NEUTRINO BREMSSTRAHLUNG IN AN INTENSE MAGNETIC FIELD

by BRAJA GOPAL MAZUMDER, *Raghunathpore, P.O. Bishalgarh, Tripura (W)*

(Received 19 August 1977; after revision 25 February 1978)

The neutrino luminosity for a completely relativistic electron gas in presence of an intense magnetic field is computed according to the Photon-Neutrino Coupling theory of weak interaction. Comparisons with other approximate results due to both Current-Current and Photon-Neutrino Couplings are made and reasons for discrepancies are discussed. Lastly, possibility for astrophysical evidence is also stressed.

INTRODUCTION

IN RECENT times, it has been considered by many authors that the importance of weak interactions in stellar evolution problem depends on neutrino emission. The reason is that the neutrino can escape from stars without being absorbed anywhere. The mean free path of 1 Mev. neutrino in lead is about 10^{18} cms, or one light year, and is much larger than the dimension of a star ($\sim 10^{11}$ cms). This characteristic of neutrino gives rise to the radiation from it which can be of great importance in certain stages of stellar evolution. Practically, neutrino processes are important in later stages of stellar evolution, when the core of the star has contracted considerably. The effect of contraction of stellar core increases the strength of the magnetic field already present in earlier stages of the stellar interiors. The free electron moving in such a magnetic field will give rise to the emission of a pair of neutrinos, which is normally forbidden in vacuum.

In this paper, we shall study the process $e^- \rightarrow e^- + \nu + \bar{\nu}$, which is forbidden in vacuum due to energy and momentum conservation laws when both the electrons are free. But the process can take place in presence of magnetic field which causes electrons to move in circular orbits. The effect of magnetic field on the electron quantizes its energy in the Z-direction of the field. Leaving the momentum p_z unaltered, this quantization replaces in the expression for the electron energy, the momenta p_x and p_y in the X and Y directions respectively by the field strength, i.e.,

$$p_x^2 + p_y^2 \rightarrow n\left(\frac{H}{H_c}\right) \text{ with } n = 0, 1, 2 \ldots, \text{ and}$$

$$H_c = \frac{m^2c^3}{e\hbar} = .4.414 \times 10^{13}$$

Gausses. Thus the process can be visualised as a bremsstrahlung process or a decay of an electron from an initial state with quantum number n to new one with n' through the emission of a pair of neutrino and antineutrino.

Landstreet (1967) and Canuto *et al.* (1970) studied this process in the light of Current-Current Coupling theory and the *V-A* theory of Weak Interactions respectively. Ray Choudhuri (1970) studied the process in the light of Photon-Neutrino Coupling theory (Bandyopadhyay 1968). Here we shall also study the process on the basis of Photon-Neutrino Coupling theory. It has been suggested by Bandopadhyay (1965) that in view of Neutrino theory of light, photon can interact weakly with neutrinos also apart from the usual electromagnetic interactions. It has been further argued that photons can interact weakly with two component neutrinos and with no other particle, charged or uncharged. The astrophysical implications of neutrino emissivity from Stars due to this Photon-Neutrino Coupling have been discussed in a series of papers (Bandyopadhyay and Ray Choudhuri 1969*a*, 1969*b*, 1970; and Bandyopadhyay 1970). The importance of studying the process in the light of Photon-Neutrino Coupling lies in the fact that it avoids nonrenormalizability and unitary catastrophe at high energy in contradiction with the conventional Current-Current Coupling theory and the Vector-Bosom theory of weak interactions. Moreover, it is shown that the neutrino luminosity calculated on the basis of Photon-Neutrino Coupling due to different neutrino generating processes takes a significant role in explaining the evolutionary processes of some Stars like White Dwarfs, Neutron Stars etc., which cannot be explained in the light of Current-Current Coupling theory.

Calculation of Energy Loss Rate

According to the Photon-Neutrino Coupling, the process can occur through a Virtual Photon in a magnetic field as shown in Fig. 1. The process is analogous to the ordinary synchrotron radiation by a free electron in a magnetic field.

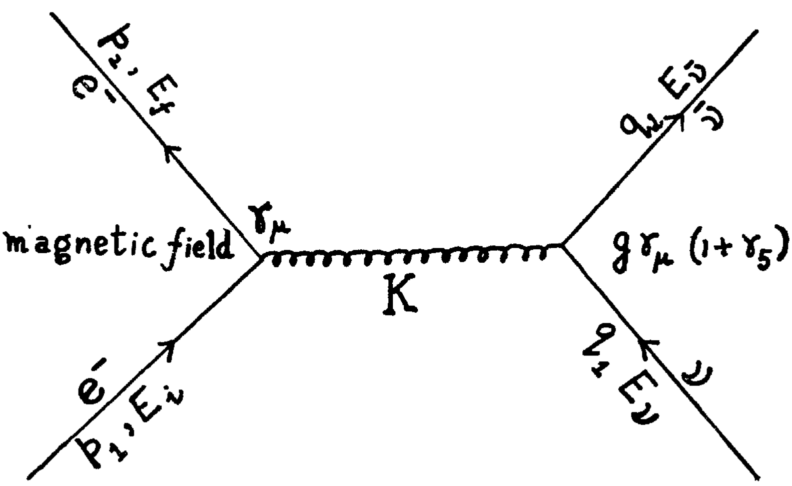

Fig. 1. The Feynman diagram for the process.

