PARAMAGNETIC FLUID FLOW THROUGH AN ANNULUS WITH POROUS WALLS

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The steady flow of an incompressible, non-conducting paramagnetic fluid in the annulus between two co-axial infinite porous tubes has been analysed here. The fluid is injected into the annulus through the walls of the inner tube and sucked out through the outer one at the same rate. A magnetic field is applied in the longitudinal direction. The problem is solved on computer using the method of adjoints. The results are discussed with the help of graphs. The magnetic field is seen to accelerate the axial flow near the outer wall and retard it near the inner wall. Midway between the two walls, it remains almost the same. The effect of suction and injection is to accelerate the flow near the inner wall and retard it near the outer one. The effect of flow is, however, to orient dipoles in a direction perpendicular to it while they are originally in the direction of applied magnetic field. Thus their direction is governed by a balance between the hydrodynamic and magnetic forces.

INTRODUCTION

MAGNETIC FLUIDS are prepared by suspending ferromagnetic grains in a non-magnetic non-conducting liquid (Rosensweig 1966). They are used in magnetic fluid seals and direct energy converters (Resler & Rosensweig 1967; and Rosensweig *et al.* 1968). The grains are generally treated with a dispersing agent which operates in conjunction with the thermal agitation of the particles to counteract the tendency of the particles to collect together and eventually precipitate from solution as a result of the mutual magnetic forces. Consequently, colloidal suspensions of this kind are ultrastable and behave for all practical purposes as non-conducting magnetizable fluids. These fluids reportedly do not show any magnetic hystereses.

Attempts (Brenner 1970; and Jenkins 1971) have already been made to understand the nature of these fluids with the help of continuum theories. Because of the marked magnetic properties of these fluids, it is possible to influence their behaviour by applying magnetic field. The application of the magnetic field induces magnetization which interacts with an inhomogenous magnetic field to produce a body force, and when the magnetization and the magnetic field are not collinear, a body couple. Jenkin's (1971) theory is in essence similar to Ericksen's theory of anisotropic fluids (Erickson 1960) with magnetization density vector replacing orientation vector $\bar{\mathbf{n}}$. The use of local magnetization as an independent variable provides us with a uniform treatment for static and dynamic situations and a natural distinction between paramagnetic and ferromagnetic fluids.

In the conservation law for moment of momentum, the density of moment of momentum is assumed to be composed of the moment of the velocity field and an additional contribution related to the spin of the magnetization vector. The component of spin parallel to the magnetization is ignored. In a similar fashion, the flux and volume supply of moment of momentum includes surface and volume couples in addition to the moments of the tractions and the body forces. For a ferromagnetic fluid, the list of independent variables includes magnetization gradients while for paramagnetic fluids it does not. Therefore, the term in which it appears as a coefficient of magnetic stress in the local form of the balance law of moment of momentum requiring the stress tensor to be asymmetric is taken as zero for paramagnetic fluids. Separate sets of governing equations for ferromagnetic and paramagnetic fluids are available in (Jenkins 1971).

In the present paper we study the flow of paramagnetic fluid through an annulus between two infinite co-axial porous cylindrical tubes when the fluid is injected into the annular space through the inner one and sucked out through the outer one at the same rate. The external field is taken to act parallel to the common axis of the tubes. The field equations are solved numerically on a computer. The results are discussed with the help of grahps and tables given at the end.

BASIC EQUATIONS

A summary of the basic equations used (Rosensweig 1966) is as follows :

(i) Equation of continuity —

$$\dot{\rho} + \rho \dot{x}_{k,k} = 0. \qquad \ldots(1)$$

(ii) Equation of motion —

$$\rho \ddot{x}_i = t_{ik,k} + \rho f_i + \rho m_k H^{(1)}_{i,k}. \qquad \ldots(2)$$

(iii) Equation governing magnetization —

$$\beta \dot{m}_i = -\frac{4\pi M_s}{\chi_0}\frac{m_i}{m_s - m} - \frac{2\alpha^2}{m}\overset{*}{m}_i + H^{(0)}_i + H^{(1)}_i. \qquad \ldots(3)$$

(iv) Constitutive equations

$$t_{ik} = -p\delta_{ik} + 2\mu_f d_{ik} - \frac{2\rho\alpha^2}{m} m_{[k}\overset{*}{m}_{i]}. \qquad \ldots(4)$$

