BEAUTY QUARK AND NEW HADRONS

by M. AHMAD, G. Q. SOFI, M. SHAFI, *and* N. A. SHAH, *Department of Physics, University of Kashmir, Srinagar 190006*

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By choosing the bottom or beauty quark (b) as the fifth quark the SU(5) symmetry scheme is presented here. The baryons and measons under this scheme are classified and the B-particles are incorporated in the mass spectrum for mesons. Gelfand and Tsetlin technique is exploited to classify the hadrons which proves to be elegant and economical in this analysis.

INTRODUCTION

THERE IS clear evidence now for the existence of four or five different flavours of leptons. Several phenomenological and theoretical considerations have recently led to the idea of neq quarks more than four. Some of the considerations which necessitates additional quarks are : the observations of the new resonances. These phenomena give rise to the idea of Truth and Beauty quarks. Scattering cross section and branching ratio analysis give rise to the confidence that the T families represent bound states of at least one nee quark. The t quark describes the T-family (9.5 FeV) and the b quark describes the B-family (5 GeV) (Herb *et al.* 1977) of mesons. These different quark states give rise to Bottomonium and Toponium structure of elementary particles.

Most properties of the low lying mesons are sensitive to the gauge theory assignment of left or right handed new quarks (b or t). The t quark carries a charge $Q\ \frac{2}{3}$ and b quark carries a charge $Q - \frac{1}{3}$ (Ellis *et al.* 1977). The charges of the newly found mesons including b quark correspond to the iso-spin doublet $B = b\bar{d}$; and $B = b\bar{u}$, while the lowest lying mesons with t quark are

$$T = t\bar{d};\ \text{and}\ T = t\bar{u}.$$

The mass of the B', B'' mesons is expected to be around 5 GeV.

The additional new quark either t or b and the quark lepton grand unification scheme which unifies strong and weak electromagnetic interactions have made SU(5) symmetry very topical in these days (Georgi & Glashow 1974).

In the present paper, the authors discuss an SU(5) beauty symmetry scheme involving the new quark "b" as the fifth quark. Here a spectroscopy is given for the hadron system involving beautiful particles. Although the proliferation of quarks and leptons have led to a new field of investigation these days, yet the hadron spectroscopy scheme is still very much alive and gives physical insights to the ever increasing puzzling situation of high energy physics. The study of hadron as a system of confined quarks is one of the biggest puzzles of Particle Physics. Quantum-Chromo-dynamics (QCD) (Appelquist *et al.* 1975) is the outcome and the candidate theory for explaining quark confinement and some of its features like asymptotic freedom. These are tested in deep-inelastic scattering and in process which are forbidden by Zweig-Iizika-rule.

In connection with the SU(5) beauty symmetry, a Gelfand and Tsetlin technique is used to classify beauty and non-beautiful hadrons. This technique is both elegant and economical in particle classification. In addition to giving the spectroscopy for mesons and baryons under this scheme, Gelfand states to physical intuitions to the states of the hadrons are also exhibited in terms of their well assigned quantum numbers.

SU(5) Generators

Any transformation in SU(5) scheme can be written as

$$U = e^{i\Sigma\alpha_i}\, A_i,$$

where A_i is hermitian and can be written as,

$$(A_{ij})_{\mu\nu} = \delta_{i\mu}\delta_{j\nu} - \tfrac{1}{5}\,\delta_{ij}\delta_{\mu\nu}. \qquad ...(1)$$

Accordingly, we have 24 matrices out of which four can be diagonalised simultaneously. The physical multiplets can be constructed as $5 \otimes 5$ mesons and $5 \otimes 5 \otimes 5$ baryons from the basic pentets

$$q = (udscb), \qquad ...(2)$$

where u, d, s and c are the conventional quarks and b is the additional quark due to SU(5) which we name as beauty quark. Let I be the unit matrix and

$$\begin{aligned}
A_{55} &= \text{diag}\,(-\tfrac{1}{5}, -\tfrac{1}{5}, -\tfrac{1}{5}, -\tfrac{1}{5}, \tfrac{4}{5})\\
A_{44} &= \text{diag}\,(-\tfrac{1}{5}, -\tfrac{1}{5}, -\tfrac{1}{5}, \tfrac{4}{5}, -\tfrac{1}{5})\\
A_{33} &= \text{diag}\,(-\tfrac{1}{5}, -\tfrac{1}{5}, \tfrac{4}{5}, -\tfrac{1}{5}, -\tfrac{1}{5})\\
A_{22} &= \text{diag}\,(-\tfrac{1}{5}, \tfrac{4}{5}, -\tfrac{1}{5}, -\tfrac{1}{5}, -\tfrac{1}{5})
\end{aligned} \qquad ...(3)$$

