TEMPORAL GROWTH OF FILAMENTATION INSTABILITY OF A GAUSSIAN LASER BEAM IN A PLASMA

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(Communicated by Prof. M. S. Sodha, F.N.A.)

(*Received 22 July 1978*)

The effect of saturating nonlinearity on the temporal growth rate of filamentation instability of a Gaussian Laser beam in homogeneous collisional and collisionless plasma has been studied. While nonlinearity in collisional plasma is because of redistribution of carriers by non uniform heating, ponderomotive force is responsible for the same in collisionless plasma. The maximum temporal growth rate for a collisionless plasma occurs at a power three orders of magnitude higher in comparison to a collisional plasma. The growth rate does not show a monotonic increase with increasing beam intensity. It is also found to be an oscillatory function of the distance of propagation in the plasma: this occurs on account of the self-focusing of the Gaussian Laser beam.

INTRODUCTION

THE interaction of an intense electromagnetic beam with plasma excites various parametric instabilities (Bingham and Lashmore-Davies 1976; Brueckner and Jorna 1974; Drake *et al.* 1974; Eidmann *et al.* 1975; Langdon and Lasinski 1975; Lee *et al.* 1974; Manheimer and Ott 1974; Nishikawa 1968; Perkins & Valeo 1974; Schwarz and Hora 1974; Simon and Thomson 1976; Valeo 1974). Amongst these filamentation instability occupies a unique place because it occurs at moderate threshold powers and has reasonably high growth rate.

Kaw *et al.* (1973) and Sodha *et al.* (1973) have studied the spatial growth of perturbations in the intensity distribution of a plane electromagnetic wave in a plasma. But in Laser plasma interactions one usually encounters a Gaussian Laser beam. Sodha *et al.* (1974) have investigated the spatial growth of filamentation instability of such beams. Recently Drake *et al.* (1974), Valeo (1974), Manheimer and Ott (1974) and Bingham and Lashmore-Davies (1976) have studied the temporal growth of filamentation instability in Laser induced plasmas. Langdon and Lasinski (1975) and Eidmann *et al.* (1975) have observed similar type of instabilities in computer simulation and Laser target experiments. However, these analyses predict only purely growing instability with increasing intensity of the beam.

Note: The paper is the result of work supported by N.S.F. (U.S.A).

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**Laser application Programme

In the present paper, the authors have investigated the effect of saturating nonlinearity of the high power Gaussian Laser beam on the temporal growth of filamentation instability in collisional and collisionless plasma. The nonlinearity in the dielectric constant of the plasma arises from the redistribution of carriers on account of (*i*) non uniform heating of electrons in a collisional plasma, in the time domain ($t >> \tau_\varepsilon$, τ_ε being the energy relaxation time) and (*ii*) the ponderomotive force in a collisionless plasma on short time scale ($t << \tau_\varepsilon$). The treatment is three dimensional and applicable to arbitrary beam intensities. It is found that the temporal growth rate, without showing a monotonic increase with the intensity of the beam, is an oscillatory function of the distance of propagation due to self-focusing of the main beam.

In the following Section, we have stated expressions for the nonlinear dielectric constant in case of collisional and collisionless plasmas in the presence of a Gaussian electromagnetic beam. In the next Section, an expression for the maximum temporal growth rate and optimum size of perturbation has been derived. A brief discussion of the results follows in the last Section.

Nonlinear Dielectric Constant of the Plasma

Following Sodha *et al.* (1975, 1976), the dielectric constant of a plasma can be written as

$$\in (EE^*) = \epsilon_0 + \Phi(EE^*),$$

where $\epsilon_0 = 1 - \dfrac{\omega_p^2}{\omega_0^2}$, $\omega_p^2 = \dfrac{4\pi N_0 e^2}{m}$.

e, m and N_0 are the charge, mass and equilibrium concentration of electrons and the expressions for the nonlinear dielectric constant are given as follows :

(*i*) *Collisional Nonlinearity :* In the case of a collisional plasma on long time scale ($t >> \tau_s$, τ_s is the heating time of the electrons) where dissipation of electron energy is due to collisions with heavy particles, the nonlinear dielectric constant is given by (Sodha *et al.* 1976)

$$\Phi(EE^*) = \frac{\omega_p^2}{\omega_0^2}\left[1 - \left\{\left(1 + \frac{\alpha_0 EE^*}{2}\right)^{(S/2-1)}\right\}\right], \qquad \text{...(2)}$$

where $\alpha_0 = e^2 M / 6m^2 \omega_0^2 k_B T_0$ is the nonlinearity parameter, M is the mass of the heavy scatterer, k_B is Boltzmann's constant, T_0 is the equilibrium plasma temperature, ω_0 is the angular frequency of the wave (whose electric vector varies as $e^{i\omega_0 t}$) and S is scattering parameter being equal to unity for electron neutral particle collisions and equal to -3 for electron-ion collisions. In eqn. (2), we have assumed that electron collision frequency (ν) is much smaller than the wave frequency i.e., $\nu << \omega_0$, so that the absorption of the beam is not significant.

