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HARMONIC OSCILLATOR IN QUARK PHYSICS : SOME ASPECTS A REVIEW

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"বহুরূপে সম্মুখে তোমার"

Bahurupe Shanmukhe Tomar (*Meaning* : God has several forms before you.)

—SWAMI VIVEKANANDA

(§ 1) INTRODUCTION AND SCOPE

QUARK physics is less than two decades old (Gell-Mann, 1964; and Zweig, 1964). Yet the growth of this subject within such a short span of time has been so phenomenal that even to keep track of the main developments is no longer an easy task, despite the existence of some excellent reviews on the subject. Perhaps the biggest single success of the model is a remarkably simple description of hadronic excited states ($\sim$ 300 pieces of data) in terms of the group SU (6) $\times$ 0 (3) (Mahanthappa & Sudarshan, 1965) and its modest relativistic counterpart in broken SU $(6)_w \times 0$ (3) (Lipkin & Meshkov, 1965), as a straight-forward extension of Wigner's supermultiplet classification (Wigner, 1937) of nuclear levels in terms of SU (4) to take care of strange particles through the enlargement SU (2) $\rightarrow$ SU (3). This part of the development came within less than two years of the original quark proposal in 1964 (*loc. cit.*) after several alternative possibilities (Dyson, 1966) represented by more complicated group structures got quickly eliminated against the experimental background through vigorous worldwide activities. If a full-fledged dynamical model of quarks did not emerge immediately afterwards, it was perhaps only because the "fashion of the day" made it a taboo to speak too vehemently about "constituent" quarks in preference to the more elusive "current" quark (Gell-Mann, 1965; Dashen & Gell-Mann, 1966; and Gell-Mann *et al.*, 1968). Nevertheless, dynamical models were proposed on the lines of two- and three-nucleon problems and one of the earliest models to receive extensive theoretical treatment and experimental checks was that of the harmonic oscillator (h.o.) (Dalitz, 1966; and Greenberg, 1974).

Indeed, the harmonic oscillator (h.o.) which perhaps represents the simplest and the most fundamental system in the whole of physics, has all along played a crucial role in the entire development of quark physics almost since its birth. No doubt, it has gone through several vicissitudes*, often changing garbs in the form of different

*Faiman & Hendry, 1968, 1969*a b*; Greenberg & Resnikoff, 1967; Fujimura *et al.*, 1970; Licht & Pagnamenta, 1970; Feynman *et al*, 1971; Horgan, 1974; Horgan & Dalitz, 1974; Jones *et al.*, 1974, 1977; Mitra, 1974, 1975, 1978; Mitra & Sen, 1974; Mitra & Sood, 1977; Gupta & Mitra, 1978; Kamath & Mitra, 1978; Le Yaouanc *et al.*, 1973, 1974, 1975*a, b*; Indira & Mitra, 1977*a, b*; and Isgur & Carl, 1978, 1979)

kinds of relativistic adaptations*, yet by virtue of its simplicity and flexibility, it has proved one of the most practical and versatile tools for plausible explanations of a very rich spectrum of observed phenomena with the fewest of assumptions. These include diverse results ranging from simple classifications and mass predictions of hadronic excited states to the non-trivial predictions of processes involving matrix elements of various currents (Mitra, 1977; and Mitra & Sood, 1977).

Early Theoretical Support for H. O.

A theoretical rationale for the h.o., apart from its obvious simplicity and ease of application, arose from its connection with the duality principle, a concept which gained currency in the late sixties (Dalitz, 1970) since the discovery of the so-called Veneziano amplitudes (Veneziano, 1968) and their subsequent construction in terms of h.o. operators (Fubini *et al.*, 1969). Though this part of the development formally tied up with quark or duality diagrams (cf. Harari, 1969; Rosner, 1969; Micu, 1969; and Carlitz & Kislinger, 1970) it nevertheless had limited ramifications on the practical, applicational front, which characterised the development of the early seventies. However, the early seventies were also the time when the twin concepts of asymptotic freedom (Politzer, 1973) and infrared slavery (Politzer, 1974; and Appelquist & Politzer, 1975) were emerging as guiding principles for quark dynamics. Asymptotic freedom (AF) means that at very short interquark distances (i.e., for large Q^2) the q-$\bar{q}$ and q-q interactions become so small as to allow them almost free movement. On the other hand, at large interquark distances (i.e., small Q^2), just the opposite happens, namely, the bonding between q-$\bar{q}$ as well as q-$\bar{q}$ pairs increases with their separation, thus justifying the terminology of "infrared slavery." The latter effect is now more commonly known as *quark confinement*, or colour confinement, as a formal theoretical response to the continued lack of observation of free quarks. Since an h.o. interaction (which increases indefinitely with separation) provides rather naturally for the confinement mechanism, revival of physical interest in the h.o. model in the seventies owes directly to the latter development. This is not to deny the existence in the literature of other competing mechanisms for confinement, especially the *linear* potential (Gunion & Willey, 1975; Harrington *et al.*, 1975, and Eichten *et al.*, 1975) which has also gained a considerable amount of phenomenological credibility in recent years in the context of *heavy* quarkonia (i.e., $c\bar{c}$, $b\bar{b}$ systems (Eichten *et al.*, 1978; Quigg & Rosner, 1978, 1979; Richardson, 1979; Ditsas *et al.*, 1978; and Henriques *et al.*, 1976). However, at the present state of the art, such confining potentials, whether harmonic or linear, must be regarded as different forms of phenomenological parametrizations of a relatively "unknown" mechanism. For, even the most respectable theory of strong interactions at the moment, viz., quantum chromodynamic (QCD) (Marciano & Pagels, 1978) is as yet unable to provide a quantitative theoretical basis for the long range part of the q-$\bar{q}$ or q-q interaction, while perturbative QCD (Buras–*to be published*) seems to account for many *short distance* phenomena (in the AF region) with considerable success.

*Lipes, 1972; Kim & Noz, 1973, 1975; Bohm *et al.*, 1973; Leutwyler & Stern, 1978; Mitra, 1980—*In press*; and Mitra & Santhanam—*to be published.*

Therefore, until such time as the formal credibility gap of QCD in accounting for the long range part of the q-q interaction is bridged, physical interest in hadron spectroscopy and other "low" and moderate energy hadronic phenomena will have to be sustained through an *effective* parametric description of this part of the interaction. The precise choice of the confining interaction under such circumstances is, therefore, largely a matter of practical expediency or applicational convenience, subject, of course, to basic compatibility with the data, and it is in such a spirit that the h.o. interaction is viewed in this article.

The harmonic interaction and Gaussian wave functions received an independent boost in the seventies in the context of the socalled quark-pair-creation model (QPC) (Le Yaouanc *et al.*, 1973), a big unifying principle for an essentially parameter-free formulation of current matrix elements or hadronic vertex functions involving arbitrarily excited hadronic states (Mitra, 1977). The QPC model (*loc. cit.*) was proposed as a quantitative translation of quark or duality diagrams (cf. Harari, 1969; Rosner, 1969; Micu, 1969; and Carlitz & Kislinger, 1970) for hadronic vertices in a spirit analogous to the writing down of matrix elements from Feynman diagrams in field theory. However, unlike the latter, the QPC matrix elements from quark diagrams which involved the (quark-level) wave functions of all the participating hardrons, necessarily had to be based on certain phenomenological considerations in the absence of a deeper theory. Nevertheless, the rules were simple and unambiguous and found most applications with h.o. wave functions (Harari, 1979; Barbieri *et al.*, 1976; Mitra, 1976*a*, *b*, *c*; Chakrabarty *et al.*, 1977; and Mitra *et al.*, 1977) for the hadrons concerned, in the empirical knowledge of a natural compatibility of the h.o. model with quark diagrams. The formulation of QPC with h.o. wave functions and the history of its applications to various hadronic decays will be one of the major topics of discussions in this article (*see* Sec. on QPC Model).

Developments of the H. O. Model : Hadron Spectra

The foregoing are meant more to illustrate than to exhaust the evidences which have provided important manifestations of the h.o. in the various stages of development of quark physics. However, the success story of h.o. albeit phenomenological, has not been without its problems, the most important ones being concerned with the understanding of the hadron spectra themselves under two broad heads :

(*i*) The inadequacies of the non-relativistic h.o. model (Greenberg, 1974; Faiman & Hendry, 1968, 1969*a*, *b*; La Yaouanc *et al.*, 1973; and Mitra, 1977) which led to various levels of relativistic formulations (Feynman *et al.*, 1971; Lipes, 1972; Kim & Noz, 1973, 1975; Bohm *et al.*, 1973; Lieutwyler & Stern, 1978; and Licht & Pagnamenta, 1970) in the seventies.

(*ii*) The high degree of h.o. degeneracy which was basically incompatible with the observed features of hadron spectroscopy and stimulated efforts to formulate different kinds of symmetry-breaking effects (cf. Greenberg & Resnikoff, 1977; Horgan, 1974; Horgan & Dalitz, 1973, 1974; Jones *et al.*, 1977; and Le Yaouanc *et al.*, 1975 *a*, 1977, 1978).

These two developments were somewhat interconnected, as evidenced by the history of attempts to tackle them. Thus while the earliest NR formulation was

already adequate for mere classification of hadronic excitations, it was soon realised that the Schrödinger equation needed to be reformulated at least to the extent of facilitating an interpretation of the "energy" as M^2, *before* a comparison with the data could be seriously attempted (Dalitz, 1966). A more substantial relativistic formulation in a four-dimensional form was made by Feynman *et al.* (1971) through a somewhat intuitive but deeply physical approach, for a comprehensive determination of both M^2-spectra as well as current matrix elements for e.m. and pseudoscalar couplings. This treatment, however, suffered not only from the M^2 degeneracy problem (as it did not have answers for the spin-and flavour-dependent effects) but also from a more serious theoretical problem of time-like excitations which rendered the wave functions unnormalizable necessitating certain *ad hoc* assumptions. Several contemporary and subsequent formulations at the four-dimensional level, but otherwise on lines analogous to FKR, were designed to tackle the problem of time-like excitations with varying degrees of success (*loc. cit.*). However, except for one approach (Lieutwyler & Stern, 1978) which had a spin-spin term in its M^2 formula, most of these h.o. oriented attempts were silent on the symmetry-breaking effects. Only recently, several symmetry-breaking effects (spin-spin, spin-orbit, quark mass and momentum dependent corrections) have been shown to appear (Mitra, 1980—*in press*; Mitra & Santhanam—*to be published*) through a Bethe-Salpeter treatment (Salpeter & Bethe, 1951; and Gell-Mann & Low, 1951) of a basically h.o. interaction between q-q and q-$\bar{q}$ pairs, albeit within the instantaneous approximation (Levy, 1952; Salpeter, 1952; and Klein & Zeemach, 1957).

Other h.o. oriented approaches to symmetry-breaking in the hadronic spectra range from the *ad hoc* introduction (Greenberg & Resnikoff, 1967; Horgan, 1974; Horgan & Dalitz, 1974; and Jones *et al.*, 1974, 1977) of various parametric terms in the Hamiltonian to the more precise formulation of short-range effects of the v^2/c^2-corrections to the one-gluon exchange potential in QCD* on the lines of the original Bethe-Salpeter (1967) article. These effects have generally been estimated through diagonalizations of the appropriate hamiltonians in terms of h.o. basis functions (Isgur & Carl, 1978, 1979) within an N. R. framework. An alternative point of view was advocated by (Mitra, 1974)† wherein the bulk of symmetry-breaking in the spectra was assumed to arise, not from the breaking of SU (6) $\times$ 0 (3), but from that of the h.o. symmetry itself. H.O. symmetry-breaking, in turn, was introduced through an exchange-type h.o. interaction, in association with direct h.o. interaction, on a 50-50 basis, this last ratio being designed to keep out the (as yet unobserved) $\underline{20}$ states. This "even-wave h.o. theory" was applied to both M^2-spectra (*loc. cit.*) as well as hadronic decays and reactions (Mitra & Sood, 1978; Kamath & Mitra, 1978; and Gupta & Mitra, 1978) with a fair amount of phenomenological success. On the othar hand, the more recent Bethe-Salpeter treatment (Mitra, 1980—*in press*, and Mitra & Santhanam—*to be published*) of the h.o. interaction has been found to exhibit several symmetry-breaking features in common with those of the even-wave theory. This

*(De Rujula *et al.*, 1980; Gromes, 1976, 1977; Barbieri *et al.*, 1976*b*; Billoire & Morel, 1978; Reinders, 1978; and Eilwanger, 1978).

†Mitra and Sen (1974), Mitra and Sood (1977*a*) and Kamath and Mitra (1978).

illustrates the possibility in principle of a deeper "understanding" of certain phenomenological assumptions made within a restricted (e.g. Schrödinger like) theoretical framework, merely through an enlargement of the framework to a more general (e.g., that of Bethe-Salpeter) form of dynamics which may already incorporate the desired features without invoking separate assumptions.

Taking account of the diverse facets of the h.o. approach to hadron physics with quarks, it may not be unrealistic to claim that the harmonic oscillator has probably received more attention in the entire development of quark physics, and has organized more data under a single head in this field—albeit at the phenomenological level—than perhaps any comparable model to-date. However, the subject of hadron physics, with quarks, which was at one time in the forefront of high energy physics, is itself showing signs of saturation in the absence of totally new and revolutionary ideas, except for the newer regions of spectroscopy which have been periodically opening up as a result of discoveries of newer flavours since the mid-seventies (Aubert *et al.*, 1974; Augustin *et al.*, 1964; and Herb *et al.*, 1977). Particle physics is now at the cross-roads, with a new high energy physics (on the 100 GeV scale) emerging, where the thrust areas are represented by quarks, gluons, jets and grand unification (cf. *Proc. XX HEP. conf.*, 1980). At this stage, it is perhaps in order to take stock of the main sequence of developments of a model whose role in the progress of hadron physics has been at least as crucial as that of the shell model in the understanding of nuclear structure. Such an exercise may be useful not only for the records but hopefully also for possible insights into unexplored areas.

Scope of the Review

The scope of this review is limited only to those aspects of hadron physics (mostly spectroscopy) which have had a rather direct bearing on different facets of the h.o. model. (Non-harmonic approaches, such as the bag model*, lattice guage theories (Wilson, 1974, 1976; and Kogut & Susskind, 1975), or linear confinement models in association with one gluon-exchange potentials (Hendry & Lichtenberg, 1978) are not intended for any quantitative discussion within the scope of this article.) For this purpose, a prior knowledge of the reader on many routine aspects of quark physics will be assumed, especially the early phases of the developments** which are documented in earlier reviews (Dalitz, 1965) as well as in certain books and monographs+. These include the SU (6) × 0 (3) basis of classification of $q\bar{q}$ and qqq states (Dalitz, 1965; Mitra & Ross, 1967; Mitra, 1968; and Mitra & Katyal, 1968) and the quark additivity model (Becchi & Morpurgo, 1966; and Mitra & Ross, 1967) for hadron couplings (Mitra, 1973—*unpublished*; and Rosner, 1974) and form factors. Since the last two items (couplings and form factors) have

*Chados *et al.*, 1974*a*, *b*; De Grand *et al.*, 1975; Jaffe & Kiskis, 1976; Jaffe, 1977, 1978; Hasenfrantz & Kuti, 1978; Hasenfrantz *et al.*, 1980; and Bardeen *et al.*, 1975).

**Gell-Mann, 1964; Greenberg, 1974; Morpurgo, 1965; Dalitz, 1965; Becchi & Morpurgo, 1966; Mitra & Mazumdar, 1966; Mitra & Ross, 1967; Mitra, 1968; Mitra & Katyal, 1968; Karl & Obryk, 1968, and Gell-Man & Nee'man, 1964.

+Gel-Mann & Nee'mann, 1964; Kokkeddee, 1969; Aly, 1970; Chand, 1970; and Mitra, 1970-*unpublished.*

been covered rather extensively in fairly recent reviews (Mitra, 1977; and Mitra & Sood, 1977) their basic formulations as *such* will not form the subject matter of this review, but the results thereof will be referred to and freely made use of wherever necessary, to the extent that these are of relevance for the central theme of this article which is the harmonic oscillator model.

A good starting point would seem to be a quantitative discussion of the harmonic oscillator model of Feynman, Kislinger and Ravndal (FKR for short) (Feynman *et al.*, 1971) as a prototype of the nature of physical and mathematical inputs that must go into an essentially intuitive formulation before it can be meaningfully expected to confront the data. To this end, the next Section is devoted to an exposition of the FKR model for $q\bar{q}$ and qqq systems, together with the mechanism of their couplings to the basic 'fields', problems of normalization of the wave functions in the presence of time-like excitations, and certain early recipes for a solution to the problem of e.m. form factors. The thind Section is intended for a brief but comparative discussion of certain developments in the seventies connected with the problem of symmetry-breaking effects on the spectra mostly in an h.o. basis. However, it also includes a short formulation of the so-called even-wave h.o. model on the lines of the FKR treatment, together with a qualitative discussion of the main results thereof. The Section on APC model deals with a particular class of applications in which the harmonic oscillator has played an especially important role requiring an essential use of the few-quark wave functions in Gaussian form, viz., the quark-pair-creation (QPC) model for hadron couplings. Though this topic had been dealt with briefly in an earlier review, (Mitra & Sood, 1977), it is not only within the scope of the present h.o. oriented article, but needs to be updated in the context of several subsequent developments (especially of baryonium states). The topics dealt with in the above-mentioned two sections are thus necessarily of a diffuse applicational nature, with rather limited support from formal theoretical premises. These are meant more to illustrate than to justify the diverse possibilities to which the h.o., by virtue of its extreme simplicity, has been harnessed.

After these two diversions to spectroscopic and hadron coupling applications of the oscillator model, we return towards efforts at finding better theoretical rationales for the h.o. within a relativistic framework in two sections viz., Theoretical Rationales of the Relativistic Oscillator and Bethe-Salpeter Approach to H. O. The former, in particular, is confined to certain formulations essentially at the quantum mechanical level (short of full-fledged field theoretical machinery) designed to find a *raison d'etre* for the relativistic oscillator, while the latter is concerned with the more orthodox Bethe-Salpeter-like approaches towards the same end at the field-theoretical level; concrete applications are, however, not discussed in these two sections. However, in a later section dealing with B. S. Equations, Harmonic Confinement and Hadron spectra, a quantitative formulation of the B. S. theory in the more practical Instananeous Approximation (Mitra, 1980—*in press*; Mitra & Santhanam—*to be published*) is sketched for a general class of few-quark systems, together with concrete solutions in closed form for $q\bar{q}$ and qqq systems under harmonic confinement. This section also gives a fairly detailed comparison of the results thus obtained with the data. In view of the considerable success

evidenced in the comparison with data of the B. S. formalism for $q\bar{q}$ and qqq system under harmonic confinement, thus specifying the spring constants of the $q\bar{q}$ and qq interactions, a section covering baryonic states and the h.o., is devoted to a corresponding formulation and solution of the B. S. equation for a $qq\bar{q}\bar{q}$ system under h.o. confinement. The results so obtained are used to predict the nature of the baryonic spectra, including location of their ground and excited states, in relation to contemporary approaches. Finally, the article is concluded with a few remarks on the h.o. approach and its limitations.

(§2) FKR Model for the Relativistic Oscillator

The relativistic harmonic oscillator (h.o.) model of Feynman *et al.* (1971), or termed FKR for short, was the first comprehensive attempt to formulate a non-trivial relativistic quark dynamics of hadrons and its applications to M^2-spectra as well as current matrix elements under a single unified framework. This paper effectively incorporated all the *relevant* aspects of quark dynamics that had been generated piece-meal in the sixties through phenomenological means, and had by and large come to be accepted. Apart from the basic group structure SU (6) $\times$ 0 (3), these included among other things (*i*) the "symmetric" quark model based on parastatistics (Greenberg, 1974; and Dalitz, 1966) and supported observationally by the *nodeless* behaviour of nucleon form factors (Mitra & Mazumdar, 1966), (*ii*) h.o. classification of hadronic states based on their plots of M^2 *vs.* N (excitation quantum number) (cf. Dalitz, 1966) and (*iii*) the mechanism of *single quark* transitions (Becchi & Morpurgo, 1966) with *quark recoil* effects generated by Galilean invariance (Mitra & Ross, 1967). Most quark model approaches of the seventies have generally recognized the FKR model as a sort of "standard" for the construction of models for hadron spectroscopy through quarks. We summarise below the main features of the model in respect of both $q\bar{q}$ and qqq systems and their currents.

qqq-Dynamics

The central point of departure of FKR from the ordinary N. R. h.o. model (Dalitz, 1966; and Kokkeddee, 1969) is the recognition that the excitation of energies of states are *not* small in comparison to the quark masses (m_q) so as to justify the use of N. R. principles. As we understand things today, this situation certainly describes the light (*uds*) quarks which are all that existed when this paper appeared. (It does not hold for the heavy (c, b) quarks discovered subsequently, and that probably explains the success of subsequent N. R. approaches to the corresponding hadrons). However, the problem of quark masses themselves was sought to be bypassed in the FKR approach through the following device. The Hamiltonian for an N. R. oscillator, viz.,

$$H_{\text{NR}} = \tfrac{1}{2}\, p^2/m + \tfrac{1}{2} m\omega_0^2\, x^2 \qquad \ldots(1)$$

may be transformed by multiplication with $2m$ as

$$2m\, H_{\text{NR}} = p^2 + \Omega^2 x^2 \qquad \ldots(2)$$

where $\Omega = m\omega_0$ is identified as the new spring constant so that the quark mass disappears from the R.H.S. Addition of a constant m^2 on both sides of (2) now makes the L.H.S. approximately equal to the squared relativistic mass operator M^2, where $M \approx m + H_{\rm NR}$, since in the N.R. limit $H_{\rm NR}{}^2$ is small. In this way, one expects the M^2 for a given hadronic state to rise *linearly* with the Ω -parameter without directly depending on m_q, except as a possible additive constant ($\sim m_q^2$).

Using such considerations, FKR were led to the following "relativistic" Hamiltonian for a qqq system with 4-coordinates $u_{a,b,c}$ and 4-momenta $p_{a,b,c}$:

$$-K_B = 3\left(p_a^2 + p_b^2 + p_c^2\right) + \text{Const}$$

$$+ \frac{1}{36}\Omega^2\left[(u_a - u_b)^2 + (u_b - u_c)^2 + (u_c - u_a)^2\right] \qquad ...(3)$$

where the numbers 3 and 36 are certain arbitrarily chosen scale factors and

$$p_a^2 = p_{a\mu}p_{a\mu} = p_{ax}^2 + p_{ay}^2 + p_{az}^2 - p_{at}^2 \qquad ...(4)$$

$$u_{a\mu} = +\, i\,\partial/\partial p_{a\mu}. \qquad ...(5)$$

To separate the internal motion from the c.m. motion described by $P = p_a + p_b + p_c$, FKR introduced the relative 4-momenta (ξ, η) as

$$\left.\begin{aligned} p_a &= \tfrac{1}{3}P - \tfrac{1}{3}\xi \\ p_{b,c} &= \tfrac{1}{3}P + \tfrac{1}{6}\xi \mp \tfrac{1}{4}\sqrt{2}\,\eta \end{aligned}\right\} \qquad ...(6)$$

and the corresponding 4 coordinates R, x,y as

$$\left.\begin{aligned} u_a &= R - 2x, \qquad (x = +\, i\partial_\xi) \\ u_{b,c} &= R + x \mp \sqrt{3}y, \qquad (y = +\, i\partial_\eta) \end{aligned}\right\} \qquad ...(7)$$

so as to obtain

$$-K_B = P^2 + M_B^2 \qquad ...(8)$$

where the squared mass operator, describing the internal motion is given by the double h.o. form

$$M_B^2 = \tfrac{1}{2}\xi^2 + \tfrac{1}{2}\eta^2 + \tfrac{1}{2}\Omega^2(x^2 + y^2) + C \qquad ...(9)$$

The interpretation of (8) is very simple : K_B^{-1} may be regarded as the *free propagator* for the composite baryon (qqq) state, whose poles correspond to the eigenvalues $-P^2$ of M_B^2. In the presence of a perturbation $-\delta K_B$, the propagator changes into $(K_B - \delta K_B)^{-1}$, which can be developed as

$$K_B^{-1} + K_B^{-1}\delta K_B K_B^{-1} + K_B^{-1}\delta K_B K_B^{-1}\delta K_B K_B^{-1} + \qquad ...(10)$$

The free propagator K_B^{-1}, in turn, may be expressed in terms of the eigenvalues M_i^2 of the mass operator (9) according to the standard formula

$$\frac{1}{P^2 + M_B^2} = \sum_i h_i(\xi, \eta) \frac{1}{P^2 + M_i^2} \bar{h}_i(\xi, \eta) \qquad ...(11)$$

where h_i is the eigenfunction corresponding to M_i^2 and $\bar{h}_i$ a suitably modified hermitian conjugate (adjoint) to preserve the necessary Lorentz structures in Dirac theory (cf. $\bar{\psi}$ versus ψ^+). Substitution of (11) in (10) tells us that the probability amplitude thot a disturbance δK_B makes a change in first order from the propagation of the state i with mass M_i to propagation in the state j with mass M_j is just the matrix element

$$N_{ji} = < \bar{h}_j(\xi, \eta) \mid \delta K_B \mid h_i(\xi, \eta) > \qquad ...(12)$$

Now the perturbations like δK_B, basically come about through e.m. and pseudoscalar interactions which usually cause transitions involving excited hadronic states. To introduce the e.m. interaction, FKR used the trick of first reinterpreting each p_a^2 in (3) as $\gamma \cdot p_a \gamma \cdot p_a$ and then using the gauge principle, viz.,

$$\gamma_\mu p_{a_\mu} \to \gamma_\mu p_{a_\mu} - e_a \gamma_\mu A_\mu \qquad ...(13)$$

where e_a is the corresponding quark charge. This yields

$$\delta K_B = 3 \sum_\alpha e_\alpha \left(\gamma \cdot p_\alpha \gamma \cdot A(u_\alpha) + \gamma \cdot A(u_\alpha)\, \gamma \cdot p_\alpha\right) - 3 \sum_\alpha e_\alpha^2 A_\mu(u_\alpha) A_\mu(u_\alpha) \qquad ...(14)$$

From eqn. (14), one infers the first order e.m. current caused by a photon of polarization ϵ_μ in the form ($\delta K_B = j_\mu \epsilon_\mu$) :

$$j_\mu^{e.m} = 3 \sum_\alpha e_\alpha \left(\gamma \cdot p_\alpha \gamma_\mu e^{iq.u_\alpha} + \gamma_\mu \gamma \cdot p_\alpha e^{iq.u_\alpha}\right) \qquad ...(15)$$

In a similar way, the modification in p_μ due to an *axial* vector field B_μ is $p_\mu - \gamma_5 B_\mu e'_\alpha$, so that the corresponding Klein-Gordon operator in "Dirac form" becomes $(\gamma \cdot p_\alpha - e'_\alpha \gamma_5 \gamma \cdot B)^2$ for each $p_\alpha^2 = \gamma \cdot p_\alpha^2$, e'_α being now the "axial charge" operator in SU (3) space. The corresponding axial current j_μ^A ($\delta K_B = j_\mu^A \epsilon_\mu^A$), or rather its divergence $\partial \mu j_\mu^A$ is given by

$$-iq_\mu j^A_\mu = 3\sum_\alpha e'_\alpha (\gamma\cdot p_\alpha \gamma_5 \gamma\cdot q\, e^{iq.u_\alpha} + \gamma_5\gamma\cdot q\, e^{iq.u_\alpha}\gamma\cdot p_\alpha) \qquad ...(16)$$

In this expression, the axial charges e'_α may be interpreted as the strengths of the *pseudoscalar* couplings of the concerned quarks in a 3 × 3 Gell-Mann matrix form (Gell-Mann & Nee'man, 1964) (to take care of non-diagonal effects).

For evaluation of the matrix elements (12) one needs a complete knowledge of the (spinor) eigenfunctions $h_i(\xi, \eta)$ of (9). The difficulties of FKR theory associated with their construction are two-fold : (*i*) problem of time-like excitations; and (*ii*) problem of too many states. The former problem (discussed in the sub-section below) was not solved in the FKR model. The latter owes its origin to FKR's second-order Klein-Gordon form for the equation of motion, viz., $-P^2 h_i = M_i^2 h_i$ instead of the first order (Dirac) form $i\gamma\cdot p h_i = -m_i h_i$ which would, e.g., distinguish a quark from an antiquark (unlike the Klein-Gordon form). In the FKR theory, such constraints are sought to be restored through the *ad hoc* postulation of the following supplementary conditions on the eigen-spinors h_i :

$$P_\mu \gamma_{\alpha\mu} h_i = iM_i h_i \qquad (\alpha = 1, 2, 3) \qquad ...(17)$$

Eqn. (17) implies that the spinors for each quark (α) are *pure quark states* in the rest system ($\vec{P} = 0$) of the excited hadron of mass M_i. As for the normalization of the spinors, the form (15) of the current j_μ necessitates the normalization $\bar{u}u = 1$ if the more usual form $i\bar{u}\gamma_\mu u$ of the current corresponds to $\bar{u}u = 2M$.

$q\bar{q}$-Dynamics

As for $q\bar{q}$ states corresponding to 4-momenta p_a and p_b, FKR proposed the following relativistic hamiltonian analogous to (3) :

$$-K_M = 2\left(p_a^2 + p_b^2\right) + \frac{1}{16}\Omega^2 (u_a - u_b)^2 + C \qquad ...(18)$$

where the spring constant Ω and the scale factors (16) are *empirically* adjusted to correspond to the same M^2 *vs.*N slope ($\sim$1 GeV^2) for both qqq and $q\bar{q}$ states as an *experimental* fact. The c.m. 4-momentum P_μ and relative 4-momentum ζ_μ are defined as

$$p_{a\mu}, p_{i\mu} = \tfrac{1}{2}P_\mu \mp \tfrac{1}{4}\sqrt{2}\,\tau_\mu \qquad ...(19)$$

and the conjugate coordinate z_μ as

$$z_\mu = +i\partial/\partial\zeta_\mu = \tfrac{1}{4}\sqrt{2}\,(u_{b\mu} - u_{a\mu}) \qquad ...(20)$$

to give

$$-K_M = P_\mu^2 + M_M^2 \qquad ...(21)$$

where

$$M_M^2 = \tfrac{1}{2}\zeta^2 + \tfrac{1}{2}\Omega^2 z^2 \qquad ...(22)$$

Similarly the e.m. and pseudoscalar currents are

$$j_\mu^{e.m.} = 2\sum_\alpha e_\alpha(\gamma\cdot p_\alpha\ \gamma_\mu + \gamma_\mu\ \gamma\cdot p_\alpha) \qquad ...(23)$$

$$-iq_\mu j_\mu^A = 2\sum_\alpha e'_\alpha\ (\gamma\cdot p_\alpha\ \gamma_5\gamma\cdot q + \gamma_5\gamma\cdot q\ \gamma\cdot p_\alpha) \qquad ...(24)$$

where the antiquark has been defined as a quark with the *opposite* charge, so as to correspond to the adjoint representation $\bar{\lambda}$ for the corresponding Gell-Mann matrices. Finally, the supplementary condition for the restriction of spinor states analogous to (17) is

$$P_\mu\gamma_\mu^{(\alpha)}\ h_i = iM_i h_i \qquad (\alpha = a, b) \qquad ...(24a)$$

where $h_i(\zeta)$ is the (spinor) eigenfunction for the $q\bar{q}$ state concerned.

