landau and Lifshitz (1971) have shown that the total energy of a particle of proper mass m_0 in a static gravitational field is given by

$$E = m_0 c^2 \, g_{44} \frac{dt}{\sqrt{g_{44}\, dt^2 - dl^2}} , \qquad \ldots(1)$$

where c is the velocity of light in empty gravitation-free space, g_{44} the (44)-component of the metric tensor and dl an invariant infinitesimal element of spatial displacement.

If we introduce the velocity

$$v = \frac{dl}{\sqrt{g_{44}}\; dt} , \qquad \ldots(2)$$

of the particle, measured in terms of the proper time, that is, by an observer located at the given point, then we obtain for the energy

$$E = \frac{m_0 c^2 \sqrt{g_{44}}}{\sqrt{1 - \frac{v^2}{c^2}}} . \qquad \ldots(3)$$

Landau and Lifshitz have also shown that the expression (3) remains valid for a stationary gravitational field if v is given by

$$v = \frac{c\, dl}{\sqrt{g_{44}} \left(dt + \frac{g_{4\sigma}}{g_{44}} dx_\sigma \right)}, \qquad \sigma = 1, 2, 3. \qquad \ldots(4)$$

In the case of photons, (3) and (4) give for the velocity

$$V(O) = \frac{dl}{\sqrt{g_{44}(O)}dt} = \sqrt{\frac{g_{44}}{g_{44}(O)}} \left(1 + \frac{g_{4\sigma}}{g_{44}} \frac{dx^\sigma}{dt} \right), \qquad \ldots(5)$$

where $\sqrt{g_{44}(O)}\, dt$ is the proper time of an observer O located at $x^\sigma (O)$.

Lewis (1932) has given the following line-element for an infinite cylinder rotating about its axis

$$ds^2 = f\, dt^2 - \rho\, (dr^2 + dz^2) - l d\phi^2 + 2m\, d\phi\, dt, \qquad \ldots(6)$$

where, f, ρ, l and m are given by the following expressions

$$\begin{aligned} f &= \gamma^2 (r^{2k} - \omega^2 r^{2-2k}) \\ l &= \gamma^2 (r^{2-2k} - \omega^2 r^{2k}) \\ m &= \gamma^2 \omega\, (r^{2-2k} - r^{2k}) \\ \rho &= A^2 r^{2k(k-1)} \end{aligned} \qquad \ldots(6a)$$

with $\gamma^2 = (1 - \omega^2)^{-1}$.

Here A and k are constants related to mass per unit length of the cylinder and ω represents its angular velocity. The expression for the velocity of photons in the field is given from (5) by

$$\left(\frac{dl}{\sqrt{f(o)}\ dt} \right) = \sqrt{\frac{f}{f(o)}} \left(1 + \frac{m}{f} \frac{d\phi}{dt} \right). \qquad \ldots(7)$$

Equation (7) shows that the angular velocity $\left(\frac{d\phi}{dt} \right)$ is an intrinsic part of the motion of photons in the field of a rotating cylinder.

The geodesic equations for photons are—

$$f \dot{t}^2 - \rho\, (\dot{r}^2 + \dot{z}^2) - l \dot{\phi}^2 + 2m\, \dot{\phi} \dot{t} = O, \qquad \ldots(8)$$

$$\ddot{Z} + \frac{\rho'}{\rho} \dot{r} \dot{Z} = O, \qquad \ldots(9)$$

$$\ddot{\phi} + \left[\frac{fl'}{r^2} - \frac{mm'}{r^2}\right] \dot{r}\,\dot{\phi} - \left[\frac{fm'}{r^2} + \frac{mf'}{r^2}\right] \dot{r}\,\dot{t} = O, \qquad \ldots(10)$$

and

$$\ddot{t} + \left[\frac{ml'}{r^2} + \frac{lm'}{r^2}\right] \dot{r}\,\dot{\phi} - \left[\frac{mm'}{r^2} - \frac{lf'}{r^2}\right] \dot{r}\,\dot{t} = O, \qquad \ldots(11)$$

where dots and dashes represent differentiations with respect to an affine parameter and r respectively.

From equations (9) and (10) above, it follows that both $\dot{Z}$ and $\dot{\phi}$ acquire extremal values at the turning points (r_o) i.e., when $\dot{r} = O$

At the turning point r_o in a Z-constant plane, (8) reduces to the quadratic form

$$l\left(\frac{d\phi}{dt}\right)^2 - 2m\,\frac{d\phi}{dt} - f = O. \qquad \ldots(12)$$

We propose to call the angular velocity given by (12) the "recoil angular velocity". Equation (12) shows that the recoil angular velocity of photons will have two different values given by

$$\left(\frac{d\phi}{dt}\right)_+ = \frac{m\,(r_o) + r_o}{l\,(r_o)} \qquad \ldots(13)$$

and

$$\left(\frac{d\phi}{dt}\right)_- = \frac{m\,(r_o) - r_o}{l\,(r_o)} \qquad \ldots(14)$$