(*ii*) *Ponderomotive Nonlinearity :* In the case of a collisionless plasma, the nonlinearity arises through the Ponderomotive force in short time scale and moderate intensities and the nonlinear dielectric constant is given by (Sodha *et al.* 1975, 1976)

$$\Phi(EE^*) = \frac{\omega_p^2}{\omega_0^2}\left[1 - \exp\left(-\tfrac{3}{4}\alpha_0 \frac{m}{M}\,\mathbf{E}\,.\,\mathbf{E}^*\right)\right]. \qquad ...(3)$$

It can be seen that in the steady state case, this nonlinearity is m/M times smaller than the collisional nonlinearity.

In the presence of the perturbation field ($\mathbf{E}_1$) the total electric field is $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$, where $\mathbf{E}_0$ is the unperturbed field; hence the dielectric constant can be expanded as

$$\epsilon(EE^*) = \epsilon(E_0E_0^*) + \epsilon'(E_0E_0^*)\,(\mathbf{E}_0^*\,.\,\mathbf{E}_1 + \mathbf{E}_1^*\,.\,\mathbf{E}_0), \qquad ...(4)$$

where

$$\epsilon(E_0E_0^*) = \epsilon_0 + \Phi(E_0E_0^*) \qquad ...(4A)$$

and

$$\epsilon'(E_0E_0^*) = \left.\frac{d\epsilon}{dEE^*}\right|_{EE^* = E_0E_0^*} \qquad ...(4B)$$

Temporal Growth of Perturbation

We consider here the propagation of a three dimensional Gaussian electromagnetic beam along the Z-direction in a plasma. The total electric field vector of the wave inside the plasma is

$$\mathbf{E} = (\mathbf{E}_0(x, y, z, t) + \mathbf{E}_1(x, y, z, t))\exp(^i\omega_0 t),$$

where $\mathbf{E}_0$ is the wave electric field vector (polarised in the Y-direction) in the absence of fluctuations. The intensity distribution of the unperturbed beam at $Z = 0$ is taken to be

$$\mathbf{E}_0\,.\,\mathbf{E}_0^*\,|_{z=0} = E_{00}^2 \exp(-r^2/r_0^2), \qquad ...(5)$$

where E_{00} and r_0 are the axial field amplitude and initial beam width of the wave and r is the radial co-ordinate of the cylindrical co-ordinate system, $\mathbf{E}_1$ is the amplitude of the fluctuations which is a slowly varying function of space and time. The total electric vector of the wave inside the plasma is governed by the wave equation

$$\nabla^2\mathbf{E} - \nabla(\nabla\,.\,\mathbf{E}) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\epsilon\,\mathbf{E}), \qquad ...(6)$$

where ϵ is the effective dielectric constant of the plasma given by eqn. (1) and c is the velocity of light in vacuum.

In the absence of any perturbation field the unperturbed wave equation is given by

$$\nabla^2 \mathbf{E}_0 + \frac{\omega_0^2}{c^2} \epsilon \mathbf{E}_0 = 0, \quad \ldots(6A)$$

where $\nabla(\nabla \cdot \mathbf{E}_0)$ term has been neglected in the W.K.B. approximation (Sodha *et al.* 1976) i.e., $\frac{c^2}{\omega_0^2} \frac{1}{\epsilon} \nabla^2 \ln \epsilon << 1$. The paraxial ray solution of the above equation for a homogeneous plasma can be written as (Sodha *et al.* 1975, 1976)

$$\mathbf{E}_0(x, y, z, t) = \mathbf{A}_0(x, y, z) \exp\left(-ik_0(z + S)\right), \quad \ldots(7A)$$

$$A_0^2(x, y, z) = \frac{E_{00}^2}{f^2(z)} \exp\left[-\frac{x^2 + y^2}{r_0^2 f^2(z)}\right], \quad \ldots(7B)$$

$$S = \tfrac{1}{2}(x^2 + y^2)\,\beta(z) + \psi(z) \quad \ldots(7C)$$

and

$$\beta(z) = \frac{1}{f} \cdot \frac{df}{dz}, \quad \ldots(8)$$

where f is the dimensionless beam width parameter given by

$$\frac{d^2 f}{dz^2} = \frac{1}{R_d^2 f^3} - E_{00}^2\, \Phi'(E_{00}^2/f^2), \quad \ldots(9)$$

where $\psi(z)$ is a function of Z alone. $k_0 = \frac{\omega_0}{c} \epsilon_0^{1/2}$ is the wave vector and $R_d = k_0 r_0^2$ is the diffraction length and Φ' represents the derivative of $\Phi(E_{00}^2/f^2)$ w.r.t. its argument.