Wave Function Normalization Problems

The difficulties of normalization of the 4-dimensional h.o. wave functions are best exhibited by the structure of the ground state function which reads as

$$\begin{aligned} h_0(\xi, \eta) &= \exp\,[-(\xi^2 + \eta^2)/2\Omega] \\ &= \exp\,\Omega^{-1}[-(p_a - p_b)^2 - (p_b - p_c)^2 - (p_c - p_a)^2] \end{aligned} \qquad ...(25)$$

and shows that while the space parts of $\xi^2 = \vec{\xi}^2 - \xi_t^2$ have the usual Gaussian forms characteristic of the non-relativistic h.o., the time-like part would give divergent matrix elements. To avoid this problem, FKR assume that only space-like excitations are meaningful in the formalism and that matrix elements should be computed for these alone. This requirement is met by putting a further projection condition on all such states h, viz.,

$$P_\mu a_\mu \mid h> = P_\mu b_\mu \mid h> = 0 \qquad ...(26)$$

where a set of creation (a_μ^*, b_μ^*) and destruction (a_μ, b_μ) operators are defined in terms of (ξ_μ, η_μ) and (x_μ, y_μ) as

$$a_\mu, a_\mu^* = (\xi_\mu \mp i\Omega x_\mu)/\sqrt{2\Omega} \qquad ...(27)$$

$$b_\mu, b_\mu^* = (\eta_\mu \mp i\Omega\, y_\mu)/\sqrt{2\Omega} \qquad ...(28)$$

with the standard commutation relations

$$\left[a_\mu,\ a_\nu^*\right] = \left[b_\mu,\ b_\nu^*\right] = -\,g_{\mu\nu}\,;\ (g_{\mu\nu} = 1, -1, -1, -1) \qquad ...(29)$$

The "wrong" sign of the time-part in (25) is directly associated with its corresponding appearance in the mass operator in terms of the a_μ and b_μ variables :

$$M_B^2 = +\Omega\left(a_\mu^* a_\mu + b_\mu^* b_\mu \right) + C \qquad ...(30)$$

and the condition (26) for space-like excitation (which is most easily checked for the rest frame $P_\mu = (m_i, 0, 0, 0)$ is a simple device to retain the positive definiteness of the hamiltonian. In a similar way for the meson system, the corresponding condition is postulated as

$$P_\mu c_\mu \mid h > = 0 \qquad ...(31)$$

where

$$c_\mu \; c_\mu^* = (\zeta_\mu \pm i\Omega z_\mu)/\sqrt{2\Omega} \qquad ...(32)$$

and $$M_M^2 = \Omega c_\mu^* \; c_\mu + C \qquad ...(33)$$

The price for such a truncation is the loss of unitarity which shows up in the violation of sum rules, e.g., for the e.m. current as a result of suppression of the time-like excitations. Such tests are conveniently carried out in terms of the adjoint functions $\tilde{h}$, introduced in eqn. (11). To define them for any excited state, note that the latter can be constructed out of the ground state h_0, eqn. (25), with an appropriate number of creation operators. The adjoint function is merely obtained by the replacements

$$a_\mu^*, \; b_\mu^*, \; c_\mu^* \Rightarrow - \sum_\nu g_{\mu\nu} \; (a_\nu, \; b_\nu, \; c_\nu) \qquad ...(34)$$

where

$$g_{00} = - g_{11} = - g_{22} = - g_{33} = + 1$$

so as to ensure the relativistically invariant forms for the norms of states. Now consider an obvious identity like

$$\sum_{\text{all } j} < 0 \mid e^{iq.x} \mid j > < j \mid e^{-iq.x} \mid 0 > = 1 \qquad ...(35)$$

in which the time-parts of j-states make negative contributions by virtue of the relation

$$< 0 \mid e^{iq.x} \mid j >^* = (-1)^{n_j} < j \mid e^{-iq.x} \mid 0 > \qquad ...(36)$$

where n_j is the number of time-like excitations in $\mid j>$. So, if and only if the space-like excitations are considered, the left hand side of (35) will *exceed unity*, thus making the matrix elements appear too large.

To compensate for this error, FKR suggested an *ad hoc* Gaussian factor of the form $e^{-c_1 \vec{Q}^2}$, analogous to what appears in the non-relativistic h.o. model, as a replacement for a quantity like $e^{c(q_i^2 - \vec{Q}^2)}$ which would appear in the (relativistic) calculation of the matrix element. Here $\vec{Q}$ is the 3-momentum of one of the decay

products in the rest system of the decaying particles. (For the actual form used by FKR, the reader is referred to their paper).

FKR have given an elaborate method for the evaluation of the e.m. and pseudoscalar transition matrix elements in terms of a four-dimensional operator formalism strongly reminiscent of similar methods in the dual theories (Fubini *et al.*, 1969). The general philosophy is still on the lines of single quark transitions using the creation and annihilation operators α_μ, α_μ^* ($\alpha = a, b, c$) described in the foregoing. While the details of these evaluations are best left to the FKR paper, it is useful to note that the "relativistic" limitations of the theory show up through the appearance of certain unphysical factors which must be clearly recognized and isolated before the structures are ready for comparison with experiment. One of them, mentioned in the earlier paragraph, is, of course, the *ad hoc* Gaussian factor needed to compensate for the neglect of time-like excitations.

Another adjustment factor which is more kinematic comes from the Dirac spinor matrix elements as follows. As in *qqq*-dynamics, the complete harmonic matrix element is determined by the interaction of one of the quarks, say *a*, and multiplying the result by *three*. Since this does not involve a direct perturbation on the other two quarks, there should really be no projection factor for a new direction. On the other hand, because of the artifice of considering only *positive* energy quark states (thus violating unitarity by leaving out the negative energy quark states), there is a projection factor and hence a spurious enhancement of the matrix element for each quark transition by a factor g, where

$$g^2 = [(m_1 + m_2)^2 - m_3^2]/4\, m_1 m_2 \qquad \ldots(37)$$

Therefore, one must recognize (and divide out by) an overall factor of g^3 or g^2 for baryonic and mesonic transitions respectively.

Apart from the above corrections, the Feynman method enjoys many formal advantages characteristic of a complete theory, e.g., for the e.m. interaction the gauge invariance is automatically built-in into their framework. In the same way, the quark-recoil effect does not need to be put in by hand, but it comes out automatically within their relativistic framework, together with the GOR factor $(m_1 - m_2)$; See eqn. (4.3) of FKR (1971).

FKN-Lines Normalization

The *ad hoc* nature of the Gaussian form factor used in FKR is perhaps the only unattractive feature in an otherwise elegant and extremely powerful scheme for the description of a wide range of hadronic phenomena. The main difficulty about the Gaussian factor is its quick damping with the momentum transfer (Q) so as to make it virtually inapplicable to processes like elastic and inelastic scattering of electrons off nucleons involving very large Q^2-values (Ravndal, 1971) almost as in the non-relativistic h.o. model (Thornber, 1968). To remedy for this defect Lipes (1972) suggested an alternative form of normalization, motivated by an earlier work of Fujimura *et al.* (1970) in connection with the problem of e.m. form factors of the relativistic nucleon. FKN had earlier noticed that the inadequacy of the N. R.

model to reproduce the dipole formula had stemmed primarily from the failure of the latter to take account of the effect of Lorentz contraction of the nucleon (*not* quark) *core*, and had thus been led to consider a relativistic harmonic oscillator wave function in a more or less *ad hoc* fashion. However, they used a different *kind of* normalization form (25), in that the time-component of their wave function was also normalizable. Specifically, they used for the ground state, the wave function

$$h_0'(\xi, \eta) = \text{Exp}\left[-\xi_t^2 - \vec{\xi}^2 - \eta_t^2 - \vec{\eta}^2 \right] \Big/ 2\,\Omega \qquad \ldots(38)$$

instead of FKR's eqn. (25) and obtained extremely good fits to the elastic form factors (including the dipole formula). However, their overall framework was much less comprehensive than FKR's and in particular their choice of the hadronic e.m. current was more phenomenological.

Lipes (1972) advocated the use of the structure (38) for the ground state wave function within the FKR framework. Since this expression is not relativistically invariant, it can at best be made covariant through a reference to the rest frame P_μ $(=M_0, 0, 0, 0)$ of the hadron, so as to look like

$$h_0'(\xi, \eta) = \text{Exp}\left\{ -\xi^2 - \eta^2 - 2\,(P.\xi)^2\, M_0^{-2} - 2\,(P.\eta)^2\, M_0^{-2} \right\} \Big/ 2\,\Omega \qquad \ldots(39)$$

Lipes otherwise followed the FKR prescription for space-like excitations only *in the rest frame of the system*, thus necessitating the use of the space-like four-vectors

$$\hat{\xi}_\mu = \xi_\mu + P \cdot \xi\, P_\mu\, M_0^{-2}\,, \quad \hat{\eta}_\mu = \eta_\mu + P \cdot \eta\, P_\mu\, M_0^{-2} \qquad \ldots(40)$$

to express the arguments of the Hermite polynomials for space-like excitations. As in FKR, Lipes had to put up with loss of unitarity, but in view of his normalization (39), he no longer needed to make a further "Gaussian adjustment" unlike the former. His most important result is the derivation of the nucleon form factor

$$G(q^2) = \left(1 + q^2/2M^2\right)^{-2} \exp\left[-\frac{1}{6}\, q^2\, \Omega^{-1}\, (1 + q^2/2M^2)^{-1} \right] \qquad \ldots(41)$$

which, in the limit $|\, q^2 \,| \gg M^2$, approaches the "dipole" form and agrees fairly well with the data upto to $+\, q^2 \approx 25\ \text{GeV}^2$.

Licht-Pagnamenta Model and Dimensional Counting

An intuitively simpler approach applicable in principle to the N.R. wave function for an arbitrary cluster (and not merely a Gaussian wave function), is a three-dimensionally relativistic formulation due to Licht and Pagnamenta (1970), which brings out rather succinctly the precise effect of Lorentz contraction on the cluster form factor as a result of its motion. In recent times, this theory has had some remarkable successes on a wide range of hadronic phenomena from low to high energies within the premises of the constituent quark model. We shall give a short account of this formalism together with a couple of recent applications.

The L-P recipe for relativization of a non-relativistic matrix element consists in the following argument. There exists a preferred frame (the Breit frame) in which

the interaction of the individual members of a cluster with an external potential (or radiation quantum) may be regarded as instantaneous to a good approximation. In the N. R. theory, one considers the emission or absorption of radiation by a cluster at rest, with its individual members in small relative motion w.r.t. the centre of mass. According to L-P, such a picture can, with a good approximation, be taken over to the Breit frame for the cluster motion as a whole, involving concomitant Lorentz-transformation effects on the cluster coordinates. Since in this picture, the relative time-coordinates among the individual members of the cluster are neglected, there is only one time coordinate for the cluster as a whole, so that formal relativistic invariance can be maintained only up to its external kinematics. However, a transition to the Breit frame from the rest frame will cause a frame dependence to show up due to the Lorentz transformation on the internal coordinates.

It is instructive to give a more quantitative description of the L-P recipe. The non-relativistic form of the internal matrix element for a transition A → B with suitably normalized internal coordinates $\vec{x_i}$ is

$$F_{AB}\left(\vec{q}^{\,2}\right) = \int \prod_{1}^{n-1} \mathrm{d}\vec{x_i}\, \psi_B^*\left(\vec{x_i}\right)\; e^{i\vec{q}.\vec{x_i}}\, \psi_A\left(\vec{x_i}\right) \qquad \ldots(42)$$

since the states A and B are now in different Lorentz frames, the internal coordinates associated with the A and B clusters must now be different, i.e.,

$$\vec{x_i} \to \vec{x}_{Ai}\ (\text{in } \psi_A);\ \vec{x_i} \to \vec{x}_{Bi}\ (\text{in } \psi_B) \qquad \ldots(43)$$

To express the coordinates $\vec{x}'_{Ai}$, $\vec{x}'_{Bi}$ in terms of $\vec{x_i}$, take the common direction of motion as the z-direction, in which case the Lorentz transformations on the z-coordinates (at $t = 0$) are expressed as

$$z'_{Ai} = z_i E_A/M_A,\ z'_{Bi} = z_i E_B/M_B \qquad \ldots(44)$$

while the transverse coordinates (x_i, y_i) remain unaffected. Note that after the replacement (44) the matrix element is now physically different from the original N.R. form, and it is precisely at this point that the L-P proposal amounts to a departure from the physical premises of N-R theory. For the elastic case, the substitution (44) in (43) leads to the replacement $F(q^2) \to F_R(q^2)$ where

$$F_R(q^2) = (M_A/E_A)^{n-1}\, F[q^2(1+\eta)^{-1}] \qquad \ldots(45)$$

with $$\eta = 1 + q^2/4M_A^2 = E_A^2\ /M_A^2 \qquad \ldots(46)$$

noting that in the Breit frame with momenta $\pm \vec{p}$, the momentum transfer $\vec{q}$ equals $2\vec{p}$.

This formula brings out the essential mechanism by which a Lorentz contraction effect on the momentum transfer q makes itself felt on the structure of the

form factor. Thus, for large q^2, $F(q^2)$ no longer vanishes but goes into a non-zero constant, while the asymptotic behaviour is determined entirely by the Lorentz-contraction factor $(M/E)^{n-1}$ in front.

It is this last feature which brings about a power behaviour ($\sim q^{1-n}$) in the form factor for large q^2. Though this falls short of the observed ($\sim q^{-4}$) behaviour of the e.m. form factor of the proton ($n = 3$), it was shown in L-P how the introduction of a VMD mechanism could effectively compensate for the above deficiency. On the other hand, more powerful techniques of field theory which boil down to dimensional scaling (Brodsky & Farrar, 1975) arguments, have been found to produce $a \sim q^{2-2n}$ behaviour, even without the VMD mechanism. It has been found that a simple variant of the L-P argument not only produces a q^{2-2n} behaviour but generalizes the L-P result for elastic form factors to the inelastic case as well. We shall briefly sketch the modified argument (Mitra & Kumari, 1977) which also seems capable of generalization to arbitrary quark or duality diagrams (including the QPC mechanism).

To bring about a more symmetric appearance of the A and B coordinates we rewrite the N.R. matrix element (42) in the alternative form

$$F_{AB}\left(\vec{q}^{\,2}\right) = \int \prod_{1}^{n-1} d\vec{x}_{Ai}\, d\vec{x}_{Bi}\ \delta\left(\vec{x}_{Ai} - \vec{x}_{Bi}\right)\ \psi_B^*\left(\vec{x}_{Bi}\right)\ e^{ip(z_{Ai}+z_{Bi})} \times \psi_A(\vec{x}_{Ai}) \quad \ldots(47)$$

which has the same physical content as (42), but facilitates the option of regarding the δ-function as a function of the mixed coordinates $(\vec{x}_A, \vec{x}_B)$ which is subject to the L.P. transformation (44) just like the wave functions. With the help of this trick, one obtains two sets of Lorentz contraction factors arising out of the integrations over both the A and B coordinates, viz.,

$$(M_A/E_A)^{n-1}\ (M_B/E_B)^{n-1} \quad \ldots(48)$$

while the damping in the overlap integral F due to increasing q^2 is arrested by the same mechanism as in (45). This result is not only symmetrical in A and B but has also the correct asymptotic behaviour expected from dimensional scaling (*loc. cit*). For the elastic case, Brodsky and Chertok (1976) proposed an interpolation formula

$$F_{el} \sim (1 + q^2/n\beta^2)^{1-n} \quad \ldots(49)$$

using a model of propagator chains. While it has the correct asymptotic behaviour ($\sim q^{2-2n}$) expected from dimensional scaling arguments, it gives excellent fits to the π, N, D etc., form factors with a universal value of $\beta^2 \simeq 0.24\ \text{GeV}^2$. A comparison of (49) with (48) reveals a strong similarity between the L-P and Brodsky-Chertok formulae if one makes the identification $M_n^2 = n\beta^2$ for the "mass" M_n of the cluster. This implies variation of M_n as $\sqrt{n}$, instead of as n (which would be indicated through quark counting). Such a behaviour of M_n need not cause alarm if this quantity is interpreted as some sort of "effective mass" (Bethe, 1956) and if

one also takes note of a similar behaviour of M^2 with the excitation quantum number in the relativistic h.o. model. Certain applications of this formalism have been discussed in a recent review (cf. Mitra & Sood, 1977).

(§3) SU (6)-Breaking, Non-H.O. and Broken-H.O. Models

The major issue with any concrete model of hadron spectroscopy with quark constituents concerns the nature and extent of symmetry-breaking needed to meet the experimental situation, while remaining within the permissible limits of a 'calculable' theory which must have a certain amount of built-in symmetry to start with. The general consensus in this respect has been that SU (6) $\times$ 0 (3) or its relativistic counterpart SU (6)$_w \times$ 0 (3) has by and large been a reasonably good symmetry, not only for mere classification of states, but also for the more substantial purpose of dynamical calculations of masses, coupling constants and form factors, being broken by only modest amounts which are amenable to perturbative calculations. In contrast, the harmonic oscillator, especially one within an N. R. framework, is a much bigger symmetry than SU (6), and it is believed to be badly broken if it is to describe hadron dynamics at all, though the amount of breaking, or more importantly the precise source of breaking, are much more open questions. Indeed to many advocates of *linear* or other non-harmonic forms of confinement, the oscillator model may not even be a good zero-order input for the dynamics. On the other hand, even among the "non-believers" one observes a frequent tendency to use an "h.o. basis" set of wave functions for the calculations of various dynamical effects (including a diagonalization of the Hamiltonian), with or without a formal commitment to a minimum h.o. dynamics which may be needed to generate such wave functions. (This last is reminiscent of the use of shell-model wave functions in nuclear physics, even though the harmonic oscillator is not the basis of the *N-N* interaction).

H.O. Model vs. Observed Baryonic States

The harmonic oscillator has had a very chequered history in quark physics. At a qualitative level, its most important success is that, in association with SU(6) $\times$ 0 (3), it provides a convenient classification seheme for all the well-established baryons (Bricman *et al.*, 1980) in a most natural fashion. In particular, it correctly predicts the alternating parity sequence up to the few excitations of the more prominent baryonic states (S = 0, −1). On the other hand, it predicts more states of higher excitations $L \geqslant 2$) than are actually found, especially 20-states of unnatural parity. Most of these features have remained by and large unchanged over the years since the Berkeley Conference (Dalitz, 1966). A current status report on the baryonic states would be found in the *Proc. XX HEP Conf. Madison* (1980)—Review on resonances.

The central issue with the h.o. model is the high degree of mass degeneracy born out of too much symmetry. The early attempts at symmetry-breaking were confined to the introduction of *ad hoc* interaction terms (Greenberg & Resnikoff, 1967), which have since been continued on a more elaborate basis (Horgan, 1974; Horgan & Dalitz, 1974; and Jones *et al.*, 1974, 1977). The general strategy has been to calculate the matrix elements of these terms (spin-orbit, tensor, SU (3)-breaking, etc.) in h.o. basis functions, much on the lines of the shell-model in nuclear physics.

This automatically leads to mixing of various states, the latter percentages being determined by the relative strengths of the interaction terms themselves. A comprehensive review on these lines has been given by Dalitz and Horgan (cf. Jones *et al.*, 1977). However, the difficulty about this kind of analysis is that it affords no insight into the possible theoretical sources of symmetry-breaking effects. Nor is it comprehensive enough to predict any definite "pattern" of symmetry-breaking in the classification of states.

Since the simple h.o. model predicts too many qqq states of $N \geqslant 2$ ($N = 2n + L$ principal quantum number) than are observed in nature, let alone the more quantitative aspects of a high degree of level degeneracy, special models have been suggested since the early stages of hadron spectroscopy to reconcile the theory with observation. One of the features of baryon excitations which had been observed fairly early in this development is the occurrence of excitations alternating in $\underline{56}$ and $\underline{70}$, as well as in parity :

$$(\underline{56}, \text{even}^+), (\underline{70}, \text{odd}^-) \qquad \ldots (50)$$

A simple dynamical explanation of this fact was offered (Mitra & Ross, 1967) on the basis of q-q forces in s-waves only. These forces were taken to be of the separable form for convenience, to facilitate use of the mathematical machinery that was being developed in the sixties for a closed form solution of the 3-body problem (Faddeev, 1961; Mitra, 1962; and Amado, 1962). Because of the assumed operation of q-q forces only in s-waves, $\underline{20}$ states were automatically ruled out because of the spatially antisymmetric structure of their wave functions. This was during the days before colour had been formally accepted (Fritzsch *et al.*, 1973) as a part of the quark culture (even though there had been at least one earlier proposal to the same effect (Han & Nambu, 1965). Since the colour wave function by itself is antisymmetric, the above basis of classification in terms of *symmetric* wave functions (Greenberg, 1974) without the colour space has remained unchanged. The kernel of the connected qqq-equation had, apart from the details of the dynamics, a proportionality factor $2\,(-1)^L$ and $(-1)^{L-1}$ for $\underline{56}$ and $\underline{70}$ states respectively. This would automatically account for the *attractive* kernels for the states (50) so as to render them low lying in a reasonable dynamical scheme. On the other hand, the complementary states

$$(\underline{56}, \text{odd}^-) \text{ and } (\underline{70}, \text{even}^+) \qquad \ldots (51)$$

would have repulsive kernels, and hence high-lying in the same scheme of dynamics.

The above mechanism gave a natural understanding of the absence of "unwanted" states otherwise predicted by the h.o. model. However, at a more quantitative level, the predicted masses (M) tapered horizontally off too fast with the excitation quantum number (N) compared with the observed (Bricman *et al.*, 1980) linear behaviour $M^2 \sim N$ (Dalitz, 1966; and Feynman *et al.*, 1971). A few years later this defect was sought to be removed by generalizing the operation of the q,q interaction from mere s-waves to all *even* waves in an otherwise h.o. model. This is discussed in the next sub-section in some detail. In the meantime, a brief mention of an alternative mechanism designed to achieve the same end is in order viz., the quark-diquark (q-d) model.

The *q-d* model (Lichtenberg, 1969) is based on an effective description of a *q-q* subsystem within a baryonic (*qqq*) system in terms of an elementary diquark (*d*). This two-body description is admittedly *less* microscopic than the orthodox three-body description which incorporates the admissible symmetries to a fuller extent, yet it effectively serves the same purpose of keeping out the unwanted states as does a truncated (e.g., *s*-wave) *q-q* interaction. The trick lies in selecting the representation $\underline{21}$ in the SU (6) break-up

$$\underline{6} \times \underline{6} = \underline{21} + \underline{15} \qquad \ldots (53)$$

The $\underline{21}$ ($= 6 \times 3 + 3 \times 1$) corresponds to a spatially symmetric *q-q* state, while $\underline{15}$ is associated with a spatially antisymmetric one, thus establishing the consistency of this alternative diquark picture with that of the *s*-wave *q-q* interaction which would lead to (50) through the separable model (Mitra, 1968; and Mitra & Katyal, 1968). In the *q-d* picture too, the assumption of strong exchange forces would lead to a lowering of the series (50) w.r.t. (51).

The *q-d* model, however, fails at the quantitative level in predicting the detailed properties of baryon states, especially their mass spectrum and e.m. excitation form factors (Mitra—*unpublished*). This is hardly surprising since the *q-d* model is a formally incomplete description of a baryon, and can at most be of limited value. Nevertheless, the model is extremely suggestive and almost "made-to-order" in certain situations. For example, baryonium ($qq\bar{q}\bar{q}$) states, which we discuss more fully in the last section before conclusion, can be compactly thought of as *dd* systems (Rosenzweig, 1976) in order to account in a natural fashion for the reluctance of such states to decay into two mesons, in preference to baryon-antibaryon pairs, despite the phase space disadvantage of the latter. For other possibilities with diquarks as well as references see Wilson (1974, 1976).

Even-Wave H.O. Model

This model was developed by Mitra and collaborators (Mitra, 1974, 1975, Mitra & Sen, 1974; Mitra & Sood, 1977*a*; and Kamath & Mitra, 1978), as a generalization of the *s*-wave *q-q* interaction (Mitra, 1968; and Mitra & Katyal, 1968) to include all *even* waves. It was motivated directly by (Mitra, 1974) the twin experimental featres of (*i*) *linear* M^2- *vs.* N-plots; and (*ii*) continued absence of $\underline{20}$ states, the first of which in the spectroscopic notation reads as $L^P = 1^+$ or as $P\,(1\,p)^2$ and arises out of a primary *p*-wave *q-q* interaction. These two features are most naturally built in through a harmonic oscillator interaction operative in even partial waves only, i.e., through a Serber-type h.o. model, in which the direct and exchange contributions are 50 per cent each. Two other experimental features which motivated a modification in the h.o. model, though not necessarily in the form of the even-wave model (EWM), were (*i*) the unusually low mass of $P\pi$ (1470) for its $N = 2$ states in the standard h.o. picture, indeed even lower than the $N = 1$ states like D_{13} (1520), etc; (*ii*) the existence of two distinct mass groups in the ($\underline{70}$, 1^-) spectrum for each of the species N, Λ, Σ. The last two features turned out to be rather automatically satisfied by the EWM which was completely specified by the Serber-type h.o. interaction, perhaps speak for its credibility as a viable model.

The EWM equations can be formulated (Mitra, 1975) on lines analogous to the FKR oscillator model wherein the energy reads essentially as M^2 but is otherwise effectively 3-dimensional to keep out the time-like excitations. In particular, it can be formulated in an SU (6)-invariant fashion, since SU (6) symmetry does not have to be necessarily broken in order to break the (much bigger) h.o. symmetry. (The mass splittings in such a model must be necessarily *orbital* in character). The essential steps are sketched below, leaving the details to Mitra (1975). An SU (6) invariant $q_i - q_j$ potential which is h. o. in *even* partial waves

$$V_{ij}^{(+)} = \left[P_\sigma^+(ij)\, P_u^+(ij) + P_\sigma^-(ij)\, P_u^-(ij) \right] P_x^+(ij) \times \left(\frac{1}{2} m\omega_0^2 r_{ij}^2 + V_0 \right) \qquad ...(54)$$

where the projection operators are defined as (cf. Mitra, 1975)

$$P^{\pm}_{\sigma,u,x} = \tfrac{1}{2}\left(1 \pm (ij)_{\sigma,u,x}\right) \qquad ...(55)$$

$$(ij)_\sigma = \tfrac{1}{2}\left(1 + \vec{\sigma}_i \cdot \vec{\sigma}_j\right) \qquad ...(56)$$

$$(ij)_u = \tfrac{1}{3} + \tfrac{1}{2} \sum_{\alpha=1}^{8} \lambda_{i\alpha}\, \lambda_{j\alpha} \qquad ...(57)$$

The complementary potential $V^{(-)}$ which is operative in *odd* partial waves is the same as (54) except for the replacement $P_x^+ \to P_x^-$ (colour indices are suppressed with the understanding that only colour-singlet states are considered).

Since the q-q potentials are SU (6)-invariant, the equations for the spatial functions are most quickly recovered by considering only the 10_q and 8_q multiplets of $\underline{56}$ and $\underline{70}$ states respectively, viz.,

$$|\, \underline{10} >_q = \psi^s \chi^s \phi^s \qquad ...(58)$$

$$\sqrt{2}\, |\, \underline{8} >_q = (\psi'\phi' + \psi''\phi'')\, \chi^s \qquad ...(59)$$

in the notation of Mitra and Mazumdar (1966). Since $P_x^+ = 1$ for ψ^s states, the $\underline{56}$-spectrum remains unaltered w.r.t. usual h.o. dynamics, so that with the identification (Feynman *et al.*, 1971) of mE as $M^2/6$, one finds, after FKR but with an alternative normalization (cf. Mitra, 1978),

$$M^2_{56}\psi^s = 6mE\, \psi^s = [\Omega(N_\xi + N_\eta + 3) - \beta]\, \psi^s \qquad ...(60)$$

$$N_\xi = a^\dagger_{\xi i}\, a_{\xi i},\ N_\eta + a^\dagger_{\eta i}\, a_{\eta i} \qquad ...(61)$$

$$\Omega = 6\alpha = 6\sqrt{3}\, m\omega_0,\ \beta = -18\, mV_0 \qquad ...(62)$$

$$(2/\alpha)^{1/2}\, \vec{\xi} = \vec{r}_3 - \vec{r}_2,\ (6/\alpha)^{1/2}\, \vec{\eta} = -2\vec{r}_1 + \vec{r}_2 + \vec{r}_3 \qquad ...(63)$$

$$\left[a_{\xi i},\ a^\dagger_{\xi j} \right] = \delta_{ij}, \text{ etc.} \qquad ...(64)$$

For the $\underline{70}$-states, there are now two coupled equations in (ψ', ψ'')

$$4K_0 \begin{pmatrix} \psi' \\ \psi'' \end{pmatrix} = \begin{pmatrix} \xi^2 - \eta^2 & 2\vec{\xi}.\vec{\eta} \\ 2\vec{\xi}.\vec{\eta} & \eta^2 - \xi^2 \end{pmatrix} \begin{pmatrix} \psi' \\ \psi'' \end{pmatrix} \quad \ldots(65)$$

$$K_0 = -\nabla_\xi^2 - \nabla_\eta^2 - \frac{2M_{70}^2 + \beta}{\Omega} + \tfrac{1}{2}(\xi^2 + \eta^2) \quad \ldots(66)$$

A solution of the $\underline{70}$ equations is facilitated in terms of the Simonov (1966) variables (ρ, γ, λ) defined by

$$\xi^2 - \eta^2 = \gamma \cos\lambda, \quad 2\vec{\xi}.\vec{\eta} = \gamma \sin\lambda \quad \ldots(67)$$

$$\rho = \xi^2 + \eta^2 \quad \ldots(68)$$

The variable λ which is like an angle has an associated angular momentum-like operator $\overset{\Lambda}{\nu}$ defined by

$$2i\overset{\Lambda}{\nu} = \xi_i \partial\,\eta_i - \eta_i \partial\,\xi_i = 2\,\partial/\partial\lambda \quad \ldots(69)$$

whose eigenvalue is a measure of the [2, 1]-symmetric content of a given $\underline{70}$-state since the other two variables (ρ, γ) are totally symmetric.

To diagonalize the RHS of equations (65), it is convenient to transform to the variables

$$\left.\begin{aligned} x_i &= \cos\tfrac{1}{2}\lambda\xi_i + \sin\tfrac{1}{2}\lambda\eta_i \\ y_i &= \sin\tfrac{1}{2}\lambda\xi_i - \cos\tfrac{1}{2}\lambda\eta_i \end{aligned}\right\} \quad \ldots(70)$$

Noting that $\sin\frac{1}{2}\lambda$ and $-\cos\frac{1}{2}\lambda$ transform as the ξ- and η-components of [2, 1] symmetry it is easy to see that x_i is "antisymmetric" and y_i is "symmetric" respectively w.r.t. the permutation group. An approximate solution to eqn. (65) was obtained by Mitra (1975) in the (x, y) representation, using the combinations :

$$\left.\begin{aligned} \phi' &= \psi' \cos\tfrac{1}{2}\lambda + \psi'' \sin\tfrac{1}{2}\lambda \\ \phi'' &= \psi' \sin\tfrac{1}{2}\lambda - \psi'' \cos\tfrac{1}{2}\lambda \end{aligned}\right\} \quad \ldots(71)$$

These functions satisfy two approximately uncoupled differential equations analogous to (65), but expressed in the (x, y) variables. The results for the M^2-spectrum are expressed as

$$M_{70}'^2 = \frac{1}{2}\Omega\left(2n_x + l_x + \frac{3}{2}\right) + \frac{1}{2}\Omega\sqrt{3}\left(2n_y + l_y + \frac{3}{2}\right) - \frac{1}{2}\beta \quad \ldots(72)$$

$$M_{70}''^2 = \frac{1}{2}\Omega\sqrt{3}\left(2n_x + l_x + \frac{3}{2}\right) + \frac{1}{2}\Omega\left(2n_y + l_y + \frac{3}{2}\right) - \frac{1}{2}\beta \quad \ldots(73)$$

where $n_{x,y}$ and $l_{x,y}$ are the radial and orbital excitations corresponding to x- and y-modes respectively.