Here, cartesian tensor notation is used. ρ is the fluid density, f_i the external body force per unit mass, m_i the magnetization per unit mass, t_{ik} the stress tensor, m_s and M_s the saturation magnetization per unit mass and per unit volume respectively, p the hydrostatic pressure, μ the viscocity, δ_{ik} the kronecker delta and α and β are constants. Comma denotes differentiation and bracketed indices are to be anti-symmeterized. Superposed dot stands for material time derivative. χ_0 is the initial susceptibility, d_{ij} the stretching tensor and $\overset{*}{m}_i$ the co-rotational derivative of m_i defined by

$$\overset{*}{m}_i = \dot{m}_i - w_{ik}m_k, \qquad \text{...(5)}$$

where w_{ik} is the spin tensor. In (2) and (4) $H_i^{(0)}$ is the external field which would be present even in the absence of the fluid and $H_i^{(1)}$ is the self-field due to the presence of the magnetic fluid. Thus the total magnetic field H_i is

$$H_i = H_i^{(0)} + H_i^{(1)}. \qquad \text{...(6)}$$

We consider the paramagnetic fluid confined in some region V with surface Σ. We regard the external magnetic field as determined by known distribution of electric current in the region exterior to V. Then the self-field $H_i^{(1)}$ satisfies the following conditions :

$$\epsilon_{ijk} H_{k,j}^{(1)} = 0 \qquad \text{...(7)}$$

everywhere, with

$$H_{k,k}^{(1)} = -4\pi M_{k,k} \qquad \text{...(8)}$$

in V, and

$$H_{k,k}^{(1)} = 0 \qquad \text{...(9)}$$

outside V.
At Σ, we should have

$$< | \epsilon_{ijk} H_k^{(1)} | > \nu_j = 0 \qquad \text{...(10)}$$

and

$$< | H_i^{(1)} + 4\pi M_i | > = 0. \qquad \text{...(11)}$$

In the above equation ϵ_{ijk} is the permutation tensor and ν_i is the outward drawn normal to Σ. $< | \ | >$ stands for the jump in the quantity enclosed, over the surface Σ.

Flow Through an Annulus with Porous Walls

We analyse the steady flow of an incompressible nonconducting paramagnetic fluid through the annulus between two co-axial, right circular porous tubes of radii a and b, $(b > a)$. The fluid is flowing in the axial direction under a constant pressure gradient $-k$. The fluid is injected into the annular space through the pores of the inner tube and sucked out through those of the outer one at the same rate (Berman 1958). A uniform external magnetic field $H^{(0)}$ is applied parallel to the common axis of the tubes. Fig. 1 below gives flow through an annulus with porous walls.

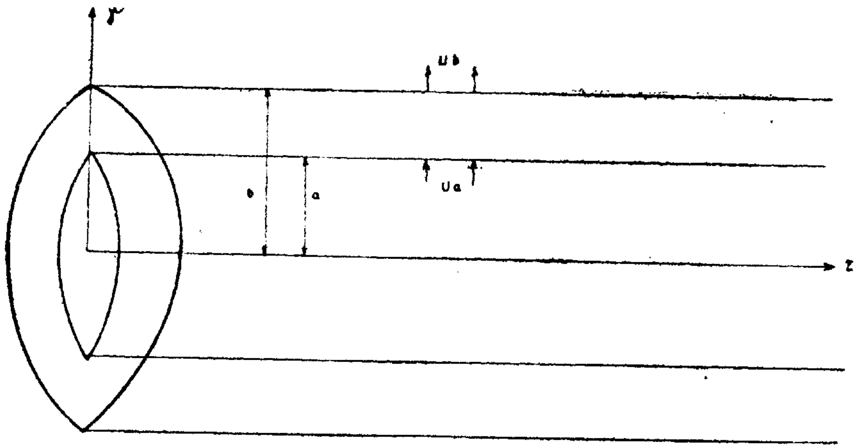

FIG. 1. Flow through a porous annulus.

We use cylindrical polar co-ordinates (r, θ, z) with Z-axis along the common axis of the tubes. The physical components of the field quantities are considered. Due to axial symmetry, all the quantities are independent of θ. Since the tubes are of infinite length, the partial derivatives of the field quantities with respect to z vanish.

We consider the velocity components u, v and w in r, θ and z-directions respectively. As there is no flow in the θ-direction,

$$v = 0. \quad \text{...(12)}$$

The boundary conditions on u give

$$u(a) = u_a \text{ and } u(b) = u_b, \quad \text{...(13)}$$

where u_a and u_b are velocities across the inner and the outer walls respectively. Using no-slip condition at the walls of the cylinders, we have

$$w(a) = w(b) = 0. \quad \text{...(14)}$$

From eqns. (1) and (13), we obtain

$$ru(r) = au_a = bu_b. \quad \text{...(15)}$$

We set $\frac{b}{a} = \sigma$ in (15) to write it in the form

$$u = \sigma \frac{au_b}{r}. \quad \text{...(16)}$$

Thus the velocity components are

$$u = \sigma \frac{au_b}{r}, \quad v = 0, \quad w = w(r). \qquad \text{...(17)}$$

The magnetization field components are taken in the form

$$m_r = m \sin \phi, \quad m_\theta = 0, \quad m_z = m \cos \phi, \qquad \text{...(18)}$$

where $m = m(r, t)$ and $\phi = \phi(r, t)$ are functions to be determined.

