BIOMECHANICS OF THE GLENOHUMERAL JOINT : MATHEMATICAL TREATMENT

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It has been established that humeral articular surface changes its point of contact in the glenoid by rolling with slipping in all movements (Saha 1948 & 1950). This requires both spinning and translatory motion with slipping when stated mathematically in relation to the three axes.
Upper end of the humerus is acted upon by forces of contraction of eight muscles and gravity. Innermost group of steerers form by their action two opposing couples in the vertical and similarly in the horizontal planes. The steerers being tangents to almost a spherical surface have no other components.
Intermediate groups consist of three muscles anteriorly and their action opposing that of *teres minor* posteriorly. They give rolling with slipping from the start and spinning when the head-neck axis is in alignment with the shaft axis.
The prime movers consist of clavicular head of pectoralis major and deltoid which by their action and with the force of gravity help rolling with slipping to raise the shoulder to various planes.
Instantaneous rotations, translations, dynamic stability and equilibrium in various stages of lifting have been worked out mathematically.

HUMERAL articular surface changes its point of contact in the glenoid by rolling with slipping in all movements (Saha 1948, 1950) (Fig. 1). This requires both spinning and translatory motion with slipping when stated mathematically in relation to the three axes. Upper end of the humerus is acted upon by forces of contraction of eight muscles and gravity which are necessary for these movements.

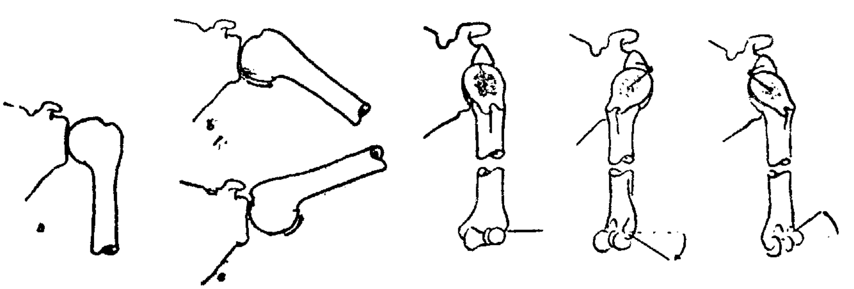


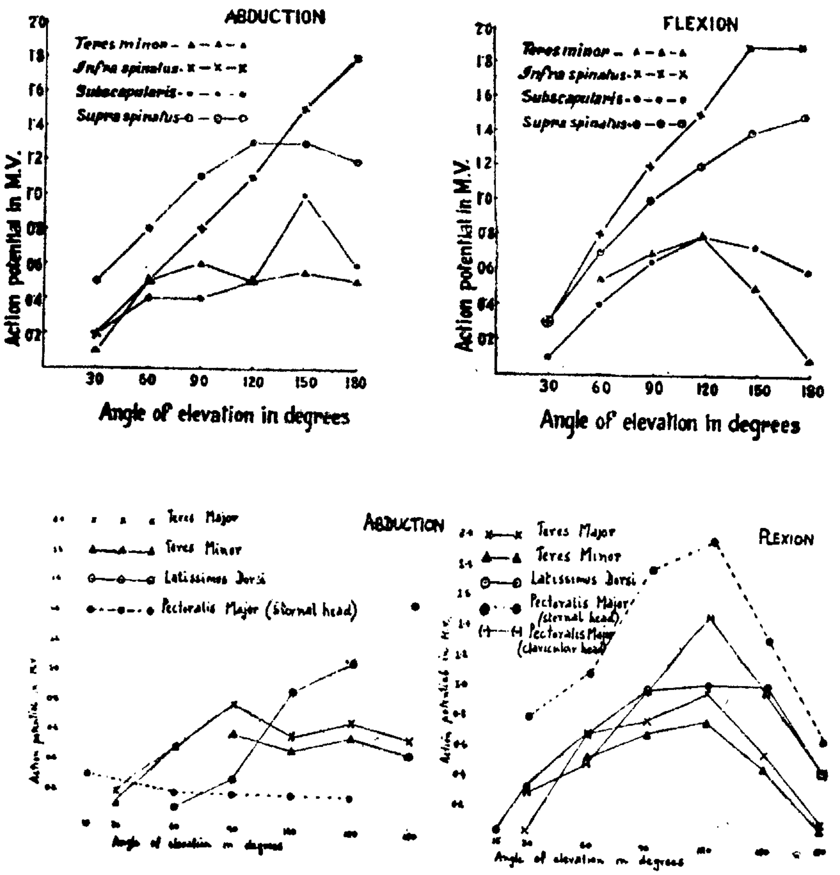
FIG. 1. Line drawing of the shoulder joint in the plane of the paper showing the vertical and horizontal rolling with slipping in different positions of the arm. Right hand drawing is an oblique horizontal section through the head-neck axis; the top portion has been removed to show rolling.