In obtaining (13) and (14) we have made use of the following property satisfied by f, l and m [Vanstockum (1937)]

$$fl + m^2 = r^2 \qquad \ldots(15)$$

Now substituting for $m\,(r_o)$ and $l\,(r_o)$ from (6a) in (13) and (14), we obtain

$$\left(\frac{d\phi}{dt}\right)_+ = \frac{y^2\omega\,(r_o^{1-2k} - r_o^{2k-1}) + 1}{y^2\,(r_o^{1-2k} - \omega^2 r_o^{2k-1})} \qquad \ldots(16)$$

$$\left(\frac{d\phi}{dt}\right)_- = \frac{y^2\omega\,(r_o^{1-2k} - r_o^{2k-1}) - 1}{y^2\,(r_o^{1-2k} - \omega^2 r_o^{2k-1})} \qquad \ldots(17)$$

We now plot curves (Figs. 1, 2 and 3) to show variation of $\left(\frac{d\phi}{dt}\right)_{\pm}$ with r_o for

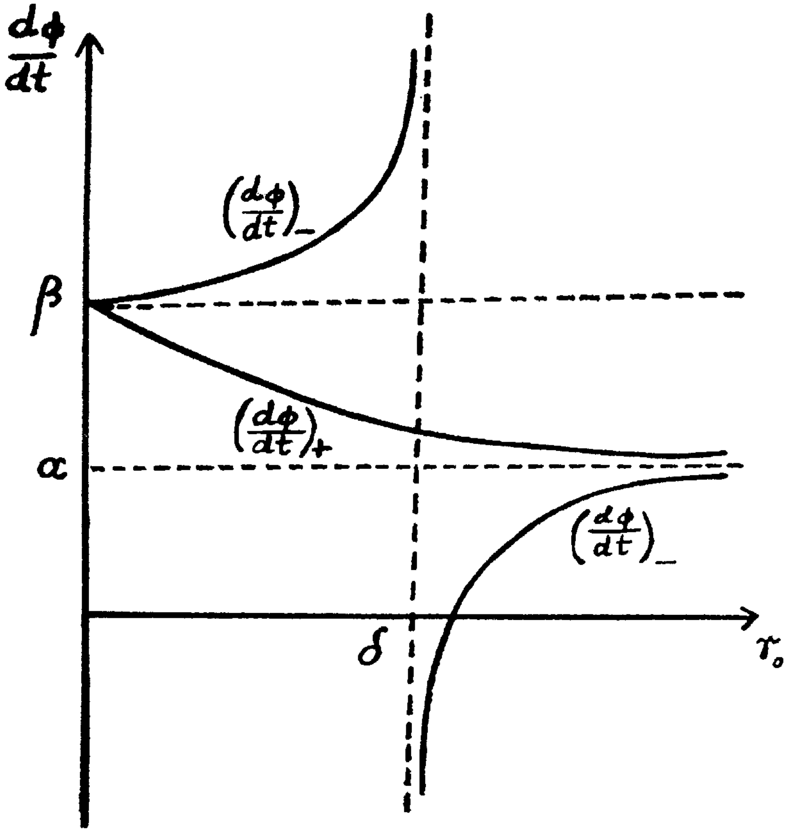

FIG. 1

TABLE 1

ω	k	α	β	δ
< 1	$\frac{1}{4}$	ω	$\frac{1}{\omega}$	ω^2
	$\frac{1}{2}$	-1	$+1$	—
	$\frac{3}{4}$	ω	$\frac{1}{\omega}$	$\frac{1}{\omega^2}$
	1	ω	$\frac{1}{\omega}$	$\frac{1}{\omega}$
> 1	$\frac{1}{4}$	$\frac{1}{\omega}$	ω	ω
	$\frac{1}{2}$	-1	$+1$	—
	$\frac{3}{4}$	$\frac{1}{\omega}$	ω	$\frac{1}{\omega^2}$
	1	$\frac{1}{\omega}$	ω	$\frac{1}{\omega}$