The magnetization field (19) has a non-vanishing divergence. A set of components of induced magnetic field satisfying eqns. (7) to (11) is given by

$$H_r^{(1)} = -4\pi\rho m \sin \phi, \quad H_\theta^{(1)} = H_z^{(1)} = 0. \qquad \text{...(19)}$$

The components rate of strain tensor, d_{ij}, and of spin tensor, w_{ij}, are

$$[d_{ij}] = \begin{bmatrix} -\sigma \frac{au_b}{r^2} & 0 & \frac{1}{2} w' \\ 0 & \sigma \frac{au_b}{r^2} & 0 \\ \frac{1}{2} w' & 0 & 0 \end{bmatrix} \qquad \text{...(20)}$$

and

$$[w_{ij}] = \begin{bmatrix} 0 & 0 & -\frac{1}{2} w' \\ 0 & 0 & 0 \\ \frac{1}{2} w' & 0 & 0 \end{bmatrix} \qquad \text{...(21)}$$

where the primes, here and throughout the paper, denote differentiation with respect to r. We neglect the inertia associated with magnetization and set $\beta = 0$ in eqn. (3). Ignoring the self-field and using the eqns. (3), (5), (18) and (21), the magnitude, m, and the direction, ϕ, of the steady magnetization vector are given by

$$2\sigma \frac{au_b}{r} \phi' + w' + \frac{H^{(0)}}{\alpha^2} \sin \phi = 0, \qquad \text{...(22)}$$

and

$$\frac{4\pi M_s}{\alpha^2 \chi_0} \frac{m}{m_s - m} + 2\sigma \frac{au_b}{r} \frac{m'}{m} = \frac{H^{(0)}}{\alpha^2} \cos \phi. \qquad \text{...(23)}$$

If we include the effect of self-field also the character of the steady field is not essentially altered. Physically the self-field directly suppresses the magnitude of the radial component of magnetization and induces a couple tending to align the magnetization vector in the direction of axis. Thus the self-field effectively augments the external magnetic field. In our analysis, we shall assume that the steady magnetization is adequately described by the above equations.

For the fluid in the porous walls, where $w \equiv 0$, if we apply eqns. (22) and (23) and also assume that $m' = \phi' = 0$, we get the boundary conditions

$$\phi = 0 \text{ and } m = \frac{\chi_0 H^{(0)} m_s}{\chi_0 H^{(0)} + 4\pi\rho m_s} \qquad \ldots(24)$$

at $r = a$ and $r = b$.
From eqn. (4), the stress components are

$$t_{rr} = -p - 2\mu_f \sigma \frac{a u_b}{r^2},$$

$$t_{\theta\theta} = -p + 2\mu_f \sigma \frac{a u_b}{r^2}$$

$$t_{zz} = -p,$$

$$t_{rz} = \left(\mu_f - \frac{\rho\alpha^2 m}{2}\right) w' - \rho\alpha^2 \sigma m \frac{a u_b}{r} \phi',$$

$$t_{zr} = \left(\mu_f + \frac{\rho\alpha^2 m}{2}\right) w' + \rho\alpha^2 \sigma m \frac{a u_b}{r} \phi',$$

and

$$t_{r\theta} = t_{\theta r} = t_{z\theta} = t_{\theta z} = 0. \qquad \ldots(25)$$

Substituting for the components of velocity, stress and magnetization in the equations of motion (4), we get

$$\frac{\partial p}{\partial r} = \frac{\rho\sigma^2 a^2 u_b^2}{r^3} - 2\pi\rho^2 \frac{\partial}{\partial r}(m \sin\phi)^2, \qquad \ldots(26)$$

$$\frac{\partial p}{\partial \theta} = 0 \qquad \ldots(27)$$

and

$$\frac{1}{r}\left[r\left\{\mu_f w' + \rho\alpha^2 m\left(\frac{w'}{2} + \sigma \frac{a u_b}{r} \phi'\right)\right\}\right]' + k = \rho\sigma \frac{a u_b}{r} w', \qquad \ldots(28)$$

where we have substituted $\dfrac{\partial p}{\partial z} = -k$.

Eqns. (26), (27) and (28) give

$$p = -2\pi\rho^2 m^2 \sin^2\phi - kz - \frac{\rho a^2 \sigma^2 u_b^2}{2r^2} + \text{constant} \qquad \ldots(29)$$

Now m and ϕ having been given by (22), (23) and (24), we can solve eqn. (28) for w, subject to the boundary conditions (14).