Simultaneous multilead eight channel electromyographs show action of almost all the muscles to varying degrees in a particular single movement e. g., abduction or flexion (Figs. 2 & 3).

Innermost group of steerers and gravity form by their action two opposing couples in the vertical and similarly two in the horizontal planes. The steerers being tangents to almost a spherical surface have no other components.

Intermediate groups consist of three muscles anteriorly and their action opposing that of teres minor posteriorly. They give rolling with slipping from the initiation of movement and spinning when the humerus is raised above 60° in the glenoid i.e., when the contact point is in alignment with the shaft axis (Fig. 4).

The prime mover consists of clavicular head of pectoralis major and deltoid which by their action and with the force of gravity help rolling with slipping to move the shoulder in various planes.



FIGS. 2 & 3. Simultaneous electromyographs of the muscles of the upper end of the arm in different stages of elevation showing most of the muscles taking part in abduction and flexion. Abscissa shows the elevation in degrees and ordinate the force in different muscles in millivolts.

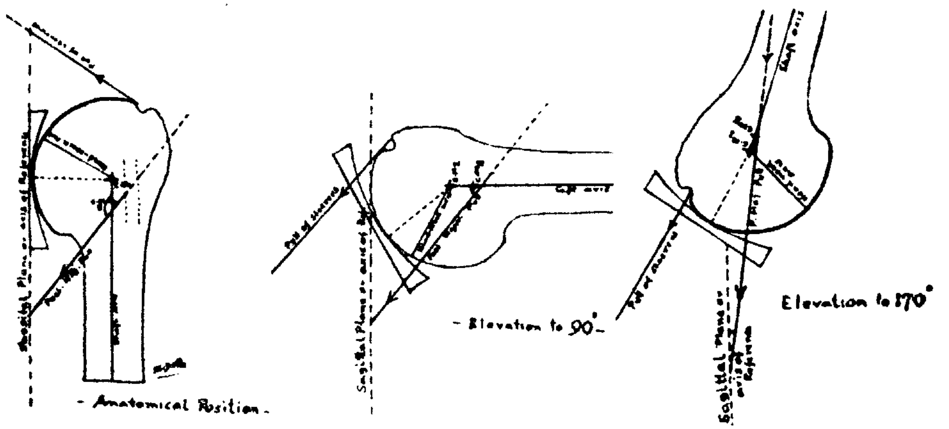


FIG. 4. Two dimensional diagram to show the segments acted on by the steerers and intermediate group of muscles. The centre of masses of these are CM_1 and CM_2.

As arm is raised or lowered in any direction through a range of nearly 180°, it requires acceleration, deceleration, equilibrium in the newly assumed position (when needed) and dynamic stability.

Variable angular acceleration at every instant during movement through an angular range π and linear accelerations at every instant during translation over nearly 4 sq. cm. (surface area) $\times$ 0.3 cm. (depth) in the glenoid in any direction make it difficult to arrive at an exact solution of the problem. Therefore, instantaneous rotations, translations, dynamic stability and equilibrium in various stages of lifting of the arm were worked out mathematically in vector notation.

Movement of the head of the humerus in the glenoid may be considered with reference to three planes (Fig. 5) : –

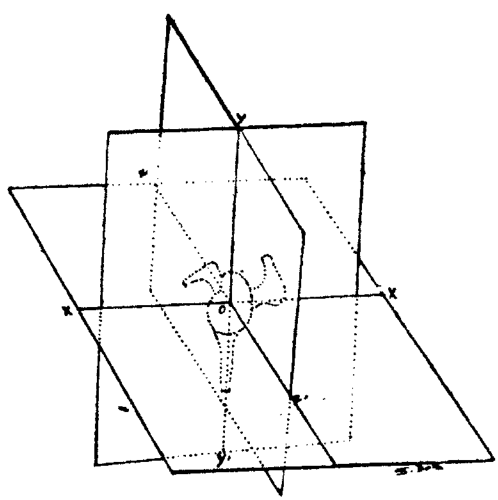


FIG. 5. Reference planes passing through the centre of glenoid. The planes cut in XX', YY' and ZZ' axes.