The linearised equation for the perturbation field $\mathbf{E}_1(x, y, z, t)$ from eqn. (6) can be written as

$$\nabla^2 \mathbf{E}_1 + \nabla\left[\mathbf{E}_0 \cdot \frac{\nabla}{\epsilon}(\epsilon' \mathbf{E}_0 \cdot (\mathbf{E}_1 + \mathbf{E}_1^*))\right] - 2i \frac{\omega_0}{c^2} \frac{\partial \mathbf{E}_1}{\partial t} +$$

$$\frac{\omega_0^2}{c^2} \epsilon' E_0 \left[\mathbf{E}_0 \cdot (\mathbf{E}_0 + \mathbf{E}_1^*) + \frac{\mathbf{E}_0}{i\omega_0} \cdot \frac{\partial}{\partial t} (\mathbf{E}_1 + \mathbf{E}_1^*)\right] = 0 \quad \ldots(10)$$

We consider the initial beam to be polarised in the Y-direction (i.e., $\mathbf{E}_0 \parallel Y$-axis) and then the linearized equation for the perturbation field $\mathbf{E}_1(x, y, z, t) = A_1(x, y, z, t)\, e^{-ik_0(z+S)}$ can be written as

$$\nabla^2 \mathbf{A}_1 - 2ik_0 \frac{\partial \mathbf{A}_1}{\partial z} + \frac{1}{r_0^2 f^2}\left(2 - \frac{x^2 + y^2}{r_0^2 f^2}\right) \mathbf{A}_1 -$$

$$2ik_0\beta \mathbf{A}_1 - 2ik_0\beta x \frac{\partial \mathbf{A}_1}{\partial x} - 2ik_0\beta y \frac{\partial \mathbf{A}_1}{\partial y} +$$

(*continued on next page*)

$$\frac{\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} \frac{\partial}{\partial y} (\mathbf{A}_1 + \mathbf{A}_1^*) + \frac{ik_0\beta y^2 \epsilon' A_0}{\epsilon(A_0^2)\, r_0^2 f^2} (\mathbf{A}_1 + \mathbf{A}_1^*) -$$

$$\frac{ik_0\beta y \epsilon' A_0^2}{\epsilon(A_0^2)} \frac{\partial}{\partial y} (\mathbf{A}_1 + \mathbf{A}_1^*) - \frac{2i\omega_0}{c^2} \cdot \frac{\partial \mathbf{A}_1}{\partial t} +$$

$$\frac{\omega_0^2}{c^2} \epsilon' A_0^2 \left[(\mathbf{A}_1 + \mathbf{A}_1^*) + \frac{1}{i\omega_0} \frac{\partial}{\partial t} (\mathbf{A}_1 + \mathbf{A}_1^*) \right] = 0, \qquad \text{...(10A)}$$

where

$$\epsilon(EE^*) = \epsilon(A_0^2) + \epsilon'(A_0^2)\, \mathbf{A}_0 \,.\, (\mathbf{A}_1 + \mathbf{A}_1^*).$$

It may be mentioned here that the main contribution of the nonlinearity lies in the Y-component of A_1 since x and y components of A_1 are affected by the nonlinearity to a much lesser extent and are hence of little interest. So we consider $A_1 = A_{1y}$ only. For simplification writing $A_{1y} = A_{1\gamma} + iA_{1i}$, we get from eqn. (10A) the following equations for $A_{1\gamma}$ and A_{1i}

$$\nabla^2 A_{1\gamma} + 2k_0 \frac{\partial A_{1i}}{\partial z} + \frac{1}{r_0^2 f^2}\left(2 - \frac{x^2 + y^2}{r_0^2 f^2}\right) A_{1\gamma} + 2k_0\beta A_{1i} +$$

$$2k_0\beta \left(x \frac{\partial}{\partial x} A_{1i} + y \frac{\partial}{\partial y} A_{1i} \right) + \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)} \frac{\partial^2}{\partial y^2} A_{1\gamma} -$$

$$\frac{6y\epsilon' A_0^2}{\epsilon(A_0^2)} \frac{\partial}{\partial y} A_{1\gamma} - \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} \left(1 - \frac{2y^2}{r_0^2 f^2}\right) A_{1\gamma} +$$

$$2 \frac{\omega_0^2}{c^2} \epsilon' A_0^2 A_{1\gamma} + 2 \frac{\omega_0}{c^2} \frac{\partial}{\partial t} A_{1i} = 0 \qquad \text{...(11)}$$

and

$$\nabla^2 A_{1i} - 2k_0 \frac{\partial A_{1\gamma}}{\partial z} + \frac{1}{r_0^2 f^2}\left(2 - \frac{x^2 + y^2}{r_0^2 f^2}\right) A_{1i} -$$

$$2k_0\beta A_{1\gamma} - 2k_0\beta\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right) A_{1\gamma} - 2 \frac{\omega_0}{c^2} \frac{\epsilon' A_0^2}{\epsilon(A_0^2)} \frac{\partial A_{1\gamma}}{\partial t} +$$

$$\frac{2k_0\beta y^2 \epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} A_{1\gamma} - 2k_0\beta y \frac{\partial}{\partial y} A_{1\gamma} \frac{\epsilon' A_0^2}{\epsilon(A_0^2)} = 0. \qquad \text{...(12)}$$