We define the following non-dimensional parameters

$$\bar{r} = \frac{r}{b}, \ \bar{m} = \frac{m}{m_s}, \ \bar{w} = \frac{\mu_f w}{kb\alpha^2},$$

$$\bar{u}_b = \frac{2\mu_f u_b}{kb^2}, \ \bar{H}^{(0)} = \frac{\mu_f H^{(0)}}{kb\alpha^2},$$

$$\lambda = \frac{\rho\alpha^2 m_s}{2\mu_f}, \ \bar{M}_s = \frac{4\pi\mu_f M_s}{kb\alpha^2\chi_0} \qquad \text{...(30)}$$

and

$$\xi = \frac{k\rho^2 b^3}{2\mu_f^2}.$$

Substituting in eqns. (22), (23) and (28) and simplifying, we get

$$m'' = \frac{rm}{u_b}\left[\left(\frac{u_b}{r}\phi' + w'\right)\phi' - \frac{M_s m'}{(1-m)^2} + u_b\left(\frac{m'}{r^2 m} + \frac{m'^2}{rm^2}\right)\right], \qquad \text{...(31)}$$

$$\phi'' = \frac{r}{u_b}\left[\phi'\left(\frac{u_b}{r^2} - \frac{M_s m}{1-m} - \frac{u_b}{r}\frac{m'}{m}\right) - w''\right] \qquad \text{...(32)}$$

and

$$w'' = -\frac{[\{r(1+\lambda m)\}' - \xi u_b]\, w' + u_b\lambda(m\phi')' + r}{r(1+\lambda m)}, \qquad \text{...(33)}$$

where we have dropped the bars above the dimensionless parameters for simplicity and primes denote differentiation with respect to the dimensionless parameter r.

The boundary conditions (14) and (24) reduce to the following:

At $r = \dfrac{1}{\sigma}$ and $r = 1$, we have

$$w = 0, \ \phi = 0 \ \text{ and } \ m = \frac{H^{(0)}}{H^{(0)} + M_s}. \qquad \text{...(34)}$$

Numerical Solutions of the Equations

The eqns. (31), (32) and (33) are solved numerically subject to the boundary conditions (34). The method of adjoints, a shooting method due to Goodman and Lance, is used. For the details of this method the reader is referred to (Roberts & Shipman 1972). The computation is done on an IBM 360 computer. The two point boundary value problem is reduced to an initial value problem by guessing the missing initial conditions. Then these guessed values are corrected by repeated iterations.