(1) A plane passing through the rim of the glenoid, more or less at right angles to the axis of the scapula depending on the tilt of the glenoid.

(2) Plane at right angles to (1), roughly parallel to the plane of the infra-glenoid part of the scapula.

(3) A horizontal plane passing through the glenoid fossa and at right angles to (1) and (2).

Let these planes cut each other in three axes XX', YY', and ZZ' at the level of the centre of the glenoid O. The articular surface of the humeral head which is taken to be a part of a sphere (though not exactly) with a radius r revolves through roughly π around its centre (Centre of mass CM_1) with reference to the above three planes.

The couples formed by steerers which revolve or spin the head of the humerus around the axis XX' are clockwise and counter-clockwise i.e., capable of neutralising each other if and when necessary. Forces of contraction of supraspinatus and gravity which individually is equal to the resultant of the reaction at the point of contact, friction in the opposite direction and soft tissue obstruction at the other end of the arm, form two couples spinning the head in opposite direction. Similarly, around YY' axis two counteracting couples are formed by subscapularis, infraspinatus-cum-upper part of *teres minor*. These couples formed by steerers have no other component than that which spin the head (Fig. 6).

In ZZ' axis, two opposing couples are formed by intermediate group of muscles which only spin the head of the humerus when it is raised above 60° in the glenoid. It has in addition other components to be described later (*vide* infra).

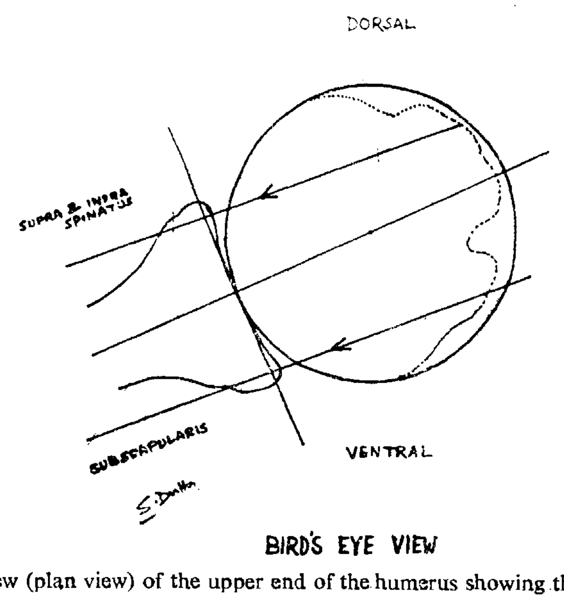


FIG. 6. Bird's eye view (plan view) of the upper end of the humerus showing the articular surface is almost a part of a sphere and the distal attachements of the steerers are tangents to it.

The angular movement when the arm is raised takes place with an angular acceleration or deceleration. It may also come to stop at any desired position of elevation. The change of angular momentum at any instant would be equal to the angular impulse of the force of contraction. Thus

$$\Sigma\tau_x \, dt = \Sigma \, (Gx—Gx_0)$$
$$\Sigma\tau_y \, dt = \Sigma \, (Gy—Gy_0)$$
$$\Sigma\tau_z \, dt = \Sigma \, (Gz—Gz_0)$$

Resultant of these in vector notation is

$$\tau dt = \mathbf{G} - \mathbf{G}_0 = \Omega,$$

where τ is the moment of the force and τdt is the instantaneous angular impulse. G_0 and G are the angular momenta at the beginning and end of that instant and subcripts x, y and z indicate the corresponding components $G_0 = I_0\omega_0$, where I_0 is the moment of inertia and ω_0 is the initial instantaneous angular velocity.

The resultant obviously is the precessional change of angular velocity (Ω) for that particular instant and has got an axis of its own passing through O.

Intermediate group of muscles inserted to the upper shaft of the humerus are *latissimus dorsi, teres major* and *pectoralis major* anteriorly in the floor of the bicipital groove and its edges form one couple and teres minor gives another in the opposite direction. These muscles being distal to the centre of mass of the upper segment (CM_2) has got both rotatory and translatory movements (Fig. 7). The spinning takes place in the ZZ' axis by the resultant of the rotatory components of the opposing