For obtaining the growth rate of the perturbations we assume the space and time variations of $A_{1\gamma,i}$ as $Re\,[\exp\{i(\omega t - k_{11} z - \mathbf{k}_1(\mathbf{x} + \mathbf{y}))\}]$ and then the dispersion relation for ω is given by

$$\omega^2 - \left\{ \frac{(2 + \epsilon' A_0^2)}{(1 + \epsilon' A_0^2)} \right\} \mathbf{K} \,.\, v_g + i\beta v_g)\, \omega + \frac{1}{(1 + \epsilon' A_0^2)} \times$$

$$\left[(\mathbf{K} \,.\, v_g + i\beta v_g)^2 - \frac{c^4}{4\omega_0^2} \left(\mathbf{K}^2 - \frac{2}{r_0^2 f^2} \right) \left\{ \left(\mathbf{K}^2 - \frac{2}{r_0^2 f^2} \right) + \right.\right.$$

$$\left.\left. \frac{2\epsilon' (\mathbf{A}_0 \,.\, \mathbf{K}_1)^2}{\epsilon(A_0^2)} - \frac{2k_0^2 \epsilon' A_0^2}{\epsilon(A_0^2)} + \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} \right\} \right] = 0, \qquad \text{...(13)}$$

where $\upsilon_g = \mathbf{k}_0 c^2/\omega_0$ is the group velocity of the wave. It can be easily seen that when the perturbation is not fluctuating in time (viz., $\omega = 0$), eqn. (13) yields the usual dispersion relation for the spatial growth of the Filamentation Instability for the case when $\mathbf{K}.\ \upsilon_g = 0$ and $\beta = 0$. The solution of eqn. (13) is

$$\omega = \frac{(2 + \epsilon' A_0^2)}{2(1 + \epsilon' A_0^2)} (\mathbf{K}\, .\, \upsilon_g + i\beta\, \upsilon_g) \pm \frac{1}{2(1 + \epsilon' A_0^2)}$$

$$\Bigg[(2 + \epsilon' A_0^2)(\mathbf{K}\, .\, \upsilon_g + i\beta\, \upsilon_g)^2 - 4(1 + \epsilon' A_0^2)\Bigg\{(\mathbf{K}.\, \upsilon_g + i\beta\, \upsilon_g - \frac{c^4}{4\omega_0^2}$$

$$\left(\mathbf{K}^2 - \frac{2}{r_0^2 f^2}\right)\left\{\left(\mathbf{K}^2 - \frac{2}{r_0^2 f^2}\right) + \frac{2\epsilon' (\mathbf{A}_0\, .\, \mathbf{K}_1)^2}{\epsilon(A_0^2)} -\right.$$

$$\left.\left.\frac{2K_0^2\, \epsilon' A_0^2}{\epsilon(A_0^2)} + \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2}\right\}\Bigg\}\Bigg]^{1/2} \quad \ldots(14)$$

The local conditions for the validity of the above equations are

$$K_{\text{II}}^{-1} << R_n \quad \text{and} \quad K_{\perp}^{-1} << r_0 f,$$

where R_n is the self-focusing length. The former implies that the scale length of perturbation in the axial direction is smaller than the self-focusing length, i.e., self-focusing is a slower process as compared to the growth of perturbation in the propagation direction. This slowness of the self-focusing process causes smaller self-induced axial inhomogeneity in K_{II} and the scale length of variation of K_{II} in the Z-direction remains larger than the characteristic length of intensity variation. The latter condition implies that the transverse size of the perturbation is much smaller than the effective beam width at any distance of propagation. This local condition is satisfied for all Z except when f is reduced to such a value that $r_0 f << K_{\perp}^{-1}$ because the self-focusing theory becomes invalid in that region. The present theory is valid in the paraxial ray approximation ($r << r_0 f$) and is therefore applicable to small scale perturbations in the vicinity of the axis (viz., $K_{\perp}^{-1} << r_0 f$). This restriction is not a very serious one, because the growth of large scale perturbations would be small due to the relatively weak intensity of marginal rays.

It can be seen that the condition for the perturbation to grow is

$$\frac{2k_0^2\, \epsilon' A_0^2}{\epsilon(A_0^2)} > \left(k^2 - \frac{2}{r_0^2 f^2}\right) + \frac{2\epsilon'(\mathbf{A}_0 . \mathbf{K}_1)^2}{\epsilon(A_0^2)} + \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} \quad \ldots(15)$$

and hence the growth rate is given by

$$\Upsilon = \frac{C^2}{2\omega_0(1 + \epsilon' A_0^2)^{1/2}} \left[\left(K_1^2 - \frac{2}{r_0^2 f^2}\right)\left\{\frac{2K_0^2\, \epsilon' A_0^2}{\epsilon(A_0^2)} - \frac{2\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} -\right.\right.$$

$$\left.\left.\left(K_1^2 - \frac{2}{r_0^2 f^2}\right) - \frac{2\epsilon'(\mathbf{A}_0 . \mathbf{K}_1)^2}{\epsilon(A_0^2)}\right\}\right], \quad \ldots(16)$$

where $K_{II} << K_{\perp}$ has been assumed, which is justified as the fastest growing perturbation has a scale length only somewhat longer than the wave length.