Results of E.W. Model

To interpret these results properly, note that the x- and y-modes are *independent* and can be considered separately. In the x-mode, the excitations bear the relations $\triangle M''^2 = \sqrt{3}\triangle M'^2$ while in the y-mode, the roles of M' and M'' get exactly interchanged. Therefore, it is enough to consider only one of them, say the x-mode, for purposes of determining the nature of the spectrum. There are three main qualitative features : (*i*) the state of the lowest excitation is now a $(\underline{70}, 0^+)$ lying *below* the $(\underline{70}, 1^-)$ states; (*ii*) the $(\underline{70}, 1^-)$ states come in two distinct bunches of excitations $\Omega/2$ and $\sqrt{3}\Omega/2$ respectively; and (*iii*) the spacings among $\underline{70}$ states are much smaller than those among $\underline{56}$ states.

A *ground* state of $(\underline{70}, 0^+)$ is a natural candidate for the P_{11} (1470) whose unusually low mass has all along been a serious problem for the usual h.o. model. The bunching of the first excited $\underline{70}$ states into two distinct types of excitation comes in rather handy for the understanding of large mass gaps among several $(\underline{70}, 1^-)$ states, e.g., between D_{13} and D_{15}.

These features which are automatic consequences of the even-wave h.o. model, have come about through an SU (6)-invariant mechanism. Thus the EW theory accounts for a major part of symmetry-breaking effects through the h.o., and not SU (6),-breaking mechanism, leaving hopefully a smaller balance to SU (6)-breaking (spin and SU (3) effects). The theory gives a quantitatively satisfactory account of the mass positions not only of non-strange baryons (Mitra, 1975) but of strange ones as well (Kamath & Mitra, 1978).

The EW theory was also developed at the level of wave functions to construct couplings with the e.m. field and pseudoscalar mesons, and the corresponding transition form factors (Mitra, 1976; Mitra & Sen, 1974; Mitra & Sood, 1977 *a*; Kamath & Mitra, 1978; and Gupta & Mitra, 1978), on lines similar to FKR. The Licht-Pagnamenta form (Licht & Pagnamenta, 1970) of 3-dimensional relativization for elastic form factors, as well as its generalization to include transition form factors (Mitra & Kumari, 1977), were employed extensively for evaluating the baryonic decay widths, photocouplings (*loc. cit.*) and structure functions in deep-inelastic scattering to test the more quantitative features of the EW model. These are based on an *ad hoc* mixing between the $(\underline{70}, 0^+)$ and $(56, 0^+)$ nucleons, the mixing being more naturally facilitated in the EW model than in the standard h.o model where the corresponding $\underline{70}$ state has a much higher mass (Le Yaouanc *et al.*, 1975 *a*). The numerical results in all these cases are also consistently better than those in the standard h.o. model. For details the reader is referred to the original papers.

SU (6)-Breaking Effects (Relativistic)

Despite their attractiveness in providing plausible explanations for important experimental facts, the above models may at best be regarded as "educated guesses" at the truth at the purely phenomenological level. To qualify as more serious candidates, a symmetry-breaking mechanism must be based on more natural theoretical premises. Now, except for SU $(6)_W$ which is a rather special type (collinear),

of relativistic group of the same size as SU (6), almost all relativisitic corrections to an N. R. dynamics imply a certain degree of SU (6)-breaking, the precise nature and extent of the breaking depending on the scope of the relativistic modification itself.

Certain relativistic effects are quite obvious. For example, the hadron c.m. motion is often relativistic, an effect which is taken care of partly through Lorentz contractions of the L-P (1976) type, and partly through spin-boost matrices acting on the quark-spinors, leading to Wigner rotation (Le Yaouanc *et al.*, 1975 *b*). Again, the internal quark motion inside the hadron is relativistic, even when the hadron is at rest, an effect for which the replacement of Pauli spinors by Dirac spinors was suggested for each quark (Le Yaouanc *et al.*, 1974). The manifestations of the L-P effect on the behaviour of elastic form factors have already been discussed in the previous section. It also ensures scaling of the spatial wave function in deep-inelastic $e^- p$ scattering through the boost prescription (Le Yaouanc *et al.*, 1975 *b*).

$$\psi_{P_z = \infty}(x_i, \vec{p}_{i\perp}) = \psi_{\vec{P}=0}(x_i m_N, \vec{p}_{i\perp}) \qquad ...(74)$$

which relates the $P_z = \infty$ frame with the $\vec{P} = 0$ c.m. frame of the nucleon, and $x_i = P_{iz}/P$ is the fraction of the longitudinal momentum carried by quark i. Similarly, the replacement of Pauli spinors by Dirac spinors for the quarks gives rise to large corrections to the g_A/g_V ratio over its non-relativistic quark value of 5/3, to bring it near the experimental value of 1.24. Similar corrections, again in the right direction, occur on the baryon magnetic moments (cf. Le Yaouanc *et al.*, 1974).

The $(\underline{56}, 0^+)$ and $(\underline{70}, 0^+)$ mixing, discussed earlier in connection with the even-wave model (Mitra & Sen, 1974), and also for the regular h.o. model (Le Yaouanc *et al.*, 1975 *a*), is more *ad hoc*, but is directly responsible for the departure of the F_2^{en} F_2^{ep} ratio in deep inelastic scattering (Feynman, 1972) from the SU (6) value of 2/3 to almost 1/3 for large x. Further effects may arise from the composite-ness assumption on quarks but these are much more tentative).

Le Yaouanc *et al.* (1977, 1978) have considered several observational manifestations of such relativistic corrections on the symmetry-breaking properties of hadrons. A particularly interesting case is represented by the Σ- mass difference ($\sim$ 80MeV), which cannot be explained by the usual SU (3)-breaking in the standard quark model brought about by the quark mass differences, for both Σ and Λ have the *same* quark content. However the relativistic correction to the mass shows up as

$$m_i \to \epsilon_i \approx m_i + \vec{p}^2(i)/2\, m_i + \ldots \qquad ...(75)$$

in which the kinetic energy term "sees" the difference between the s- and u-quark masses and leads to Σ-Λ splitting through the difference in the expectation values of this operator, w.r.t. the *mixed* $\underline{56}$-$\underline{70}$ oscillator model wave functions for the Σ, Λ ground states. Σ-Λ mass difference has also been considered by Cabibbo and Testa as well as De Rujula *et al.* (1975). The former relate the Σ-Λ splitting to SU (6)-breaking in deep inelastic electroproduction in the $P_z = \infty$ frame, the relevant quantity being $\langle x_i^{-1} \rangle$. However, the $P_z = \infty$ frame results can be recovered in

terms of the more familiar c.m. wave function through the correction (74). The DGG (1975) is described below.

SU (6)-Breaking (QCD)

Ideas about the nature of SU (6)-breaking faces inside the hadrons were suggested in a most interesting paper due to De Rujula *et al.* (1975) (abbreviated as DGG). Invoking the asymptotic freedom of colour-gauge theories (Politzer, 1973, 1974; and Appelquist & Politzer, 1975), they proposed that the short-distance q-q forces should be dominated by one-gluon-exchange and obtained the familiar Fermi-Breit (δ-function) interaction in the v^2/c^2-order relativistic correction to the one-gluon-exchange kernel within a Bethe-Salpeter (1967) framework. The resulting Hamiltonian for a $q\bar{q}$ or qqq system (colour neutral) under such assumptions is of the form (De Rujula *et al.*, 1975).

$$\overline{H} = H_L(\vec{r}_1, \vec{r}_2 \ldots) \sum_i (m_i + \vec{p}_i^{\,2}/2m_i + \ldots) + \sum_{i>j} (\alpha\, Q_i Q_j + k\alpha_s)\, S_{ij} \qquad \ldots(76)$$

Q_i = charge of quark i; $k = -2/3$ for q-q and $-4/3$ for q-$\bar{q}$; H_L is the universal confining interaction which is taken to be SU (6) symmetric. The S_{ij}-term is given by ($\vec{r} = \vec{r}_{ij}$):

$$\begin{aligned} S_{ij} = {} & \frac{1}{r_{ij}} - \frac{1}{2m_i m_j}\left(\frac{\vec{p}_i \cdot p_j}{r} + \frac{\vec{r}\,.\,(\vec{r}\,.\,\vec{p}_i)\,\vec{p}_j}{r^3}\right) \\ & - \frac{\pi}{2}\,\delta^3(\vec{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\,\vec{\sigma}_i \cdot \vec{\sigma}_j}{3\,m_i m_j}\right) \\ & - \frac{1}{4r^3}\left(\frac{1}{m_i^2} + \vec{r} \times \vec{p}_i \,.\, \vec{\sigma}_i - \frac{1}{m_j^2}\,\vec{r} \times \vec{p}_j \,.\, \vec{\sigma}_j\right) \\ & - \frac{1}{2r^3}\,\frac{1}{m_i m_j}\left(\vec{r} \times \vec{p}_i \,.\, \vec{\sigma}_j - \vec{r} \times \vec{p}_j \,.\, \vec{\sigma}_i \right. \\ & \qquad \left. + 3\,\vec{\sigma}_i \,.\, \hat{r}\,\vec{\sigma}_j \,.\, \hat{r} - \vec{\sigma}_i \,.\, \vec{\sigma}_j\right) \end{aligned} \qquad \ldots(77)$$

This gives rise to an effective mass formula for $L = 0$ baryons of the form

$$\begin{aligned} M = {} & M_0 + \sum_i \left[m_i - m_u + a\left(m_i^{-1} - m_u^{-1}\right)\right] \\ & + \sum_{i>j}\left(\alpha\, Q_i\, Q_j - \frac{2}{3}\alpha_s\right)\left[b - \frac{c}{m_i m_j}\right. \\ & \left. - d\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\,\vec{\sigma}_i \,.\, \vec{\sigma}_j}{3\,m_i m_j}\right)\right] \end{aligned} \qquad \ldots(78)$$

where M_0 is the unperturbed mass due to the SU (6) invariant hamiltonian

$$H_0 = H_L(\vec{r}_1, \vec{r}_2, \ldots) + \sum_i \left(m_\mu + \frac{\vec{p}_i^{\,2}}{2m_\mu} \right) \qquad \ldots(79)$$

and (a, b, c, d) are the expectation values of the various symmetry-breaking terms; in particular d is the Fermi-Breit term

$$d = \frac{\pi}{2} < \psi_0 \mid \delta^3(\vec{r}_{12}) \mid \psi_0 > \qquad \ldots(80)$$

Apart from reproducing the well-known sum rules, DGG obtained some specialized relations all crucially dependent on the Fermi-Breit term and hence could be termed typical hyperfine effects. Some examples are

$$\Sigma - \Lambda = \tfrac{2}{3}(1 - m_\mu/m_s)(\Delta - N) \qquad \ldots(81)$$

$$\mu(\Lambda) = -\tfrac{1}{3} m_\mu m_s^{-1} \mu(p) \qquad \ldots(82)$$

$$\frac{K^* - K}{\rho - \pi} + \frac{2(\Sigma^* - \Sigma)}{2\Sigma^* + \Sigma - 3\Lambda} = \frac{m_\mu}{m_s} \qquad \ldots(83)$$

The roles of the spin-orbit and tensor terms were understandably neglected for the ground state hadrons, but are presumably more important for the excited states. DGG also applied this model to the charm sector, but the results were less clean, as illustrated by the rather small prediction

$$\psi - \eta_c \approx 27 \text{ MeV} \qquad \ldots(84)$$

However, the DGG approach should be viewed more from the wider perspective of giving an entirely new and fundamental orientation to the issue of SU (6)-breaking than hitherto evidenced, than from the narrower angle of quantitative agreement with the data as such. In the same spirit, the question of actual values of the parameters (a-d) (78), which would involve a formal commitment to the precise form of the wave function $\mid \psi_0 >$ for a given $q\bar{q}$ or qqq state, was not so relevant as comparisons among certain mass combinations, e.g., 81–83. And in the absence of a formal commitment to the forms of $\mid \psi_0 >$ and other excited states, the nature of mixing brought about by their colour gauge theory remained unanswered at that stage. However, subsequent developments which were to throw light on these questions were considerably conditioned by the h.o. model, insofar as the calculations were made in terms of the oscillator basis functions. Evidence of such efforts is to be found in the work of Gromes (1977) and Reinders (1978) for the qqq system, using a more elaborate classification than DGG, of v^2/c^2-correction terms (Bethe & Salpeter, 1967) to the general two-body kernel which may be symbolically expressed as (Gromes, 1977)

$$\hat{K} = \hat{S}\, 1 \otimes 1 + \hat{P}\, \gamma_5 \otimes \gamma_5 + \hat{V} \gamma_\mu \otimes \gamma_\mu + \hat{A}\, \gamma_\mu \gamma_5 \otimes \gamma_\mu \gamma_5 + \tfrac{1}{2} \hat{T}\, \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \qquad \ldots(85)$$

A more complete investigation on these lines was carried out by Isgur and Karl (1978, 1979) for both non-strange and strange baryons with $L \geqslant 1$ excitations, using the hyperfine and tensor terms of the DGG corrections (77), but neglecting the L.S and momentum-dependent ones. They used an h.o. basis set of wave functions for the detailed predictions. The latter were based on the N. R. Hamiltonian (Isgur & Karl, 1978, 1979)

$$H_{HO} = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{1}{2m'}p_3^2 + \tfrac{1}{2}K(r_{12}^2 + r_{23}^2 + r_{31}^2) \qquad ...(86)$$

effectively "imbedding" in the confining term, the Coulomb part of the one-gluon-exchange q-q interaction. For the strange baryons they allowed for *two* distinct spring constants which would be generated from (86) as a result of $m \neq m'$ (despite the equality of K). They claimed a reasonable pattern of numerical values for the masses of the baryons for both $L = 1$ and $L = 2$ supermultiplets, but with an effective hyperfine interaction strength of ~ 300 MeV, a value much too large for a perturbative calculation to look comfortable. Some formal objections which have been raised (Bhaduri *et al.*, 1980; Stanley & Robson, 1980; and Hey, 1980) on this approach bear essentially on the questionable validity of an N.R. treatment of a basically relativistic system which *light* quarks constitute.

A relativistic treatment of $q\bar{q}$ and qqq systems (Mitra & Santhanam—*to be published*) based on a Bethe-Salpeter (B.S) framework for harmonic confinement with the one-gluon-exchange effects treated perturbatively, is given in the seventh Section. On the other hand, an Isgur-Karl type N.R. treatment for a $qq\bar{q}\bar{q}$ system in its ground state has been given recently by Kalman *et al.* (1980) by imbedding the Coulomb term in the confining h.o. interaction and carrying through the analysis of hyperfine splitting for all the possible baryonium states. We defer discussion on this subject till the 8th Section, where a B.S. treatment for the $qq\bar{q}\bar{q}$ system under harmonic confinement is outlined on lines similar to Section 7, except for the parenthetical remark at this stage that the same type of objection about an N.R. treatment as has been raised for the qqq system (Bhaduri *et al.*, 1980; and Stanley & Robson, 1980) should presumably hold for the $qq\bar{q}\bar{q}$ system as well.

We conclude this section with a brief account of non-harmonic approaches to quark confinement and in particular, a few applications of Coulomb plus linear potentials for heavy quark systems.

Non-Harmonic Models : Bag, Lattice-Gauge

These approaches can be roughly classified under three heads as evidenced from the nature of the activities in these areas :

(*i*) Bag models

(*ii*) Lattice-gauge theories

(*iii*) One-gluon exchange plus linear confinement.

These topics are not quite within the purview of this h.o. oriented review, so only brief indications of their developments are given for comparison.

The MIT bag model (Chados *et al.*, 1974 *a*, *b*; De Grand *et al.*, 1975; Jaffe & Kiskis, 1976; Jaffe, 1977, 1978; Hasenfrantz & Kuti, 1978; and Hasenfrantz *et al.*,

1980) describes the motion of quarks bound in a hadron, in terms of the free Dirac equation inside a region with a boundary called the bag surface. A crucial ingredient in this model is the boundary condition which comprises the following elements.

(*i*) All vector currents vanish at the bag surface.

(*ii*) No energy or momentum crosses the surface.

The model is characterized by a universal constant B (the bag constant), an adjustable but crucial parameter designed to *confine* the quarks as well as the coloured gluons to the inside of the bag. It can be interpreted as a positive potential energy inside the bag, and its presence ensures that an infinite amount of energy is necessary to expand the volume to infinity, which amounts to "freeing" the quarks or gluons. There exist several calculations in which the bag is treated semi-classically (Hasenfrantz & Kuti, 1978; and Hasenfrantz *et al.*, 1980). The decay of a hadron in the bag model is pictured as a "fissioning" of a colour neutral (singlet) bag into two or more bags each of which must be a colour singlet in turn, otherwise colour gluon flux lines would cut across the bag surface, violating the boundary conditions.

The MIT model does not specify any physical mechanism by which the "bag" constant is generated in the first place—it is content with essentially putting it in by hand. An alternative formulation of the bag model—the SLAC bag model (Bardeen *et al.*, 1975)—starts from the (apparently) more fundamental premises of a field theory wherein coloured quarks and gluons are described by appropriate canonical fields treated semi-classically. In the SLAC bag model, there is no bag constant (B) representing extra energy density as such, but the same effect seems to be generated by the structure of the field equations themselves. More specifically, in the scalar coupling theory, the solutions to the field equations in the strong coupling limit, lead to effective bound quark masses which are much lower than the *bare* quark masses which are large. (The situation is reminiscent of the concept of effective nucleon masses in nuclear matter—Bethe, 1956—as distinct from their free counterparts). The "bag" aspect in the SLAC version seems to come about from the field equations themselves, viz., the quarks are confined to a thin-shell of a certain surface characterizing the solution to the equations, *albeit* by a variational method.

The lattice gauge (Jaffe, 1977, 1978) theories ostensibly represent a more fundamental yet tractable approach to the problem of confinement, short of the (as yet intractable) machinery of a full-fledged non-abelian gauge field theory. In a lattice gauge type theory, quarks are supposed to be confined to points on a lattice, the lattice spacing being a free parameter of the theory. The theory contains coloured quarks, as well as "coloured strings". The latter effectively plays the role of the gauge field which behaves like a string with colour at one end and anticolour at the other. The strings are necessary to conserve colour at each lattice site, for otherwise an isolated coloured quark would not be able to move from one site to another. A colour-neutral object like a $q\bar{q}$ attached to a string, or a qqq attached to three strings (in a Y-shaped fashion), can of course move without violating colour conservation at any site, thus facilitating normal hadronic reactions.

The lattice gauge theory has been shown to be asymptotically free in the limit of zero lattice spacing. Further, for sufficiently large coupling, quarks have been shown to be "confinable" with $V(R) \sim R$ in a two-dimensional lattice gauge theory

(Bethe, 1956), but it is not known as yet if a corresponding result holds in the limit when the lattice constant tends to zero, or whether a similar result holds in four dimensions. For details, the reader is referred to Wilson's review (*loc. cit.*)

QCD-cum-Linear Confinement

Another kind of non-harmonic method which has been extensively employed in the literature is an effective potential description with the one-gluon-exchange term motivated directly from QCD, and the confining term partly motivated by 2-dimensional lattice gauge theories (Bethe, 1956) and partly from the elongated "bag" picture. Most of the work on these premises has concerned *heavy meson* systems ($c\bar{c}$, $b\bar{b}$), which warrant two kinds of simplification, (*i*) viability of the N.R. approximation with relativistic corrections up to 0 (v^2/c^2) at most and (*ii*) a wider range of validity of the one-gluon-exchange potential than for light meson systems. Thus to a first approximation one would expect a hydrogen-like spectrum, and the deviations therefrom (which are in practice significant) would represent the effect of the confining interaction.

The calculations have varied from the earliest (Eichten *et al.*, 1975) Schrödinger-like approaches with *ad hoc* non-central terms, to more formal consideration of the latter as v^2/c^2 corrections arising from a B.S. reduction (Bethe & Salpeter, 1967) of the spin-effects due to one-gluon-exchange between two fermion quarks (Barbieri *et al.*, 1976; Billoire & Morel, 1978; and Gromes, 1976). Later, these equations were "improved" through considerations of "dressed" quark-gluon vertices in terms of renormalization group equations, and use of special gauges before resorting to the N.R. approximation (Celmaster & Henyey, 1978). More complete relativistic formulations without resorting to the v^2/c^2 approximation were also made through reduction of the B.S. equations in the instantaneous approximation (Levy, 1952; Salpeter, 1952; and Klein & Zemach, 1957) which would then read as (cf. Henriques *et al.*, 1976).

$$(E - 2W)\,\phi(\vec{q}) = \Lambda^a + \Lambda^b + \beta^a\beta^b \int V(\vec{q}, \vec{q}')\,\phi(\vec{q}')\,\mathrm{d}^3q' \qquad \ldots(87)$$

An interesting though *ad hoc* extrapolation from the Coulomb-region to the confining region was given by Richardson (1979) through the *ansatz* :

$$\tilde{V}(q^2) = -\frac{4}{3}\,\frac{12\pi}{33 - 2n_f}\,\frac{1}{q^2}\,\frac{1}{\ln(1 + q^2)/\Lambda^2)} \qquad \ldots(88)$$

with the standard logarithmic argument (q^2/Λ^2) *augmented* by *unity*. This simple prescription, when transformed to the coordinate space representation and plugged into the N.R. Schrödinger equation gives a rather good description of the heavy $\bar{q}q$ (charmonium, upsilon) spectra. The form of $V(r)$ for small r is "softer" than Coulomb, viz., $\sim r^{-1}\,(\ln r)^{-1}$, while for large r it goes like $\sim r \ln r$. A pure logarithmic potential was proposed by Quigg and Rosner (1978) though the theoretical logic behind such parametrization is not very clear. A detailed review of various quantum mechanical techniques (upper and lower bounds, WKB approximation and so on) with different types of potentials in $q\bar{q}$ systems, may be found in Quigg and Rosner (1979), which also gives a long list of up-to-date references (Martin, 1980; and Willis

et al., 1977). This article as well as Hendry and Litchtenberg (1978) give a fairly detailed account of the numerical results for heavy qq systems. Results with the oscillator model for $q\bar{q}$ systems are less in evidence in these approaches.

For a *relativistic* treatment of $q\bar{q}$ and qqq systems with QCD plus h.o. terms, see § 7.

(§ 4) Quark-Pair-Creation Model (QPC)

The quark-pair-creation (QPC) model which has a direct topological connection with the quark or duality diagrams proposed in the late sixties (Harari, 1969; Rosner, 1969; Micu, 1969; and Carlitz & Kislinger, 1970), represents a big unifying principle designed to relate a very large class of hadronic couplings (involving unexcited *and* excited hadrons) within a single theoretical framework consistent with duality arguments. Though the subject of hadron couplings as such is not within the purview of this article, the subject of QPC nevertheless seems to have a natural relevance in a review whose emphasis is particularly oriented to the oscillator approach, insofar as most of the QPC applications to-date have made essential use of h.o. wave functions. The model was originally conceived in a phenomenological spirit (LOPR, 1973), but was (later) found to admit of a formal field-theoretical justification under conditions of large quark masses (Bohm *et al.*, 1973), a discussion of which aspect is given more fully in (§ 6). In keeping with the general scope of this article, we shall summarize the basic principles which have traditionally governed hadron coupling structures and indicate how QPC fits into this picture and enlarges the scope of applications with a high degree of parametric economy.

Hadron Couplings : RQ vs. QPC

The traditional ingredient for the construction of hadron couplings has been the so-called radiation quantum (RQ) hypothesis in the additive quark model (Becchi & Morpurgo, 1966) wherein one quark constituent of a hadron at a time emits or absorbs 'radiation' (pion, photon, and so on), thus causing the hadron to undergo a transition between two quantum states, in much the same way as one pictures a nuclear transition due to emission or absorption of radiation. Certain powerful dynamical rules, such as the quark recoil effect (cf. Mitra & Ross, 1967), as well as some important selection rules for e.m. transitions (Moorhouse, 1966; and Copley *et al.*, 1969) were discovered through the RQ mechanism. The RQ hypothesis may be employed for a systematic construction of hadron couplings to pions, photons (Van Royen & Weisskopf, 1967) and V-mesons (Mitra, 1973—*unpublished*) (of both 'magnetic' and 'charge' types), in which the transition form factors arise as overlap integrals of the initial and final hadronic wave functions (*loc. cit.*) in an SU (6) x 0 (3) framework. These couplings may also be relativistically adapted (1967) VW, a necessary condition for meaningful hadronic applications. Using a bigger unifying principle which may be termed as chiral SU (3) $\times$ SU (3) or SU $(6)_W$, or better partial symmetry (Schwinger, 1967; and Weisskopf, 1967) it is possible to obtain an SU $(6)_W \times 0$ (3) invariant structure for the entire coupling scheme (Mitra, 1969), wherein the P- and V-meson (charge and magnetic) couplings of a given SU (6) species at the quark level, are calculable in terms of a single overall coupling const-

ant, the necessary connections being expressed, e.g., by the SU (4) meson coupling matrices (Mitra, 1977 *a*; and Mitra & Sood, 1977 *b*).

$$M_{\pm} = \frac{g_\rho}{2m_\rho}\,\bar{q}\left[\left(\partial_i\pi_a \pm \rho^a_{Hi}\right)\sigma_i\tau_a + (\partial_i\eta \pm \omega_{Hi}) + m_\rho\,\rho^a_0\,\tau_a + m_\omega\omega_0\right] \qquad ...(89)$$

The partial symmetry principle has also been extended (Mitra, 1973—*unpublished*; Mitra, 1977 *a*; and Mitra & Sood 1977 *b*) to the couplings of L-excited mesons at the quark level, through a suitable generalization of eqn. (89).

The only parameters in such a description are the so-called supermultiplet form factors (*loc cit.*) $\mathcal{F}_{LL'}(q^2)$ governing transitions between entire supermultiplet states (L, L'). These form factors in turn require for their evaluation, the detailed knowledge of the hadronic wave functions involved, in the absence of which these quantities must be regarded as free parameters. On the other hand, a specific model, such as the h.o. model, has the necessary gradients for an explicit calculation of all these quantities within its own framework.

The RQ hypothesis has been enormously successful in the correlation of data (Mitra, 1973—*unpublished*), with or without a specific model such as the oscillator. Nevertheless, it has certain logical shortcomings insofar as the radiation quantum, despite its $q\bar{q}$ compositeness, is effectively treated as an 'elementary' object, while its other hadronic counterparts that undergo transitions, are treated as quark composite ($q\bar{q}$ or qqq). The anomaly is illustrated through a comparison of the two diagrams (Fig. 1A *vs* 1B) drawn under the RQ and QPC hypotheses respectively. The situation is particularly anomalous for a purely mesonic transition $M \to M + M$, where an RQ diagram fails to bring out the especially symmetric character of the coupling that is revealed by QPC (Fig. 1C).

QPC is best regarded as a rearrangement model, wherein a $q\bar{q}$ pair created anywhere within the hadronic matter mixes with the initial qq or qqq system and gives rise to a final state by a rearrangement process (see Figs. 1B or 1C). In a QPC process, the two members (q and $\bar{q}$) of the vacuum $q\bar{q}$ pair do not suffer any change in their quantum numbers, and of course, they together carry the quantum numbers of the vacuum. It is thus meaningful to speak of a $^3P_0$-pair, as was originally proposed (Micu, 1969), with an SU (3) *singlet* structure, viz.,

$$\phi_0 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{S}S) \qquad ...(90)$$

Further, only connected diagrams like (1D) make physical sense, while those like (1E) do not, as was emphasized by Okubo (1963), Zweig (1964) and justified by Iizuka (1966) from a neutral vector model. This OZI rule is extremely well obeyed by hadronic processes. Also, Carlitz and Kislinger (1970) showed that such diagrams carry information about spin in a very simple way and imply SU $(6)_w$ as a vertex symmetry.

The applicational prospects of QPC were considerably enhanced by a concrete proposal of Le Yaouanc *et al.* (1973) to calculate a QPC matrix element as a direct

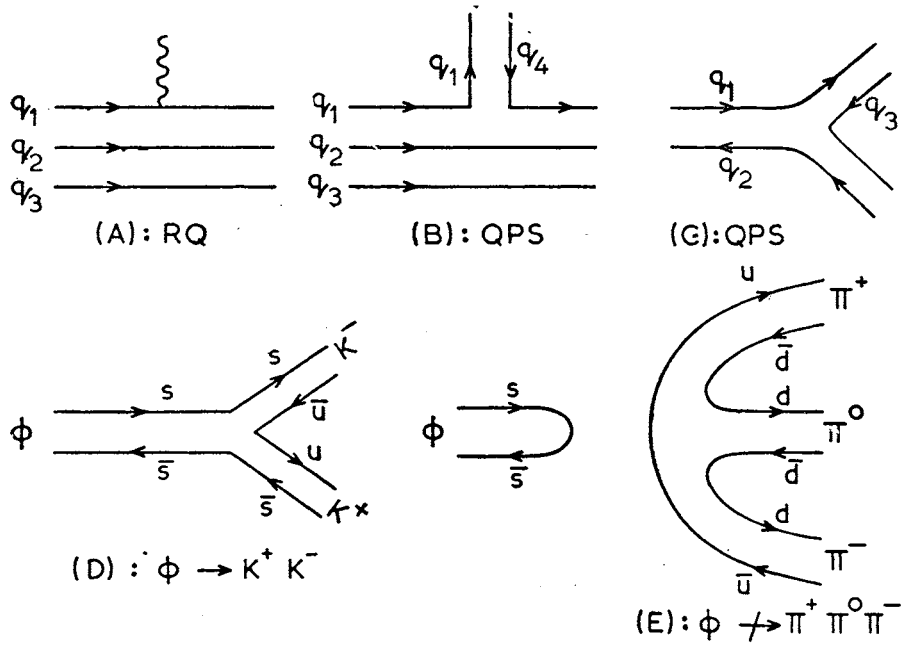

FIG. 1. RQ *versus* QPC diagrams for baryon and meson couplings. (*D*) and (*E*) refer to *OZI* allowed and forbidden processes respectively.

overlap of the (h.o.) wave functions of the three sets of quark lines involved in the diagram under consideration (Fig. 1). The correlative powers of QPC may be expressed by Fig. 2 which is a schematic diagram illustrating the types of couplings predicted by QPC, under one single head. This diagram shows how three distinct

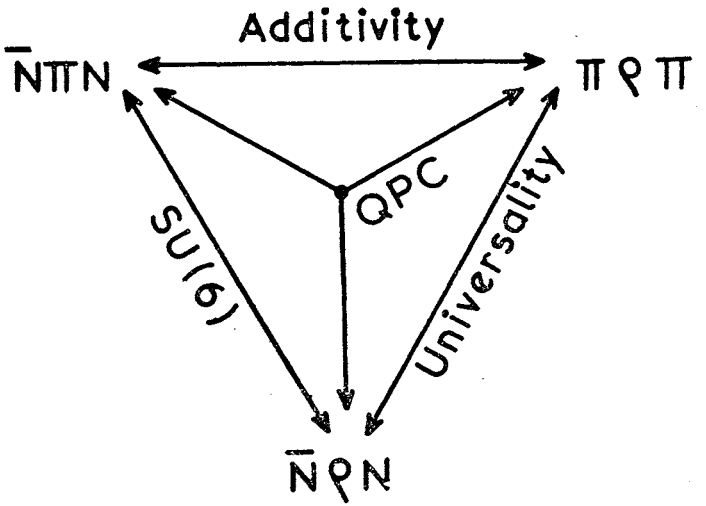

FIG. 2. Schematic diagram (after Le Yaouanc *et al.*, 1973) showing the correlative powers of QPC illustrated by certain coupling connections.

principles of (*i*) universality of *V*-meson couplings (Licht & Pagnamenta, 1970) (which comes about through the "gauge principle", in modern jargon), (*ii*) quark additivity (Becchi & Morpurgo, 1966) and (*iii*) SU (6) or partial symmetry (Schwinger 1967; and Weisskopf, 1967), which individually connect certain coupling constants in pairs, in turn get connected at the deeper QPC level in the manner indicated. More importantly, QPC provides, a formal framework for relating the couplings of arbitrary *L*-excited states with others of $L = 0$ (or $\neq 0$) in terms of an overall coupling constant G and the *known* wave functions $\psi_{A,B,C}$ corresponding to a hadronic process $A \to B + C$ the choice of h.o. wave functions for this purpose is optional, but is the one most extensively employed in the literature.