The martix element for the process is

$$S = \sum [\bar{\psi}_e(p_1)\, 0_i \psi_e(p_2)] \frac{1}{K^2} [\bar{\psi}_\nu(q_1)\, F_i \psi_\nu(q_2)], \quad \text{...(1)}$$

where $0_i = \gamma_\mu$ refers to the vectorial part of the interacting current between initial and final electrons in argument with the V-A theory.

$F_i = \frac{g}{\sqrt{2}} \gamma_\mu(1 + \gamma_5)$ refers to a part of weak interaction between neutrino and antineutrino of the form $\{\bar{\nu}_e \gamma_\mu(1 + \gamma_5)\, \nu_e\}$, g is the Photon-Neutrino Weak coupling constant. Here p_1, p_2; q_1, q_2 are the momenta of the initial, final electron, neutrino and antineutrino respectively $K = p_1 - p_2 = q_1 + q_2$, the virtual Photon-propagator shown in Fig. (1) by the wavy-line. The electron wave function in magnetic field is calculated from the solution of the Dirac equation,

$$\left[\gamma_\mu \frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar} + \frac{ie}{\hbar c} \gamma_\mu A_\mu\right] \psi_D = 0, \quad \text{...(2)}$$

where m refers the mass of the electron, e the charge, c the velocity, A_μ the external electromagnetic potential, ψ_D the 4-dimensional spinor.

Following Canuto *et al.* (1970), we can compute eqn. (2) as follows :

In presence of an external magnetic field $\vec{H}$, we have

$$A_4 = 0,\ \vec{A} = \tfrac{1}{2} \vec{H} \times \vec{r}, \quad \text{...(3)}$$

where $\vec{H}$ is considered in Z-direction, $H_X = H_Y = 0$; $H_Z = H$

and

$$A_4 = 0,\ A_X = -\tfrac{1}{2} YH,\ A_Y = \tfrac{1}{2} XH,\ A_Z = 0 \quad \text{...(4)}$$

Let us define the Gauge transformation

$$\psi = \psi_D \exp\,[if(X, Y)], \quad \text{...(5)}$$

where $f(X, Y)$ is chosen in such a way that

$$A_4 = 0,\ A_X = -YH,\ A_y = A_z = 0. \quad \text{...(6)}$$

From eqn. (2), we find

$$f(X, Y) = -\tfrac{1}{2}\left(\frac{e}{\hbar c}\right) XYH. \quad \text{...(7)}$$

As shown before (2) – (3), the spinor ψ is given by

$$\psi(\vec{r}, t) = \exp\,(-i\eta Et/\hbar)\, \psi(\vec{r})$$

$$\psi(\vec{r}) = \exp\,[i(k_x X + k_z Z)] \exp\,[-\xi^2/2]\, u_n(\xi). \quad \text{...(8)}$$

where

$$U_n(\xi) = \begin{bmatrix} C_1 \bar{H}_n(\xi) \\ C_2 \bar{H}_{n-1}(\xi) \\ C_3 \bar{H}_n(\xi) \\ C_4 \bar{H}_{n-1}(\xi) \end{bmatrix}$$

with

$$E = mc^2 \sqrt{1 + X^2 + 2n\theta} = mc^2 \epsilon(X), \; n = 0, 1, 2, \ldots X = \frac{p_z}{mc} \quad \ldots(9)$$

$$H_c = \frac{m^2c^3}{e\hbar} = 4.414 \times 10^{13} \text{ Gauss}, \; \theta = \frac{H}{H_c}$$

$$\xi = y\sqrt{\gamma} + k_x/\sqrt{\gamma}, \; \gamma = \theta \lambda_e^{-2}, \; \lambda_c = \frac{\hbar}{mc} \quad \ldots(10)$$

$$C_1 = aA, \; C_2 = SaB, \; C_3 = \eta SbA, \; C_4 = \eta bB$$

$$a = \sqrt{\tfrac{1}{2}(1 + \eta\epsilon^{-1})}, \; b = \sqrt{\tfrac{1}{2}(1 - \eta\epsilon^{-1})};$$

$$A = \sqrt{\tfrac{1}{2}\left(1 + S\frac{X}{\sqrt{X^2 + 2n\theta}}\right)}, \quad B = \sqrt{\tfrac{1}{2}\left(1 - S\frac{X}{\sqrt{X^2 + 2n\theta}}\right)} \cdot \quad \ldots(11)$$

$\eta = \pm 1$ stands for positive and negative energy eigenvalues.