Different quantum numbers are then given as,

$$\begin{aligned}
\text{Charge } Q &= A_{22} - \frac{2}{15}\, I && = \text{diag}\,(-\tfrac{1}{3}, \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3})\\
\text{Hypercharge } Y &= -A_{33} + \frac{2}{15}\, I && = \text{diag}\,(\tfrac{1}{3}, \tfrac{1}{3}, -\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{3})\\
\text{Charm } C &= A_{44} + \tfrac{1}{5} I && = \text{diag}\,(0, 0, 0, 1, 0)\\
\text{Beauty } B &= A_{55} + \tfrac{1}{5} I && = \text{diag}\,(0, 0, 0, 0, 1),
\end{aligned} \qquad ...(4)$$

where

charge $Q = I_3 + \dfrac{Y}{2} - \dfrac{B + C}{2}$, which is then the modified Gellmann Nishijima formula. The quantum numbers of quarks are accordingly given in Table I.

TABLE I

Quantum numbers of quarks

Quarks	I_z	I	Y	C	B	Q
d	– 1/2	1/2	1/3	0	0	– 1/3
u	1/2	1/2	1/3	0	0	2/3
s	0	0	– 2/3	0	0	– 1/3
c	0	0	1/3	1	0	– 1/3
b	0	0	1/3	0	1	– 1/3

The beautified mesons that we get from this mode are,

$$b\bar{u} = B',\ b\bar{d} = B'',\ b\bar{s} = B''',\ b\bar{c} = B''''$$

$$\bar{b}u = \bar{B}',\ \bar{b}d = \bar{B}'',\ \bar{b}s = \bar{B}''',\ \bar{b}c = \bar{B}''''$$

$$b\bar{b} = B^0 \qquad \ldots(5)$$

These mesons are pseudoscalar mesons ($J^p = 0^-$)

Similarly baryons can be constructed from $5 \otimes 5 \otimes 5$. The 35-plot of baryons found from this beauty model of which the SU(4) 15-plot forms a part is also shown in Table II.

GELFAND PATTERN

The mode '*k*' for group theoretic and symmetric techniques in quantum physics has always been that of angular momentum and this can be generalised to particle physics. There are many ways in which one might introduce angular momentum technique; the most widely used method is that of Schwinger, based on Boson calculus. Instead of following this technique, we can interpret this in the most elegant and economic way in terms of Gelfand calculus which diagrams the states. We associate a uniqe Gelfand pattern to every state by imposing the condition of lexical ordering.

To simplify the calculation for mass splitting, we follow this pattern technique and represent all the states of SU(5) in terms of quantum numbers I, I_z, Y, C and B according to Gelfan pattern. Here for SU(5), we associate a unique orthonormal state of an irreducible representation $(P, Q, R, S, 0)$ to the Gelfand pattern as :

$$\begin{pmatrix} P & & Q & & R & & S & & 0 \\ & m_{14} & & m_{24} & & m_{34} & & m_{44} & \\ & & m_{13} & & m_{23} & & m_{33} & & \\ & & & m_{12} & & m_{22} & & & \\ & & & & m_{11} & & & & \end{pmatrix} \qquad \ldots(6)$$

In terms of usual I, I_z, Y, C and B, variables, the general Gelfand pattern for SU(5) reads,

$$| (P, Q, R, S, 0), I, I_z, Y, C, B > =$$

$$\left(\begin{array}{ll} P & Q \\ & I - \frac{Y}{2} + \frac{C-B}{2} + \frac{P+Q+R+S}{4} \\ & I + \frac{Y}{2} + \frac{C-B}{2} + \frac{P+Q+R+S}{8} \\ & -I + \frac{Y+C+B}{2} \end{array} \right.$$

$$\left. \begin{array}{lll} R & S & 0 \\ I + \frac{Y}{2} - \frac{C+B}{2} + \frac{P+Q+R+S}{8} & -I + \frac{Y}{2} + \frac{C-B}{2} + \frac{P+Q+R+S}{8} & 0 \\ I - \frac{Y}{2} - \frac{C+B}{2} + \frac{P+Q+R+S}{8} & -I + \frac{Y}{2} - \frac{C+B}{2} + \frac{P+Q+R+S}{8} & \\ -I + \frac{Y+C-B}{2} + \frac{P+Q+R+S}{8} & & \\ I_z + \frac{Y+B+C}{2} + \frac{P+Q+R+S}{8} & & \end{array} \right) \qquad ...(7)$$