QPC also determines the strength of the 'recoil' term to that of the 'direct' term (Le Yaouanc *et al.*, 1973). The mechanics of this connection lies in the basic $^3P_0$ structure of the vacuum pair with momenta $\pm \vec{q}_3$ (Fig. 1C) which manifests itself through the combination $\pi \vec{\sigma} \cdot \vec{q}_3$, for P-meson coupling, or the scalar $\vec{\sigma} \cdot \hat{\nu} \times \vec{q}_3$ for *V*-meson coupling, to the adjoining quark lines forming the $\bar{q}q$ current. When expressed in terms of the pion momentum $\vec{k}$ and that of the emitting quark—$\vec{q}_2$, the P-meson operator, e.g., becomes $\pi \vec{\sigma} \cdot (\vec{k} + \vec{q}_2)$, which thus identifies the 'recoil' term as $\vec{\sigma} \cdot \vec{q}_2$ without extra parametrization. There are certain problems (Mitra & Sood, 1977*b*) of compatibility of this description with the so-called GOR effect (Gell-Mann, 1965; Dashen & Gell-Mann, 1966; and Gell-Mann *et al.*, 1968) but these can be sorted out through an alternative coupling formulation in terms of the so-called N_+-matrix (Schwinger, 1967; Weisskopf 1967; and Santhanam *et al.*, 1979).

Hadron Couplings in QPC Model

The QPC model was originally defined directly in terms of the **T**-matrix, viz. (cf. Le Yaouanc *et al.*, 1973)

$$S_{fi} = \delta_{fi} - 2\pi i\, \delta(E_f - E_i)\, \mathbf{T}_{fi} \qquad \ldots(91)$$

$$T = \mathbf{I}_1 \otimes \mathbf{I}_2 \otimes \mathbf{I}_3 \otimes \mathbf{T}_{vac} \qquad \ldots(92)$$

The I's are identity matrices standing for the "quasi-free" propagation of quarks as "spectators", and (see Fig. 1B) with the labels q_i representing also the 3-momenta of the quarks,

$$< \bar{q}_4\, q_5/\mathbf{T}_{vac} \mid 0 > = \delta\,(\vec{q}_4 - \vec{q}_5)\, \gamma \sum_m (-1)^m\, y_1^m\, (\vec{q}_4 + \vec{q}_5)$$

$$\times\, \chi_1^{-m}\, \phi_0 \qquad \ldots(93)$$

The $^3P_0$ structure of the matrix element is evidenced from the scalar product of the spatial (y_1^m) and spin χ_1^{-m} functions, while γ is a dimensionless parameter representing the pair-creation strength. The matrix element of the full operator T between hadronic states is

$$< BC \mid \mathbf{T} \mid A > = \gamma \sum_{m} (-1)^m < \Phi_B \Phi_C \mid \Phi_A \phi_0 \chi_1^{-m} > \mathbf{I}_m (A; BC)$$

...(94)

where Φ are SU (6) wave functions, ϕ_0 = is given by (90) and for mesons (Fig. 1C) :

$$\mathbf{I}_m = \int \prod_{i=1}^{3} d\vec{q}_i \prod_{ABC} \delta (\vec{q}_1 - \vec{q}_2 - \vec{k}_A) \psi_A (\vec{q}_1, \vec{q}_2) y_1^m (2\vec{q}_3) \qquad ...(95)$$

with the normalization (Le Yaouanc *et al.*, 1973)

$$\int d\vec{q}_1 \, d\vec{q}_2 \mid \psi_A (\vec{q}_1, \vec{q}_2) \mid^2 = 1 \qquad ...(96)$$

In the c.m. system ($\vec{q}_A = 0$) for mesons, with $\vec{q} = \vec{q}_2 + \vec{q}_3$

$$\mathbf{I}_m = \frac{1}{8} \delta (\vec{k}_B + \vec{k}_C) \int d\vec{q} \, y_1^m \vec{q} - \vec{k}_B) \; \psi_A(\vec{q} + \vec{k}_B) \; \psi_B(\vec{q}) \; \psi_C(\vec{q}). \qquad ...(97)$$

Similar expressions hold for a baryonic transition, except for two internal momenta. These integrals can be readily evaluated for Gaussian wave functions (Le Yaouanc *et al.*, 1973).

The entire QPC coupling framework described above can be formulated with equal ease in the tensorial language (Mitra & Sood, 1977*b*) which facilitates a more direct comparison with the RQ hypothesis. The necessary connection, which has been explained in Le Yaouanc *et al.* (1973), stems from the recognition that the operator $\vec{\sigma} \cdot \vec{k}$ that refers to the emission of meson C in the RQ language, has the QPC adaptation $\vec{\sigma} \cdot \vec{q}_3 = \vec{\sigma} \cdot (\vec{k} + \vec{q}_2)$ which (*see* Fig. 1C) takes into account the *interplay* of the vacuum *pair* ($\pm \vec{q}_3$) with the quark line q_2 that stands for the "emitting" quark in the hadronic transition $A \to B$(A, B being quark *composites*). For a fuller identification, the *matrix element* of $\vec{\sigma} \cdot (\vec{k} + \vec{q}_2)$ between $\mid A>$ and $\mid B >$ states is just the spin part on the RHS of the full QPC matrix element (94). The considerations for the SU (3) part $\vec{\tau} \cdot \vec{\pi}$ are identical.

Now the tensorial language (Mitra & Sood, 1977*b*) of couplings has the advantage that the effects of successive dynamical inputs (which merely affect the scalar form factors) can be isolated from the more general aspects of the coupling structures which remain unaffected. Since QPC represents a dynamical assumption, its effect is best exhibited in terms of the scalar form factors within the tensorial framework itself. This programme has already been outlined in Mitra & Sood (1977*b*), together with a description of certain QPC applications. To avoid unnecessary duplication, we shall, therefore, skip most of these details except those which are essential for maintaining a reasonable continuity in presentation, and for facilitating the discussion of certain later applications not covered by Mitra & Sood (1977*b*).

Consider the case of an L-excited meson C coupling to two $L = 0$ mesons A and B as in Fig. 1C. The h.o. wave functions in momentum space for mesons of the same species, are (cf. Mitra & Sood, 1977*b*)

$$\psi_{A,B} = N_0 \exp\left[-\tfrac{1}{2}(\vec{\zeta} + \sqrt{2}\,\vec{k}_{B,A})^2/\Omega_{A,B}\right] \qquad ...(98)$$

$$\psi_C = \sqrt{4\pi}\, Y_{LO}(\hat{\zeta})\, \zeta^L N_L e^{-\frac{1}{2}\zeta^2/\Omega_C}$$

$$\equiv \sqrt{p_L}\, \zeta_{i_1} \ldots \zeta_{i_L} \ldots B_{i_L} \ldots B_{i_L}\; N_L\, e^{-\frac{1}{2}\zeta^2/\Omega_C} \qquad ...(99)$$

$$N_L^2 p_L\, L\,! = \pi^{-3/2} 2^L, \quad N_0^2 = \pi^{-3/2} p_L\, L\,! = (2L+1)\,!! \qquad ...(100)$$

Here the quark momenta of Fig. 1C are re expressed (Feynman *et al.*, 1971) as :

$$\vec{q}_2 + \vec{q}_3 = \frac{1}{\sqrt{2}}\vec{\zeta},\ \vec{q}_3 - \vec{q}_2 = \vec{k}_c,\ \vec{q}_1 - \vec{q}_3 = \vec{k}_B,\ \vec{q}_1 - \vec{q}_2 = \vec{k}_A \qquad ...(101)$$

The QPC matrix element, expressed as a matrix in the SU (6) space of A and B mesons, while C plays the role of RQ, is

$$J_{A\to BC} \equiv \sum_m \sigma_m (-1)^m I_m(A\,;\,BC)\,(\vec{\pi}_c \cdot \vec{\tau}) \qquad ...(102)$$

$$= \delta(\vec{k}_A - \vec{k}_B - \vec{k}_C)\, V_{A\to BC}\, \vec{\sigma} \cdot \left(\frac{1}{\sqrt{2}}\vec{\zeta} + \vec{k}_c\right)\left(\vec{\pi}_c \cdot \vec{\tau}_3\right) \qquad ...(103)$$

$$V_{A\to BC} = G \int \mathrm{d}\vec{\zeta}\ \psi_B^*\left(\frac{1}{\sqrt{2}}\vec{\zeta} + \vec{k}_A\right)\ \psi_C^*\left(\frac{1}{\sqrt{2}}\vec{\zeta}\right) \otimes \psi_A\left(\frac{1}{\sqrt{2}}\vec{\zeta} + \vec{k}_B\right) \qquad ...(104)$$

where $2\vec{q}_3 = \frac{1}{\sqrt{2}}\vec{\zeta} + \vec{k}_C$ from Fig. IC and (101), and the connection with (95) of LOPR (1973) is explicit. The SU (6) part of the matrix element, which must now be taken between the SU (6) states of $|A>$ and $|B>$, can be elegantly expressed in the form of a trace (Colglazier & Rosner, 1971) by noting that a P-meson state has a representation of $\frac{1}{\sqrt{2}} I$ and a V-meson of $\frac{1}{\sqrt{2}}\vec{\sigma} \cdot \vec{\epsilon}$ in spin space. Likewise an $I = 1$ or $I = 0$ meson is $\frac{1}{\sqrt{2}}\vec{\pi} \cdot \vec{\tau}$ or $\frac{1}{\sqrt{2}} I$ respectively in isospin space. Thus, e.g., for a $V_A^{I=0} \to \pi_C + V_B^{I=1}$ transition, the full SU (6) matrix element would involve the trace factor

$$\tfrac{1}{2}\,\mathrm{Tr}\left[\vec{\sigma}\cdot\vec{\epsilon}_B\,\vec{\sigma}\cdot\left(\frac{1}{\sqrt{2}}\vec{\zeta} + \vec{k}_C\right)\vec{\sigma}\cdot\vec{\epsilon}_A\right] \otimes \tfrac{1}{2}\,\mathrm{Tr}\,(\vec{\tau}\cdot\vec{\pi}_C)(\vec{\tau}\cdot\vec{\rho}_B) \qquad ...(105)$$

The *orbital* part (104) can in turn be recast in the tensorial form (Mitra & Sood, 1977 *b*).

$$V_{A\to BC} = \bar{k}_{i_1} \ldots \bar{k}_{i_L} B^L_{i_1} \ldots B_{i_L} F_L; \quad 2\bar{k}_i = k_{Ai} + k_{Bi} \qquad \ldots(106)$$

where F_L is a scalar form factor (Mitra &Sood, 1977*b*)

$$F_L = G(2/3)^{L+3/2}\pi^{-3/4} \frac{2^L}{\sqrt{L!}} \exp\left[-\frac{1}{3}\left(\vec{k}_A^2 + \vec{k}_B^2 + \vec{k}_C^2\right)\Big/ \Omega\right] \qquad \ldots(107)$$

in the special case of equal quark mass kinematics, viz.,

$$\Omega_A = \Omega_B = \Omega_C = \Omega \qquad \ldots(108)$$

The tensorial factor in (106) can be combined with the spin matrix element arising from (105), and the direct product expressed as a Clebsch-Gordon series in accordance with the techniques (Fronsdal, 1958) developed in Mitra and Sood (1977*b*). The resultant structure may be recast in a relativistically invariant form for the *external* kinematics in the standard manner explained in Mitra and Sood (1977*b*). The complete structures of several hadron couplings of major experimental interest have also been given (*loc. cit.*) under conditions of partial symmetry, e.g.,

$$\pi_B^\dagger M_L \pi_A; \; \pi_B^\dagger M_L(\rho_A, \omega_A); \; \bar{N}_B M_L(N_A, \Delta_A), \qquad \ldots(109)$$

with an appropriate scalar form factor multiplying each coupling type. The latter can be explicitly identified and calculated under QPC dynamics, as in (104)–(107).

Applications of QPC (N.R.)

The applications of QPC can be classified under two broad heads which involve N. R. and relativistic systems respectively. The N. R. applications are physically valid for intrinsically *heavy* quark systems and do not require any further relativization prescriptions on the scalar form factors F_L, so that the dynamical framework outlined in the foregoing is taken to be directly employed without modification for such applications. These are particularly relevant for $c\bar{c}$ physics where QPC has been extensively employed for the study of various decay processes below and above threshold for the production of charmed mesons. The processes below threshold are mainly radiative transitions of the El and Ml types :

El $\qquad \psi' \to \chi_J \gamma, \quad \chi_J \to \psi\gamma \qquad \ldots(110)$

Ml $\qquad \psi \to \eta_c\gamma, \quad \psi' \to \left(\eta_c, \eta_c'\right)\gamma, \; \eta_c' \to \psi\gamma \qquad \ldots(111)$

These processes are amenable to QPC treatment through a generalized VMD mechanics which amounts to the e. m. substitutions (Schwinger, 1967; Weisskopf, 1967; and Mitra, 1979)

$$\rho_\mu^0, \omega_\mu, \phi_\mu, \psi_\mu \to e\left(g_\rho^{-1}, \tfrac{1}{3} g_\rho^{-1}, -\frac{\sqrt{2}}{3} g_\phi^{-1}, \frac{2\sqrt{2}}{3} g_\psi^{-1}\right) \qquad \ldots(112)$$

To calculate the amplitudes for all such processes, one needs two types of radially excited cc vector wave functions which can couple to a photon:

$$^3S_1(n) : \psi_{ni}(\zeta) = \mathcal{V}_i N_{on} L_n^{1/2}(\zeta^2/\Omega_c) \; e^{-1/2\,\zeta^2/\Omega_c} \qquad \ldots(113)$$

$$^3D_1(n=0): \phi_{0i}(\zeta) = \frac{N_{20}}{\sqrt{15\Omega_c^2}} (2\zeta_i\zeta_j V_j + \zeta^2 V_i)\, e^{-1/2\zeta^2/\Omega_c} \quad ...(114)$$

$$^3D_1(n): \phi_{ni}(\zeta) = \frac{N_{2n}}{N_{20}}\, \phi_{0i}\, L_n^{5/2}(\zeta^2/\Omega_c) \quad ...(115)$$

$$N_{Ln} = (\pi\Omega_c)^{-3/4} [2\Gamma(L+3/2)\Gamma(n+1)/L!\ \Gamma(n+L+3/2)]^{1/2}$$

The parametrization $\Omega_c \approx 2$ GeV^2 and

$$M_n^2(\psi) \approx \Omega_c(2n + L) + \beta_c \quad ...(116)$$

gives an effective description of the first few excited states. These processes have been described in some detail in Mitra and Sood (1977*b*) together with their experimental states and a list of references (Joshi & Mitra, 1978; Barbieri *et al.*, 1975, 1976; Fritzch & Jackson, 1977; Arafune *et al.*, 1977; Chakrabarty, *et al.*, 1977; and Jackson, 1976). The El transitions are fairly well understood, but some of the Ml transitions which were then found to be anomalously large in relation to the data, have since been reconciled to the data after the SLAC crystal ball experiments found the η_c-mass to be only about 100 MeV smaller than the ψ-mass (Bloom, 1979) The El results remain essentially unchanged.

Certain processes amenable to QPC treatment refer to decays of n-excited ψ-states *above* the charm threshold, viz. $\psi''(4030)$ and $\psi'''(4440)$. Since these can decay into DD, $D^*\bar{D}$ etc., states, through the creation of vacuum $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs, one should expect them to have typical strong decay widths ($\sim$100 MeV). Indeed, in their original analysis of charmed mesons, De Rujula, Georgi and Glashow (1976, 1977) had predicted certain geometrical ratios for the decays

$$\psi''(4030) \to D\bar{D};\ D\bar{D}^*;\ +\ \bar{D}D^*;\ \bar{D}^*D^* \quad ...(117)$$

based on relative *spin*-populations of the respective final states involved. Since the experimental data, on the other hand, showed large deviations from these ratios, this fact led Le Yaouanc *et al.* (1977*b*, *c*) to reinvestigate the problem in terms of the QPC model, using for simplicity a *common* radius parameter for the h.o. wave functions of all the states involved (ψ'', D, D^*). They found a remarkably close agreement of the calculated ratios with the observed values, in sharp contrast to the predicted DGG ratios (De Rujula *et al.*, 1976, 1977) and attributed the difference to the dynamical effect of nodal structure in the ψ'' wave function ($n = 2$) on the resulting form factors in relation to the available phase space in each case. While the assumption of equal radii for all the wave functions may by questionable, it is unlikely that a small difference on this count would vitiate their predicted pattern to any significant extent.

A related effect of the nodal structure of n-excited ψ-states seems to show up in the unusually low total width ($\sim$ 30 MeV) of ψ'''. Here again, QPC calculations (Le Yaouanc *et al.*, 1977 *c*) reveal large cancellations in the decay amplitudes which have understandably been attributed to an identical mechanism.

For systems made up of light quarks, it is not clear if the non-relativistic QPC mechanics is meaningful to the same degree, yet several such applications have been

in evidence, especially on systems with small phase space for decays. Some of these which have been reviewed by Mitra and Sood (1977*b*) include the helicity ratios (cf. Le Yaouanc *et al.*, 1973) $B \to \omega\pi$ and $A_1 \to \rho\pi$ which are explained by the QPC model. Very recently, QPC in its original form (Kamath & Mitra, 1978) has been employed for the evaluation of $\bar{N}N$ decay widths from baryonium states regarded as $d\bar{d}$ (d = diquark) composites using a q-d model for the baryon (Ader *et al.*, 1979). This model seems to give considerably smaller widths than those predicted by the more conventional $d\bar{d}$ picture based on the classification "true" and "mock" baryonium states (Chan & Hogaasen, 1977). The structure of baryonium states is more fully discussed in Sec. 8 which contains other references as well.

Most other applications, ordinary [e.g. $B^* \to BP$; $M^* \to MP$] according to Kim and Noz (1973) as well as specialized [e.g., e.m. masses of hadrons (Mitra, 1976*b*, *c*), $\eta \to 3\pi$ decay (Chakrabarty *et al.*, 1977)], have involved *relativistic* assumptions on the scalar form factors in varying degrees. Some of these methods which had been developed up to 1977 have been reviewed earlier (Mitra & Sood, 1977*b*) and will not be discussed further in this article. Instead, we shall outline, in what follows, a fairly comprehensive set of relativistic prescriptions which were formulated subsequent to the earlier review for a unified description of mesonic two and three body processes, radiative and non-radiative, involving mainly light quark systems (Santhanam *et al.*, 1979; Chakrabarty and Chakrabarty *et al.*, 1979).

Applications of QPC (Relativistic)

Most of the relativisation prescriptions on quark-level couplings needed for applications to hadronic processes are detailed by Mitra and Sood (1977*b*) and these are directly amenable to other allied processes as well. However, for a unified coverage of a wider class of phenomena, some of these recipes need to be extended.

The first point concerns the choice of the RQ, for though QPC is *prima facie* symmetrical w.r.t. the choice of RQ, there are nevertheless certain residual aspects of the latter which violate crossing symmetry, especially the relativistic normalization factors needed for the non-RQ mesons (Van Royen & Weisskopf, 1967). This RQ choice is specified by giving precedence to V-mesons ($S = 1$), whether primary ($L = 0$) or L-excited ($J = L + 1$), over P-mesons ($S = 0$). Further, in a process involving more than one V-meson, the RQ is the *lighter* one in an *s-channel* process, or the *exchanged* one in a *t-channel* process. On the other hand, the RQ claim of a *lower-ranking* L-excited V-meson ($J = L$ or $L - 1$) is greatly reduced compared to its $J = L + 1$ partner, thus relatively enhancing the corresponding claim of a P-meson. (Further specifications are sometimes necessary for more specialized applications).

The next ingredient of partial symmetry, viz. (89), remains the same as described by Mitra and Sood (1977*b*), but may be re-expressed more sharply for any (V, P) meson system interacting with a $q\bar{q}$ current (suppressing the flavour indices) as (cf. Mitra, 1979)

$$M_q = \frac{g_V}{2\,m_V}\,\bar{q}\,[-i\,\vec{\sigma}\,.\,\vec{V}\times\vec{k} + m_V V_0 + \vec{\sigma}\,.\,\vec{k}\,P]\,q \qquad \ldots(118)$$

The e.m. interaction for each quark flavour is in turn governed by the substitution rule (111).

The quantity g_V which may be regarded as a "universal" coupling constant for the V-meson carrying a particular "flavour" to *all* the quarks "possessing" the same flavour, must now be determined in the QPC language. To do this, the scalar form factor (106) first need to be "suitably relativised". In the earlier phases of the application (Mitra & Sood, 1977*b*), this was done in a 3-dimensional fashion, but the result was necessarily frame-dependent (cf. Licht & Pagnamenta, 1970). To remove this ambiguity a simple prescription which is relativistically invariant, was recently proposed (*see* Santhanam *et al.*, 1979; and Chakrabarty, 1979), namely

$$\mathrm{Exp}\left[-\frac{1}{3}\sum_{ABC}\vec{k}_A/\Omega\right] \Rightarrow \mathrm{Exp}\left[-\frac{1}{3}\sum_{ABC} k_{A\mu}\, k_{A\mu}/\Omega\right] \qquad ...(119)$$

where $K^2_{A\mu} = -m^2_A$ for an on-shell meson, or $K^2_{C\mu} = -t$ it is *off-shell* (e.g., in the *exchange* channel). This prescription needs to be used with caution, since its range of validity (it certainly has a small N. R. domain of validity to start with) must be severely limited by unitarity. A conservative estimate is $\left| K^2_\mu \right| \gtrsim 1$ GeV^2 which is already adequate for many physical applications. Fortunately, the new prescription (119) which replaces the older, L-P oriented recipe (Mitra & Sood, 1977*b*), leaves exactly unaltered the "good" results already obtained for the hadron e.m. masses (Mitra, 1976*b*, *c*). Using (119) as the scalar form factor the universal ρ-coupling constant carrying the (*ud*) flavour may be related to the calculation of a standard VVV coupling ($\rho\rho\rho$) at the "reference" point

$$m^2_A = m^2_B = m^2_\rho\,,\ k^2_{C\mu} = -t\ (t \to 0) \qquad ...(120)$$

and this leads to the identification

$$g_\rho = 2\, m_\rho\, G''\left(\frac{2}{3\sqrt{\pi}}\right)^{3/2} \exp\left(\frac{2}{3}\, m^2_\rho/\Omega\right) \qquad ...(121)$$

G being the QPC coupling constant for (*ud*) flavoured quarks. Likewise, the universal coupling constant g_ϕ carrying the *s*-flavour may be related to the *s*-quark QPC constant $\bar{G}'$ by considering the standard $K^*K^*\phi$ coupling in this QPC model, with $K^2_\phi \to 0$, leading to the identification

$$g_K = 2\, m_K\, G''\left(\frac{2}{3\sqrt{\pi}}\right)^{3/2} \exp\left(\frac{2}{3}\, m^3_{K^*}/\Omega\right) \qquad ...(122)$$

In a similar way, the coupling constant g_{K^*} may be defined in terms of $K^*\rho K^*$ coupling with one of the K^* masses off-shell at $t = 0$, to yield

$$g_{K^*} = 2\, m_{K^*}\, G''\left(\frac{2}{3\sqrt{\pi}}\right)^{3/2} \exp\frac{1}{3}\left(m^2_P + m^2_{K^*}\right)/\Omega \qquad ...(123)$$

In eqns. (120–133) no distinction has been made for the spring constants of the h.o. wave functions involving (*uds*) quarks. However, the inclusion of heavy (*c*) quarks in the analysis necessitates a different spring constant viz., $\Omega_C \approx 2\,\Omega$, for the ψ-states.

and another $\Omega_D = 1.5\ \Omega$ for the D, D^* states. Then if considerations similar to above are employed for the determination of the coupling constants g_ψ, g_{D^*}, one obtains (cf. Chakrabarty, 1979)

$$g_\psi = 2\, m_\psi\, G_C \left(\frac{2}{3\sqrt{\pi}}\right)^{3/2} \exp\left[\, 2\, m_{D^*}^2 / 2\,\Omega_C + \Omega_D \,\right] \qquad ...(124)$$

from $D^*D^*\psi(t = 0)$ coupling, and

$$g_{D^*} = 2\, m_{D^*}\, G_C' \left(\frac{2}{3\sqrt{\pi}}\right)^{3/2} \exp\left[\, \frac{m_{D^*}^2}{2\,\Omega + \Omega_D} + \frac{m_P^2}{2\Omega + \Omega_\rho^2 / \Omega} \,\right] \qquad ...(125)$$

from $D^*\rho D^*(t = 0)$ coupling, where G_C and G_C' represent the corresponding QPC constants. The 'universality' assumption now consists in the statement

$$g_\rho = g_\rho = g_\phi = g_{K^*} = g_\psi = g_{D^*} \qquad ...(126)$$

instead of the equality of the corresponding QPC constants. Finally for VMD substitution, one employs the *off-shell* photon mass at the value m_V.

For the relativistic normalizations of a generalized VW (1967) type, the prescriptions are identical to those already formulated. The widths of $V \to P\gamma$ or $P \to V\gamma$ are *independent* of g_V, while the amplitudes for $P \to \gamma\gamma$ decays are inversely proportional to g_V. Some sample results are shown in Table I, with $g_\rho = \sqrt{12\pi}$ which yields the following values (in MeV)

$$\Gamma_{\rho \to \pi\pi} = 110,\ \Gamma_{k^* \to k\pi} = 47.5,\ \Gamma_{\phi \to k^+k^-} = 1.94$$

TABLE I

$V \to P\gamma$, $P\gamma \to V\gamma$, $\gamma\gamma$ decays of $L=0$ mesons

$A \to B\gamma$	Γ_{th}(KeV)	Γ_{expt}
$P^\circ \to \pi^\circ\gamma$	42	37 ± 10
$\omega \to \pi^\circ\gamma$	470	$890(600 \pm 200)$
$\phi \to \eta\gamma$	71	66 ± 8
$K^{*\circ} \to K^\circ\gamma$	117	75 ± 35
$K^{*+} \to K^+\gamma$	72	$\lesssim 80$
$\eta' \to \rho^\circ\gamma$	116	105 ± 20
$\pi^\circ \to \gamma\gamma$	4.5×10^{-3}	7.8×10^{-3}
$\eta \to \gamma\gamma$	0.38	0.33 ± 0.05
$\eta' \to \gamma\gamma$	7.3	7.0 ± 0.5

Other applications of the above QPC formalism have been made to certain decays involving charmed mesons, namely

$$\psi' \to D\bar{D}, D^* \to D\pi, D^{*+} \to D^+\gamma$$

Some of the results, together with the data of Bricman *et al.* (1980) are (cf. Chakrabarty, 1979)

$$\Gamma(D^{*0} \to D^0\pi^0)/\Gamma(D^{0*} \to D^0\gamma) = 1.06\,(1.15 \pm 0.20)$$

$$\Gamma(D^{*+} \to D^0\pi^+)/\Gamma(D^{*+} \to D^+\gamma + D^+\pi^0) = 1.30\,(1.5 \pm 0.3)$$

Certain three-body decays are amenable to this formalism, a particularly interesting case being $\eta' \to \eta\pi\pi$ whose experimental value (*see* Bricman *et al.*, 1980) seems to be too "large" compared with the "expected" value based on PCAC constraints which manifest as Adler zeros (*see* Weinberg, 1966). The process is supposed to go through *via* δ and ϵ intermediate states requiring couplings of the types PP' S where S is a *scalar* ($^3P_0$) meson. Recently, it was suggested by Deshpande and Truong (1978) that while PCAC constraints would be satisfied by derivative couplings like $\partial\mu P\, \partial\mu P' S$, these couplings would nevertheless exhibit amplitude enhancement in the *physical* region of $\eta' \to \eta\pi\pi$. Now such coupling structures arise as a natural consequence of the so-called $\mathbf{N}_+$ (negative parity meson) matrix of Partial Symmetry (cf. Schwingar, 1967; and Weisskopf, 1967), defined by

$$\mathbf{N}_+ = \tfrac{1}{2}\, g_\rho\, m_\rho^{-1} \left[(m_\rho + k_{\rho_0})\, \rho_i^a\, \sigma_i + (m_\pi + k_{\pi_0})\, \pi^a \right] \tau_a \qquad \ldots(127)$$

The coupling structures generated by $\mathbf{N}_+$ were recently employed (Santhanam *et al.*, to make a g_ρ dependent (but otherwise parameter free) prediction of the $\eta' \to \eta\pi\pi$ width at the value

$$\Gamma_{\eta' \to \eta\pi\pi} = 265 \text{ KeV} \qquad \ldots(128)$$

which, together with the $\eta' \to \gamma\gamma$, $\rho^0\gamma$ width (Table I) make up $\Gamma_{\text{tot}} = 388$ KeV, in general agreement with data (Bricman *et al.*, 1980), as well as in excellent agreement on the branching ratios for all the above modes. The same $\mathbf{N}_+$ mechanics gives $\Gamma_{B \to \omega\pi} = 132$ MeV (125 ± 10) as well. The derivative coupling structure which is common to all these cases is a general feature of coupling structures revealed by the $\mathbf{N}_+$ matrix whenever in a PPV coupling at least one of the mesons is L-excited and the transition occurs in a *lower* wave than the allowed maximum. For details, see Santhanam *et al.* (1979).

To conclude this section, we have tried to outline a mere cross-section of the rich applicational potential of QPC in association with the h.o. model. Some of the applications do indeed show the possibilities of non-trivial quantitative predictions with little or no parametrization. Yet, if the depth and breadth of applications of such a simple and attractive model have so far been rather limited, it is presumably because of a lack of adequate theoretical rationale despite its topological appeal and its obvious connection with duality. Certain attempts at field-theoretic justifications for QPC are questionable, but have nevertheless produced some very interesting applications (Misra *et al.*, 1980).

The Bethe-Salpeter framework in the Wick-rotated formulation indicates a topological justification in the limit of large quark masses (Bohm *et al.*, 1973) but such a formulation suffers from other practical problems. Nevertheless, Joos and

collaborators have applied the BJK model to a number of physically interesting processes (Bohm & Kramer, 1974, 1975, and Joos *et al.*, 1974), ($\omega \to \pi^0\gamma$, pion e.m. form factor and strong decays) in an essentially QPC spirit, mainly to illustrate the methods of application, and obtained certain qualitatively correct results. Kim and Noz have also employed QPC techniques in a 4-dimensional h.o. form to strong decays of hadrons (Kim & Noz, 1973, 1975). The details of the Kim-Noz and BJK models are discussed in (§5) and (§6) respectively.