$S = \pm 1$ stands for the projection of the momentum component along the spin. $\bar{H}_n(X)$ are the Hermite Polynomials normalised to unity. We have

$$\bar{H}_n(X) = \sqrt{\frac{\gamma^{1/2}}{\pi^{1/2} 2^n n!}} H_n(X) \quad \ldots(12)$$

ψ_ν is free-particle wave-function, i.e.,

$$\psi_\nu = \Omega^{-1/2} e^{i\vec{q}\,\vec{r}/\hbar} . e^{-iE_\nu t/\hbar} u_\nu(q); \quad \ldots(13)$$

$u_\nu(q)$ satisfies the eqn.

$$\left(\gamma_\mu q_\mu + \frac{m_\nu C}{\hbar}\right) u_\nu(q) = 0, \quad m_\nu = 0$$

Substituting Eqns. (8) and (13) into (1) and integrating over t, we get

$$S = \Omega^{-1}\delta(E_i - E_f - E_\nu - E_{\bar{\nu}}) \sum_i [\bar{u}_\nu(q_1)\, F_i u_{\bar{\nu}}(q_2)].$$

$$\frac{1}{K^2}\int d^3r \bar{\psi}_{e^-}(r)\, 0_i \psi_{e^-}(r)\, e^{-(\vec{q}_1 + \vec{q}_2)\cdot\vec{r}/\hbar}$$

$$= \Omega^{-1}\delta(E_i - E_f - E_\nu - E_{\bar{\nu}})\frac{1}{K^2}\sum_i [u_\nu(q_1)\, F_i u_{\bar{\nu}}(q_2)] \langle 0_i \rangle \quad \ldots(14)$$

The Square-modulus of S with the use of $\delta^2(E) \to \frac{T}{2\pi\hbar}\,\delta(E)$ follows as

$$| S |^2 = \Omega^{-2}\,\frac{T}{2\pi\hbar}\,\delta(E_i - E_f - E_\nu - E_{\bar{\nu}})\,\frac{1}{K^4}\sum_{ij}[\bar{u}_\nu(q_1)\,F_i u_{\bar{\nu}}(q_2)$$

$$\bar{u}_\nu(q_2)\,\tilde{F}_j u_{\bar{\nu}}(q_1)]\,\langle 0_i\rangle\,\langle 0_j\rangle^\dagger, \qquad \ldots(15)$$

where $\tilde{F}_j = \gamma_4 F_j^\dagger \gamma_4$, T represents the time-differential operator.

Following Landstreet (1967) we obtain after some algebraic calculation,

$$\sum_{r,S=1} [\bar{u}_\nu^{(r)}(q_1)\,F_i\,u_{\bar{\nu}}^{(s)}(q_2)\,\bar{u}_\nu^{(s)}(q_2)\,\tilde{F}_j u_{\bar{\nu}}^{(r)}(q_1)]$$

$$= \tfrac{1}{2}\,g_i g_j\,\frac{C_{q_1\alpha}C_{q_2\beta}}{4E_{q_1}E_{q_2}}\,\text{Trace}\,\lambda_{ij}^{\alpha\beta},$$

where

$$\lambda_{ij}^{\alpha\beta} = i\gamma_\alpha 0_i(1+\gamma_5)\,i\gamma_\beta(1-\gamma_5)\,\gamma_4 0_j^\dagger\gamma_4, \qquad \ldots(16)$$

Here

$$E_{q_1} = C\,|\,\bar{q}_1\,|,\quad E_{q_2} = C\,|\,\bar{q}_2\,|,\quad \text{since } m_\nu = 0.$$

Using

$$\sum_{q_1} \to \frac{\Omega}{(2\pi\hbar)^3}\int d^3q_1;\quad \sum_{q_2} \to \frac{\Omega}{(2\pi\hbar)^3}\int d^3q_2$$

and Lenard relation

$$\int d^3q_1\,\frac{C_{q_1\alpha}}{2E_{q_1}}\int d^3q_2\,\frac{C_{q_2\beta}}{2E_{q_2}}\,\delta^4(Q = q_{1\mu} + q_{2\mu})$$

$$= \frac{\pi}{24}\,(Q_\mu^2\,\delta_{\alpha\beta} + 2Q_\alpha Q_\beta)\,\theta(Q_0)\,\theta(Q_\mu^2).$$

With the identity $\int d^3Q\,\delta^3(\vec{Q} = \vec{q}_{1\mu} + \vec{q}_{2\mu}) = 1$ and four-vector

$$Q_\mu = (\vec{Q}, iQ_0) = \left[\vec{q}_1 + \vec{q}_2;\ \frac{i}{C}\,(E_{q_1} + E_{q_2})\right],$$

we obtain

$$| S |^2 = \frac{T}{2\pi\hbar}\cdot\frac{1}{h^6}\,\frac{1}{2C}\cdot\frac{\pi}{24}\,\frac{1}{K^4}\int d^3Q\,F(Q)\,\theta(Q_0)\,\theta(Q_\mu^2) \qquad \ldots(17)$$

with

$$F(Q) = \sum_{ij} g_i g_j (Q_\mu^2 \delta_{\alpha\beta} + 2Q_\alpha Q_\beta) \langle O_i \rangle \langle O_j \rangle^\dagger \text{ Trace } \lambda_{ij}.$$