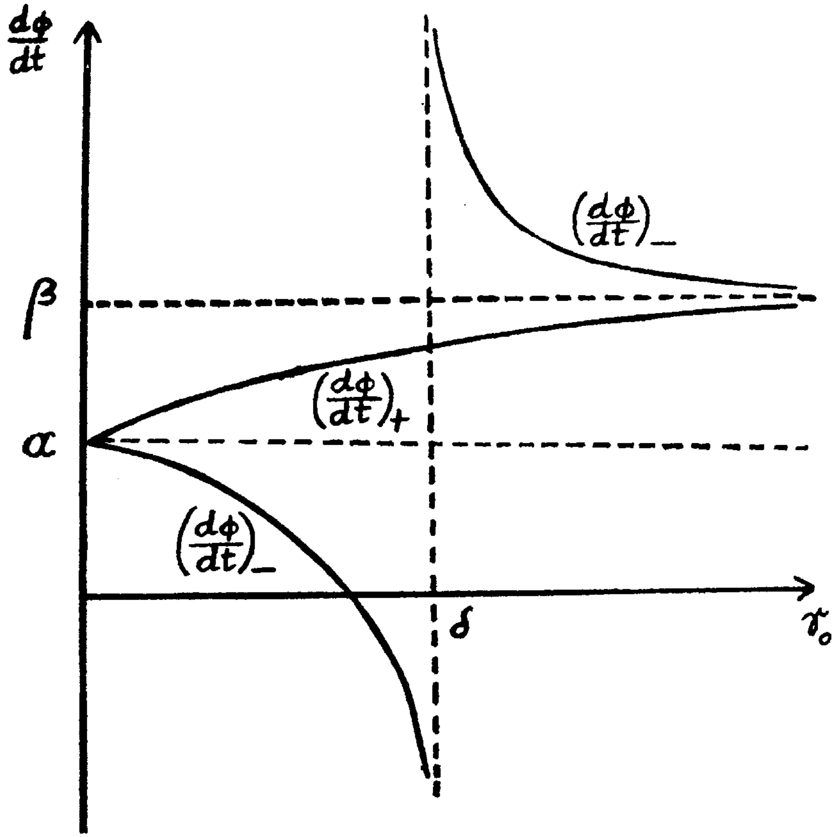

FIG. 2

TABLE II

Figures	Conditions
1	(i) $\omega < 1, k < \frac{1}{2}$
	(ii) $\omega > 1, k > \frac{1}{2}$
2	(iii) $\omega < 1, k > \frac{1}{2}$
	(iv) $\omega > 1, k < \frac{1}{2}$
3	(v) $\omega < 1$ or $> 1, k = \frac{1}{2}$

various values of k and ω. For this purpose, we have made use of Table I. Since we are interested only in the restrictions on the motion of photons caused by the rota-

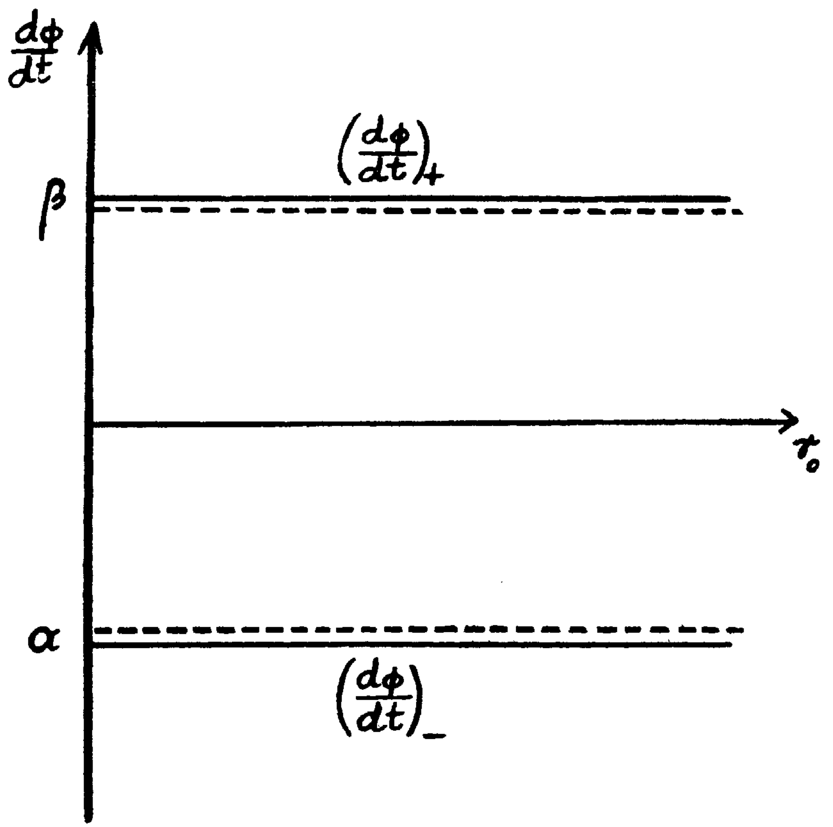

FIG. 3

tion of the cylindrical body, the curves in Figs. 1 and 2 have been plotted *only broadly* (not exactly). Table II explains briefly the figures.

CONCLUSIONS

We may derive now some conclusions of physical significance from the three figures and Table II.

Fig. 1 shows that photons having a recoil angular velocity near-about α may escape from the rotating cylinder or reach it from infinity. Also this figure indicates that photons with a recoil angular velocity near-about β may escape from a cylinder of finite radius only.

Fig. 2 shows that photons having a recoil angular velocity near-about β may escape from the cylinder or reach it from infinity. Also this figure indicates that photons with a recoil angular velocity near-about α may escape from a cylinder of finite radius only.

Finally, Fig. 3 shows that for $\omega < 1$ or > 1, $k = \frac{1}{2}$, photons may escape from the cylinder to infinity or reach it from infinity.

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