To discuss the behaviour of the growth rate we consider two interesting cases :

(*i*) *When* $\mathbf{E}_0 \cdot \mathbf{K}_{\perp} = 0$ viz., the wave vector of the perturbation is at right angle to the electric vector of the main beam and the perturbation propagates as *TE* waves. The growth rate is maximum for an optimum value of K_1(for $\mathbf{E}_0 \cdot \mathbf{K}_{\perp} = 0$) given by

$$k_{1\text{opt}}^2 = \frac{\omega_0^2}{c^2}\,\epsilon' A_0^2 + \frac{2}{r_0^2 f^2} - \frac{\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2} \qquad \ldots(17)$$

and hence the corresponding maximum growth rate is

$$\gamma_{\max} = \frac{C^2 \epsilon' A_0^2/\epsilon(A_0^2)}{2\omega_0(1 + \epsilon' A_0^2)^{1/2}}\left[K_0^2 - \frac{1}{r_0^2 f^2}\right]. \qquad \ldots(18)$$

Using eqns. (2) – (4), we get :

$$\gamma_{\max} = \frac{C^2(1 - S/2)\dfrac{\omega_p^2}{\omega_0^2}\dfrac{\alpha_0 E_{00}^2}{2}\left\{1 + \dfrac{\alpha_0 E_{00}^2}{2}\right\}^{(S/2-2)} \bigg/ \left\{1 - \dfrac{\omega_p^2}{\omega_0^2}\left(1 + \dfrac{\alpha_0 E_{00}^2}{2}\right)^{(S/2-1)}\right\}}{2\omega_0\left\{1 + \dfrac{\omega_p^2}{\omega_0^2}(1 - S/2)\dfrac{\alpha_0 E_{00}^2}{2}\left(1 + \dfrac{\alpha_0 E_{00}^2}{2}\right)^{(S/2-2)}\right\}^{1/2}} \times \left[K_0^2 = \frac{1}{r_0^2 f^2}\right] \qquad \ldots(18\text{A})$$

and for a collisionless plasma :

$$\gamma_{\max} = \frac{C^2 \dfrac{\omega_p^2}{\omega_0^2}\left(\frac{3}{4}\alpha_0 \dfrac{m}{M} E_{00}^2\right)\exp\left(-\frac{3}{4}\alpha_0 \dfrac{m}{M} E_{00}^2\right)\bigg/\left[1 - \dfrac{\omega_p^2}{\omega_0^2}\exp\left(-\frac{3}{4}\dfrac{m}{M}\alpha_0 E_{00}^2\right)\right]}{2\omega_0\left\{1 + \dfrac{\omega_p^2}{\omega_0^2}\left(\frac{3}{4}\dfrac{m}{M}\alpha_0 E_{00}^2\right)\exp\left(-\frac{3}{4}\dfrac{m}{M}\alpha_0 E_{00}^2\right)\right\}} \times \left[K_0^2 - \frac{1}{r_0^2 f^2}\right] \qquad \ldots(18\text{B})$$

(*ii*) *When* $\mathbf{E}_0 \cdot \mathbf{K}_{\perp} = E_0 K_{\perp}$ — Here the wave vector of the perturbation is parallel to the electric vector of the main beam. The growth rate is maximum for the optimum value of K_1 given by

$$K_{1\text{opt}}^2 = \frac{\left\{\dfrac{\omega_0^2}{c^2}\,\epsilon' A_0^2 + \dfrac{2}{r_0^2 f^2} + \dfrac{\epsilon' A_0^2}{\epsilon(A_0^2)\, r_0^2 f^2}\right\}}{(1 + 2\epsilon' A_0^2/\epsilon(A_0^2))} \qquad \ldots(19)$$

and the corresponding maximum growth rate in this case is given by

$$\gamma_{\max} = \frac{c^2 \epsilon' A_0^2/\epsilon(A_0^2)}{2\omega_0(1 + \epsilon' A_0^2)^{1/2}}\left(K_0^2 - \frac{3}{r_0^2 f^2}\right). \qquad \ldots(20)$$

Using eqns. (2)–(4), we obtain :

$$\gamma_{max} = \frac{c^2(1 - S/2)\, \frac{\omega_p^2}{\omega_0^2} \frac{\alpha_0 E_{00}^2}{2} \left\{1 + \frac{\alpha_0 E_{00}^2}{2}\right\}^{(S/2-2)} \Big/ \left\{1 + \frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\alpha_0 E_{00}^2}{2}\right)^{(S/2-1)}\right\}}{2\omega_0 \left\{1 + \frac{\omega_p^2}{\omega_0^2}(1 - S/2)\frac{\alpha_0 E_{00}^2}{2}\left(1 + \frac{\alpha_0 E_{00}^2}{2}\right)^{(S/2-2)}\right\}^{1/2}} \times \left[K_0^2 - \frac{1}{r_0^2 f^2}\right] \quad \text{...(20A)}$$

and for a collisionless plasma

$$\gamma_{max} = \frac{c^2 \frac{\omega_p^2}{\omega_0^2}\left(\frac{3}{4}\frac{m}{M}\alpha_0 E_{00}^2\right)\exp\left(-\frac{3}{4}\frac{m}{M}\alpha_0 E_{00}^2\right) \Big/ \left[1 - \frac{\omega_p^2}{\omega_0^2}\exp\left(-\frac{3}{4}\frac{m}{M}\alpha_0 E_{00}^2\right)\right]}{2\omega_0 \left\{1 + \frac{\omega_p^2}{\omega_0^2}\left(\frac{3}{4}\frac{m}{M}\alpha_0 E_{00}^2\right)\exp\left(-\frac{3}{4}\frac{m}{M}\alpha_0 E_{00}^2\right)\right.} \times \left[K_0^2 - \frac{3}{r_0^2 f^2}\right] \quad \text{...(20B)}$$