Summing over the indices ij, we get after some tedious calculations

$$F(Q) = 32\dot{g}^2 (Q_\mu^2 \delta_{\alpha\beta} - Q_\alpha Q_\beta)(-1)^{1+\delta_{\beta 4}} \langle \gamma_\alpha (1 + \gamma_5) \rangle \langle \gamma_\beta (1 + \gamma_5) \rangle^\dagger,$$

where we have used the notation

$$\tilde{\gamma}_r = \gamma_4 \gamma_r^\dagger \gamma_4 = (-1)^{1+\delta_{r4}} \gamma_r$$

$$\widetilde{(i\gamma_r\gamma_5)} = i\gamma_r\gamma_5(-1)^{\delta_{r4}}$$

Now eqn. (17) reads

$$| S |^2 = \frac{T}{2\pi\hbar} \frac{1}{h^6} \frac{\pi}{2C} \frac{32g^2}{24} \frac{1}{K^4} \int d^3q (q_\mu^2 \delta_{\alpha\beta} - q_\alpha q_\beta) \theta(q_0) \theta(q_\mu^2)$$
$$(-1)^{1+\delta_{\beta 4}} R_\alpha R_\beta^\dagger, \qquad ...(18)$$

where

$$R_\mu = \int d^3r \, \psi_f^*(\vec{r}) \, \Gamma_\mu \psi_i(\vec{r}) \exp (i\vec{q} \, . \, \vec{r}/\hbar), \qquad ...(19)$$

with $\Gamma_\mu = \gamma_4\gamma_\alpha(1 + \gamma_5)$. It stands for $\langle O_i \rangle \langle O_j \rangle^\dagger$ in eqn. (17) which represents the final form of the electron wave function and its complex conjugate in magnetic field. Substituting eqn. (8) into eqn. (19) we obtain

$$R_\mu = \frac{\delta[(p_x - p_{x'} - q_x)/\hbar]}{L_X} \cdot \frac{\delta[(p_2 - p_{z'} - q_z)/\hbar]}{L_Z} I_\mu$$

$$I_\mu = \int_{-\infty}^{+\infty} d_y \exp [- \tfrac{1}{2} (\xi^2 + \xi'^2)] \, u_{n'}^\dagger (\xi') \, \Gamma_\mu u_n(\xi) \exp (iq_y \, . \, y/\hbar) \qquad ...(20)$$

Eqn. (18) turns to

$$| S |^2 = \frac{T}{2\pi\hbar} \frac{32g^2\pi}{2C\hbar^6 24} \cdot \frac{1}{K^4} \frac{\hbar^2 L_X^2 L_Z}{L_X^2 L_Z^2} \int_{-\infty}^{+\infty} dq_x \int_{-\infty}^{+\infty} dq_y \int_{-\infty}^{+\infty} dq_z$$

$$\delta(p_z - p_{z'} - q_z) \, \delta p_x = p_{x'} + q_x \, . \, \theta(q_0) \, \theta(q_\mu^2)$$

$$[q_\mu^2 \delta_{\alpha\beta} - q_\alpha q_\beta] (-1)^{1+\delta_{\beta 4}} I_\alpha I_\beta^\dagger$$
$$...(21)$$

We have used the notation $\delta^2(X) = L^{-1}\delta(X)$ which follows from the fact that if X denotes the dimension of length, we can change the usual δ-function on the variable p_x into a Kronecker delta function using the relation

$$\delta(X) = L^{-1}\,\delta_x.$$

Using the polar Co-ordinate $q_x = q\cos\varphi$, $q_y = q\sin\varphi$ and integrating over q_z we obtain

$$|S|^2 = \frac{T}{2\pi\hbar}\,\frac{32\pi g^2}{2.24Ch^6}\cdot\frac{\hbar^2 L_x^2 L_2}{L_x^2 L_z^2}\cdot\frac{1}{K^4}\int_0^\infty q\,dq \int_0^{2\pi} d\varphi\,\theta(q_0)$$

$$\theta(q_0^2 - q^2 - q_z^2)\,F(q,\varphi), \qquad \ldots(22)$$

where

$$F(q,\varphi) = [q_\mu^2\,\delta_{\alpha\beta} - q_\alpha q_\beta]\,(-1)^{1+\delta_{\beta 4}}\, I_\alpha I_\beta^\dagger \Bigg|_{\substack{q_x = q\cos\varphi \\ q_y = q\sin\varphi \\ q_z = p_z - p_{z'}}} \qquad \ldots(23)$$

The function θ indicates that integration on q^2 can go only upto $q_M^2 = q_0^2 - q_z^2$.

Introduction of $\rho = 1 - \left(\frac{q^2}{q_M}\right)$ reduces the eqn. (22) to

$$|S|^2 = \frac{1}{L_z}\,S_0\,\frac{T}{K^4}\int_0^1 d\rho\, I(\rho, X, X'), \qquad \ldots(24)$$

where

$$S_0 = \frac{1}{2\pi\hbar}\,\frac{32\pi g^2\hbar^2}{2.24Ch^6} \qquad \ldots(25)$$

and

$$I(\rho, X, X') = \frac{q_M}{\sqrt{1-\rho}}\,F\left(\rho, \varphi = \frac{\pi}{2}\right). \qquad \ldots(26)$$

All the momenta entering in eqn. (20) are measured in Units of

mC, i.e., $\frac{P_z}{mc} = X$, $\frac{P_{z'}}{mC} = X'$ etc.