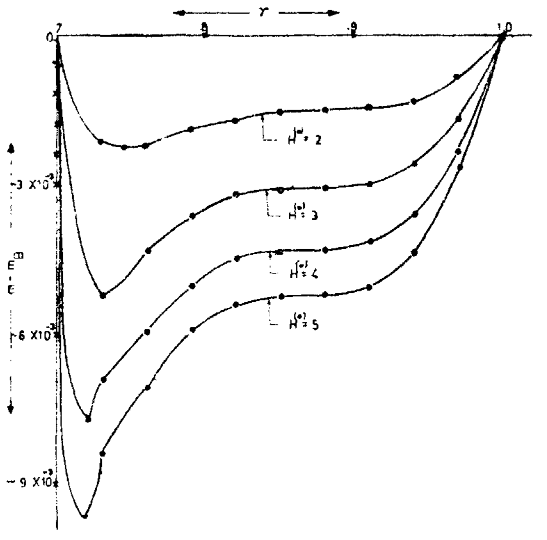

FIG. 2. $m - m_B$ *vs.* r for different values of $H^{(0)}$

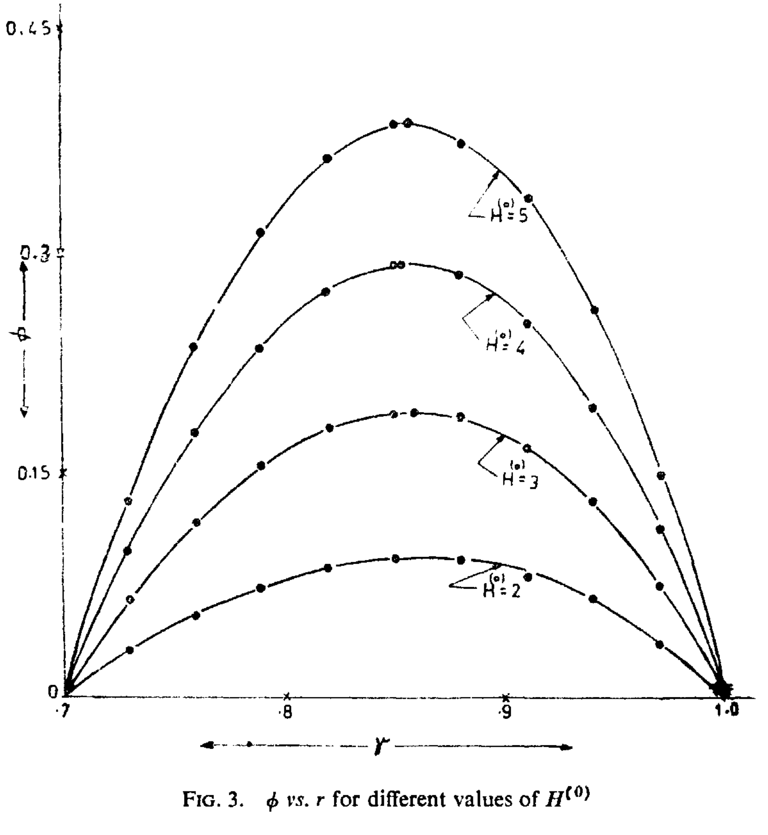

FIG. 3. ϕ *vs.* r for different values of $H^{(0)}$

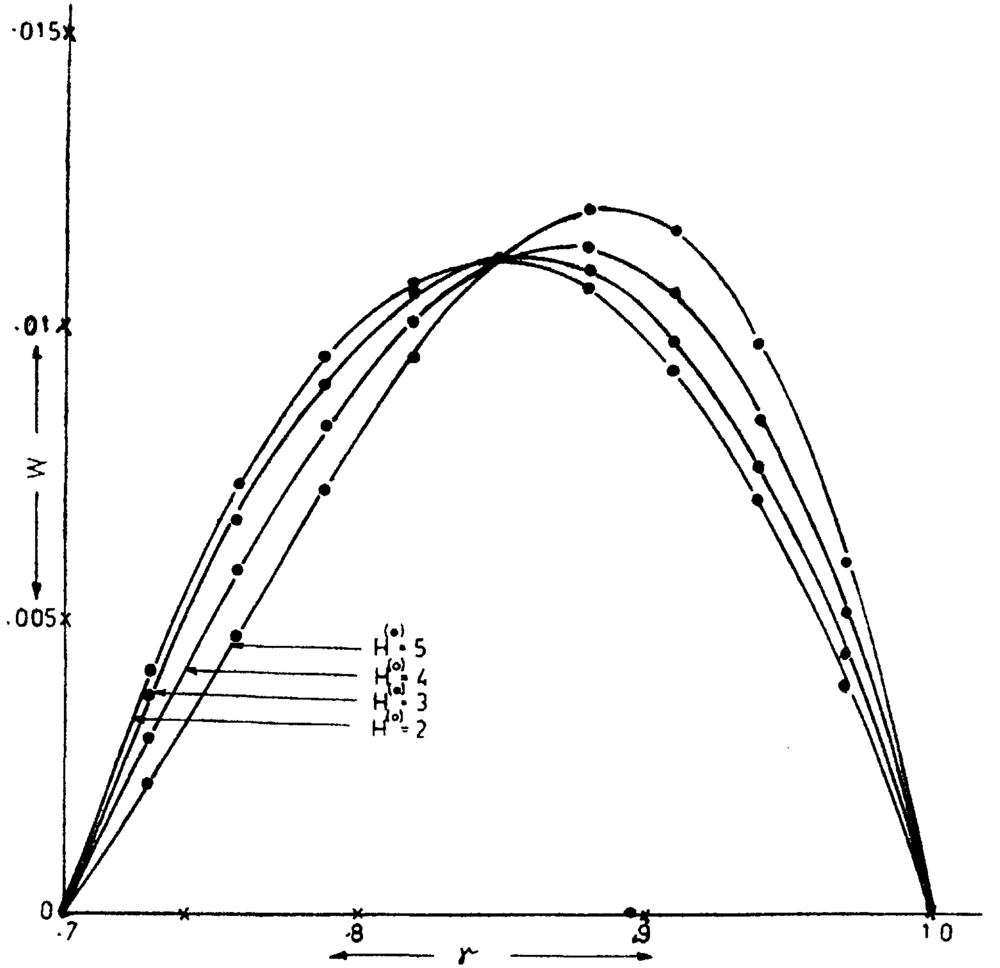

FIG. 4. w vs. r for different values of $H^{(0)}$

The three equations are first solved for m'', ϕ'' and w'', in terms of m, ϕ, w, and their first derivatives m', ϕ', and w'. By usual procedure this is reduced to a system of six first order differential equations. The integration involved in the process of solving is performed by Runge-Kutta method (Scarborough 1971).