Discussion of Results

The perturbation in the intensity distribution of a Gaussian electromagnetic beam renders the medium more inhomogeneous and attracts power from its surroundings. The temporal growth rate of instability also depends on the extent of self-induced inhomogeneity created in the medium. It is found that at low powers of the main beam (before the dielectric constant has attained the saturation), the perturbation grows at a fast rate. While at higher powers of main beam, the plasma is depleted or redistributed from the axial region by non-uniform carrier heating/ponderomotive force in a collisional/collisionless plasma and hence any intensity perturbation in the beam does not effectively alter the nonuniformity of the medium and cannot grow appreciably. Thus the temporal growth rate of perturbation is not monotonically increasing and manifests a maximum for some optimum value of the intensity of the main beam. This behaviour of the temporal growth rate is similar to the uniform plane beam case in collisional plasmas but not in a collisionless plasma where for a uniform plane beam the ponderomotive force is zero and hence the growth rate merely saturates with the increase of the incident beam intensity. The other interesting feature of the temporal growth is that it decreases with the focusing of the beam and hence manifests as oscillatory behaviour with the distance inside the plasma.

To have a numerical appreciation of the results the maximum temporal growth rate has been computed as a function of the incident intensity of the beam and the distance of propagation inside the plasma for the following set of parameters :

$\omega = 2 \times 10^{14}$ Rad/Sec (CO_2 Laser : $\lambda_0 = 10.6$)

$$\frac{\omega_p^2}{\omega_0^2} = 0.4,\ 0.6$$

$N_0 = 5.0 \times 10^{18}\ \text{cm}^{-3}$

$T_0 = 10^7\ {}^\circ K$

$r_0 = 6 \times 10^{-3}$ cm (60μ)

$\alpha_0 E_{00}^2 = (1.0 : -\ 10^4)$

Fig. 1 shows the variation of the maximum temporal growth rate (maximised w.r.t. the size of the perturbation) with the incident intensity of the beam in a weakly ionized plasma (where collisional loss is dominant in energy transfer) for different values of the scattering parameter (S) and ω_p^2/ω_0^2. It is found that the temporal growth rate of perturbation in a weakly ionized plasma shows maxima at a higher value of the beam intensity for a electron-neutral particle collision dominated plasma ($S = 1$) while it shows maxima at a lower value of the beam intensity for a electron-ion collision dominated plasma ($S = -\ 3$) as compared to

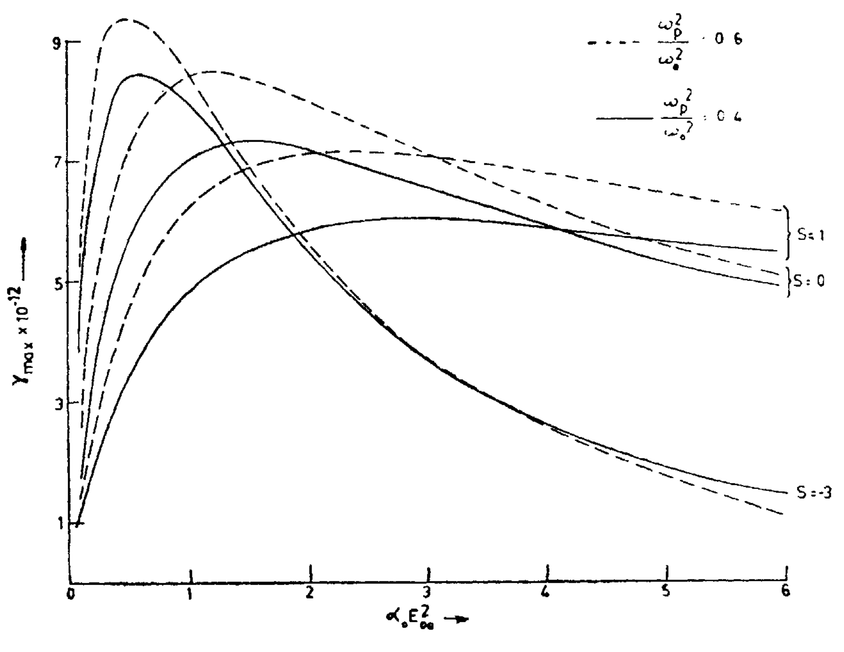

FIG. 1. Variation of the maximum temporal growth rate of perturbation (γ_{max}) with the intensity of the beam ($\alpha_0 E^2_{00}$) in a weakly ionized plasmas for different scattering parameters (viz., $S = 0$, 1 and $-$ 3) and for different $\omega^2_p/\omega_0^2 = \cdot 4,\ \cdot 6$.

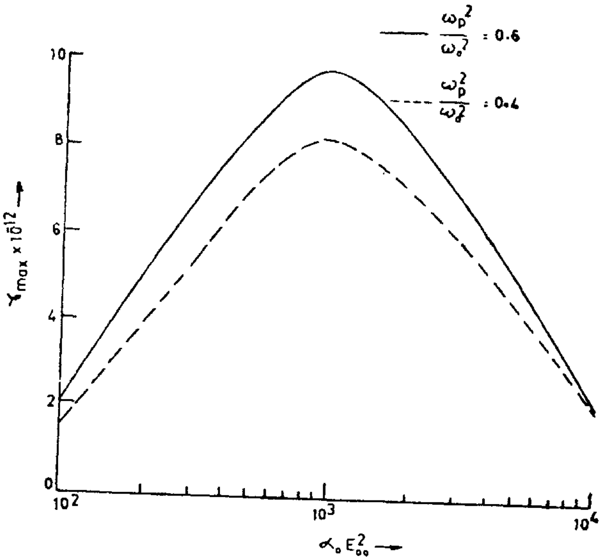

FIG. 2. Variation of the maximum temporal growth rate (γ_{max}) with the intensity of the beam ($\alpha_0 E^2_{00}$) in a collisionless plasma for different ω^2_p/ω^2_0 = ·4, ·6.