The probability per unit time and volume, P is given by

$$P = \frac{|S|^2}{L_X L_y L_z T} = \frac{1}{L_X L_y L_z^2} \frac{1}{K^4} S_0 \int_0^1 dP\, I(P, X, X') \qquad ...(27)$$

The neutrino-luminosity L is defined as

$$L = \sum_i \sum_f (E_i - E_f)\, f(E_i)\, [1 - f(E_f)]\, P, \qquad ...(28)$$

where

$$E_i - E_f = mC^2\, [\epsilon(X) - \epsilon(X')]$$

is the energy carried away by the neutrino and $f(E)$ is the Fermi distribution function.

$$f(E) = \left[1 + \exp\left(\frac{\epsilon - \mu}{\varphi}\right)\right]^{-1}, \quad \varphi = \frac{KT}{mC^2}$$

The summation of eqn. (28) are given by

$$\sum_i \rightarrow \sum_n \frac{L_z}{2\pi\hbar} \int \mathrm{d}p_z = \frac{L_z}{2\pi\lambda_C} \sum_n \int_{-\infty}^{\infty} \mathrm{d}X \qquad ...(29)$$

$$\sum_f \rightarrow \sum_{n^1} \sum_{P_{z'}} = \sum_{n^1} \frac{L_z}{2\pi\hbar} \int \mathrm{d}P_{z'} N(P_{z'})$$

$$= \frac{L_z}{2\pi\hbar} \frac{1}{2\pi} \frac{H}{H_C} \frac{L_X L_y}{\lambda_C^2} \sum_{n'} \int \mathrm{d}P_{z'}$$

$$= \frac{L_x L_y L_z}{(2\pi)^2 \lambda_C^3} \frac{H}{H_C} \sum_{n'} \int_{-\infty}^{\infty} \mathrm{d}x'. \qquad ...(30)$$

Substituting eqns. (27), (29) and (30) into (28) we obtain

$$L = L_0 \frac{H}{H_C} \sum_n \sum_{n'} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}X'\, [\epsilon(X) - \epsilon(X')]\, f(X)\, [1 - f(X')]$$

$$\int_0^1 \mathrm{d}_\rho\, I(X, X', \rho), \qquad ...(31)$$

where

$$L_0 = \frac{1}{6} \frac{1}{(2\pi)^8} \cdot \frac{g^2 mC^2}{C\hbar^2 \lambda_C^8} \frac{1}{K^4} \cdot (C\hbar)^6$$

$$\simeq \cdot \frac{1}{T^4}\, [6.59 \times 10^{56} \text{ ergs/cm}^3 \text{ sec.}].$$

Here T denotes temperature in Kelvin. To make the integral I entering in eqn. (31) dimensionless we have multiplied L_0 by $(C\hbar)^6$

Evaluation of the Integral

With the use of Pauli-Dirac γ matrices eqn. (23) can be rewritten as

$$F(q, \varphi) = (q_\mu^2\, \delta_{\alpha\beta} - q_\alpha q_\beta)\, I_\alpha L_\beta,$$

where I_α is given by eqn. (20) and $L_\alpha = I_\alpha(\xi \to \xi',\ q_\varphi \to -\, q_\varphi)$. After a very lengthy and tedious algebraic calculation, the function $I(X, X', \rho)$ can be reduced to the form

$$I(X, X', \rho) = q_M^4\,(A + \rho B) + q_M^2\,(q_3 C + q_0 D)^2, \qquad \ldots(32)$$

Where

$$q_0 = \xi + \xi',\ q_M^2 = (\xi + \xi')^2 - (X - X')^2$$

$$(A \cdot B) = \omega_1^2 \varphi_1^2 + \omega_2^2 \varphi_2^2 \mp 4\omega_3\omega_4\varphi_3\varphi_4$$

$$(C \cdot D) = \omega_3\varphi_3 \mp \omega_4\varphi_4$$

$$\varphi_1 = \Phi\left(\frac{n}{n' - 1}\right)$$

$$\varphi_2 = \Phi\left(\frac{n - 1}{n'}\right)$$

$$\varphi_3 = \Phi\left(\frac{n}{n'}\right)$$

$$\varphi_4 = \Phi\left(\frac{n - 1}{n'}\right)$$

Summing over the spin index the final form of $\omega'S$ turns out to be

$$\omega_1^2 = \left[1 - \frac{X}{\epsilon(X)}\right]\left[1 + \frac{X'}{\epsilon(X')}\right]$$

$$\omega_2^2 = \left[1 + \frac{X}{\epsilon(X)}\right]\left[1 - \frac{X'}{\epsilon(X')}\right]$$

$$\omega_3^2 = \left[1 - \frac{X}{\epsilon(X)}\right]\left[1 - \frac{X'}{\epsilon(X')}\right]$$

$$\omega_4^2 = \left[1 + \frac{X}{\epsilon(X)}\right]\left[1 + \frac{X'}{\epsilon(X')}\right]$$

$$\omega_1\omega_2 = -\,\omega_3\omega_4 = 2\,\frac{H}{Hq}\,\frac{\sqrt{nn'}}{\epsilon(X)\,\epsilon(X')} = 2\,Ⓗ\,\frac{\sqrt{nn'}}{\epsilon(X)\,\epsilon(X')}$$

$$\omega_1\omega_3 = \left[1 - \frac{X}{\epsilon(X)}\right] \frac{\sqrt{2n'\text{Ⓗ}}}{\epsilon(X')}$$

$$\omega_2\omega_3 = \left[1 - \frac{X'}{\epsilon(X')}\right] \frac{\sqrt{2n\text{Ⓗ}}}{\epsilon(X)}$$

$$\omega_1\omega_4 = \left[1 + \frac{X'}{\epsilon(X')}\right] \frac{\sqrt{2n\text{Ⓗ}}}{\epsilon(X)}$$

$$\omega_2\omega_4 = \left[1 + \frac{X}{\epsilon(X)}\right] \frac{\sqrt{2n'\text{Ⓗ}}}{\epsilon(X')}$$