Throughout computation work the values $M_s = 1$ and $\lambda = \xi = .1$ are considered. The numerical calculations are done for different values of $H^{(0)}$, u_b and σ. The results are shown by graphs and tables.

Discussion of Results

For any fixed set of values of the parameters, m changes, in the annulus, only very slightly from its value on the boundaries. Thus, in the graphs, $m - m_B$ where m_B is the common value of m on the two boundaries is plotted against r.

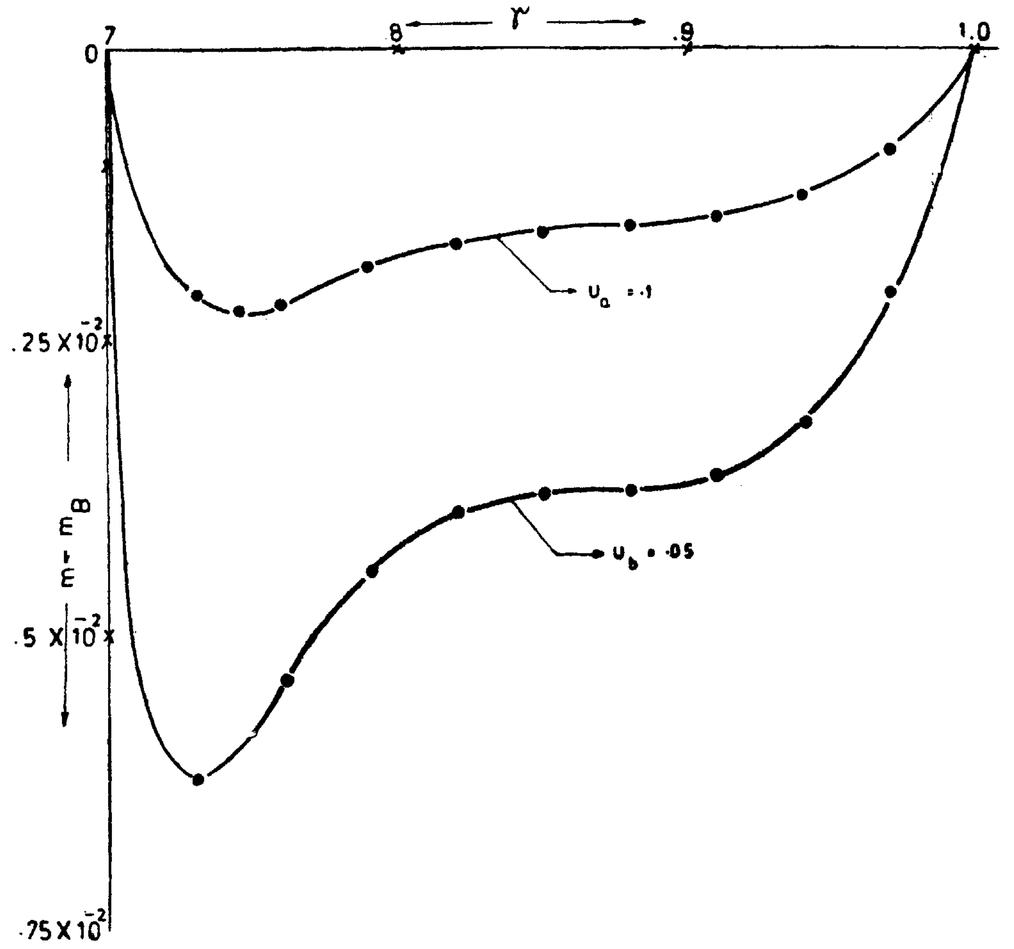

FIG. 5. $m - m_B$ *vs.* r for different values of U_b

Figs. 2, 3 and 4 show the effect of $H^{(0)}$ on m, ϕ and w respectively. The magnetization density falls sharply near the inner boundary. The more the applied field, the more the fall. Then it gradually increases to attain the initial value on the outer wall. The profiles of ϕ and w are parabolic. Fig. 3 shows that in the region of axial flow ϕ increases with $H^{(0)}$. An increase in $H^{(0)}$ retards the axial flow near the inner wall and accelerates the flow near the outer one, as is shown by Fig. 4. Midway between the tubes it is almost unaffected.

The effect of an increase in suction and injection, u_b, on m and ϕ is opposite to that of $H^{(0)}$ as is evident from Figs. 5 and 6. Fig. 7 shows that the effect of suction and injection is to retard the axial flow near the inner tube and accelerate it near the outer one.