To summarize, QPC-cum-H.O. has proved a most valuable phenomenological tool for first hand studies of many hadronic processes looked upon as planar, few-quark systems, and thus have brought out yet another dimension to the historical role of the harmonic oscillator in the development of quark physics. The subsequent sections are oriented to more formal theoretical rationales for the oscillator model, though QPC as such remains largely at the effective phenomenological level.

§ Theoretical Rationales for the Relativistic Oscillator

The intuitive approach of FKR (1971) to the construction from an essentially phenomenological level, of a relativistic harmonic oscillator model and its instant quantitative impact on the understanding of diverse hadronic data stimulated certain attempts at finding a better rationale for such relativistic dynamics in terms of systems with a *finite* number of degrees of freedom instead of using the full-fledged machinery of field theory with infinite number of degrees of freedom. The main problems in this regard have understandably been (*i*) the effects of Lorentz contraction of wave functions in moving frames and (*ii*) the "unwelcome" feature of time-like excitations, especially their effects on the normalization of the wave functions. The intuitive attempts of Fujimura *et al.* (1970) as well as of Licht-Pagnamenta (1970) and its subsequent modification (Mitra & Kumari, 1977) discussed in Sec. 2, are particularly relevant in this context, insofar as they serve as effective guiding principles to the shape of things to expect from these non-field theoretic approaches.

The central issue rests on (*i*) how to view the harmonic oscillator within a relativistic framework and (*ii*) what constitutes a minimal basis for a relativistic framework. The latter question which is even more fundamental than the former is especially relevant in the context of various attempts on 3-dimensional relativistic formulations in preference to the more natural 4-dimensional ones, characterizing, e.g., the Bethe-Salpeter form of dynamics. In this relatively narrow review which deals mainly with the role of the oscillator model in quark physics, we shall be concerned with this question only in the limited h.o. context, and not in its full generality. In particular, we shall consider the main features of *two* related approaches to the covariant h.o. problems, one due to Kim-Noz (1973, 1976) and another due to Leutwyler and Stern (1978*a*,*b*).

The theoretical motivation for the 4-dimensional h.o. as a possible model for strong interaction dynamics dates back to 1953 when Yukawa (1953), in a fundamental paper, proposed an oscillator-like Hamiltonian on the basis of Born's reciprocity relation between coordinates and momenta. However, the covariance requirement forced him to introduce time-like excitations with negative energies (as also done later by FKR in 1971). In order to eliminate this undesirable feature,

Yukawa introduced coupling with an external 4-momeneum P_μ, leading to the two-body wave function

$$\psi(z, P) = \text{Exp}\{-\tfrac{1}{2}\Omega\,(z^2 + 2\,(P.z)^2 M^{-2})\} \qquad ...(129)$$

in the FKR (1971) normalization, which agrees exactly with the FKN (1970) wave function, eqn. (38) of (§2).

Kim-Noz Model

The main trick in this approach is to express the 3-dimensional h.o. wave functions in the hyperplane perpendicular to the total 4-momentum P_μ, so that the coordinates y_μ in this plane are related to the general 4-coordinates z_μ by

$$y_\mu = \theta_{\mu\nu} z_\nu,\ \theta_{\mu\nu} = \delta_{\mu\nu} + P_\mu P_\nu M^{-2} \qquad ..(130)$$

$$P_\mu y_\mu\ 0,\ y_\mu^2 = z_\mu^2 + (P.z)^2 M^{-2} \qquad ...(131)$$

In particular if P_μ is in the z_3-direction,

$$y_\mu^2 = y_1^2 + y_2^2 + y_3^2 \qquad ..(132)$$

where

$$y_1 = z_1,\ y_2 = z_2,\ \beta = P_3/P_0 \qquad ...(133)$$

$$y_3 = \gamma(z_3 - \beta z_0),\ \gamma^2(1 - \beta^2) = 1 \qquad ...(134)$$

Thus the ground state h.o. wave function in this hyperplane is

$$\phi\,(z, \beta) = \text{Exp}\left\{-\tfrac{1}{2}\Omega\left(y_\mu^2 + y_0^2\right)\right\} \qquad ...(135)$$

where

$$y_0 = \gamma(z_0 - \beta z_3) = -P.z/M \qquad ...(136)$$

This expression is still different from (129) on two counts : (*i*) the $(P.z)^2/M^2$ term appears in (129) and (135) with coefficients 2 and 1 respectively; and (*ii*) eqn. (129) is integrated over d^4z, while eqn. (135) has only d^3y for its phase space.

Now if eqn. (135) is multiplied with $e^{-\frac{1}{2}\Omega y_0^2}$, it does *not* change the character of the hyperplane oscillator since the variable y_0 is perpendicular to the 3 hyperplane variables y_1, y_2, y_3, and indeed an integration over the *extra variable* y_0 *restores the* form (135). On the other hand, the extra factor $e^{-\frac{1}{2}\Omega y_0^2}$ not only renders (135) exactly equal to (129), but now the phase space of (135) increases from d^3y to d^4y which is equal to, and as invariant as, d^4z of the Yukawa form (129).

Similarly, one can write down the corresponding excited wave functions belonging to the same hyperplane in the ψ – and ϕ – representations, the connection of their inner products being

$$<\psi_n, \psi_m> = \int e^{-\Omega y_0^2}\,\phi_n^*\,\phi_m\,d^4z \qquad ...(137)$$

The essential point in this picture is that the entire 4-space integration be performed, thus giving an essential role to the time-like variable y_0 or equivalently to z_0. Thus

the factor $e^{-\frac{1}{2}\Omega y_0^2}$ is not merely a universal weight factor common to all wave functions for purposes of inner products like (137), but must be regarded as intrinsic to *each* hadron concerned, carrying with it information on its own 4-momentum P_μ, in the sense of (129). Thus for two wave functions defined in two different hyperplanes corresponding to the 4-momenta $P_{1\mu}$ and $P_{2\mu}$, the inner product is

$$<\psi_1, \psi_2> = \int \mathrm{Exp}\left[-\tfrac{1}{2}\Omega\,(P_1 z)^2/m_1^2 - \tfrac{1}{2}\,\Omega\,(P_2 \cdot z)^2\, m_2^2 \right] \mathrm{d}^4 z$$

$$\times\, \phi_1^*(z, \beta_1)\, \phi_2(z, \beta_2) \qquad \ldots(138)$$

In particular, if this quantity is calculated in the *Breit* frame for the initial and final wave functions, one recovers exactly the Lipes form factor (41) based on the FKN (1970) wave functions. The Licht-Pagnamenta form (42) or its modification (47) on the other hand, which is intrinsically *3-dimensional* in character corresponds to the "instantaneous approximation", $t = z_0 = 0$ in the corresponding overlap integral.

To recapitulate the main points of the Kim-Noz model, they express to FKR equation for $q\bar{q}$, viz.,

$$\left(-\tfrac{1}{2}\,\Box_z^2 + \tfrac{1}{2}\,\Omega^2 z^2 \right) \psi(P, z) = \lambda\, \psi(P, z) \qquad \ldots(139)$$

in the hyperplane coordinates (y_1, y_2, y_3, y_0) defined in eqns. (133–136), and obtain the excitation as

$$-P^2 = M^2 = \Omega\,(n_1 + n_2 + n_3) + C \qquad \ldots(140)$$

corresponding to the wave functions

$$\psi_n(P, z) = N \prod_{i=1}^{3} H_{n_i}(y_i)\, \psi_0 \qquad \ldots(141)$$

$$\psi_0 = \mathrm{Exp}\left[-\tfrac{1}{2}\,\Omega \left(y_1^2 + y_2^2 + y_3^2 + y_0^2 \right) \right]$$

$$= \mathrm{Exp}\left[-\tfrac{1}{2}\, z^2 - \frac{(zP)^2}{M^2} \right] \Omega \qquad \ldots(142)$$

Their elimination of time-like excitations in (141) follows exactly the FKR pattern, viz.,

$$P_\mu\, (\Omega\, z_\mu - \partial/\partial z_\mu)\, \psi\,(P, z) = 0 \qquad \ldots(143)$$

The orthogonality relation between two wave functions ψ_n, ψ_m in two *different* hyperplanes corresponding to β_1 and β_2 may be straight-forwardly worked out from (138) and (141) as

$$\int \psi_n^*\,(P, z)\, \psi_m(P, z)\, \mathrm{d}^4 z = \prod_{i=1}^{3} \delta_{n_i\, m_i} \left(1 - \beta_{12}^2 \right)^{\frac{1}{2}(n_3 + 1)} \qquad \ldots(144)$$

$$\beta_{12} = (\beta_1 - \beta_2)/(1 - \beta_1\beta_2) \qquad \ldots(145)$$

The limiting form of eqn. (142) in the infinite momentum frame ($P_3 \to \alpha$) becomes

$$\psi_0 \approx e^{-\frac{1}{2}\Omega(z_1^2 + z_2^2)} \exp\left[-\frac{\Omega}{16}\left(\frac{M}{P_3}\right)^2 (z_0 + z_3)^2 - \Omega\left(\frac{P_3}{M}\right)^2 (z_0 - z_3)^2\right]$$

Thus the $(z_0 + z_3)$ dependence is rather insensitive, but the $(z_0 - z_3)$ dependence which is controlled by the last factor, becomes a δ-function in $(z_0 - z_3)$, for

$$\text{Exp}\left[-\Omega\left(\frac{P_3}{M}\right)^2 (z_0 - z_3)^2\right] \approx \frac{\sqrt{\pi}}{\Omega}\frac{M}{P_3}\delta(z_0 - z_3) \qquad ...(146)$$

Such δ-function behaviour in the $P_3 \to \infty$ limit seems to be very similar to what obtains also for the Bethe-Salpeter (BS) equation corresponding to a Yukawa-interaction, whose Green's function G for $\beta \to 1$ takes the form

$$G(z, P) \approx \delta(z_0 - \beta z_3)\, K_0(\mu R)/8\pi P_0$$

$$R^2 = z_1^2 + z_2^2 + (1 - \beta^2)\, z_3^2 \qquad ...(147)$$

The detailed comparison of the two wave functions has been given in a recent Kim-Noz paper. The main point seems to concern the crucial role of the relative time-like variable on the respective wave functions. While in the B. S. formalism, this variable stands in the way of a simple probabilistic interpretation to the wave function, a corresponding interpretation, at least in the sense of eqn. (144), still seems to go through in the relativistic h.o. case. However, judged from a more fundamental point of view, as distinct from an effective description, any exaggerated theoretical claim bearing on the very foundations of quantum mechanics must be guarded against. In particular, despite the crucial role of the variable y_0 (or z_0) on the probability interpretation, the very attempt, represented by eqn. (143), to eliminate the "unphysical" time-like excitations after FKR) formally destroys the mathematical unitarity of the group representation (Celmaster & Henyey, 1978; and Martin, 1980). The same defect is of course, contained in FKR as well, but they are well aware of this fact and do not make any theoretical claims beyond their "Shadow of Truth" statement. Nevertheless, the Kim-Noz formalism represents an important effort to "justify" the h.o. model well beyond the simplest N. R. premises.

Kim and Noz (1973) have also carried out several hadronic applications of their formalism, especially the quark-pair-creation (QPC) model for hadronic decays and structure functions for deep-inelastic electron scattering. However, in most of these the role of the time-like variable in calculations appears to be marginal, and had in fact been obtained earlier by Le Yaouanc *et al.* (1973, 1975*a*) either in terms of their non-relativistic formulation of QPC for hadron decays, or their 3-dimensional Lorentz-boosts for structure functions (See § 4 for further details).

Leutwyler-Stern Covariance Model

Using an ingenuous approach based on Hamiltonian dynamics for investigating the properties of relativistic systems with a *finite* number of degrees of freedom (*not*

full-fledged field theory), Leutwyler and Stern (1978*a*,*b*) have established a general criterion of "covariance" in the form of an (operator) relation among the "mass" and "spin" operators for the system, and found the FKR (1971) confinement model to provide a non-trivial solution, as an interesting alternative to other possible ones based on local field theory. Without going into details for which the interested reader is referred to the original papers (Leutwyeler & Stern, 1978*a*, *b*) we summarize below the essential idea as well as the main steps of the L-S argument, leading to the desired result.

The Hilbert space of a Lorentz invariant quantum theory must carry a unitary representation of the Poincaré group whose generators determine how, e.g., a state prepared at a given time, *evolves* with time. This is the typical *Hamiltonian* approach which can be (but has hardly been) profitably pursued, independently of the bigger issues of quantum field theory. Indeed more than 3 decades ago, Dirac (1949) pointed out that a relativistically invariant Hamiltonian theory can be based on different classes of initial surfaces Σ : space-like planes (z_0 = const.), null planes ($z_3 = z_0$) or hyperboloids ($z^2 = -a^2 < 0$; $z_0 > 0$). The structure of the theory is strongly dependent on these forms of relativistic dynamics. Now, out of the 10 generators (P_μ, $M_{\mu\nu}$) of the Poincaré group, those that leave the initial surface *invariant* (i.e. generate the *stability* group of this surface) are "kinematical" quantities, i.e., their representations are independent of the dynamical properties of the system. The remaining ones are the "hamiltonians" of the theory containing information about the specific dynamical characteristics of the system. The stability groups for the three surfaces Σ above are 6, 7, 6 respectively, thus giving the highest "score" (7) to the null-plane dynamics ($z_0 = z_3$) whose "kinematical" generators are the following :

$$\mathcal{K} = (P_1, P_2, P_+, E_1, E_2, K_3, J_3) \qquad \ldots(148)$$

$$2P_\pm = P_0 \pm P_3, \quad 2E_1 = K_1 + J_2, \; 2E_2 = K_2 - J_1 \qquad \ldots(149)$$

$$J_i = \tfrac{1}{2}\,\epsilon_{ijk}\, M_{jk}, \; K_i = M_{bi} \qquad \ldots(150)$$

The elements (148) form a closed algebra :

$$\begin{aligned} &[K_3, E_r] = -iE_r, \qquad [K_3, P_+] = -iP_+ \\ &[J_3, E_r] = i\epsilon_{rs}\, E_s, \qquad [J_3, P_r] = i\epsilon_{rs}\, P_s \\ &[E_r, P_s] = -i\delta_{rs}\, P_+ \; ; \quad (r, s = 1, 2) \end{aligned} \qquad \ldots(151)$$

On the other hand, there are 3 "Hamiltonian" or dynamical elements which can be chosen in one of several different ways. For this purpose, it is necessary to introduce certain 'rotation' or 'spin' operators $\mathcal{J}_i$ defined by $\mathcal{J}_i \mid n > = J_i \mid n >$ on a *rest* system $\mid n >$, but extended to states in motion defined by

$$\mid \vec{p}, n > \; = \mathrm{Exp}\,(-i\beta_1 E_1 - i\beta_2 E_2 - i\beta_3 K_3) \mid n > \qquad \ldots(152)$$

$$\beta_r = p_r/p_+, \; \beta_3 = \ln\,(2\,p_+/M) \qquad \ldots(153)$$

such that $\vec{\mathcal{J}}$ commutes with the algebra $\mathcal{K}$. More explicitly,

$$\mathcal{J}_i \mid \vec{p}, n > = \text{Exp}\,[-i\beta_1 E_1 - i\beta_2 E_2 - i\beta_3 K_3]\, J_i \mid n > \quad \ldots(154)$$

with

$$[\mathcal{J}_i, \mathcal{J}_j] = i\epsilon_{ijk} I_k \quad \ldots(155)$$

$$[M, \mathcal{J}_i] = 0 \quad \ldots(156)$$

In particular,

$$\mathcal{J}_3 = J_3 + (E_1 P_2 - E_2 P_1)/P_+ \quad \ldots(157)$$

$$\equiv (W_0 + W_3)/P_+ \quad \ldots(158)$$

where

$$W_\mu = \tfrac{1}{2}\,\epsilon_{\mu\nu\alpha\beta} P^\nu M^{\alpha\beta} \quad \ldots(159)$$

is the Pauli-Lubanski relativistic spin operator, and

$$M\mathcal{J}_r = W_r - P_r W_+/P_+, \quad (r = 1, 2) \quad \ldots(160)$$

One thus arrives at the 'dynamical' group

$$D = \{M, \mathcal{J}_i\}, \qquad \text{or} \qquad \{M^2, M\mathcal{J}_i\} \quad \ldots(161)$$

which has the structure of U (2) because of (156). [Of these members or D, $\mathcal{J}_3$ really belongs to $\mathcal{K}$ because of (157). For a physical idea of the significance of $\vec{\mathcal{J}}$, an appeal to the structure of the N.R. Galilei-invariant algebra generated by the momentum $\vec{P}$ and Galilei boosts $\vec{K}$, yields

$$\vec{\mathcal{J}} = \vec{J} + m^{-1}\vec{K} \times \vec{P} \quad \ldots(162)$$

Now Galilei invariance of this system is equivalent to the condition that the corresponding dynamical algebras constitute a unitary representation of U(2). In the relativistic case, there is a superficial similarity to the U (2) structure of D, but unlike the N. R. case, only $\mathcal{J}_3$ is now 'kinematical' by virtue of (157) while $\mathcal{J}_{1,2}$ are 'dynamical' in origin and do not have explicit representations by themselves. This gap is sought to be bridged in the L-S approach through the *covariance* requirement which goes beyond mere Lorentz invariance and implies an *angular condition* for the operators $\mathcal{J}_i$ in the form

$$z_1 M\mathcal{J}_1 + z_2 M\mathcal{J}_2 + z_L \mathcal{J}_3 = 0 \quad \ldots(163)$$

$$z_L = p_1 z_1 + p_2 z_2 + p_+ z_+ \quad \ldots(164)$$

which can be shown to be valid on the null plane $z_- = 0$.

A distinctive feature of the L-S formalism is a *compact support* for the longitudinal part of the relative 4-momentum for a composite two-body system (e.g., $q\bar{q}$). Such a system is characterized by the requirement that under the action of the stability group it transforms like the direct product of two unitary representations corresponding to the two elementary sub-systems. Thus, for a direct product representation of the two constituents, one has

$$\vec{P} + \vec{P}' + \vec{P}'', \quad J_3 = J_3' + J_3'' \quad \ldots(165)$$

and likewise for the boosts E_T and K_3. In particular, the transverse and longitudinal components of the internal momentum operator

$$\vec{k}_T = \left(P''_+ \vec{P}'_T - P'_+ \vec{P}''_T \right) / P_+ \qquad ...(166)$$

$$K_L = \left(P'_+ - P''_+ \right) / 2P_+ \qquad ...(167)$$

are invariant under the action of the kinematical algebra $\mathcal{K}$. The *positivity* of the two operators P'_+, P''_+ in turn implies from (167) the compactness condition for the support of the variable k_L

$$-\tfrac{1}{2} < k_L < +\tfrac{1}{2} \qquad ...(168)$$

The *internal* wave function ϕ defined by

$$< \vec{p}', \vec{p}'' \mid \vec{p}, \phi > = (2\pi^3 2) p_+ \delta^3(\vec{p}' + \vec{p}'' - \vec{p})\, \phi(\vec{K}_T, K_L) \qquad ...(169)$$

where ϕ is a matrix in helicity (h', h'') space and the small letters denote the eigenvalues of the corresponding momentum operators, has the norm

$$< \phi \mid \phi > = \tfrac{1}{2}(2\pi)^{-3} \int \frac{d^2\vec{k}_T dk_L}{\frac{1}{4} - k_L^2} \sum_{h'h''} \left| \phi_{h'h''}(\vec{k}_T, k_L) \right|^2 \qquad ...(170)$$

the measure in (170) is related to the internal 3-momentum $\vec{k}$ of two equal mass (m_0) constituents by

$$\vec{k} = (k_1, k_2, Mk_L) \qquad ...(171)$$

$$\vec{L} = -i\vec{k} \times \vec{\nabla}_k, \quad M^2 = 4(m_0^2 + \vec{k}^2) \qquad ...(172)$$

$$\frac{d^2\vec{k}_T dk_L}{\frac{1}{4} - k_L^2} = \frac{4d^3\vec{k}}{M} \qquad ...(173)$$

FKR Model and Covariance Conditions

For the FKR equations (139) and (143), with $\lambda = \frac{1}{4} M^2 - m_0^2$ L-S take the "natural" solution for the ground state wave function

$$\Phi_0(z) = e^{-\frac{1}{2}\Omega z_\mu^2}, \quad z_\mu^2 = \vec{z}^2 - z_0^2 \qquad ...(174)$$

despite its unnormalizability, and the excitations

$$M_n^2 = 8\Omega(n_1 + n_2 + n_3 + 2) + 4m_0^2 \qquad ...(175)$$

described by the wave functions

$$\Phi_n(z) = \prod_{i=1}^{3} H_{n_i}(\sqrt{\Omega}\, z)\, \Phi_0 \qquad ...(176)$$

The contrast of the solution Φ_0 from the FKN or K-N wave function (129) may be noted. In the null-plane formalism, the FKR equation reads as

$$\left(\frac{1}{4} + \nabla_L^2\right) M^2 \phi = \left[- \nabla_T^2 + \Omega^2 z_T^2 + 2\,\Omega\, \nabla_L z_L + m_0^2 \right] \phi \quad ...(177)$$

where ∇_L, $\vec{\nabla}_T$ $(= \nabla_{1,2})$ denote partial derivatives w.r.t. $z_L = \vec{P}.\vec{z}$ and $\vec{z}_T$ respectively.

The null-plane eigenstates of (177) are

$$\phi_n(\vec{z}_T, z_L) = \prod_{i=1,2} H_{n_i}(z_i\sqrt{\Omega})\, H_{n_3}\left(\frac{\sqrt{\Omega}\, z_L}{M_n}\right) e^{-\frac{1}{2}\Omega z_T^2} \quad ...(178)$$

Eqn. (177) implicitly defines the M^2 operator in terms of $\vec{z}$ and $\vec{\nabla}_Z$, while the "spin" operators I_i are given by

$$M\,\mathcal{J}_r = i\epsilon_{rs}\,(z_L\,\nabla_S - z_S\,\nabla_L\,M^2 + \Omega\, z_S z_L \quad ...(179)$$

$$\mathcal{J}_3 = -\,i\,(z_1\nabla_2 - z_2\nabla_1) \quad ...(180)$$

after using the constraint (143).

These expressions explicitly obey the angular condition (163), thus verifying the FKR model as a possible solution of the L-S criterion of covariance under conditions of null plane dynamics for two spin-0 constituents. There remains the requirement that the Hamiltonians are self-adjoint in a suitable norm in relation to the question of whether the null plane eigenstates (178) constitute a *complete* set of functions with the *orthogonality* property satisfied for different values of M^2, $\vec{\mathcal{J}}^2$, $\mathcal{J}_3$. A "natural" norm with this property is one in which the integral is taken in the *rest* frame of the system over z_1, z_2, $z_3 = z_L/M_n$, with the weight factor $e^{-\Omega z_3^2}$. Comparing this statement with the corresponding statements in the FKN or Kim-Noz formulations, the effective L-S wave function, taking account of the *square root* of the norm factor $e^{-\Omega z_3^2}$, is

$$\psi_{L\text{-}S} = \psi_{FKN}\ \exp\left[+ \frac{1}{2}\,\Omega\,\frac{(z\,.\,P)^2}{M^2}\right] \quad ...(181)$$

However, there is a difference in the philosophy of L-S from that of K-N. In the L-S formulation, the weight factor modulating the phase space is *common* to all wave functions, while in the K-N scheme the corresponding quantity is an *integral part* of the wave function (and hence depends on the *frame* of reference). Moreover, the integrals in the L-S formalism are only 3-dimensional, while those in the K-N scheme are intrinsically 4-dimensional in content.

Leutwyler and Stern (1978*a*) investigated not only the sufficiency of the angular condition for a covariant FKR-like oscillator model, but also its *necessity* up to a point. For the latter purpose, they were led to investigate the structure of a *pair* of covariant equations which yield a manifestly covariant representation more naturally than does a single equation (by the FKR analogy). Specifically they considered the pair

$$[-\Box^2 + U(z)]\,\phi(z) = \tfrac{1}{4} M^2\,\phi(z) \qquad \ldots(182)$$

$$p_\mu \partial_\mu \phi(z) = V(z)\,\phi(z) \qquad \ldots(183)$$

the FKR model amounting to the values

$$U(z) = \Omega\ z^2 + m_0^2\,,\ V(z) = \Omega p_\mu z_\mu \qquad \ldots(184)$$

Imposing the angular condition, they were led to the general solutions :

$$V = \pm\,[a(p\ .\ z)^2 + b]^{1/2},\ U = az^2 - V' + c \qquad \ldots(185)$$

For $b = 0$, one gets the oscillator model, the $\pm$ signs in V corssesponding to the FKR (1971) and FKN (1970) models respectively. For $b \neq 0$, the motion is still harmonic in the transverse direction, but anharmonic in the longitudinal one.

Other solutions of the angular condition considered by L-S (Leutwyler & Stern, 1978 *a*) are in the domain of *local field theory* with Bethe-Salpeter (B-S) amplitudes as the starting point. These are, however, beyond the scope of this review. However, using the B-S language they were able to develop a suitable generalization of eqn. (182–183) so as to incorporate the spin of the quark.

The latter equations are of the form (cf. Leutwyler & Stern, 1978*b*)

$$\gamma_\mu\,\partial_\mu^x\ \psi(x, y) = U\,\psi(x, y) \qquad \ldots(186)$$

$$\partial_\mu^y\ \psi(x, y)\,\gamma^\mu = V\,\psi(x, y) \qquad \ldots(187)$$

where the wave function $\psi(x, y)$ is a 4×4 matrix or a B-S like amplitude of the form $< 0 \mid q(x)\,\bar{q}(y) \mid p_\mu, n >$ so that translational invariance would imply

$$\psi(x, y) = \mathrm{Exp}\,[\tfrac{1}{2}\,ip\ .\ (x + y)]\,\psi(z, p_\mu) \qquad \ldots(188)$$

Considerations similar to, but more involved than the scalar case led them to a harmonic confinement model of $q\bar{q}$ pairs, and a realization of a spontaneously broken chiral symmetry. Without going into details, the most important result of their investigation was the appearance of a hyperfine splitting term (spin-spin interaction) in the (mass)^2 spectrum of mesons with a strength determined by the same universal slope which characterises linear Regge trajectories. In the N. R. approximation, the content of this important result may be expressed by the Hamiltonian

$$m_0 H_{\mathrm{NR}} = -\nabla_z^2 + \lambda^2 z^2 + 2\lambda\ \vec{S}_1\ .\ \vec{S}_2 + \tfrac{1}{2}\lambda \qquad \ldots(189)$$

This model had some successes in the light quark (u, d, s) region, but predicted too large ψ-η_c splittings which QCD corrections (Jersak *et al.*, 1978) did not improve. Though it was not pursued much further at the applicational level, its theoretical significance in providing a strong rationale for oscillator-like models can hardly be overestimated.

(§6) Bethe-Salpeter (BS) Approach to H.O.

In §5, we discussed certain theoretical attempts to 'rationalize' FKR-like (Feynman *et al.*, 1971) relativistic h.o. formulations for quark confinement within certain quantum mechanical frameworks dealing with a *finite* number of degrees of freedom, but stopped short of making use of the full-fledged machinery of field theory. This section is intended for comparative discussion of attempts to formulate the confinement problem within the more formal field-theoretic framework, and also to check its performance at the *observational* level. There are two main aspects to the issue, the first one obviously concerning the basic language of dynamics and the second dealing with the more difficult question of whether confinement itself can at all come about from the more fundamental premises of a universally acceptable theory at the present stage of development. The answer to the first question lies in the Bethe-Salpeter (BS) framework (Salpeter & Bethe, 1951) which arises most naturally from the premises of quantum field theory (cf. Gell-Mann & Low, 1950). As to the second question, even the most attractive theory of strong interactions at the moment viz., QCD, is as yet unable to generate a confinement mechanism (a long distance or infrared phenomenon), so that at this stage of the art there is little alternative to a phenomenological description, say, through an effective kernel put in by hand in the BS equation.

Bethe-Salpeter Preliminaries

The BS equation of 1951 has a 3-decade old history having found its early applications in e^+e^- physics in the fifties (Karplus & Klein, 1972) and in $q\bar{q}$ dynamics since the late sixties (Narayanaswamy & Pagnamenta, 1968; Smith, 1969, Nakanishi, 1969; and Sundaresan & Watson, 1970). However, these $q\bar{q}$ attempts, though mathematically elegant, did not find enough *quantitative* contacts with the details of hadron spectroscopy, and were soon overshadowed by the spectacular success of the FKR model (1971) to both $q\bar{q}$ and qqq systems. The revival of BS dynamics in the seventies came about mainly from two directions. On the one hand, Joos and collaborators (Bohm *et al.*, 1973) formulated a new 4-dimensional approach to B. S. dynamics within a Wick-rotated (cf. Wick, 1954) framework to take care of time-like excitations, and obtained equations similar to the FKR model under the assumption of infinitely heavy quark masses. They also considered several applications of this theory, mostly to light $q\bar{q}$ systems, with varying degrees of success (Bohm & Krammer, 1974, 1975; and Joos *et al.* 1974). On the other hand, the discovery of heavy quark flavours ($\bar{c}c$, $\bar{b}b$) in the seventies (Aubert *et al.*, 1974; Augustin *et al.*, 1974; and Herb *et al.*, 1977), stimulated a more restricted application of BS dynamics based on the N. R. approximation (*see* De Rujula *et al.*, 1975; Gromes, 1976, 1977; Barbieri *et al.*, 1976; Billoire & Morel, 1978; Reinders, 1978; Eilwanger, 1978; Bethe & Salpeter, 1967), with corrections of 0 (v^2/c^2).

For heavy quark systems which are dominated by the short-distance interaction, the N.R. approximation is quite reasonable, amounting to a Schrödinger-like equation. There is also the added facility of an unambiguous form of the interaction valid at these distances—the Coulombic one-gluon exchange potential attenuated by asymptotic freedom corrections—with comparatively smaller effects arising from

the confining region (cf. Eichten *et al.*, 1978; Quigg & Rosner, 1978, 1979; Richardson, 1979; Ditsas *et al.*, 1978;Henriques *et al.*, 1976; and Marciano & Pagels, 1978). Some of these attempts have already been described in context in Sec. 3 in which were summarized certain approaches to quark spectroscopy within a basically non-relativistic framework with modest relativistic correlations. However, within the bigger framework of BS dynamics, such N.R. attempts which are necessarily 3-dimensional in character, are only a particular case of the so-called *Instantaneous Approximation* (IA) to the B. S. equations which have also been known in the literature (Levy, 1952; Salpeter, 1952; and Klein & Zemach, 1957) since the fifties. The latter approximation, which involves setting the relative time coordinate equal to zero in the B. S. kernel, obviously goes well beyond a mere N. R. approximation, insofar as the velocities of the constituents are not necessarily small, so that an expansion in powers of (v/c) is no longer warranted. (The situation is analogous to the Dirac equation which yields a fully *relativistic* mass formula for the hydrogen atom despite the use of the Coulomb potential which, e.g., represents the instantaneous approximation to the Moller interaction.)

Now this latter aspect of the B.S. framework, viz., the Instantaneous Approximation for a 3-dimensional description but otherwise complete with relativistic kinematics is probably more relevant to *light* quark physics (which should not warrant a v/c-expansion), than a mere Schrödinger-like N.R. framework with v^2/c^2 corrections, earlier employed with success for heavy quark systems. However, unlike heavy quark systems where the extrapolation of the one-gluon-exchange potential to the confining region (Richardson, 1979) probably plays a modest role, the latter region is presumably more dominant for light quarks, thus necessitating a more consciously phenomenological choice of the confining potential in this case. At the present stage of development, this amounts to the introduction of at least one free parameter which would have to play a crucial role in quark physics if light hadron spectroscopy must be pursued without waiting for the final QCD outcome at large distances. This point of view has been developed recently by Mitra *et al.* (*to be published*) with the choice of a harmonic potential for the confining q-$\bar{q}$ and q-q interactions within an otherwise relativistic B.S. framework except for the Instantaneous Approximation (IA). The results of this formalism which have been applied to $q\bar{q}$, qqq and $qq\bar{q}\bar{q}$ systems with a considerable degree of experimental success, are directly comparable to those of the FKR model and reveal the crucial role of B. S. dynamics in bringing about certain major symmetry-breaking effects (spin- and momentum-dependent) on the mass spectra without individual parametrizations.