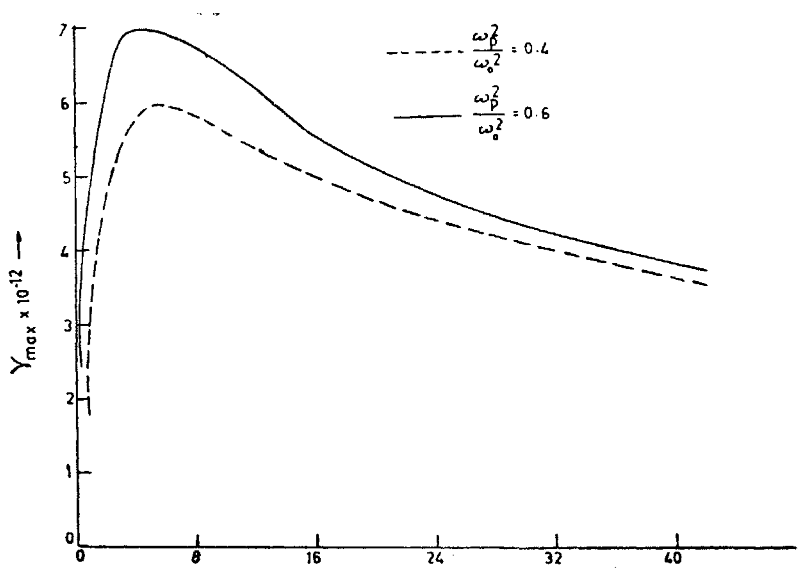

FIG. 3. Variation of the optimum size of perturbation ($a_{opt} = K_1^{-1}$) with the intensity of the beam ($\alpha_0 E^2_{00}$) in a collisional and collisionless plasma for ω^2_p/ω^2_0 = ·4, (i) collisional plasma —— (ii) collisionless plasma. - - - -

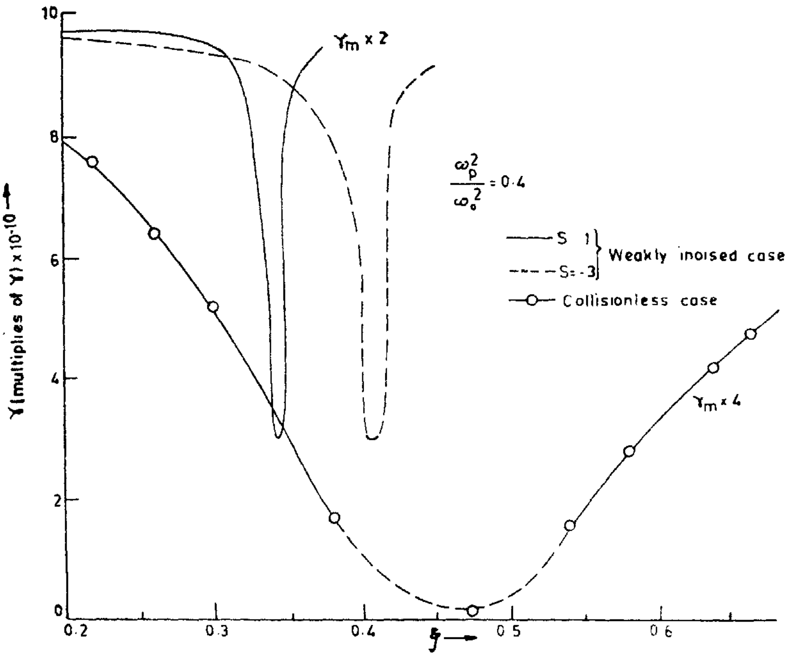

FIG. 4. Variation of the maximum temporal growth rate (γ_{max}) with the distance of propagation ($\xi = z/R_d$) in collisional and collisionless plasmas when $\omega^2_p/\omega^2_0 = \cdot 4$; (i) Collisional plasma (when $\alpha_0 E^2_{00} = 0.01$) weakly ionized case $S = 1$ (abscissa multiplied by ·4) - - - -. strongly ionized case $S = -3$ —— (ii) Collisionless plasma ($\alpha_0 E^2_{00} = 10^4$) (abscissa multiplied by ·4)

the case when $S = 0$. This is due to the fact that in a weakly ionized electron-neutral particle collision dominated plasma the dielectric constant tends to a saturation value at a slower rate while the nonlinear dielectric constant of a electron-ion collision dominated plasma tends rapidly to a saturation value. The growth rate is higher for higher values of the electron density because this corresponds to the high non-linearity in the dielectric constant of the plasma and hence the higher growth rate of intensity perturbation in the plasma.