Figs. 8, 9 and 10 show the effect of a change in the width of the annulus on magnetization vector and velocity. The fall in m near the inner boundary is more

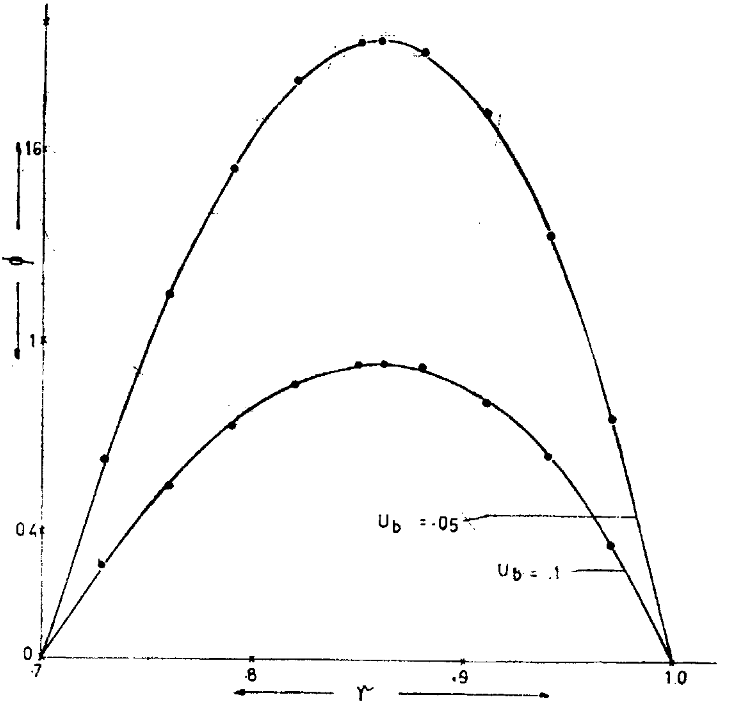

FIG. 6. ϕ *vs.* r for different values of U_b

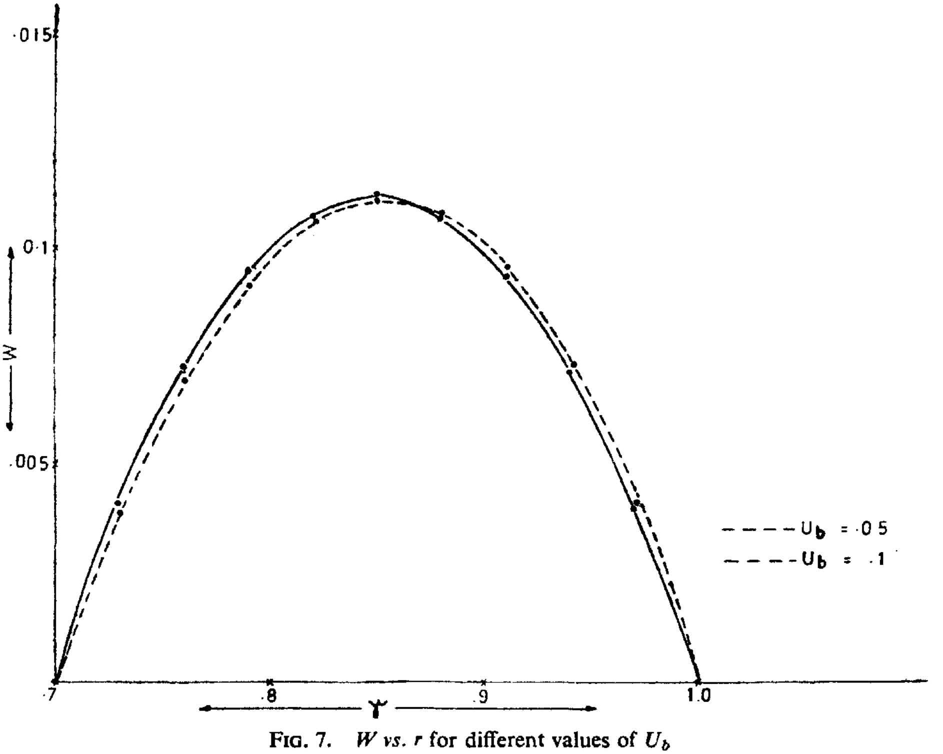

FIG. 7. W *vs.* r for different values of U_b

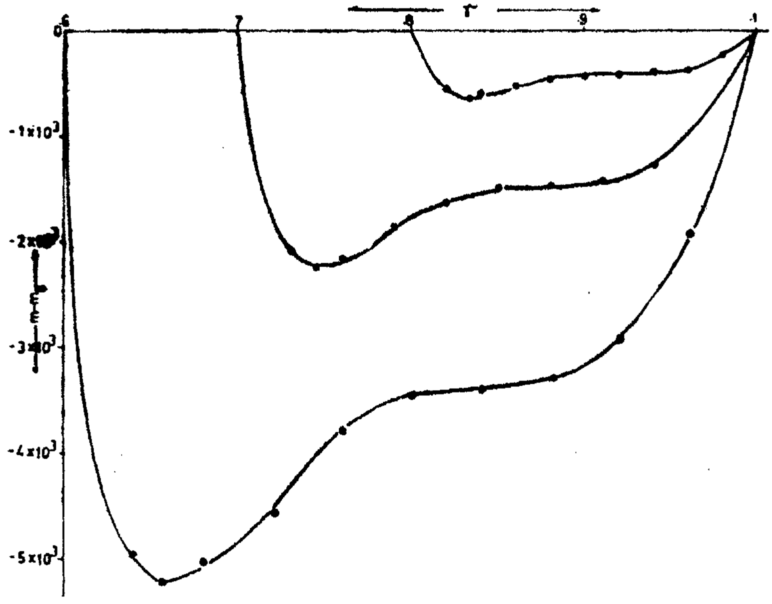

FIG. 8. $m - m_B$ *vs.* r for different radii ratio.

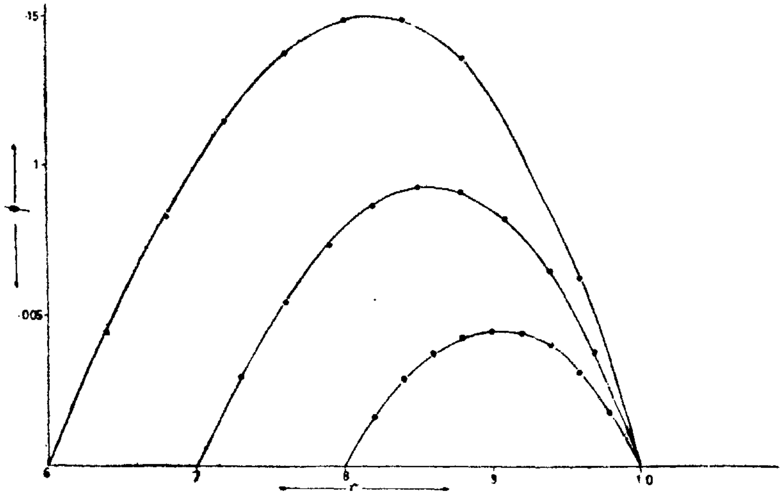

FIG. 9. ϕ *vs.* r for different radii ratio.