In this section, we give a brief review of the main points of the 4-dimensional Wick-rotated formalism of Joos *et al.* (1974). This is followed in §7 by a rather general formulation of the B. S. dynamics for few-quark systems in the IA for any two-body kernel, and the applications of this formalism to $q\bar{q}$ and qqq systems for harmonic confinement.

Wick-rotated B. S. Model for Confinement

In a long paper, Joos *et al.* (BJK, 1973) formulated a formal field-theoretic model of few-quark systems in terms of the BS equation which for a $q\bar{q}$ system had

earlier been explored by several authors (Narayanaswamy & Pagnamenta, 1968; Smith, 1969; Nakanishi, 1969; and Sundaresan & Watson, 1970) but had not been pursued by them in a sufficiently quantitative form for applicational purposes. Now from the purely conceptual point of view, it is perhaps an esoteric question as to whether quarks, which cannot exist in free states, obey the same kind of field-theoretic rules as do the more conventional "fields" (e^-, γ, π, etc.). In any case, none of these authors seemed to have bothered about these questions but merely took the formal tools of field theory for granted in the cases of quarks as well. And of course, by the time the BJK papers appeared (Bohm *et al.*, 1973; and Okubo, 1963), the "statistics" problem for quarks had been "resolved" in favour of the standard Fermi statistics, thanks to the discovery of the so-called "colour" degrees of freedom (Firtzsch *et al.*, 1973).

In the modern view, quark confinement is supposed to find its rationale in colour confinement, which in turn is believed to be brought about through coloured gluon couplings to quarks in a gauge-theoretic fashion (cf. Marciano & Pagels, 1978; Buras —*to be published*; and Harari, 1979). However, this theory of "quantum chromodynamics" (QCD) which represents an SU(3) generalization (*a la Yang-Mills*) of the more familiar QED, does not now seem to be developed enough to "explain" colour confinement within its own framework, not to speak of the time when the BJK series of papers started appearing (*see* Bohm *et al.*, 1973). At this pre-QCD stage, Joos indeed preceded by Sundaresan and Watson (1980), took the simplest available path, viz., the approximation of *infinitely heavy* quarks as a formal means to generate an h.o. type confining interaction.

BJK Model for Scalar Quarks

In the BJK (1973) and earlier approaches, only the simplified *equal* mass kinematics for $q\bar{q}$ systems were considered. In the following summary, we give an integrated overview of the main points of these approaches only for the equal quark masses, while the unequal mass case is developed more systematically in Sec. 7. The BS amplitude $\phi\,(P, q)$ in momentum space for two scalar quarks which is defined by $(x = x_1 - x_2)$ (cf. Smith, 1969).

$$\delta^4(p_1 + p_2 - P)\,(2\pi)^8\,\phi\,(P, q) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2 \exp\left[\tfrac{1}{2}\,iP\,.\,(x_1 + x_2) + iq\,.\,x\right] \times \langle 0 \left| T\left(\phi\left(\frac{x}{2}\right)\bar{\phi}\left(-\frac{x}{2}\right)\right)\right| P, M\rangle \qquad \ldots(190)$$

satisfies an equation of the form

$$\phi\,(P, q) = i\,\Delta_F(p_1)\,\Delta_F(p_2) \int \mathrm{d}^4q^1\,K\,(q, q^1)\,\phi(P, q^1) \qquad \ldots(191)$$

where

$$P = p_1 + p_2,\ 2q = p_1 - p_2 \qquad \ldots(192)$$

If the kernel is a superposition of Yukawas, viz.,

$$K(q, q') = \sum_i \alpha_i \left[(q - q')^2 + \mu_i^2 - i\epsilon\right]^{-1} \equiv V(q - q') \qquad \ldots(193)$$

a Wick-rotation in the q_0 -variable, $q_0 \to iq_4$ leads to the Euclidean form (cf. Bohm *et al.*, 1973).

$$\left[\left(q^2 + m_q^2 - M^2/4 \right)^2 + q_4^2 M^2\right] \phi(q) = -\int d^4q' \; V(q-q') \; \phi(q') \quad ...(194)$$

But for the q_4^2 term on the LHS, which would vanish for a zero mass bound state ($M = 0$), the equation would be invariant w.r.t. the 0(4) group and admit of an expansion of the form (cf. Bohm *et al.*, 1973)

$$\phi(q, \Omega_4) = \sum_{n, l, m} \phi_{lm}^{(n)} (q^2) \; y_{lm}^{(n)} (\Omega_4) \quad ...(195)$$

where y's are 0(4)-spherical harmonics. The solutions to (194) take a particularly simple form in the *oscillator* approximation. The latter comes about partly through the ansatz (cf. Bohm *et al.*, 1973)

$$V(k) = \delta^4(k) \left(\alpha - \beta \Box^2_{k+...}\right) ; \; k = q - q' \quad ...(196)$$

implying a 'smoothness' of the coordinate transform $V(R)$ near $R = 0$:

$$V(R) = \alpha + \beta R^2 + ... \quad ...(197)$$

and partly through the conditions $M^2 \ll 4\, mq^2$ and $<q^2> \ll m_q^2$ which imply the consideration of only low-lying states. Under these conditions eqn. (194) reduces to the form

$$\left(2q^2 + m_q^2 - \tfrac{1}{2} M^2\right) \phi(q) = m_q^{-2} \left(-\alpha + \beta \Box_q^2\right) \phi(q) \quad ...(198)$$

whose general solution is expressed by the M^2-spectrum :

$$M^2 = 2\left(\alpha + m_q^4\right) \Big/ m_b^2 + \left[4\sqrt{2\beta}/m_q\right] \sum_{i=1}^{4} (n_i + \tfrac{1}{2}) \quad ...(199)$$

where n_i are non-negative integers, including the time-like component n_4. Eqn. (199) shows that for small M, the quark mass term must be compensated by the depth α of the potential. The effect of the Wick-rotation shows up through the positive contribution from time-like excitation (n_4) as well, in contrast to the FKR and allied approaches (Secs. 2 and 5) which were designed to eliminate these excitations as unphysical. In the same vein, there is no normalization difficulty with the wave functions either, since all the four coordinates are now on the same footing.

A corresponding result obtains for the qqq problem as well (Mayer, 1974). In this case, the BS amplitude for scalar quarks is defined analogously to (190) as

$$(2\pi)^{12} \delta^4(p_1 + p_2 + p_3 - P) \; \phi_B(p_1 p_2 p_3) = \int d^4x_1 d^4x_2 d^4x_3 \times \exp\left(ip_1 \cdot x_1 + ip_2 \cdot x_2 + ip_3 \cdot x_3\right) < 0 \mid T(\phi(x_1)\phi(x_2)\phi(x_3)) \mid P; B > \quad ...(200)$$

To write down the corresponding BS equation, define (cf. Mayer, 1974)

$$p_{1,2} = \tfrac{1}{3}P + \tfrac{1}{2}q_3 \pm k_3,\ p_3 = \tfrac{1}{3}P - q_3 \qquad \ldots(201)$$

with two analogous sets of eqations obtained by cyclic permutations of the indices. The B. S. equation is now of the form (cf. 95) :

$$\prod_{\alpha=1}^{3} \Delta_F^{-1}(p_\alpha)\,\phi_B(p_1p_2p_3) = \sum_{\alpha=1}^{3} \Delta_F^{-1}(p_\alpha)\int \mathrm{d}k'_\alpha\, V(k_\alpha - k_\alpha')\phi_B\left(\ p'_\alpha\ \right)$$

$$+ \text{3 body force term} \qquad \ldots(202)$$

This equation must also be viewed in a Wick-rotated fashion for the time-like components of k_α, k'_α. The oscillator approximation on the 3 kernels in (202) can similarly be made as in (196) and the conditions

$$< q^2 > \sim < k^2 > \ll m_q^2, \quad M^2 \ll 9m_q^2 \qquad \ldots(203)$$

can be employed to simplify the LHS of (202). Ignoring further the 3-body term for the moment, eqn. (202) finally reduces to $k_3 \equiv \xi$, $q_3 \equiv \eta$) :

$$m_q^2\left(2\xi^2 + \frac{3}{2}\eta^2 + m_q^2 - \frac{1}{3}M^2\right)\phi_B = \left[-3\alpha + \beta\left(\frac{3}{2}\,\Box_\xi^2 + 2\Box_\eta^2\right)\right.$$

$$\left. - \alpha\left(2\xi^2 + \frac{3}{2}\eta^2\right) + \tfrac{1}{3}\alpha m_q^{-2}M^2\right]\phi_B(\xi, \eta) \qquad \ldots(204)$$

whose M^2-spectrum is then of the *linear* form

$$M^2 = \Omega_0 + \Omega \sum_{i=1}^{4}(n_i + m_i) \qquad \ldots(205)$$

The two sets of *non-negative* integers n_i, m_i (including the components $i = 4$) now correspond to the two *independent* 4-dimensional excitation modes (ξ, η), so that the time-like excitations are no longer an embarrassment either for the eigenvalues or for the eigenfunctions.

B. S. Equations for Fermion Quarks

A similar formalism exists for Fermion qq or $q\bar{q}$ pairs whose BS amplitudes $\psi_{\alpha\beta}$ are 16-component bi-spinors expressible either as $(4\times1) \otimes (4\times1)$, ($\alpha, \beta = 1, 2, 3, 4$) or as 4×4 matrices. These two representations (ψ, $\tilde{\psi}$) correspond to the following equation forms respectively (Smith, 1969)

$$(2\pi)^8\psi(P, q) = S_F^{(1)}(p_1)\,S_F^{(2)}(p_2)\int V(q, k)\,\psi(P, k)\mathrm{d}^4k \qquad \ldots(206)$$

$$(2\pi)^8\,\tilde{\psi}(P, q) = S_F(p_1)\int V(q, k)\,\tilde{\psi}(P, k)\,S_F(-p_2)\mathrm{d}^4k \qquad \ldots(207)$$

These amplitudes are related by the charge conjugation matrix

$$\bar{\psi}(P, q) = C_B C\, \psi^T(P, -q) C^{-1} \qquad \ldots(208)$$

for a state with charge conjugation parity C_B, where ψ^T is the $(4\otimes 1) \times (4\times 1)^T$ form of ψ, and of course

$$C\gamma_\mu^T\, c^{-1} = -\gamma_\mu \qquad \ldots(209)$$

These B. S. amplitudes obviously have much richer structures than those made up of scalar quarks. These structures have been discussed extensively in the BJK series (cf. Bohm & Krammer, 1975, and Joos *et al.*, 1974) corresponding to the different states (S, P, V, A, T) as well as the corresponding varieties of kernels. For completeness we also state without proof, the normalization condition (*see* Smith, 1968) for the BS amplitude $\tilde{\psi}$ of eqn. (207) :

$$(2\pi)^{-4}\,\mathrm{Tr}\int\!\!\int \bar{\tilde{\psi}}(P, q)\left[\frac{\partial}{\partial P_\mu} S_F^{-1}(q + \tfrac{1}{2}P)\, S_F^{-1}(q - \tfrac{1}{2}P)\,\delta^4(q - k)\right.$$
$$\left. - V(q, k)\right]\tilde{\psi}(P, k)\mathrm{d}^4q\mathrm{d}^4k \qquad \ldots(210)$$

which is basically an expression for the conservation of baryon number [*See* e.g., N. Nakanishi (1969) for details].

The Bethe-Salpeter language is ideally suited for providing a rationale for the QPC model discussed in (§4). This is best illustrated by comparing the matrix elements corresponding to the diagrams 3(a) and 3(b), each representing the process A→B + C. Apart from irrelevant factors these matrix elements for these mesonic transitions are respectively proportional to

$$\int \psi_B^*(p_1 + p_3)\,\psi_C^*(p_3 + p_2)\,\psi_A(p_1 + p_2) \qquad \ldots(211)$$

and

$$\int \phi_B^*(p_1 + p_3)\;\Delta_F^*(p_1)\,\phi_A(p_1 + p_2)\,\Delta_F^{-1}(p_2)\,\phi_C^*(p_2 + p_3)\,\Delta_F^{-1}(p_3)$$
$$\ldots(212)$$

where the integrations are carried out over all *internal* momenta. A similar expression holds for Fermion quarks with each Δ_F^{-1} replaced by an S_F^{-1} and an overall trace operation is implied, as e.g., in eqn. (210). The extra factors Δ_F^{-1} or S_F^{-1} arise from the normal Feynman-Dyson rules of field theory, taking account of the fact that a B. C. amplitude is a product of the vertex factor and two adjacent propagators. These inverse propagators are obviously insensitive to the momenta for large m_q, as is indeed the case in the BJK model.

Similar expressions can be written down for the matrix elements of several other elementary processes which can be handled by B. S. amplitudes, e.g., $\pi, k \to ll$; $\rho^\circ, \omega, \phi \to e^+e^-$ and $\pi^\circ, \eta \to 2\gamma$ (*see* Smith, 1969). Some of these cases are discussed quantitatively in the next section as applications of the BS formalism in the IA.

We conclude this section with a few remarks on the philosophy *vs.* applications of the Wick-rotated h.o. model. Its great advantage is a formal resolution of both the time-like excitation and the wave function normalizability problems. However, this is at a heavy cost. The Wick rotation is a one-way process and affects *both* the internal (virtual) and external (physical) momenta. There seems to be no rational way to disentangle the latter from the former, thus preventing a meaningful contact with observational situations, except in very special cases. This fact has stood in the way of further development of the theory, despite its obvious elegance and simplicity for a few individual applications to certain decay processes (Bohm & Krammer, 1975; and Joos *et al.*, 1974). There has been little interest in systematic attempts to define the quantitative parameters of the theory and in particular no applications to high energy hadron scattering or allied processes has been possible, as these processes are basically incompatible with the very structure of a Euclidean metric.

Other B.S. Approaches to H. O. Model

Nevertheless, the BJK theory has proved a strong stimulus for more intense investigations on the theoretical basis for harmonic confinement within the framework of field theory (Ader *et al.*, 1979). For example, though two-dimensional gauge theories (Schwinger, 1962; and Lasher *et al.*, 1974) as well as lattice gauge theories (Kiskis, 1975; and Blaha, 1974) seem to favour a potential $V(R) \sim R$ for large R, four dimensional theories have stimulated certain models (Karplus & Klein, 1952) which do not rule out an R^2 behaviour. A particularly interesting approach is due to Alibaso and Schierholtz (1977) who consider the twin questions of quark confinement and the hadron spectrum from field-theoretical premises. Their basic ingredient is the Bethe-Salpeter (BS) kernel K which governs *not only* the $q\bar{q}$ bound state spectrum, eqn. (191), *but also* the Schwinger-Dyson equation for the quark propagator, the latter being of the form

$$\Delta_F^{-1}(q) = Z\,\Delta_0^{-1}(q) - i\int \mathrm{d}^4 k\, K(q, k)\,\Delta_F(k) \qquad \ldots(213)$$

The B.S. kernel in turn is inferred from the β-function of (QCD) (cf. Marciano & Pagels, 1978) which is assumed to have no infrared stable zero. Their confinement mechanism and the absence of free quark states are inferred from the result that the quark propagator Δ_F is a *constant* in momentum space, i.e., a δ-function in coordinate space. Further, the bound states are found to lie on *linear* Regge trajectories implying an effective harmonic confinement.

A model of the BS harmonic oscillator for spinless quarks without time-like excitations or other anomalies has also been suggested by Henyey (1979) recently.

(§7) B. S. Equations, Harmonic Confinement and Hadron Spectra

In the last section, certain B.S. approaches to harmonic confinement qq systems in 4-dimensional form were discussed. Because of the known problem of probabilistic

interpretation of the BS amplitude in 4-dimensional form, efforts had been afoot since the beginning of B.S. theory to make certain 3-dimensional formulations (Levy, 1952; Logunov & Tavkhelidze, 1963; Fishbane & Namyslowski, 1980; Blankenbecler & Sugar 1966; Kady chevsky, 1968; and Weinberg, 1966) to preserve the probability interpretation more naturally and spontaneously. The Instantaneous Approximation (IA) mentioned in (§6) was the earliest one of this type (*see* Levy, 1952). It reduces to the N. R. approximation in the appropriate limit, but goes well beyond the former for a relativistic system. As noted in (§6), the IA should be especially suited to light quark (*uds*) system for which the N. R. approximation is highly questionable.

In this section, we review briefly the formulation of the B. S. equations in the IA by Mitra *et al.* (1980—*in press*) for $q\bar{q}$, qqq systems with unequal mass kinematics, and with the inclusion of spin and colour degrees of freedom. Their approach is motivated by : (*i*) a conscious effort to avoid a v/c-expansion of the relativistic effects which are large; (*ii*) an h.o. parametrization of the confining interaction within the B.S. framework with a view to obtaining explicit symmetry-breaking terms (spin and momentum dependent) affecting the mass spectrum; and (*iii*) a desire to identify a reduced spring constant $\tilde{\omega}$ whose significance should hopefully extend to several distinct regions of spin and flavour (as a result of dynamics) without a separate parametrization for each. In the language of FKR (1971), this last requirement amounts to the possibility of understanding the variation of their effective spring constant Ω over the different flavour regions :

$$\Omega_{ud} \approx 1.10,\ \Omega_s \approx 1.25,\ \Omega_c \approx 2.0\text{—}2.2,\ \Omega_b \approx 5.5 \qquad \ldots(214)$$

In this regard, they conjectured that the B. S. dynamics would incorporate that part of the variation which is an M-dependent effect, while the balance representing m_q-dependence would need to be put in by hand. Their results on the mass spectra of $q\bar{q}$ and qqq, which are also reviewed briefly in this section, show that such an expectation is indeed fulfilled by the data over a fairly wide range of spectroscopy involving at least light quarks. However, their approach breaks down rapidly for heavy quarks beyond the cc region.

B.S. Equations for Few-Quark Systems

The basic assumption of the MS approach is a *common* spin and colour dependence of the pairwise q-$\bar{q}$ as well as q-q interactions for both the short-distance one-gluon-exchange and the long-distance confining regions through the following ansatz for the B.S. kernel

$$<p_1 p_2 \mid K_{12} \mid p_1' p_2' = \delta^4 \left(P_{12} - P_{12}' \right) F_{12} \gamma_\mu^{(1)} \gamma_\mu^{(2)} < q_{12} \mid V_{12} \mid k_{12} > \qquad \ldots(215)$$

Here the colour and spin dependence of the (12)-interaction are expressed by the factors

$$F_{12} = \tfrac{1}{2} \lambda^{(1)} . \tfrac{1}{2} \lambda^{(2)} \qquad \ldots(216)$$

and $\gamma_\mu^{(1)}$ $\gamma_\mu^{(2)}$ respectively, while V_{12} is a scalar function of the relative 4-momenta (q, k) defined through the following relations (suppressing the tensor indices)

$$p_{1,2} = m_{1,2}\, P_{12}/m_{12} \pm q_{12} \qquad ...(217)$$

$$p'_{1,2} = m_{1,2} = P'_{12}/m_{12} \pm k_{12} \qquad ...(218)$$

$$m_{12} = m_1 + m_2 \qquad ...(219)$$

The scalar function V_{12} for $q\bar{q}$ interaction is split up into a gluon-exchange plus a confining h.o. term as

$$< q \mid V \mid k > \; = 4\pi\, \alpha_s (q - k)^{-2} + \frac{3}{4}\,(2\pi)^3\, \omega_{q\bar{q}}^2 \nabla_k^2\, \delta(\vec{q} - \vec{k}) \qquad ...(220)$$

Similar definitions of the various momenta and the potentials hold for pairwise qq interactions but with $\omega_{qq} \to \omega_{\bar{q}\bar{q}}$ for the confining region. However, while for a 2-body system in the c.m. frame, $\vec{P}_{12} = 0$, the same is not true of two-body subsystems of a qqq system for which the following quantities are also defined :

$$P(\equiv P^\mu) = p_1 + p_2 + p_3,\; m_0 = m_1 + m_2 + m_3 \qquad ...(221)$$

With these inputs, the B.S. equation for the $q\bar{q}$ amplitude is given (Fig. 3*a*) by

$$S_F^{-1}(p_1)\, S_F^{-1}(p_2)\; \Psi(P, q) = -\,(2\pi)^{-4}\, F_{12} \int d^4k\, \gamma_\mu^{(1)}\, \gamma_\mu^{(2)} \times < q \mid v \mid k > \Psi\,(P, k) \qquad ...(222)$$

$$S_F^{-1}(p_i) = +\, i\,(m_i + i\,\gamma^{(i)} \cdot p_i) \qquad ...(223)$$

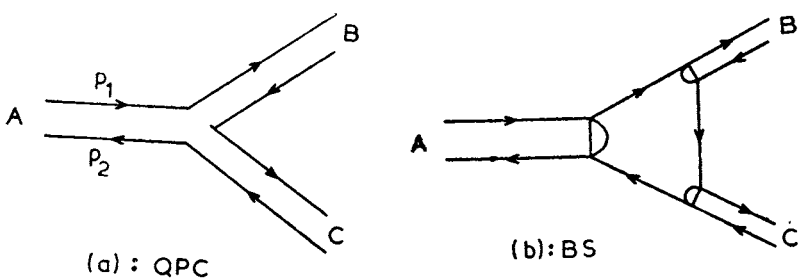

FIG. 3. Topological similarity of QPC (*a*) and Bethe-Salpeter (*b*) diagrams.

This formulation emphasizes the $(4\times1) \otimes (4\times1)$ representation instead of the 4×4 one for the BS amplitude. The corresponding equation for the qqq system (Fig. 3*b*) is :

$$\prod_{i=1}^{3} S_F^{-1}(p_i)\,\Psi\,(p_1 p_2 p_3) = -\sum_{123} S_F^{-1}(p_3)\,F_{12}\,(2\pi)^{-4}\int \mathrm{d}^4 k_{12}$$

$$\times < q_{12} \mid v_{12} \mid k_{12} > \gamma_\mu^{(1)}\,\gamma_\mu^{(2)}$$

$$\Psi\left(\; p_1'\;\; p_2'\; p_3 \;\right) \qquad \ldots(224)$$

in terms of a 64×1 representation for Ψ. Eqn. (224) does not contain a 3-body force term (Fig. 4*c*, *see p.* 232) since its colour factor

$$G = f_{abc}\;\lambda_a^{(1)}\;\;\lambda_b^{(2)}\;\;\lambda_c^{(3)} \qquad \ldots(225)$$

vanishes for a singlet qqq state (Mitra, 1979). The colour factors F_{12} are —4/3 and —2/3 in (222) and (224) respectively. Likewise, the $qq\bar{q}\bar{q}$ amplitude satisfied the BS equation

$$\Psi(p_1 p_2 p_3 p_4) = -\sum_{6} S_F(p_1)\,S_F(p_2)\,F_{12}(2\pi)^{-4}\int \mathrm{d}^4 k_{12}$$

$$\times < q_{12} \mid V_{12} \mid k_{12} > \gamma_\mu^{(1)}\;\gamma_\mu^{(2)}\,\Psi\left(\;p_1'\;\;p_2'\,p_3 p_4\;\right) \qquad \ldots(226)$$

Spin-Reduction

The next step is a reduction of the spin-structures of these equations in a covariant form by rationalizing their left hand sides with multiplication by the 'conjugate' S_F^{-1} factors to the forms $\Pi\left(\; m_i^2 + p_i^2\;\right)$. On the other hand, the spin-structures on the right hand sides are contained in the factors $V_\mu^{(1)}\;V_\mu^{(2)}$ where

$$\pm\, V_\mu^{(1,2)} = (m_{1,2} \mp i\gamma^{(1,2)} \cdot p_{1,2})\,\gamma_\mu^{(1,2)} \qquad \ldots(227)$$

A general technique for the reduction of such factors is the so-called Gordon reduction (Bjorken & Droll, 1964) which amounts to the on-mass-shell replacements

$$V_\mu^{(1,2)} \Rightarrow \mp\, 2m_{1,2}\, i\gamma_\mu^{(1,2)} \Rightarrow \left(p_{1,2}^{\mu} + p_{1,2}^{'\mu}\right) - i\sigma_{1,2}^{\mu\nu}\left(p_{1,2}^{\nu} - p_{1,2}^{'\nu}\right) \qquad \ldots(228)$$

For off-shell conditions, there is a correction factor (Mitra, 1980 *b*)

$$h_{12} = \tfrac{1}{2} + \tfrac{1}{2}\, m_{12}\, M_{12}^{-1} \qquad \ldots(229)$$

with each factor V_μ. The simplified $q\bar{q}$ and qqq equations are respectively :

$$\prod_{i=1}^{2}\left(m_i^2+p_i^2\right)\Psi(P,q)=\left(-\frac{4}{3}\right)\int \mathrm{d}^4k(2\pi)^{-4} < q\mid V\mid k > V_\mu^{(1)}\,V_\mu^{(2)}\,\Psi(P,k) \qquad ...(230)$$

$$\prod_{i=1}^{3}\left(m_i^2+p_i^2\right)\Psi(p_1p_2p_3)=\left(-\frac{2}{3}\right)\sum_{123}\left(m_3^2+p_3^2\right)\int \mathrm{d}^4k_{12} \times < q_{12}\mid V_{12}\mid k_{12} > V_\mu^{(1)}\,V_\mu^{(2)} \Psi\left(p_1'p_2'p_3\right) \qquad ...(231)$$

where, for any general pair ($q_1\bar{q}_2$ or q_1q_2),

$$V_\mu^{(1)}\,V_\mu^{(2)}=\tau_{12}\,P_{12}^2-(q_{12}+k_{12})^2-2i\left(\sigma_1^{\mu\nu}+\sigma_2^{\mu\nu}\right)q_{12}^\mu k_{12}^\nu + 2P_{12}^\mu\,m_{12}^{-1}\Big[\,i\left(m_1\,\sigma_2^{\mu\nu}-m_2\,\sigma_1^{\mu\nu}\right)\left(q_{12}^\nu-k_{12}^\nu\right) + (m_2-m_1)\left(q_{12}^\mu+k_{12}^\mu\right)\Big] + \sigma_1^{\mu\nu}\sigma_2^{\mu\lambda}\left(q_{12}^\nu-k_{12}^\nu\right)\left(q_{12}^\lambda-k_{12}^\lambda\right) \qquad ...(232)$$

$$\tau_{12}=4m_1m_2\,m_{12}^{-2} \qquad ...(233)$$

The on-shell assumption is quite good for light quark systems, while the off-shell correction (229) is more appropriate for heavy ones.

Instantaneous Approximation (IA)

Further reduction of these equations from four-to-three-dimensional form is achieved through the IA which consists in setting in the kernel of the two-body equation (230) and defining the "Schrödinger wave function" (cf. Levy, 1952)

$$\Psi(\vec{q})=\int \mathrm{d}q_0\,\Psi(P,q) \qquad ...(234)$$

For the 3-body equation (231), the above definition of the IA for pair-wise kernels must be supplemented by an additional assumption on the time-component of the *spectator* momentum variable in each term, since it cannot have a natural place in a

basically 3-dimensional framework. The simplest assumptions on P_{12}^0 and p_3^0 when p_1 and p_2 are in mutual interaction are

$$P_{12}^0 \Rightarrow m_{12}M/m_0,\ p_3^0 \Rightarrow m_3 M/m_0 \qquad ...(235)$$

Also the 3-body analogue of (234) is

$$\Psi(\vec{p}_1\vec{p}_2\vec{p}_3) = \Psi(\vec{q}_{12}, \vec{p}_3) = \int \mathrm{d}q_{12}^0\, \mathrm{d}p_3^0\, \Psi(p_1p_2p_3) \qquad ...(236)$$

Using these ingredients equations (230) and (231) reduce eventually to the forms (see Harari, 1969)

$$\psi(\vec{q}) = \left(-\frac{4}{3}\right)\int \mathrm{d}\vec{k}(2\pi)^{-3} < \vec{q} \mid V \mid \vec{k} < h_{12}^2 \sum_{n=0}^{2} T_n^{(12)}\ I_n^{(12)}\ \psi(\vec{k}) \qquad ...(237)$$

$$\psi(\vec{p}_1\vec{p}_2\vec{p}_3) = (-\tfrac{2}{3}) \sum_{123} \int \mathrm{d}\vec{k}_{12}(2\pi)^{-3} < \vec{q}_{12} \mid V_{12} \mid \vec{k} > h_{12}^2 \times \sum_{n=0}^{2} T_n^{(12)}\ I_n^{(12)}\ \psi(\vec{p}_2'\vec{p}_2'\vec{p}_3) \qquad ...(238)$$

together with a similar equation for the $qq\bar{q}\bar{q}$ system. The integrals I_n arise from the q_0-integrations on the RHS, and are defined by

$$2\pi i I_n^{(12)} = \oint \mathrm{d}q_0 q_0^n\, (m_1^2 + p_1^2)^{-1}(m_2^2 + p_2^2)^{-1} \qquad ...(239)$$

The T_n-factors in turn correspond to contributions of the different terms of (232) to these q_0-integrals in accordance with the classification of q_0-powers $n = 0, 1, 2$. For details of various symbols, *see* Mitra (1980 *b—in press*).