Fig. 2 displays the variation of the maximum temporal growth rate with the incident intensity of the beam in a collisionless plasma (where the nonlinearity arises through the redistribution of carriers by the ponderomotive force). It is found that the perturbation grows at a faster rate as the beam intensity increases and then decreases at high beam intensities due to the redistribution of carriers from the axial region by the ponderomotive force.

It must be mentioned here that the optimum beam intensities for maximum growth rate γ_{max} are of the order $\sim 10^{11}\,W/\text{cm}^2$ for a collisional plasma and $\sim 10^{13}\,W/\text{cm}^2$ for a collisionless plasma, for the Laser plasma parameters chosen in this paper.

Fig. 3 shows the variation of the optimum size of perturbation ($a_{\mathrm{opt}} = K_{\perp}^{-1}$) with the beam intensity for collisional and collisionless plasmas; a_{opt} increases with the beam intensity beyond a certain value of the beam intensity. This is due to the fact that for a Gaussian plane beam multiple hot spots are produced up to certain beam intensity so that $a_{\mathrm{opt}} < r_0$. When the beam intensity is more than this critical intensity, a_{opt} becomes greater than r_0. Fluctuations of a high large dimension cannot grow and consequently the optimum size of perturbation increases with the beam intensity. This results in disappearance of the multiple hot spots and only a single hot spot is formed due to the main beam.

Fig. 4 displays the variation of the maximum temporal growth rate of perturbation with the distance of propagation inside the plasma in each case (weakly ionized and strongly ionized collisional plasmas and collisionless plasmas). The temporal growth rate decreases with the focusing of the beam and then increases with the distance due to diffraction effects. Thus the growth rate is found to be oscillatory in nature with the distance of propagation due to the self-focusing effects.

Acknowledgements

The authors are thankful to Prof. A. K. Ghatak and Dr. D. P. Tewari for various stimulating discussions.

References

Bingham, R., and Lashmore-Davies, C. N. (1976). Self-Modulation and filamentation of electromagnetic waves in plasmas. *Nuclear Fusion*, **16**, 67–72.

Brueckner, K. A., and Jorna, S. (1974). Laser driven fusion. *Rev. mod. Phys.*, **46**, 325–367.

Drake, J. F., Kaw, P. K., Lee, Y. C., Schmidt, G., Liu, C. S., and Rosenbluth, M. N. (1974). Parametric instabilities of electromagnetic waves in plasmas. *Phys. Fluids*, **17**, 778–785.

Eidmann, K., Kessel, C. V., Key, M. H., Mulser, P., and Sigel, R. (1975). Laser plasma interaction and hydrodynamic behaviour of laser-irradiated plane targets. *Plasma Phys. Controlled Nuclear Fusion Res* **2**, 357–364.

Kaw, P. K., Schmidt, G., and Wilcox, T. (1973). Filamentation and trapping of electromagnetic radiation in plasmas. *Phys. Fluids*, **16**, 1522–1525.

Langdon, A. B., and Lasinski, B. F. (1975). Filamentation and subsequent decay of laser light in plasmas. *Phys. Rev. Lett.* **34**, 934–937.

Lee, Y. C., Liu, C. S., Chen, H., and Nishikawa, K. (1974). Self-focusing and filamentation of laser light in plasmas. *Plasma Phys. Controlled Nuclear Fusion Res.*, **3**, 207–215.

Liu, C. S., and Kaw, P. K. (1976). Parametric instabilities in homogeneous unmagnetized plasmas, *In*: *Advances in Plasma Physics* by Simon, A. and Thomson, W. B. (1976). John Wiley & Sons, New York, 83–121.

Manheimer, W. M., and Ott, E. (1974). Parametric instabilities induced by the coupling of high and low frequency plasma modes. *Phys. Fluids*, **17**, 1413–1421.

Nishikawa, K. (1968). Parametric excitation of coupled waves I-general formulation p. 916–922, II-Parametric plasmon-photon interaction p. 1152–1158. *J. phys. Soc. Japan*, **24**.

Perkins, F. W., and Valeo, E. (1974). Thermal self-focusing of electromagnetic waves in plasmas. *Phys. Rev. Lett.*, **32**, 1234–1237.

Schwarz, H., and Hora, H. (1974). *Laser Interactions and Related Plasma Phenomena.* Plenum Press, New York, **3**...

Sodha, M. S., Kumar, A., Tripathi, V. K., and Kaw, P. K. (1973). Hot carrier diffusion induced instabilities of electromagnetic waves in plasmas and semiconductors. *Opto Electronics*, **5**, 509–516.

Sodha, M. S., Sharma, R. P., and Tripathi, V. K. (1974). Instability of intensity distribution of a laser beam in a plasma. *J. phys. D. appl. Phys.*, **7**, 2471–2478.

Sodha, M. S., Ghatak, A. K., and Tripathi, V. K. (1975). *Self-focusing of Laser Beams in Dielectrics, Plasmas and Semiconductors*. Tata McGraw Hill Publishing Co. Ltd., New Delhi.

———— (1976). Self-focusing of laser beams in plasmas and semiconductors. *Progr. Opt.*, **13**, 171–265.

Valeo, E. (1974). Stability of filamentary structures. *Phys. Fluids*, **17**, 1391–1393.