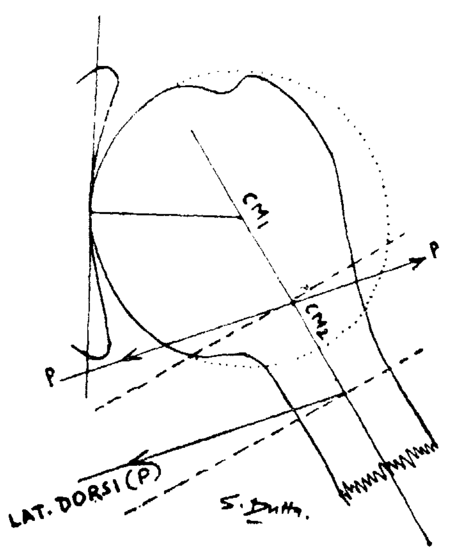


FIG. 7. Two dimensional diagram shows the pull of the intermediate group of muscles acting off the centre of mass of the segment proximal to it (CM_2). This pull has two components; a clockwise couple and a linear movement downwards.

couples formed by the external and internal rotators belonging to this intermediate group.

The translatory movement changes the point of origin simultaneously. As the point of origin moves in the glenoid to O', the axes shift and are assumed to be parallel to the original axes passing through O. The rotatory component which is a function of moment of inertia is increased by $M(x^2+y^2+z^2)$, where M=mass; and x, y and z are the shortest distances of the new origin from the three original axes.

The moment of the force τ' now gets an instantaneous change of axis owing to simultaneous translatory movement; moment of inertia being increased by $M(x^2+y^2+z^2)$. Thus the equations of rotation are

$$\Sigma\tau'_x \, dt = \Sigma (G'_x - G'x_0)$$
$$\Sigma\tau'_y \, dt = \Sigma (G'_y - G'y_0)$$
$$\Sigma\tau'_z \, dt = \Sigma (G'_z - G'z_0) = \Sigma\tau_z \, dt \text{ (vide supra).}$$

In ZZ' axis spinning is only effective when the humerus is raised to above 60° because the head-neck axis bears an angle of 120° (average) with the shaft axis.

The resultant of above three changes of angular momenta in vector notation is

$$\tau' dt = \mathbf{G}' - \mathbf{G}'_0 = \mathbf{\Omega}$$

For instantaneous change of translatory velocity, instantaneous impulse of force in three axes will be

$$\Sigma P_x dt = \Sigma M dv_x$$
$$\Sigma P_y dt = \Sigma M dv_y$$
$$\Sigma P_z dt = \Sigma M dv_z$$

where linear acceleration $a = \frac{dv}{dt}$ and P the force; subscripts x, y and z being the respective components.

In vector notation their resultant is

$$\mathbf{P} dt = \mathbf{M} \, dv \qquad \ldots \quad (1)$$

Similarly, the outer prime mover, deltoid-cum-clavicular head of *pectoralis major* roll then head of the humerus to elevate the limb against gravity.

Resultant equation for rotation will be $\tau'' \, dt = \mathbf{G}'' - \mathbf{G}''_0 = \mathbf{\Omega}''$

Resultant equation for instantaneous change of translatory velocity will be

$$P' dt = M' dv \qquad \ldots \quad (2)$$

Translatory movement is limited and most of it is wasted in slipping. It is neutralised by resultant of increased friction, force of reaction and soft tissue resistance.

The vector sum of these three precessional movements $\mathbf{\Omega}$, $\mathbf{\Omega}'$, $\mathbf{\Omega}''$, give the final resultant instantaneous angular velocity round a new axis at a particular instant. In that instant the origin of the new axes moves with a translatory acceleration which is the resultant vectors of (1) and (2) through which the parallel axes pass.

Thus for any movement in a specific direction, precessionals pinning takes place around an axis passing through the origin of the mobile three axes which has a resultant linear motion. Simultaneous electromyographs prove this (Fig. 2 & 3).

Equilibrium at any stage will be when $\Omega + \Omega' + \Omega'' = 0$ and acceleration $a=0$.

Stability is maintained by anatomical determinants (within normal range) and the force of balanced contraction of muscles which give spinning and translatory movement with slipping.

Conclusion

Final resultant of instantaneous angular impulse of six pairs of couples opposing each other in three axes if exceeds zero allows a precessional spinning around an axis whose initial origin moves simultaneously in the direction of the resultant of six instantaneous linear impulse. The latter linear movement takes place with slipping and is to a great extent checked by friction, reaction and soft tissue obstruction.

When the resultants of both instantaneous angular impulse and linear impulse are zero at a particular position the shoulder joint comes to halt in that position. The dynamic stability is maintained by the structural determinants and balancing action of both the instantaneous angular and linear impulse which bring the limb to stop due to stretch reflex in the capsule of the normal joint.

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