Solution to the $q\bar{q}$ Equation

A practical method of solution to the above equations has been developed by MS (*to be published*) treating the confining h. o. term exactly and the gluon-exchange QCD term perturbatively. While referring to MS for details, certain features are nevertheless worth pointing out as these bear on the M-dependence of various effects which characterize the B. S. dynamics. Thus, in contrast to the usual (v/c) expansion of the non-central QCD corrections (cf. De Rujula *et al.*, 1975), these effects are best exhibited for the $q\bar{q}$ equation for $m_1 = m_2$ in the following form *before* resorting to the perturbation approximation

$$\tfrac{1}{2}M\left(4m_q^2 - M^2 - 4\vec{\Delta}_r^2\right)\psi(\vec{r}) = (K_{QCD} + K_{HO})\psi(\vec{r}) \qquad ...(240)$$

where for any general (12) pair ($r = r_{12}$) :

$$K_{QCD}^{(12)}\,\tilde{\psi} = (-F_{12})\,\alpha_s\Big[\,M_{12}^2\,r^{-1} + 4\pi\delta(\vec{r})(1 - \frac{2}{3}\vec{\sigma}_1\,.\,\vec{\sigma}_2 + 2M_{12}/m_{12})$$

$$+ 2r^{-3}(3\vec{\sigma}_1\,.\,\hat{r}_2\,\vec{\sigma}\,.\,\hat{r} - \vec{\sigma}_1\,.\,\vec{\sigma}_2) - 2r^{-3}\vec{L}_{12}\,.\,(\vec{\sigma}_1 + \vec{\sigma}_2)$$

$$- 4r^{-1}\vec{\nabla}_r^2 + 4(1 + M_{12}/m_{12})\,r^{-2}\hat{r}\,.\,\vec{\nabla}_r\Big]\psi(\vec{r}) \qquad \ldots(241)$$

K_{HO} is the corresponding contribution of the h.o. term which is specified more quantitatively below. In the N.R. approximation which amounts to $M \approx 2m_q$, these QCD terms can be shown to reduce to the usual form (De Rujula *et al.*, 1975). If, in a perturbative spirit, the QCD term is neglected, eqn. (240) in *momentum* space reads as

$$\Big[\vec{q}^2 + m_q^2 - \tfrac{1}{4}M^2 - \tfrac{1}{2}M\,\omega_{qq}^2\,\vec{\nabla}_q^2\Big]\psi(\vec{q})$$

$$= \tfrac{1}{2}M^{-1}\omega_{qq}^2\,[-\hat{L}_q - 8\,\vec{J}\,.\,\vec{S} + \hat{Q}_q]\psi(\vec{q}) \qquad \ldots(242)$$

$$\hat{L}_q = 4M\,m_q^{-1}\Big(\vec{q}\,.\,\vec{\nabla} + \frac{3}{2}\Big) \qquad \ldots(243)$$

$$\hat{Q}_q = 4\vec{q}^2\vec{\nabla}_q^2 + 8\vec{q}\,.\,\vec{\nabla}_q + 6 \qquad \ldots(244)$$

which brings out two remarkable terms viz., $\vec{J}\,.\,\vec{S}$ and $\hat{Q}_q$, in a *non-perturbative, relativistic* form. $\vec{J}\,.\,\vec{S}$, which is a generalization of the spin-spin term of Leutwyler and Stern (1980*b*), is a direct consequence of the spin-structure of the BS dynamics. Likewise, the important momentum dependent correction Q_q arises from the same dynamical package, without extra assumptions. The involved M-dependence of eqn. (242) is an evidence of its essentially relativistic structure despite its N. R. looks. The spring constant ω_{qq}^2 is *assumed* to have the following m_q-dependence

$$\omega_{qq}^2 = (m_1 + m_2)\tilde{\omega}^2 \qquad \ldots(245)$$

where the 'reduced spring constant' $\tilde{\omega}$ is expected to be a universal confining parameter (Novikov *et al.*, 1978; and Joshi & Anderson, 1979). The solution of eqn. (242) was shown in MS (*to be published*) to be expressible as a 'universal' (spin-and flavour-independent) h.o. spectrum with unit excitations :

$$F(M) \equiv \Big(M^2 - m_{12}^2\Big)\Big/\Omega_M - \tfrac{1}{4}\Omega M^{-2}\gamma^{-2}\,(2\vec{J}\,.\,\vec{S} - 3 - Q_N) + F_{QCD}$$

$$\ldots(246)$$

$$= N + 3/2; \quad (N = 2n + L) \qquad \ldots(247)$$

$$\Omega_M = 8\sqrt{Mm_q}\gamma\tilde{\omega}, \quad \gamma^2 = 1 + 4\tilde{\omega}^2/Mm_q \qquad ...(248)$$

$$< \tfrac{1}{4}\hat{Q}_q > \equiv Q_N = (N^2 + 3N - 3)/6 - (N + 3/2)^2 \qquad ...(249)$$

F_{QCD} is the corresponding contribution obtained perturbatively from the gluon exchange term (241), Ω_M is directly comparable with FKR's (1971) Ω-parameter but now with its (M, m_q)-dependence simulated mainly from B. S. dynamics and partly from the *ansatz* (245). The quantity $F(M)$, a complicated function of M, may be regarded as a sort of *central* (mass)^2, corrected for spin-and momentum-dependent symmetry-breaking effects as well as quark masses, and measured in units of the FKR-like spring constant Ω_M. This function is predicted to exhibit the highly degenerate h.o. spectrum for different flavour regions. Since F_{QCD} is taken perturbatively, the formula is expected to break down for heavy quark spectra.

The true $q\bar{q}$ wave function is not quite $\psi(\vec{q})$ but rather a related quantity $\phi(\vec{q})$ which incorporates current conservation (Nakanishi, 1969) :

$$\phi_0(\vec{q}) = \text{Exp}\ (-q^2/Mm_q)\ \psi_0(\vec{q}) \qquad ...(250)$$

$$\phi_0(\vec{q}) = (\pi\beta^2)^{-3/4}\ \exp\ (-\tfrac{1}{2}\vec{q}^2\beta^{-2}) \qquad ...(251)$$

$$8\beta^2\gamma^2 = \Omega_M \qquad ...(252)$$

Comparison of the spectral predictions with experiment was made by calculating $F(M)$ for the *observed* values of the relevant meson masses for the following inputs (in GeV)

$$\begin{aligned} &\tilde{\omega} = 0.15 \pm 9.91, \qquad m_{ud} = 0.28 \pm 0.01 \\ &m_s = 0.35 \pm 0.01, \qquad m_c = 1.40 \pm 0.02 \\ &\alpha_s(uds) = 0.30 \qquad \alpha_s(c) = 0.20 \end{aligned} \qquad ...(253)$$

Table II which exhibits $F(M)$ versus M^2 (Gev^2 units) for several typical meson states brings out rather well the principal features of the predictions, (*i*) spin and flavour degeneracy of $q\bar{q}$ supermultiplet members at the $F(M)$ level, despite huge variations in M^2 and (*ii*) fulfilment of the unit spacing rule $\Delta F = 1$ for successive h.o. supermultiplets; (*i*) is illustrated by the $(\pi, \rho; K, K^*; \phi)$ F-values; (*ii*) is illustrated by

$$(\rho, A_2, \rho_2, A_4)\ ;\ (k^*, k^{**}, k^{***});\ (\phi, f') \qquad ...(254)$$

However, the model fails for the radial excitation of $c\bar{c}$ states. The $F(M)$ degeneracy of (P, V) values for a given flavour species leads to the prediction

$$m(\psi) - m(\eta_c) \approx 100 \pm 20\ \text{MeV} \qquad ...(255)$$

in good agreement with the latest data (Bloom, 1979).

TABLE II

$q\bar{q}$ spectra with harmonic confinement and perturbative QCD

(Here $F_1 = (M^{-2} - m^2)/\Omega_M$; $F_2 = -\frac{1}{4}\,\Omega_M \gamma^{-2} M^{-2}$ (2 J. S-3-Q_N). For other notations, *see text*

Resonance	M^2(GeV^2)	Ω_M	F_1	F_2	F_{QCD}	$F(M)$
ρ (775)	0.601	0.665	+0.432	—0.734	0.116	—0.186
π (140)	0.020	0.431	—0.692	+0.417	+0.146	—0.118
δ (980)	0.960	0.724	+0.893	—0.438	+0.302	+0.757
A_1 (1100)	1.210	0.757	+1.184	0.615	+0.229	+0.798
A_2 (1310)	1.716	0.811	+1.729	—0.862	+0.105	+0.973
B (1235)	1.525	0.792	+1.530	—0.318	+0.163	+1.374
ρ_2 (1685)	2.839	0.899	+2.808	—1.070	+0.111	+1.849
ρ' (1600)	2.560	0.880	+2.552	—0.865	—0.042	+1.646
A_4 (2040)	4.162	0.976	+3.944	—1.243	+0.113	+2.804
K^* (892)	0.795	0.739	+0.539	—0 633	+0.118	+0.024
K (495)	0.246	0.603	—0.250	+0.093	+0.146	—0.011
K^* (1430)	2.045	0.892	+1.848	—0.793	+0.106	+1.162
K^* (1790)	3.204	0.980	+0.848	—1.024	+0.113	+1.937
ϕ (1020)	1.040	0.802	+0.686	—0.577	+0.117	+0.225
f' (1515)	2.295	0.945	+1.910	—0.799	+0.108	+1.219
ψ (3100)	9.610	2.401	+0.737	—0.230	+0.055	+0.562
χ_0 (3410)	11.63	2.000	+1.574	—0.157	+0.131	+1.548
χ_1 (3510)	12.32	2.409	+1.860	—0.245	+0.099	+1.714
χ_2 (3550)	12.60	2.410	+1.976	—0.428	+0.049	+1.597
$\psi'_D(3775)_{L=2}$	14.25	2.418	+2.651	—0.255	+0.096	+2.492

$P \to l\bar{l}$, $V \to e^+e^-$ Decays

Other tests are provided by $P \to l\bar{l}$ amplitudes like $f_{\pi,k}$ and the e^+e^- widths of V-mesons. These were obtained by MS (*to be bublished*) in terms of the elementary VW formulae (Van Royen & Weisskopf, 1967) corrected for the colour effect (Fritzch *et al.*, 1973)

$$f_p = \sqrt{3/m_p}\left(\beta_p^2/\pi\right)^{3/4} \qquad \ldots(256)$$

$$\Gamma_{V\to e^+e^-} = 16\,\alpha^2 m_V^{-2}\; e_Q^2\; \beta_V^3/\sqrt{\pi} \qquad \ldots(257)$$

These values are 5–15 per cent short of the experimental numbers.

More recently, Mitra (In HEP Symposia—*to be published*) has found improved results for these processes, taking account of the relativistic spinor structures of the $q\bar{q}$ wave functions and the full B.S. normalization in the instantaneous approximation. The latter from a 4 × 4 B.S. amplitude $\phi(\vec{q})$ is given by Smith (1969).

$$\int d_q^3\, \mathrm{Tr}\Big[\Lambda_-^{(2)}(-\vec{q})\,\phi^+(\vec{q})\,\Lambda_+^{(1)}(\vec{q})\,\phi(\vec{q}) - \Lambda_+^{(2)}(-\vec{q})\,\phi^+(\vec{q})\,\Lambda^{(1)}(\vec{q})\,\phi(\vec{q})\Big] = \frac{2M}{(2\pi)^3} \qquad \ldots(258)$$

where

$$\wedge_{\pm}(\vec{q}) = (E_q \pm \vec{\alpha} \cdot \vec{q} \pm \gamma_4 m)/2E_q$$

For P-and V-states the spin-structures are given by (*loc. cit.*)

$$\phi_P(\vec{q}) = A\gamma_5(1 + i\gamma.P/M)\,\phi_0(\vec{q}) \qquad \ldots (259)$$

$$\phi_V^{\mu}(\vec{q}) = A(\gamma_\mu + \gamma.PP_\mu/M^2)(1 + i\gamma.P/M)\,\phi_0(\vec{q})$$

where A is determined from (258) to give

$$2A \approx \sqrt{M}\,(2\pi)^{-3/2}\left[1 + \frac{1}{2}\,\beta^2\, m_q^{-2}\right]^{1/4} \qquad \ldots (260)$$

in the equal mass case.

The relativistic $P \to l\bar{l}$ and $V \to e^+e^-$ amplitudes are defined by (cf. Smith, 1969)

$$f_\pi P_\mu = \sqrt{3}\,(3/5\; G_A/V_A)\;\mathrm{Tr}\int \Psi_P(P, k)\, i\gamma_\mu\gamma_5 \mathrm{d}^4k$$

$$\frac{eM_V^2}{g_V}\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{M^2}\right) = \sqrt{3}\sum_i g_i^V e_i \int \mathrm{Tr}\,\Psi_V^{\mu}(P, k)\gamma_\nu \mathrm{d}^4k$$

which are in turn expressible in terms of 3-dimensional B. S. amplitudes at zero distance. The resulting formulae are the same as (256) and (257) except for the factors $(1 + \epsilon)^{1/4}$ and $(1 + \epsilon)^{1/2}$ respectively, where $\epsilon = \frac{1}{2}\beta^2 m_q^{-2}$. These yield the final $P \to l\bar{l}$ numbers in MeV

$$f_\pi = 93\ (95)\ ;\quad f_k = 107\ (109) \qquad \ldots (261)$$

the figures in parenthesis being the data (cf. Bricman *et al.*, 1980).

Similarly the $V \to e^+e^-$ widths in (KeV) are

$$\Gamma_\rho = 6.61\ (6.54 \pm 0.3),\ \Gamma_\omega = 0.74\ (0.76 \pm 0.1)$$

$$\Gamma_\phi = 1.33\ (1.27 \pm 0.10);\ \Gamma_\psi = 4.0\ (4.8 \pm 0.7) \qquad \ldots (262)$$

In a similar manner, but with more involved calculations, $P \to \gamma\gamma$ widths can be calculated with this model, using the diagrams in Fig. 4. The results are

$$\Gamma_{\pi^0} = 8.8\ \mathrm{eV}\ (7.5 \pm 0.5)$$

$$\Gamma_\eta = 380\ \mathrm{eV}\ (325 \pm 50) \qquad \ldots (263)$$

Results for qqq System

A solution to this problem has been given in MS so far for equal mass kinematics on very simialr line to the $q\bar{q}$ system but with much heavier algebra. The starting point in this case is eqn. (238) and these are *two* independent internal variables $(\vec{\xi}, \vec{\eta})$ defined by

$$\vec{p}_1 = -\vec{\eta},\quad \vec{p}_{2,3} = \tfrac{1}{2}\vec{\eta} \mp \tfrac{1}{2}\sqrt{3}\,\vec{\xi} \qquad \ldots (264)$$

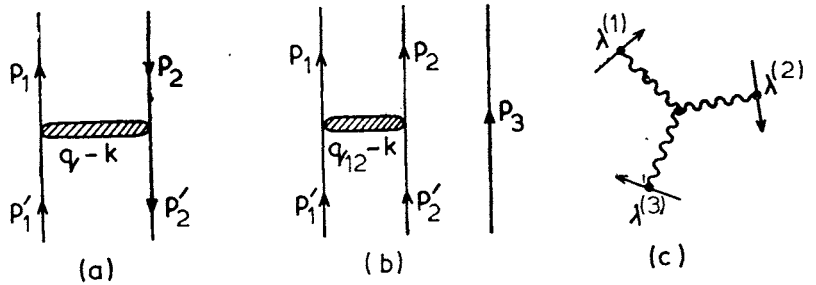

FIG. 4. Diagrams (*a*) and (*b*) are for qq and pairwise qq interactions respectively; (*c*) stands for a three-way qqq interaction through an effective 3-gluon vertex.

For the spring constant ω_{qq} in this case, MS make the plasuible *ansatz:*

$$\omega_{qq}^2 = \tfrac{1}{3}\,\omega_{\bar{q}\bar{q}}^2 = \frac{2}{3} m_q \tilde{\omega}^2 \qquad \ldots(265)$$

which facilitates a solution to the qqq problem in terms of the *same* parameters $(\tilde{\omega}, m_q)$ as employed for the $q\bar{q}$ problem. Without the QCD term which is again small. The qqq equation in the quadratic approximation to the (ξ, η) variables is of the form

$$\left[\frac{1}{9}\left(m_0^2 - M^2\right) + \frac{3}{2}\left(\vec{\xi}^2 + \vec{\eta}\right)\right]\psi\left(\vec{\xi}, \vec{\eta}\right)$$

$$= \frac{3}{4}\,\omega_{qq}^2\, M^{-1}\left[\frac{4}{9}M^2\left(\vec{\Delta}_{\xi}^2 + \vec{\nabla}_{\eta}^2\right) - 4\,\vec{J}.\vec{S} + 9 - \tfrac{1}{2}\hat{L}_{\xi} - \tfrac{1}{2}\hat{L}_{\eta}\right.$$

$$\left. + \tfrac{1}{2}\hat{Q}_B + \tfrac{1}{2}\hat{Q}'_B\right]\psi\left(\vec{\xi}, \vec{\eta}\right) \qquad \ldots(266)$$

where $\hat{L}_{\xi,\eta}$ are as in (243) and $\hat{Q}_B$ a sum of three $\hat{Q}_q$ terms as in (244) corresponding to $\hat{q}_{12}$, $\hat{q}_{23}$ and $\hat{q}_{31}$ and $\hat{Q}'_B$ certain additional terms similar to $\hat{Q}_B$ arising in the three-body case in a consistent quadratic approximation to the $(\vec{\xi}, \vec{\eta})$ variables. The mass formula derivable from (265) on lines similar to (242), is now

$$F_B(M) = N + 3, \quad (N = N_{\xi} = N_{\eta}) \qquad \ldots(267)$$

$$F_B(M) = \left(M^2 - m_0^2\right)\Big/\Omega_B - \tfrac{1}{4}\,\Omega_B\, M^{-2}\gamma_B^{-2}$$

$$\left(\tfrac{2}{3}\,\vec{J}.\vec{S} - \frac{3}{2} - Q_N - Q'_N\right) + F_{QCD} \qquad \ldots(268)$$

$$\Omega_B = 6\tilde{\omega}\gamma_B\sqrt{m_0 M}\;;\; \gamma_B^2 = 1 + 9\tilde{\omega}^2/8Mm_0 \qquad \ldots(269)$$

$$27\,\beta_B^2\,\gamma_B^2 = \Omega_B$$

$$\hat{Q}_B \Rightarrow 6\bar{Q}_N = 8Q_N \qquad \ldots(270)$$

$$\hat{Q}'_B \Rightarrow 6Q'_N \approx -\frac{4}{3}\left(1 + m_0^2 M^2\right)\left(\tfrac{7}{2}Q_N + \tfrac{9}{2} - \tfrac{9}{2}(N + 3/2)^2\right) \qquad \ldots(271)$$

The "true" wave function ϕ_0, related to ψ_0 in a manner similar to (250), is

$$\phi_0\left(\vec{\xi}, \vec{\eta}\right) = \left(\pi\beta_B^2\right)^{-3/2} \exp\left[-\tfrac{1}{2}\left(\vec{\xi}^2 + \vec{\eta}^2\right)\beta_B^{-2}\right] \qquad \ldots(272)$$

The F_{QCD} term is small, as in (246) too.

Comparison of (268) with the data (only on *non-strange* baryons because of equal mass kinematics) was made exactly on the same basis as employed for (246) w.r.t. the $q\bar{q}$ data, and with the *same* inputs (253), in view of the assumed connection (265) of ω_{qq} with $\tilde{\omega}$. This is exhibited in Table III for the prominent non-strange baryons which are classified according to the *standard* supermultiplet structures (Greenberg, 1964; and Dalitz, 1966). The following features are noticeable :

(*i*) The crucial role of the $\vec{J}.\vec{S}$. term (in the $\bar{Q}_N$, Q'_N background) is reflected in the proximity of F_B values for several pairs, especially (N, Δ), (D_{13}, D_{15}), (F_{15}, F_{37}), (D_{35}, G_{19}), (H_{19}, $H_{3,11}$), despite huge differences in their M^2-values in FKR ($\approx$1 Gev^2) units.

TABLE III

Light (ud) qqq spectrum under on shell harmonic confinement and perturbative QCD

(Here $\bar{F}_1 = \left(M^2 - m_0^2\right)/\Omega_B$; $\bar{F}_2 = -\frac{1}{4}\,\Omega_B M^{-2}\gamma_B^{-2}\left(\frac{2}{3}\vec{J}.\vec{S}. - \frac{3}{2}\bar{Q}_N - Q'_N\right)$. (For symbols *see text*)

Resonance	M^2	Ω_B	$\bar{F}_1$	$\bar{F}_2$	F_{QCD}	$F_B(M)$
N (1938)	3.756	0.824	0.211	0.728	0.100	1.038
Δ (1236)	1.5277	0.939	0.875	0.031	0.041	1.947
D_{13} (1520)	2.310	1.037	1.548	0.596	0.071	2.215
S_{11} (1510)	2.280	1.034	1.523	0.715	0.072	2.309
D_{15} (1670)	2.789	1.085	1.920	0.220	0.050	2.190
D_{33} (1690)	2.856	1.085	1.971	0.457	0.066	2.493
S_{31} (1640)	2.690	1.076	1.844	0.589	0.068	2.501
F_{11} (1470)	2.161	1.020	1.426	1.419	0.162	3.007
F_{15} (1690)	2.856	1.091	1.970	0.849	0.063	2.982
F_{37} (1960)	3.802	1.170	2.648	0.428	0.042	3.118
D_{35} (1950)	3.842	1.172	2.674	1.315	0.056	4.045
G_{19} (2200)	4.840	1.240	3.234	0.634	0.042	4.009
H_{19} (2200)	4.840	1.240	3.334	1.347	0.050	4.731
Δ_4 (2420)	5.856	1.299	3.965	0.850	0.037	4.852
$I_{1,11}$ (2580)	6.656	1.345	4.423	1.370	0.044	5.837
Δ_6 (2880)	8.294	1.414	5.365	1.234	0.033	6.632
Δ_8 (3350)	11.222	1.523	6.904	1.576	0.013	8.494

(*ii*) The $\bar{Q}_N$, Q'_N terms are particularly visible through the remarkable closeness of F_B for P_{11} with F_{15} and F_{37} despite the unusually low mass of P_{11} thus lending credence to the $N = 2$ status for this state, as the first radial h.o. partner of F_{15} ($L = 2$).

(*iii*) Deviations of (70, 1^-) states from standard unit h.o. spacings, exhibited e.g., by $D_{13}^2 - N^2 \gg F_{15}^2 - D_{13}^2$ are considerably regularized at the F_B-level.

(*iv*) The successive Regge excitations (N_L, $\triangle_L$) all the way upto $L = 8$, not only obey the predicted $\triangle F_B = 2$ rule rather well, but even the (N, $\triangle$) degeneracy is also restored at the F_B level, despite huge $\triangle_L - N_L$ differences.

(*v*) The state D_{35} which has been the subject of some recent controversy (Horgan, 1980) seems to fit in rather well with the simple $L = 3$ picture without any additional assumptions. The mass spectra in the qqq case, as well as in the earlier $q\bar{q}$ case, show a $M^{3/2} \sim N$ variation, reminiscent of a *linear* potential (Quigg & Rosner, 1979) rather than $M^2 \sim N$ as in FKR's (1971) h.o. model. This feature, it may be noted, has come about from the *relativistic* framework of BS dynamics, even though the input is an h.o. kernel.

To conclude this section, a few words of comparison with the results of some contemporary approaches discussed in §3 are in order. Thus, e.g., several features of the even-wave (Mitra 1974, 1975) h.o. model made with an FKR-like framework seem to be effectively simulated by the usual h.o. assumption made within the wider framework of BS dynamics. Again, the smallness of the QCD term in both the $q\bar{q}$ and qqq cases is sharply at variance with the conclusions of (Isgur & Carl, 1978, 1979). Nevertheless, the small numerical results not only justify the perturbative QCD assumption *afortiori*, but are also not unwelcome in the overall context of agreement with experiment which now comes about through certain equivalent symmetry breaking terms originating from the confining (h.o.) interaction. The B. S. treatment is also free from the formal objections (Bhaduri *et al.*, 1980; Stanley & Robson, 1980; and Hey, 1980), to a N.R. treatment, such as given by Isgur & Carl (1970), for *light* quark systems. Finally, the intuitive assumptions (245) and (265) which have proved crucial for providing a very economical description of a large class of hadronic phenomena, seem to be amply justified from fairly sensitive comparisons with a large variety of relevant data. The next section is devoted to a similar model for $qq\bar{q}\bar{q}$ systems.

§8 Baryonium States and the H. O.

Baryonium mesons which are believed to $qq\bar{q}\bar{q}$ states are the next non-trivial example of a few-quark systems which have been extensively studied in recent years in different models (Rosensweig, 1976; Reinders, 1978; Eilwanger, 1978; Van Royen & Weisskopf, 1967; *Proc int. HEP Conf.*, 1980; Kamae *et al.*, 1980; and Carroll *et al.*, 1980). Their reluctance to decay into two mesons gave some credibility to the "diquark" picture (Rosensweig, 1976; and Mitra 1979) which would facilitate their decay into $B\bar{B}$ by picking up a $q\bar{q}$ loop through the QPC mechanism (*see* §4). Early observation of certain sharp peaks in the 2–3 GeV rank in $N\bar{N}$ annihilation experi-

ments (Carter *et al.*, 1977; and Martin & Pennington, 1979) had also stimulated interest in the decays of $qq\bar{q}\bar{q}$ states through concrete models (Ader *et al.*, 1979; and Barbour & Ponting, 1979) with conflcting results until the results of later experiments with better statistics have revealed a disappearance of most of these states. This fact has brought to sharp focus the question of the very existence of these states, together with the related issues of their mass positions and observability through their couplings with suitable probes.

Now any serious investigation of such a fundamental question as the nature of $qq\bar{q}\bar{q}$ states requires a sufficiently reliable dynamical model which goes beyond mere classification schemes and N. R. models (Ader *et al.*, 1979; and Chan & Hogaasen, 1978). Bag models (Jaffe, 1977, 1978; and Fukugita *et al.*, 1978, 1979) have their own limitations insofar as their mass and width predictions are no longer consistent with the "disappearance" of states in the 1.9–2.5 GeV range, even though the result of phase shift analyses (cf. Wong & Liu, 1980) are not inconsistent with the bag picture. An N.R. framework cannot be basically acceptable for a $qq\bar{q}\bar{q}$ description of such states in terms of light quarks. It should, therefore, be of interest to discuss the predictions of a BS framework for $qq\bar{q}\bar{q}$ systems under harmonic confinement in view of its successes on $q\bar{q}$ and qqq systems described in (§7). These have recently been obtained by Mitra using the *same* spring constants for $q\bar{q}$ and qq interactions as employed for meson and baryon spectra, the main result now being that even the lowest baryonium state is well above 2.2 GeV so that all of them must be highly unstable against strong decay and hence difficult to observe. We sketch briefly the general feature of the model and the main results thereof.

The $qq\bar{q}\bar{q}$ Model : Colour Effects

The standard classification of $qq\bar{q}\bar{q}$ states into T (true) and M (mock) baryoniums (Chan & Hogaasen, 1977) is based on an interplay of the colour index and the Pauli principle within the overall restriction of a colour singlet state. Now a qq pair can have colour representations 6 (symmetric) and $\bar{3}$ (antisymmetric) respectively. The singlet structure demands the associations $(\bar{3}, 3)$ and $(6, \bar{6})$ which are called T corresponding representations of the $\bar{q}\bar{q}$ pair $\bar{6}$ and 3 respectively The overall colour and M states respectively (*loc. cit.*), the obvious reason for such nomenclature being that only a qq pair in the $\bar{3}$ state can combine with a quark (3) to produce a *physical* baryon. (A 6-state of qq *cannot* give a baryon by combining with a third quark).

The Pauli principle operates separately on (qq) and $(\bar{q}\bar{q})$ systems. To see this effect more clearly and in particular the selection rules it produces on the quantum numbers of the overall $qq\bar{q}\bar{q}$ states, let the indices (1, 2) and (3, 4) denote the (qq) and $(\bar{q}\bar{q})$ pairs respectively. Define further the internal momenta $\vec{\xi}, \vec{\eta}$ and the separation momentum $\vec{\zeta}$ between (qq) and $(\bar{q}\bar{q})$ for the *equal* mass case, as follows :

$$\sqrt{2}\vec{\xi} = \vec{p}_1 - \vec{p}_2, \quad \sqrt{2}\,\vec{\eta} = \vec{p}_3 - \vec{p}_4 \qquad \ldots(273)$$

$$2\vec{\zeta} = \vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4 \qquad \ldots(274)$$

Now since for T-states, the colour wave function is antisymmetric in each pair (qq and $\bar{q}\bar{q}$), the spin-isospin-cum-space functions should be symmetric for each. Thus *even* l_ξ must go with $S_{12} = 1$ and $I_{12} = 1$; or $S_{12} = 0$ and $I_{12} = 0$. Similarly, *odd* l_ξ is associated with $S_{12} = 0$ and $I_{12} = 1$, or $S_{12} = 1$ and $I_{12} = 0$. Identical statements hold for l_η versus S_{34} and I_{34} associations for T-states. For M-states which are symmetric $(6, \bar{6})$ in the colour wave functions of qq and $\bar{q}\bar{q}$ pairs, statements exactly complementary to above should hold of even and odd l_ξ; likewise for l_η. As for l_r excitations, corresponding to the separation of (12) from (34), there are no selection rules inhibiting them. In particular, for the *ground* states which correspond to $l_\xi = l_\eta = 0$, T-states have $S_{12} = S_{34} = 1$ or $S_{12} = S_{31} = 0$, but M-states have $S_{12} = 1$ and $S_{34} = 0$ or $S_{12} = 0$ and $S_{34} = 1$.

With these guidelines, the spin and colour dependence of the various pairs of interactions can be simplified for specified states T or M. The colour structures of the latter may be expressed in terms of colour wave vectors $\vec{u}$ corresponding to the three colour states (R, G, B) for each index. Thus the T-state colour function may be expressed in the obvious vector notation

$$\phi_T = (\vec{u}_1 \times \vec{u}_2).(\vec{u}_3 \times \vec{u}_4) \qquad \ldots(275)$$

while the corresponding M-state function requires a tensor (i, j) representation

$$\phi_M = u^{(12)}_{ij} u^{(34)}_{ij} \qquad \ldots(276)$$

$$u^{(12)}_{ij=} \tfrac{1}{2} u_{1j} u_{2i} + \tfrac{1}{2} u_{1i} u_{2j} - \tfrac{1}{3} u_1, \vec{u}_2 \delta_{ij} \qquad \ldots(277)$$

The colour dependence of the diagonal potentials (12) and (34) can be immediately simplified.

$$F_{12} \phi_T = F_{34} \phi_T = -\tfrac{2}{3} \phi_T \qquad \ldots(278)$$

$$F_{12} \phi_M = F_{34} \phi_M = +\tfrac{1}{3} \phi_M \qquad \ldots(279)$$

For the four pairs of $q\bar{q}$ interaction (13, 24, 14, 23), there are in general 'non-diagonal' effects. A practical method of handling this problem is to make the following replacements in these :

$$F_{13}, F_{24}, F_{23}, F_{14} \Rightarrow \tfrac{1}{2} \vec{\Lambda}_{12} \cdot \vec{\Lambda}_{34} \qquad \ldots(280)$$

$$2 \vec{\Lambda}_{12} = \vec{\lambda}_1 + \vec{\lambda}_2, \; 2 \vec{\Lambda}_{34} = \vec{\lambda}_3 + \vec{\lambda}_4 \qquad \ldots(281)$$

The structure (280) which is now diagonal in T as well as M states, can be calculated in terms of the general SU (3) formula (*see* Lichtenberg, 1978)

$$F^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \qquad \ldots(282)$$

so that

$$\tfrac{1}{2}\vec{\Lambda}_{12} \cdot \tfrac{1}{2}\vec{\Lambda}_{34} \begin{pmatrix} \phi_T \\ \phi_M \end{pmatrix} = \begin{pmatrix} -\frac{1}{3}\phi_T \\ -\frac{5}{6}\phi_M \end{pmatrix} \qquad ...(283)$$

Reduction of the $qq\bar{q}\bar{q}$ Equation

The general B.S. equation for the $qq\bar{q}\bar{q}$ system is given in (226). The principle of reduction of this equation first due to spin-structures by Gordon techniques and then to the instantaneous form, follows the qqq pattern, except for the presence of *two* spectators in each of the six pair-wise interaction kernels on the RHS. For the elimination of the two time-like momenta p_k^0, p_l^0 in the (ij) kernel, an obvious extension of the recipe (235) in the *equal* mass case is

$$p_k^0,\ p_l^0 \Rightarrow (\tfrac{1}{4}M, \tfrac{1}{4}M) \qquad ...(284)$$

The equal mass assumption is obviously adequate for non-strange baryonium states. Further, the definitions (217–218) for the momentum pairs (12) may be taken to hold for all the six pairs, with an obvious extension of the notation. Finally, the quadratic approximation which was employed in § 7 for the qqq system is probably better if anything for the $qqqq$ system since the expansion parameter in either case is *not* q_{ij}/m_q, but rather q_{ij}/M, where M is quite large.