The integral I_μ can be written as

$$\langle n/n' \rangle = \int_{-\infty}^{\infty} \exp\left[-\tfrac{1}{2}(\xi^2 + \xi'^2)\right] \bar{H}n'(\xi')\, Hn \exp\left(\frac{iq_y y}{\hbar}\right) dy.$$

For $n \geqslant n'$

$$I\left(\frac{n'}{n}\right) = \Phi\left(\frac{n}{n'}\right)(-1)^{n'} e^{-i(n-n')\varphi}$$

with $$\Phi\left(\frac{n}{n'}\right) = \Phi\left(\frac{n'}{n}\right) = [n!\,n'!]^{-1/2}\, e^{-t/2}\, t^{n+n'/2}\,.\,2F_0(-n', -n, -t^{-1}) \quad \ldots(33)$$

$$\varphi = \tan^{-1}\left(\frac{q_y}{q_X}\right),\ t = \frac{q_X^2 + q_y^2}{2\gamma\hbar^2}$$

$$2F_0(a, b; X) = 1 + \sum_{K=1}^{\infty} (a)_K\,(b)_K\,\frac{X^K}{K!}$$

$$(a)_K = \prod_{S=1}^{K} (a + S - 1) \quad \ldots(34)$$

For any n and n' where $(n - n' = N)$ the result is

$$\langle n/n' \rangle = \Phi(n/n')\{e^{-iN\varphi}(-1)^{n'}\theta(n - n') + e^{iN\varphi}(-1)^n\theta(n' - n) - (-1)^n\delta_n^{n'}\},$$

where $\theta(X) = 1$ for $X \geqslant 0$, $\theta(X) = 0$ for $X < 0$. The argument t of eqn. (33) can also be written in the form

$$t = \frac{1}{2\text{Ⓗ}}\, q_M^2\,(1 - \rho). \quad \ldots(35)$$

Using the hypergeometric function given in eqn. (34) and integrating over ρ we get

$$\int_0^1 I(X, X', \rho)\, d\rho = 4Ⓗ\, q_M^2 \Big[\omega_1^2 K\Big(\frac{n}{n'-1}\Big) + \omega_2^2 K\Big(\frac{n-1}{n'}\Big) +$$

$$\omega_3\omega_4 M_1\Big(n, \frac{n'}{n-1, n'-1}\Big)\Big] - 4Ⓗ^2\Big[\omega_1^2 L\Big(\frac{n}{n'-1}\Big) +$$

$$\omega_2^2 L\Big(\frac{n-1}{n'}\Big) + 4\omega_3\omega_4 M_2\Big(n, \frac{n'}{n-1, n'-1}\Big)\Big] +$$

$$2Ⓗ\Big\{\omega_3^2\,[(\epsilon-\epsilon') + (X-X')]^2 K\Big(\frac{n}{n'}\Big) +$$

$$\omega_4^2\,[(\epsilon-\epsilon') - (X-X')]^2 K\Big(\frac{n-1}{n'-1}\Big)\Big\}. \qquad \ldots(36)$$

The general expressions for the functions K, L, M_1 and M_2 are the followings

$$\Big(\alpha = \frac{qM^2}{2Ⓗ},\ Ⓗ = \frac{H}{H_C}\Big).$$

$$K\Big(\frac{N}{M}\Big) = (N!M!)^{-1}\Big\{(N+M)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M}\frac{\alpha^m}{m!}\Big] +$$

$$2\sum_{k=1}^{\infty}\frac{(-N)_k(-M)_k(-1)^k}{k!}(N+M-k)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M-k}\frac{\alpha^m}{m!}\Big] +$$

$$\sum_{l=2}^{\infty}(-1)^l C_l(N+M-l)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M-l}\frac{\alpha^m}{m!}\Big]\Big\}.$$

$$L\Big(\frac{N}{M}\Big) = (N!\,M!)^{-1}\Big\{(N+M+1)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M+1-k}\frac{\alpha^m}{m!}\Big] +$$

$$2\sum_{k=1}^{\infty}\frac{(-N)_k(M)_k(-1)^k}{k!}(N+M+1-k)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M+1-k}\frac{\alpha^m}{m!}\Big] +$$

$$\sum_{l=2}^{\infty}(-1)^l C_l(N+M+1-l)!\Big[1 - e^{-\alpha}\sum_{m=0}^{N+M+1-l}\frac{\alpha^m}{m!}\Big]\Big\}$$

$$M_1\left(\frac{N, M}{N-1, M-1}\right)=$$

$$[(N-1)!(M-1)!]^{-1}\left\{(N+M-1)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M-1}\frac{\alpha^m}{m!}\right]+\right.$$

$$\sum_{k=1}^{\infty}\frac{(-N)_k(-M)_k+(-N+1)_k(-M+1)_k}{k!}$$

$$(-1)^k(N+M-k-1)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M-1-k}\frac{\alpha^m}{m!}\right]+$$