Here $(P, Q, R, S, 0)$ denotes a frame which corresponds to a unique SU(5) irreducible representation; and to every Gelfand pattern belonging to the frame $(P, Q, R, S, 0)$ there corresponds a unique state. The next row denotes how the frame $(P, Q, R, S, 0)$ is to be filled in order to specify a single vector. To achieve this the symbols occurring in the pattern (6) should obey the between-ness relation,

$$P \geqslant m_{14} \geqslant m_{13} \geqslant m_{12} \geqslant m_{11} \geqslant m_{22} \geqslant m_{33} \geqslant m_{44} \text{ etc.} \qquad ...(8)$$

In the familiar terms we get,

$$I = \tfrac{1}{2}(m_{13} - m_{12})$$

$$Y = m_{13} - \left(m_{14} - \frac{P+Q+R+S}{8}\right)$$

$$C = m_{14} - \frac{P+Q+R+S}{4}; \quad B = \tfrac{3}{8}(P+Q+R+S) - m_{14} \qquad ...(9)$$

TABLE II

Quantum numbers of 3/2 baryons (beautified)

Beauty	State	I_z	I	Y	Q	C	Particle
1	*ddb*	– 1	1	1	– 1	0	Δ^{+}
	uub	1	1	1	1	0	Δ^{0}
	dsb	– 1/2	1/2	0	– 1	0	Δ_{o}^{*+}
	dcb	– 1/2	1/2	1	– 1	1	$\Delta_{b}^{**\bar{c}}$
	usb	1/2	1/2	0	0	0	Δ_{b}^{*0}
	udb	0	1/2	1	0	0	Δ_{b}^{0}
	ucb	1/2	1/2	1	0	1	Σ_{b}^{*0}
	scb	0	0	0	– 1	1	Ξ_{b}^{*-}
	ssb	0	0	– 1	– 1	0	Σ_{b}^{*-}
	ccb	0	0	1	– 1	2	Ξ_{b}^{*0}
2	*dbb*	– 1/2	1/2	1	– 1	0	$\Sigma^{-}{}_{bb}$
	ubb	1/2	1/2	1	0	0	$\Sigma^{0}{}_{bb}$
	sbb	0	0	0	– 1	0	$\Xi^{-}{}_{bb}$
	cbb	0	0	1	– 1	1	$\Xi^{*-}{}_{bb}$
3	*bbb*	0	0	1	– 1	0	$\Xi^{-}{}_{bbb}$

MASS FORMULA

From SU(3), it is known that the mass splitting in lowest order is proportional to the matrix elements of A_8, which singles out the 8-direction (hypercharge) hence A_8 gives hypercharge as generator. Besides A_8, there is another octet operator, which singles out the hypercharge direction that being the D operator D_8. Here all the vectors are linear combinations of 2 independent types, which cannot be rotated into each other. We now generalise the hypothesis to SU(4) and SU(5) where, in addition to the above operators, we have the operators A_{15}, A_{24} which singles out charm and beauty.

We now evaluate Gellmann's hypothesis for SU(5) quantitatively and write the mass splitting by the mass operator M as,

$$M = M_0 + M_1 < m \mid A_8 \mid m > + M_2 < m \mid D_8 \mid m > + M_3 < \mid A_{15} \mid m > + M_4 < m \mid A_{24} \mid m >, \quad \ldots(10)$$

m being the Gelfand pattern specifying the state with M_0, M_1, M_2, M_3, M_4 constants.

The first matrix element in eqn. (10) is just proportional to Y. Hence,

$$< m \mid A_8 \mid m > \propto Y = \frac{Y}{2} \qquad \text{(after normalization)} \qquad \ldots(11)$$

The second operator is given by the general definition

$$D_8 = d_{8jk} A_j A_k.$$

Hence by this general definition, the D operator is quadratic in the generators; the only quadratic operators available are, $I(I+1)$, Y and the invariant operator I_z. In this way $< m \mid D_8 \mid m >$ takes the form,

$$< m \mid D_8 \mid m > = \delta I(I+1) + \beta Y^2 + \gamma Z, \qquad \text{...(12)}$$

where Z should be found out. To evaluate this, we make use of the general principle that tensor operators acting on multiplets having only singly occupied points in weight space show multiplicity. Hence, when Gelfand pattern is of the form,

$$\begin{pmatrix} P & 0 & 0 & 0 & 0 \\ & m_{14} & 0 & 0 & 0 \\ & & \cdots & & \\ & & \cdot\cdot & & \\ & & \cdot & & \end{pmatrix}, \text{ we have}$$

$$\left\langle \begin{pmatrix} P & 0 & 0 & 0 & 0 \\ & m_{14} & 0 & 0 & 0 \\ & & \cdots & & \\ & & \cdot\cdot & & \\ & & \cdot & & \end{pmatrix} \middle| D_8 \middle| \begin{pmatrix} P & 0 & 0 & 0 & 0 \\ & m_{14} & 0 & 0 & 0 \\ & & \cdots & & \\ & & \cdot\cdot & & \\ & & \cdot & & \end{pmatrix} \right\rangle = \alpha Y$$