for a wider annulus. The increases in ϕ and w in the annulus are also more for wider annulus.

Thus the motivation here is to study the interaction of magnetic and mechanical forces in the case of paramagnetic fluid flow through an annulus with porous walls.

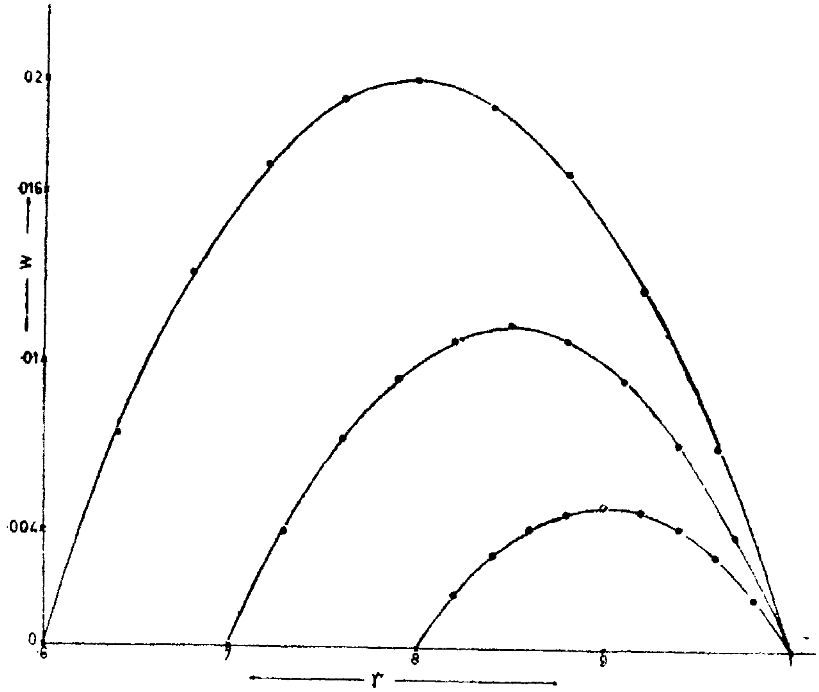

FIG. 10. *W vs. r* for different radii ratio.

This has application in the treatment of cancer by means of magnetic fluids and also in oil seals made of magnetic fluids.

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