$$\left.\sum_{l=2}^{\infty}(-1)^l C_l'(N+M-l-1)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M-1-l}\frac{\alpha^m}{m!}\right]\right\}$$

$$M_2\left(\frac{N, M}{N-1, M-1}\right)=$$

$$[(N-1)!(M-1)!]^{-1}\left\{(N+M)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M}\frac{\alpha^m}{m!}\right]+\right.$$

$$\sum_{k=1}^{\infty}\frac{(-N)_k(-M)_k+(-N+1)_k(-M+1)_k}{k!}$$

$$(-1)^k(N+M-k)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M-k}\frac{\alpha^m}{m!}\right]+$$

$$\left.\sum_{l=2}^{\infty}(-1)^l C_l'(N+M-l)!\left[1-e^{-\alpha}\sum_{m=0}^{N+M-l}\frac{\alpha^m}{m!}\right]\right\},$$

with

$$C_l=\sum_{\substack{k+k'=l\\ k>0\\ k'>0}}\frac{(-N)_k(-M)_k(-N)_k(-M)_{k'}}{k!\,k'!}$$

$$C_l'=\sum_{\substack{k+k'=l\\ k>0\\ k'>0}}\frac{(-N)_k(-M)_k(-N+1)_{k'}(-M+1)_{k'}}{k!\,k'!}$$

$$(\alpha)_k \prod_{S=1}^{k} (\alpha + S - 1)$$

L_0 can be evaluated as follows :

$$L_0 = \frac{1}{24} \cdot \frac{1}{(2\pi)^5} \cdot \frac{g^2 mC^2}{C\hbar^2 \lambda_C^8} \cdot \frac{1}{K^4} (C\hbar)^6,$$

where

$$\lambda_C = \frac{\hbar}{mC}$$

$$K = q_1 + q_2$$

$$K^2 = (q_1 + q_2)^2$$

$$\simeq 2E_\nu^2 (\cos\theta - 1)$$

$$\therefore \quad K^4 = 4E_\nu^4 (\cos\theta - 1)^2$$

$$\simeq 4E_\nu^4 \times \frac{14}{3} \cdot 2\pi$$

$$\simeq \frac{112\pi}{3} (K'T)^4$$

Here K' represents the Boltzmann Constant.

T is the temperature in Kelvin. Taking $g \simeq 10^{-10}\, e$ (Bandyopadhyay 1968) L_0 takes the form

$$L_0 = \frac{1}{24} \cdot \frac{1}{(2\pi)^5} \cdot \frac{mC^2}{C\hbar^2 \lambda_C^8} \frac{10^{-20}}{137} \cdot \frac{112\pi}{3} \frac{1}{(K'T)^4}$$

$$\simeq \frac{1}{T^4} [6.59 \times 10^{56} \text{ ergs/cm}^3 \text{ sec}]$$

Results and Discussion

In this paper we present some numerical results of the eqn. (31). The results are tabulated in Tables I to IV for $T = 5.9303 \times 10^7$ °K, $T = 3.7418 \times 10^8$ °K, $T = 5 \times 10^8$ °K and $T = 9.3988 \times 10^8$ °K for different values of densities. The results are also compared with those obtained in the light of Current-Current Coupling due to Canuto *et al.* (1970) and Landstreet (1967). For the sake of verification, the neutrino luminosity due to NSR (Ray Choudhuri 1970) obtained in the light of Photon-Neutrino Coupling is also displaced in the 4th Column of each table.

TABLES I & II

Neutrino-luminosity as a function of density ρ_6
($T = 5.9303 \times 10^7$ °K; $T = 3.7418 \times 10^8$ °K)

ρ_6	Neutrino-Luminosity (ergs/cm³ sec) due to Current-Current Coupling		Neutrino-Luminosity (ergs/cm³ sec) due to Photon-Neutrino Coupling	
	L (Landstreet)	L (Canuto *et al.*)	L (Present Work)	L (Ray Choudhuri)
	TABLE I; $T = 5.9303 \times 10^7$ °K			
3.7146	7.5923×10^4	0.26037×10^{-1}	2.401×10^5	2.213×10^{12}
5.598	9.1104×10^4	0.4998	4.480×10^6	2.661×10^{12}
5.8417×10	2.5834×10^5	4.054×10^5	2.712×10^{12}	7.551×10^{12}
2.0736×10^2	4.5366×10^5	1.9397×10^7	1.776×10^{14}	1.324×10^{13}
	TABLE II; $T = 3.7418 \times 10^8$ °K			
4.802×10^{-1}	0.3565×10^{10}	1.2063×10^{12}	7.616×10^{15}	6.761×10^{13}
1.44	0.581×10^{10}	5.092×10^{12}	3.248×10^{16}	1.114×10^{14}
3.7146	0.885×10^{10}	2.107×10^{13}	1.334×10^{17}	1.687×10^{14}
5.598	1.062×10^{10}	3.7116×10^{13}	2.352×10^{17}	1.972×10^{14}

TABLES III & IV

Neutrino-luminosity as a function of density ρ_6
($T = 5 \times 10^8$ °K; $T = 9.3988 \times 10^8$ °K)