For $\begin{pmatrix} P & 0 & 0 & 0 & 0 \\ & m_{14} & 0 & 0 & 0 \\ & & \cdots & & \\ & & \cdot\cdot & & \\ & & \cdot & & \end{pmatrix}$, we have

$$I = \tfrac{1}{2}\, m_{13} = \tfrac{1}{2}\left[\left(m_{14} - \frac{P}{8}\right) - Y\right]$$

$$C = \left(m_{14} = \frac{P}{4}\right) \qquad \text{...(13)}$$

$$B = \left(\tfrac{3}{8}\, P - m_{14}\right). \qquad \text{...(14)}$$

Hence from eqn. (12) we get,

$$\delta I(I+1) + \beta Y^2 + \gamma Z$$

$$= \delta \tfrac{1}{4}\left[\left(m_{14} - \frac{P}{8}\right) - Y\right]\left[\left(m_{14} - \frac{P}{8}\right) - Y + 2\right] + \beta Y^2 + \gamma Z$$

(*equation continued on next page*)

$$= \delta\Big[\tfrac{1}{4}\Big\{\Big(m_{14} - \frac{P}{8}\Big)^2 - Y\Big(m_{14} - \frac{P}{8}\Big)$$

$$+ 2\Big(m_{14} - \frac{P}{8}\Big) - Y\Big(m_{14} - \frac{P}{8}\Big) + Y^2 - 2Y^2\Big\}\Big] + \beta Y^2 + \gamma Z$$

$$= \delta\Big[\tfrac{1}{4}\Big(m_{14} - \frac{P}{8}\Big) \frac{-2Y}{4}\Big(m_{14} - \frac{P}{8} + 1\Big)$$

$$\tfrac{1}{2}\Big(m_{14} - \frac{P}{8}\Big) + \frac{Y^2}{4}\Big] + \beta Y^2 + \gamma Z. \qquad ...(15)$$

For (13) to hold good, quadratic terms in (15) must vanish.

Hence, $\delta = 1$, $\beta = -\tfrac{1}{4}$ and

$$-\gamma Z = \tfrac{1}{4}\Big(m_{14} - \frac{P+}{8}\Big)^2 + \tfrac{1}{2}\Big(m_{14} - \frac{P+}{8}\Big).$$

After the necessary adjustments, we get the value of

$$-\gamma Z = (C^2 + C) + (B^2 + B) -$$

$$\Big[\tfrac{7}{4} m_{14}^2 + \frac{51P^2}{256} - \frac{19}{16} m_{14}P + \frac{3P}{16} - \frac{m_{14}}{2}\Big]$$

$$= C(C + 1) + B(B + 1) +$$

$$\Big[-\tfrac{7}{4} m_{14}^2 - \frac{51}{256} P^2 + \frac{19}{16} m_{14}P - \frac{3P}{16} + \frac{m_{14}}{2}\Big]$$

or finally, we get

$$-\gamma Z = C(C + 1) + B(B + 1) + K. \qquad ...(16)$$

Thus using normalization factor, we get,

$$< m \mid D_8 \mid m > = \frac{M_2}{6}\Big[I(I + 1) - \frac{Y^2}{4} - C(C + 1) - B(B + 1) - K\Big]. \qquad ...(17)$$

Now A_{15} is used for generating charm quantum number, so we write

$$< m \mid A_{15} \mid m > = \frac{1}{12} C. \qquad ...(18)$$

A_{24} is used for generating beauty quantum number, so we have,

$$< m \mid A_{24} \mid m > = \frac{1}{20} B. \qquad ...(19)$$

Then we write the generalised linear mass formula for beautified baryons ($\tfrac{3}{2}$) as,

$$M = M_0 + \frac{M_1}{2} Y + \frac{M_2}{6}\left[I(I+1) - \frac{Y^2}{4}\right] +$$
$$\left[\frac{M_3}{12} C - \frac{M_2}{6} C(C+1)\right] + \left[\frac{M_4}{20} B - \frac{M_2}{6} B(B+1)\right] + F.$$

For beautified mesons, we have the mass squared formulas as,

$$M^2 = M_0'^2 + \frac{M_1^2}{6}\left[I(I+1) - \frac{Y^2}{4}\right] - \frac{M_1'^2}{6} C +$$
$$\left[\frac{M_2'^2}{20} B - \frac{M_1'^2}{6} B(B+1)\right] + F \quad \ldots(20)$$

Mass Spectrum for Baryons

Using the value of constants as :

$M_0 = 1.015$ (GeV); $M_1 = -0.380$ (GeV); $M_2 = 0.180$ (GeV)
$M_3 = 0.480$ (GeV), $M_4 = 10$ (GeV) $F = 8.0$ (GeV)
for beautified baryons.

For the beautified baryons, we get the mass spectrum as shown in Table III.

Table III

Quantum numbers of mesons (beautified)

Beauty	State	I_z	I	Y	C	Q	Particle
1	$b\bar{u}$	− 1/2	− 1/2	0	0	− 1	B'
	$b\bar{d}$	1/2	− 1/2	0	0	0	B''
	$b\bar{s}$	0	0	1	0	0	B'''
	$b\bar{c}$	0	0	0	− 1	0	B''''
−1	$\bar{b}u$	1/2	1/2	0	0	1	$\bar{B}'$
	$\bar{b}d$	− 1/2	1/2	0	0	0	$\bar{B}''$
	$\bar{b}s$	0	0	− 1	0	0	$\bar{B}'''$
	$\bar{b}c$	0	0	0	1	0	$\bar{B}''''$
0	bb	0	0	0	0	0	B^0

Mass Spectrum for Mesons

Using the value of constants as

$M_0'^2 = 0.819$ (GeV); $M_1'^2 = -0.9998$ (GeV) $\frac{M_2'^2}{20} = -5.0$ (GeV); and $F = 24.0$ (GeV) for beautified mesons and $F = -2.65$ (GeV) for charmed baryons. We get the mass spectrum of mesons as shown in the Table IV *a*, *b*.

TABLE IV*a*

Mass spectrum of beautified baryons

Q & Lable		Mass (GeV)
Δ^0	*uub*	9.3175
$\Delta_b^{**\bar{c}}$	*dcb*	9.5075
Σ^0_{bb}	*ubb*	9.4700
Ξ^{*-}_{bb}	*cbb*	9.7398
Ξ^-_{bbb}	*bbb*	9.9600

TABLE IV*b*

Mass spectrum of beautified mesons

Lable		Mass (GeV)
B'	$b\bar{u}$	4.434
B''	$b\bar{d}$	4.444
B'''	$b\bar{s}$	4.461
B''''	$b\bar{c}$	4.469
B^0	$b\bar{b}$	5.0

CONCLUSION

The SU(5) beauty symmetry, that is presented herein depicts the B meson (5 GeV) and its families, the B' and B''. Hadrons under SU(3) and SU(4) scheme are also easily identified from the spectrum. The spectrum obtained from our model for bottomonium particles depicts a level structure which copes with the conventional analysis of Eichten and Gottfried (1977), which predicts the same for a $q\bar{q}$ system, as a function of mass of heavy quark. All these new baryons and mesons which are identified in the present scheme, await experimental verification in future.

REFERENCES

Ahmad, M., and Zadoo, T. K. (1977). SU(4) Symmetry and hadron classification. *Czech. J. Phys.*, B, **27**, 1337-1345.

Appelquist, T., De Rujula, A., Politzer, H., and Glashow, S. L. (1975). Spectroscopy of the new mesons. *Phys. Rev. Lett.*, **34**, 63, 365-369.

Ellis, J., Gaillard, M. K., and Nanopohlos, D. V. (1977). *A Phenomenological Profile of the Higgs Boson.* Cern preprint Th-2093-Cern July 1977.

Eichten, E., and Gottfried, K. (1977). Heavy quark in e^+e^- annihilation. *Phys. Lett.*, **66**, B, 3, 286-290.

Georgi, H., and Glashow, S. L. (1974). Unity of all elementary particle forces. *Phys. Rev. Lett.*, **32**, 8, 438-441.

Herb, S., Hom, D. C., Lederman, L. M., Sen, J. C., Snyder, H. D., Yoh, J. K., Appel, J. A., Brown, B. C., Brown, C. N., Innes, W. R., Ueno, K., Yamanouchi, T., Ito, A. S., Jostlein, H., Kaplan, D. M., and Kephart, R. D. (1977). Observation of a dimuon resonances at 9.5 (Gev) in 400 Gev Proton-Nucleus collisions. *Phys. Rev. Lett.*, **39**, 5, 252-255.