ρ_6	Neutrino-Luminosity ergs/cm³ sec) due to Current-Current Coupling		Neutrino-Luminosity (ergs/cm³ sec) due to Photon-Neutrino Coupling	
	L (Landstreet)	L (Canuto *et al.*)	L (Present Work)	L (Ray Choudhuri)
	TABLE III; $T = 5 \times 10^8$ °K			
6.7082×10^{-2}	7.4905×10^{10}	1.64071×10^{13}	3.384×10^{16}	5.445×10^{13}
3.7146	0.5507×10^{11}	1.7649×10^{14}	3.960×10^{17}	3.236×10^{14}
5.598	0.6661×10^{11}	2.675×10^{14}	5.411×10^{17}	3.882×10^{14}
5.8417×10	0.1888×10^{12}	7.37×10^{14}	1.512×10^{18}	8.531×10^{14}
	TABLE IV; $T = 9.3988 \times 10^8$ °K			
6.7082×10^{-2}	1.4076×10^{11}	1.659×10^{15}	2.688×10^{17}	2.415×10^{14}
3.7146	3.0223×10^{12}	8.51×10^{15}	1.344×10^{18}	1.435×10^{15}
5.598	3.6266×10^{12}	1.0586×10^{16}	1.688×10^{18}	1.722×10^{15}
5.8417×10	1.0284×10^{13}	3.1804×10^{16}	5.040×10^{18}	3.873×10^{15}

From the tables it is seen that neutrino luminosity due to Photon-Neutrino Coupling is greater than that due to Canuto *et al.* (1970) by 3 to 4 order of magnitude at $T = 3.7418 \times 10^8$ °K, $T = 5 \times 10^8$ °K and $T = 9.3988 \times 10^8$ °K but the difference is large at low densities for $T = 5.9303 \times 10^7$ °K. It is very interesting to note that the discrepancies between Landstreet's and our result is unexpectedly large at low density for $T > 10^7$ °K. The reason for the discrepancies is apparently due to various approximations introduced by Landstreet. The final form of the neutrino energy loss rates due to Landstreet are:

$$L^{I} = 10^{-44} H_8^6 T_7 \rho^4 (\text{if } H_8 \rho^{2/3} \leqslant 8 \times 10^6 T_7) \quad L^{II} = 2 \times 10^{-7} T_7^{19/3} \rho^{4/9} H_8^{2/3},$$

which he obtained by replacing the sums over states by integrals with the densities of a free electron gas in the absence of magnetic field i.e.,

$$\sum_n \rightarrow \int \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \simeq \int \frac{E^2 dE}{2\pi^2 \hbar^3 C^3}$$

The effects of magnetic field energy levels have also been neglected. Practically, this sort of approximation holds good for high density and large quantum numbers but not for small quantum numbers and low density. Also the neutrino luminosity ($L = 7.5 \times 10^3 T_7^{7/3} \rho^{4/9} H_8^{2/3}$) due to NSR (Neutrino synchrotron radiation) calculated by Ray Choudhuri (1970) in the light of Photon-Neutrino Coupling is smaller by 2 to 4 order of magnitude in comparison with our result except for low density at $T = 5.9303 \times 10^7$ °K, because of the fact that he had also made approximations similar to Landstreet for the summation over the electron states. Computing the total neutrino luminosity of several model of White Dwarfs Landstreet (1965) concluded that Neutrino synchrotron radiation from White Dwarfs is unimportant in the evolution of White Dwarfs. But one can conclude by studying the process in the light of Photon-Neutrino Coupling theory that total neutrino luminosity can play a significant role in stellar evolution process. In a future paper we like to study the implication of this process to the evolution of White Dwarfs and Neutron Stars.

ACKNOWLEDGEMENT

The author is greatful to Prof. P. Bandyopadhyay for helpful discussion. He also likes to thank R. Kundu for Programming and executing the numerical calculations.

REFERENCES

Bandyopadhyay, P. (1965). Weak interaction of photons. *Nuovo. Cim.*, **38**, 1912.

———— (1968). Possible weak interaction of photons and emission of neutrinos from stars. *Phys. Rev.*, **173**, 1481-1484.

———— (1970). Theories of weak interaction and astrophysical evidence. *Nuovo Cim.*, 1A, 680-690.

Bandyopadhyay, P., and Ray Choudhuri, P. (1969*a*). Possible pulsation of neutron stars and neutrino emission. *Nuovo. Cim.* (*Lettre*), 3498-3502.

———— (1969*b*). Neutrino energy density of the Universe. *Nuovo. Cim.*, **66A**, 238-242.

———— (1970). Supermassive stars and neutrino emission. *Nuovo. Cim.* (*Lettre*), **3**, 269-271.

Canuto, V., Chiu, H. Y., and Chou, C. K. (1970). Neutrino Bremsstrahlung in an intense magnetic field. *Astrophys. Space. Sci.*, **7**, 407.

Landstreet, J. D. (1965). Dissertation. Columbia University.

———— (1967). Electromagnetic synchrotron radiation. *Phys. Rev.*, **153**, 1372.

Ray Choudhuri, P. (1970). Neutrino synchrotron radiation I: Application to White Dwarfs. *Astrophys. Space. Sci.*, **8**, 432-447.