Now the modes of excitation of T and M states involve certain special combinations of the spring constants ω^2_{qq} and $\omega^2_{q\bar{q}}$ governed by eqn. (278–283). These can be further classified into ζ-type (external) and ξ,η-type (internal) excitations, in accordance with the C. G. coefficients with which they appear by virtue of the relations (273–274). Taking account of the relations (245) and (265), these results may be summarized in the form

$$\omega^2_{ET};\ \omega^2_{IT} = \left(\frac{1}{2};\ \frac{5}{12}\right) m_q\tilde{\omega}^2 \qquad ...(285)$$

$$\omega^2_{EM};\ \omega^2_{IM} = \left(\frac{5}{4};\ \frac{13}{24}\right) m_q\tilde{\omega}^2 \qquad ...(286)$$

Using all these ingredients, the master equation satisfied by the 256-component multispinor $\psi(\vec{p}_1\,\vec{p}_2\,\vec{p}_3\,\vec{p}_4)$ takes the form

$$\Big[\left(m_q^2 - \frac{1}{16}M^2\right)M + \left(\frac{3}{2}M + 4\omega_E^2/m_q^2\right)\zeta^2 + \left(\frac{3}{2}M\,4\omega_I^2/m_q^2\right)(\xi^2+\eta^2) - \omega_E^2 M^2 \nabla_\zeta^2 - \omega_I^2 M^2\left(\nabla_\xi^2 + \Delta_\eta^2\right) - (SS) + (SO) - (MSO) + \hat{Q}_\varpi\Big]\begin{pmatrix}\psi_I \\ \psi_M\end{pmatrix} = 0 \qquad ...(287)$$

$$\hat{Q}_\omega = (\hat{Q}_{12} + \hat{Q}_{34})\begin{pmatrix} 1/2 \\ -1/4 \end{pmatrix} \omega_{qq}^2 + (\hat{Q}_{13} + \hat{Q}_{24} + \hat{Q}_{14} + \hat{Q}_{23})\begin{pmatrix} 1/4 \\ 5/8 \end{pmatrix} \omega_{q\bar{q}}^2 \quad \ldots(288)$$

$$SO = 8\omega_E^2 \vec{S} \cdot \vec{L}_\zeta + 8\omega_I^2 \vec{S} \cdot (\vec{L}_\xi + \vec{L}_\eta) \quad \ldots(289)$$

$$SS = 8\left(S_{12}^2 + S_{34}^2 - 3\right)\begin{pmatrix} 1/2 \\ -1/4 \end{pmatrix} \omega_{qq}^2 + 16\, \vec{S}_{12} \cdot \vec{S}_{34} \begin{pmatrix} 1/4 \\ 5/8 \end{pmatrix} \omega_{q\bar{q}}^2 \quad \ldots(290)$$

$$MSO = \begin{pmatrix} 1/3 \\ 17/6 \end{pmatrix} 2\, m_q \omega^2 \left[\vec{\sigma}_\zeta \cdot (\vec{L}_\xi - \vec{L}_\eta) + i\vec{\sigma}_\xi \cdot \vec{\zeta} \times \vec{\partial}_\xi - i\vec{\sigma}_\eta \cdot \vec{\zeta} \times \vec{\partial}_\eta\right] \quad \ldots(291)$$

$$2\vec{S} = \vec{\sigma}_1 + \vec{\sigma}_2 + \vec{\sigma}_3 + \vec{\sigma}_4 \quad \ldots(292)$$

$$2\vec{S}_{12} = \vec{\sigma}_1 + \vec{\sigma}_2,\ 2\vec{S}_{34} = \vec{\sigma}_3 + \vec{\sigma}_4 \quad \ldots(293)$$

$$2\vec{\sigma}_\zeta = \vec{\sigma}_1 + \vec{\sigma}_2 - \vec{\sigma}_3 - \vec{\sigma}_4,\ \vec{L}_\zeta = i\vec{\zeta} \times \vec{\partial}_\zeta \quad \ldots(294)$$

$$\sqrt{2}\vec{\sigma}_\xi = \vec{\sigma}_1 - \vec{\sigma}_2,\ \sqrt{2}\vec{\sigma}_\eta = \vec{\sigma}_3 - \vec{\sigma}_4 \quad \ldots(295)$$

$\hat{Q}_{ij}$, for each ij pair, is an operator of the form (244) corresponding to $\vec{q}_{ij}$, together with certain additional correction terms that arise in the quadratic approximation, as in the qqq case.

The Baryonium Spectrum

Except for the (MSO) term which is non-diagonal, most of the terms in (287) are either diagonal in the (12), (34) representation as is the case with SS and SO, or their diagonal parts can be extracted, as is the case for the $\hat{Q}_\omega$ operator using techniques similar to MS (*to be published*). Dropping the (MSO) term, an approximate but in practice highly accurate (mass)^2 spectrum is derivable from (287) on lines similar to the $q\bar{q}$ and qqq cases in the form

$$G(M) = \omega_E\left(\frac{3}{2}M + 4\omega_E^2/m_q^2\right)^{\frac{1}{2}}(N_\zeta + 3/2) + \omega_I\left(\frac{3}{2}M + 4\omega_I^2/m_q^2\right)^{\frac{1}{2}}(N_\xi + N_\eta + 3) \quad \ldots(296)$$

$$G(M) \approx \left(M^2 - 16\, m^2\right)/32 + [\langle \hat{Q}_\omega \rangle - \langle SO \rangle - \langle SS \rangle]/2\, M + G_{QCD} \quad \ldots(297)$$

$$<\hat{Q}_{12} + \hat{Q}_{34}> = 4\,Q_{N\xi} + 4\,Q_{N\eta} + 2\,(Q_{N\xi\zeta} + Q_{N\eta\zeta}) - \frac{8}{5}(7/8)^4\left(1 + 16\,m_q^2/M^2\right)(Q_{N\xi} + Q_{N\eta} + 3 - 3\,Q_{N\xi\eta} - 5/4\,Q_{N\zeta\xi} - 5/4\,Q_{N\eta\zeta}) \quad ...(298)$$

$$<\hat{Q}_{13} + \hat{Q}_{24} + \hat{Q}_{14} + \hat{Q}_{23}> = 4\,Q_{N\zeta} - 2\,Q_{N\xi\zeta} - 2\,Q_{N\eta\zeta} - 4\left(\frac{7}{8}\right)^4\left(1 + 16\,m_q^2/M^2\right)\left(Q_{N\zeta} + \frac{1}{2}\,Q_{N\xi} + \frac{1}{2}\,Q_{N\eta} - \frac{3}{2}\,Q_{N\xi\zeta} - \frac{3}{2}Q_{N\xi\eta} - \frac{3}{2}\,Q_{N\eta\zeta} + \frac{12}{5}\right) \quad ...(299)$$

$< \hat{Q}_{\omega} >$ is calculable from (288) in terms of (298–299). Calculation of $< SO >$ and $< SS >$ only needs the knowledge of the spins of the (12) and (34), and the orbital excitations $\vec{L}_{\xi,\eta,\zeta}$ in accordance with the selection rules outlined under colour effects of the $qq\bar{q}\bar{q}$ model. $Q_{N\xi}$ is given by (249) with $N \to N_\xi$, etc. Further

$$Q_{N\xi\eta} = (N_\xi + 3/2)\,(N_\eta + 3/2) \quad ...(300)$$

G_{QCD} is a small term contributed by one gluon-exchange effects, and is totally negligible for $qq\bar{q}\bar{q}$ systems in the mass range of $\sim$ 2 GeV.

The general features of the spectrum (296) are similar to the meson and baryon cases, but there are now two independent spring constants $\Omega_{E,I}$ for each type (T or M) of baryonium. However, unlike the $q\bar{q}$ and qqq systems, where the prior experimental knowledge of the masses had suggested a comparison of theory with data in terms of a direct check on the universality of the $F(M)$ functions, one needs a different kind of strategy in the $qq\bar{q}\bar{q}$ case where the very masses must be "determined" from the formula (296) for appropriate quantum states and excitations. For this purpose, the values of m_q and $\tilde{\omega}$ are 'known' from the $q\bar{q}$ and qqq data with a reasonable degree of confidence. However, the zero-point h.o. values 3/2 and 3 on the RHS of (296) cannot be trusted with an equal degree of confidence for a self-consistent determination of M using both sides of the equation.

In this respect, the comparison of $q\bar{q}$ and qqq mass formulae with the data (Tables II and III) provide some valuable clues to the "empirical" values of these zero-point constants. For the non-strange (ud) $q\bar{q}$ case, this value is very nearly zero (Table II) while for non-strange qqq baryons (Table III) it is nearly unity. These values represent short-falls of 3/2 and 2 from the standard h.o. values 3/2 and 3 for these cases respectively. Extrapolating on this basis one should expect a total short-fall of about 5/2 units in the $qq\bar{q}\bar{q}$ systems made up of (ud) quarks. This number must be divided between the internal (ξ, η) and the external (ζ) contributions to mass formula (296), but in view of the near equality of ω_I and ω_E, any uncertainty on this

account should not be significant. Further, since the intention is to obtain a *lower* bound of these masses for reasons which will be clearer below, a further lowering of this figure, say from 5/2 to 3, would facilitate, an even more conservative estimate from the 'lower' bound point of view.

The reduction 3 has been arbitrarily divided into 3/2 and 3/2 for the ω_I and ω_E terms, for a concrete estimate of the masses. Using the suffixes s and t for the intrinsic spin-values (singlet or triplet) of each of the (12) and (34) clusters, the spin structures of the ground states of T and M may be termed at T_{tt}, T_{ss}; M_{st}, M_{ts} in accordance with the selection rules discussed under $qq\bar{q}\bar{q}$ model. Now these T_{tt} states can be further classified in terms of $J = 0$, 1, 2, while T_{ss} and M_{ts}/M_{st} states correspond only to $J = 0$ and $J = 1$ respectively. The mass splittings of all these ground states are governed mainly by the SS term of (297), since the (SO) term vanishes for such states.

With these ingredients, the masses of the various ground states are as follows :

State	$= T_{tt}$	T_{tt}	T_{tt}	T_{ss}	M_{st}	M_{ts}
J	$= 0$	1	2	0	1	1
M (GeV)	$= 2.27$	2.40	2.55	2.50	2.54	2.54

A typical first excited $L = 1$ state of T_{tt} is around $M = 3.4$ GeV.

These results indicate that the ground states of these exotic mesons are rather highly placed, in marked contrast, e.g., to the findings of the bag model which tend to place these states at the masses of the scalar mesons (Jaffe, 1977, 1978; and Fukugita *et al.*, 1978, 1979). On the other hand, it has also been admitted that the bag model does not conform to certain vital data (Bricman *et al.*, 1980) on the scalar $\pi\pi$-resonances. The dual unitarization model (Chew & Rosenzweig, 1978) on its own part, places these resonances at some intermediate values, starting around 1.5 GeV.

If these predictions of the BS model are taken seriously, they do offer a plausible explanation of their elusive nature, since these resonances would decay very quickly into lighter hadrons largely unencumbered by centrifugal effects. For the fact that in some of the model calculations, there are indications of relatively "small" widths (Ader *et al.*, 1979; and Barbour & Ponting, 1979) may in part be due to the sensitivity of these models to the angular momentum barrier effects to the "size" of the $qq\ \bar{q}\bar{q}$ states, since these models take for granted the "bag"-like picture of *low ground state* masses. Further the nearness of these "ground" states (27–55), togther with their generally large widths, would tend to produce a strongly overlapping and blurring effect, making it very difficult to resolve these individual states into a clean pattern. The next bunch of baryonium states would not be expected until around 3.4 GeV, in the region of cc excitations, where there would appear many more of such states again. The details of these predictions are given by Mitra (*to be published*).

(§9) Conclusions : An Overview of H.O.

In this article, we have chosen to review a rather limited cross section of quark physics, namely those aspects of its developments which have had a direct bearing on the harmonic oscillator (h.o.) model. In retrospect, it appears that while the role of h.o. (as the simplest soluble model) in the "understanding" of quantum

mechanics is almost text-book material, it is indeed amazing how crucial a role it has been playing since the development of "subatomic" physics. In nuclear physics in particular, this is evidenced through the extensive use of shell model wave functions as an extremely successful basis for correct predictions of diverse bound state properties of nuclear systems. This fact has been largely independent of the deeper theoretical question of validity—or better the "range of validity"—of the h.o. concept at the *dynamical* level.

Indeed, in nuclear applications of the h.o. concept, a conscious distinction has all along been made between its purely "applicational" aspects in terms of h.o. oriented basis wave functions, and the more sensitive dynamical question of whether (and to what extent) these shell model wave functions are "formally derivable" in a dynamical fashion from a pairwise N-N interaction of the h.o. type. The main lesson of the shell-model seems to have been that, while the latter aspect may or may not be valid (probably less likely to be valid than not), the extremely successful use of shell model wave functions as *basis* states for a wide ranging class of nuclear problems, constitutes a sufficiently strong *afortiori* evidence of their *practical* validity at the applicational level.

In quark physics, the tentacles of the h.o. model seem to have reached out at least a step or two further. While its initial contact with hadron spectroscopy had been motivated by its obvious simplicity (Greenberg, 1974; and Dalitz, 1966), the startlingly successful results of application at the full-fledged dynamical level, howsoever naive, have had the effect of a powerful impetus towards more ambitious formulations (Feynman *et al.*, 1971). A possible theoretical reason for this extra enthusiasm for h.o. in quark physics compared to its "nuclear" counterpart could perhaps be sought in the obvious relevance of so-called quark or duality diagrams whose rationale seemed to lie in the h.o. model as the message of the Veneziano (1968) model had indicated.

A second reason which became, more prominent in the seventies could best be expressed in terms of the compulsions of *confinement* which obviously requires a rising potential with distance, the question of "linear" *vs.* "square" form being of a lesser relevance in the absence of a formal *theoretical* resolution of the issue. Since a formal confinement mechanism has not yet come about after several years of QCD, calculational convenience has obviously played an important part in determining the natural preference for the h.o. model to other phenomenological forms. Indeed so great have been the advantages of h.o. for calculational purposes, without faring at any worse at the observational level than its other counterparts of a "confining" type, that efforts to find theoretical rationales for FKR-like approaches had engaged the attention of theorists all through the seventies. These efforts have ranged from the purely quantum mechanical level with a *finite* number of degrees of freedom, to the full-fledged field-theoretical level with no bar on the number of degrees of freedom. In regard to the latter, the Bethe-Salpeter equation which represents the most natural language for field-theoretical dynamics of strong interactions, has all along played a crucial role, even where the systems considered have often involved only a *finite* number of "active" degrees of freedom (especially $q\bar{q}$ systems with effective "potentials").

In the present states of the art in quark physics, when the confining part of the interaction between $q\bar{q}$ and qq pairs still needs to be put in by hand, the Bethe-Salpeter equation has been playing an increasingly important role in determining the dynamical framework. Now even within an N. R. framework governed by the Schrödinger equation, the calculational advantage of the h.o. over, e.g., the *linear* potential becomes increasingly felt for hadron systems consisting of *more* than two quarks (e.g., qqq, $qq\,\bar{q}\bar{q}$, etc.), as the number of quarks in the system increases. For *relativistic* n-quark systems which necessitate a Bethe-Salpeter like treatment, the relative perference for h.o. would have to be still more marked, and must rapidly increase with n. In particular, it has been seen from §7 and §8 that mass-spectra are at least expressible in a closed form for such systems with input h.o. forces. At the present state of computer technology, it would be extremely optimistic even to think of equally ambitious calculations with alternative forms of confinement (e.g., linear) for n-quark systems with $n \geqslant 3$, short of drastic approximations, perhaps defeating in the process the very purpose behind such alternatives, unless of course the latter are "formally derivable" from a deeper theortical principle as QCD.

The bigger issue of a formal 'rationale' for confinement, and in particular the place of h.o. in such a scheme, is perhaps a much deeper question than can be properly answered at the present stage of theoretical development. In principle, one would be inclined to believe a non-abelian gauge theory to have all the necessary ingredients of confinement, as evidenced, e.g., by efforts (Callan *et al.*, 1978) to make use of the '*instanton*' (Polyakov, 1975) mechanism in this context, or say, two-dimensional lattice gauge theories towards the same end (Wilson, 1974, 1976), but in practice no concrete and viable "theory" of confinement to be in sight as yet. Indeed, even non-gauge on the lines of the general theory of relativity with f-gravity replacing ordinary gravity in an appropriate de-Sitter space, have been suggested as a possible mechanism for confinement and found to lead to h.o. like potentials (Salam & Strathdee, 1977*a*,*b*). More theoretical models for confinement would presumably be forthcoming in the near future with possible ramifications on field theory, but until a general consensus is reached on the issue, a confining interaction would presumably have to be handled at an effective phenomenological level. And as long as such a situation exists, the h.o. will continue to enjoy an obvious preference because of the obvious simplicity it offers in applications without coming into visible conflict with data.

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References

Ader, J., Bonnier, B., and Sood, S. (1979) *Phys. Lett.*, **84B**, 468.
Alibaso, C., and Schierholtz, G. (1977) *Nucl. Phys.*, **B126**, 461.
Aly, H. (1970) *Lectures in Particles and Fields.* Gordon and Breach, New York.
Amado, R. D. (1962) *Phys. Rev.*, **132**, 485.
Appelquist, T., and Politzer, H. D. (1975) *Phys. Rev. Lett.*, **34**, 43.

Arafune *et al.* (1977) *Phys. Rev.*, **D16**, 772.
Aubert, J. J. *et al.* (1974) *Phys. Rev. Lett.*, **33**, 1404.
Augustin, J. E. *et al.* (1974) *Phys. Rev. Lett.*, **33**, 1406.
Barbieri, R., Gatto, R., and Kogerler, R. (1975) *Phys. Lett.*, **56B**, 477.
——— (1976*a*) *Phys. Lett.*, **60B**, 183.
——— (1976*b*) *Phys. Lett.*, **B105**, 125.
——— (1976*c*) *Phys. Lett.*, **60B**, 183.
Barbour, I., and Ponting, D. (1979) *Nucl. Phys.*, **B149**, 547.
Bardeen, W. *et al.* (1975) *Phys. Rev.*, **D11**, 1094.
Becchi., C., and Morpurgo, G. (1966) *Phys. Rev.*, **149**, 1284.
Bethe, H. A. (1956) *Phys. Rev.*, **103**, 1353.
Bethe, H. A., and Salpeter, E. E. (1967) *Handbuch der Physik*. Springer Verlag, **35**, 88.
Bhaduri, R. K. *et al.* (1980) *Phys. Rev. Lett.*, **44**, 1369.
Billoire, A., and Morel, A. (1978) *Nucl. Phys.*, **B135**, 131.
Bjorken, J. D., and Droll, S. D. (1964) *Relativistic Quantum Mechanics*. McGraw Hill, N. Y., U.S.A
Blaha, S. (1974) *Phys. Rev.*, **D10**, 4268.
Blankencecler, R., and Sugar, R. (1966) *Phys. Rev.*, **142**, 1051.
Bloom, E. (1979) In : *Proc. Fermilab Conf. on Photon-Hadron Interactions*, August.
Bohm, M., Joos, A., and Krammer, M. (1973*a*) *Nucl. Phys.*, **B50**, 397. Referred to as B J K.
——— (1973*b*) *Acta Phys. Austr.*, Suppl. **XI**, 3.
——— (1975) *Acta Phys. Austr.*, **41**, 401.
Bohm, M., and Krammer, M. (1974) *Phys. Lett.*, **50B**, 457.
Bricman, C. *et al.* (1980) Particle data group. *Rev. Mod. Phys.*, **52**, No. 2.
Brodsky, S., and Chertok, B. (1976) *Phys. Rev.*, **D14**, 3003.
Brodsky, S., and Farrar, G. (1975) *Phys. Rev.*, **D11**, 1309.
Buras, A. J. (*To be published*) *Rev. mod. Phys.* (L. Smith C. H., in *Proc. Madison Conf*).
Callan, C. G. *et al.* (1978) *Phys. Rev.*, **D17**, 2717.
Carlitz, R., and Kislinger, M. (1970) *Phys. Rev.*, **D2**, 336.
Carroll *et al.* (1980) *Phys. Rev. Lett.*, **44**, 1572.
Carter, A. A. *et al.* (1977) *Phys. Lett.*, **67B**, 117.
Celmaster, W., and Henyey, F. (1978) *Phys. Rev.* **D17**, 5269.
CERN (1979) *Proc. Int. HEP Conf.*, July.
Chados, A. *et al.* (1974 *a*) *Phys. Rev.*, **D9**, 3471.
——— (1974*b*) *Phys. Rev.*, **D10**, 2599.
Chakrabarty, S., Mitra, A. N., and Sen, Sutapa (1977*a*) *Phys. Lett.*, **67B**, 110.
——— (1977*b*) *Phys. Lett.*, **71B**, 211.
——— (1979*a*) *Phys. Lett.*, **84B**, June 1.
——— (1979*b*). *Ph. D. Thevis*, Delhi Univ.
Chan, H. M., and Hogaasen, H. (1977) *Phys. Lett.*, **72B**, 400.
——— (1978) *Nucl. Phys.*, **B136**, 401.
Chand, R. (1970) *Symmetries and Quark Models*. Gordon & Breach, New York.
Chew, G., and Rosenzweig, C. (1978) *Phys. Rev.*, **41C**, 263.
Colglazier, E., and Rosner, J. (1971) *Nucl. Phys.*, **B27**, 349.
Copley, L. *et al.* (1969) *Nucl. Phys.*, **B13**, 303.
Dalitz, R. H. (1965) In : *High Energy Physics.* (*Ed* : M. Jacob & B. Dewitt). Gordon & Breach New York.
——— (1966) In : *Proc. XIII Int. Conf. HEP*, Berkeley. Univ. Calif. Press, Berkeley, U.S.A.
——— (1970) Discussion on duality. *Proc. R. Soc.*, **A138**, 243.
Dashen, R., and Gell-Mann, M. (1966) *Phys. Lett.*, **17**, 142.
De Grand, T. *et al.* (1975) *Phys. Rev.*, **D12**, 2060.
De Rujula, A., Georgi, H., and Glashow, S. L. (1975) *Phys. Rev.*, **D12**, 147.
De Rujula A. *et al.* (1976) *Phys. Rev. Lett.*, **37**, 398.
——— (1977) *Phys. Rev. Lett.*, **38**, 317.

Deshpande, N., and Truong, P. (1978) *Phys. Rev. Lett.*, **41**, 1579.
Dirac, P. A. M. (1949) *Rev. mod. Phys.*, **21**, 392.
Ditsas, P., Dougall, M., and Moorhouse, R. G. (1978) *Nucl. Phys.*, **B146**, 191.
Dyson, F. J. (1966) *Symmetry Groups in Elementary Particles.* W. A. Bengamin & Sons, Inc., New York.
Eichten, E. *et al.* (1975) *Phys. Rev. Lett.*, **34**, 369.
——— (1978) *Phys. Rev.*, **D17**, 3090.
——— (1979) Also Reprint/Preprint CLNS—425, June.
Eilwanger, U. (1978) *Nucl. Phys.*, **B139**, 422.
Faddeev, L. D. (1961) *Sov. Phys. JETP*, **12**, 1014.
Faiman, D., and Hendry, A. W. (1968) *Phys. Rev.*, **173**, 1720.
——— (1969*a*) *Phys. Rev.*, **180**, 1572.
——— (1969*b*) *Phys. Rev.*, **180**, 1609.
Feynman, R. P., Kislinger, M., and Ravndal, F. (1971) *Phys. Rev.*, **D3**, 2706 (Referred to in the text as FKR).
Feynman, R. P. (1972) *Photon-Hadron Interaction.* W. A. Benjamin & Sons, Inc., N.Y.
Freund, P., Maheshwari, A., and Schonbay, E. (1967) *Phys. Rev.*, **159**, 1232.
Fishbane, P., and Namyslowski, J. (1980) *Phys. Rev.* **D21**, 2406.
Fritzsch, H., Gellmann, M., and Leutwyler, H. (1973) *Phys. Lett.*, **B47**, 365.
Fritzch, H., and Jackson, J. D. (1977) *Phys. Lett.*, **66B**, 365.
Fronsdal, C. (1958) *Nuovo Cim. Suppl.*, **9**, 416.
Fubini, S., Gordon, D., and Veneziano, G. (1969) *Phys. Lett.*, **B29**, 679.
Fujimura, K., Kobayasi, T., and Namiki, M. (1970) *Progr. theor. Phys.*, **44**, 193 (Referred to as FKN).
Fukugita, M. *et al.* (1978) *Phys. Lett.*, **74B**, 261.
——— (1979) *Phys. Lett* , **84B**, 493.
Gell-Mann, M. (1964) *Phys. Lett.* **8**, 214.
——— (1965) *Phys. Lett.*, **14**, 77.
Gell-Mann, M., and Low, F. E. (1951) *Phys. Rev.*, **84**, 350.
Gell-Mann, M., Oakes, R., and Ranner, B. (1968) *Phys. Rev.*, **175**, 2198 (Referred to as GOR).
Gell-Mann, M., and Nee'man, Y. (1964) *The Eightfold Way.* W. A. Benjamin & Sons, Inc., New York.
Greenberg, O. W. (1974) *Phys. Rev. Lett.*, **13**, 598.
Greenberg, O. W., and Resnikoff, M. (1967) *Phys. Rev.* **163**, 1844.
Gromes, D. (1976) *Nucl. Phys.*, **B112**, 213.
——— (1977) *Nucl. Phys.* **B131**, 80.
Gunion, J. F., and Wiley, R. S. (1975) *Phys. Rev.*, **D12**, 174.
Gupta, M., and Mitra, A. N. (1978) *Phys Rev.*, **D18**, 1585.
Han, M., and Nambu, Y. (1965) *Phys. Rev.*, **139B**, 1006.
Harari, H. (1969) *Phys. Rev. Lett.*, **22**, 562.
——— (1979) SLAC Preprint SLAC-PUB-2254.
Harrington, B. J. *et al.* (1975) *Phys. Rev. Lett.*, **34**, 168.
Hasenfrantz, P., and Kuti, J. (1978) *Phys. Rep.*, **40C**, 75,
Hasenfrantz, P. *et al.* (1980) CERN-TH-2837, 2838.
Hendry, A. W., and Lichtenberg, D. B. (1978) *Rep. Progr. Phys.*, **41**, 1707.
Henley, J. R. (1979) *Phys. Rev.*, **D20**, 2532.
Henriques, A. *et al.* (1976) *Phys. Lett.*, **64B**, 85.
Herb, S. W. *et al.* (1977) *Phys. Rev. Lett.*, **38**, 252.
Hey, A. H. G. (1980) In : *Proc. Baryon-80. Conf., Toronto*
Horgan, R. R. (1974) *Nucl. Phys.*, **B71**, 514.
——— (1980) CERN-TH-2916 (Other references therein).
Horgan, R. R., and Dalitz, R. H. (1973) *Nucl. Phys.*, **B66**, 135.
——— (1974) *Nucl. Phys.*, **B71**, 546.

Iizuka, I. *et al.* (1966) *Progr. theor. Phys.*, **35**, 1061.
Isgur, N., and Karl, G. (1978) *Phys. Rev.*, **D18**, 4187.
——— (1979) *Phys. Rev.*, **D19**, 2653.
Jackson, J. D. (1976) *Phys. Rev. Lett.*, **37**, 1107.
Jaffe, R. L. (1977) *Phys. Rev.*, **D15**, 267.
——— (1978) *Phys. Rev.*, **D17**, 1444.
Jaffe, R. L., and Kiskis, J. (1976) *Phys. Rev.*, **D13**, 1355.
Jersak *et al.* (1978) *Phys. Lett.*, **77B**, 399.
Jones, M., Dalitz, R., and Horgan, R. (1977) *Nucl. Phys.*, **B129**, 45.
Jones, M., Dalitz, R. H., and Horgan, R. R. (1977) *Nucl. Phys.*, **B129**, 45.
Jones, M., Levi, Setti R., and Lasinski, T. (1974) *Nuovo Cim.*, **19A**, 365.
Joos, H. *et al.* (1974) *Nucl. Phys.*, **B69**, 349.
Joshi, G. C., and Anderson, R. (1979) *Phys. Rev.*, **D20**, 736.
Joshi, G. C., and Mitra, A. N. (1978) *Lett. Nuovo Cim.*, **5**, 85.
Kadychevsky, V. (1968) *Nucl. Phys.*, **B6**, 125.
Kalman, C. S., Hall, R. L., and Misra, S. K. (1980) *Phys. Rev.*, **D21**, 1908.
Kamae, T. *et al.* (1980) *Phys. Rev. Lett.*, **44**, 1439.
Kamath, S. G., and Mitra, A. N. (1978) *Phys. Rev.*, **D17**, 340.
Karl, G., and Obryk, E. (1968) *Nucl. Phys.*, **B8**, 609,
Karplus, R., and Klein, A. (1952) *Phys. Rev.*, **87**, 848.
Kim, Y. S., and Noz, M. (1973) *Phys. Rev.*, **D8**, 3521.
——— (1975) *Phys. Rev.*, **D12**, 122, 129.
Kiskis, J. E. (1975) *Phys. Rev.*, **D11**, 2178.
Klein, A., and Zemach, C. (1957) *Phys. Rev.*, **108**, 126.
Kogut, J., and Susskind, L. (1975) *Phys. Rev.*, **D11**, 395.
Kokkeddee, J. J. J., (1969) *Quark Model.* W. A. Benjamin & Sons, Inc., New York.
Krammer, M., and Kraseman, H. (1979) *Quarkonia*, DESY 79/20.
Kumari, Indra, and Mitra, A. N. (1977*a*) *Nuovo Cim.*, **38A**, 89.
——— (1977*b*) *Nuovo Cim.*, **38A**, 89.
Lasher, A. *et al.* (1974) *Phys. Rev.*, **D10**, 732.
Leutwyler, H., and Stern, J. (1978*a*) *Ann. Phys.* (N.Y.), **112**, 94.
——— (1978*b*) *Phys. Lett.*, **73B**, 75.
Le Yaouanc, A., Oliver, L., Pene, O., and Raynal, J. C. (1973) *Phys. Rev.*, **D8**, 2223 (Referred to as LOPR).
——— (1974) *Phys. Rev.*, **9D**, 2636.
——— (1975*a*) *Phys. Rev.*, **D11**, 680.
——— (1975*b*) *Phys. Rev.*, **D12**, 2137.
Le Yaouanc, A. *et al.* (1977*a*) *Phys. Rev.*, **D15**, 844.
——— (1977*b*) *Phys Lett.*, **71B**, 397.
——— (1977*c*) *Phys. Lett.*, **72B**, 53.
——— (1978) *Phys. Rev.*, **D18**, 1591.
Levine, R., and Tomozawa, Y. (1979) *Phys. Rev.*, **D19**, 1572.
Levy, M. (1952) *Phys. Rev.*, **88**, 72.
Licht, A. L., and Pagnamenta, A. (1970*a*) *Phys. Rev.*, **D2**, 1150. (Referred to as L P.).
——— (1970*b*) *Phys. Rev.*, **D2**, 1156.
Lichtenberg, D. B. (1969) *Phys. Rev.*, **178**, 2197.
——— (1978). *Unitary Symmetry and Elementary Particles.* Academic Press, N.Y. U.S.A.
Lipes, R. (1972) *Phys. Rev.*, **D5**, 2849.
Lipkin, H., and Meshkov, S. (1965) *Phys. Rev. Lett.*, **14**, 670.
Logunov, A., and Tavkhelidze, A. (1963) *Nuovo Cim.*, **29**, 380.
Mahanthappa, K. T., and Sudarshan, E. C. G. (1965) *Phys. Rev. Lett.*, **14**, 143.
Marciano, W., and Pagels, H. (1978) *Phys. Rep.*, **36C**, 137.
Martin, A. (1980) *Phys. Lett.*, **93B**, 338.
Martin, A., and Pennington, M. (1979) *Phys. Lett.*, **86B**, 93.

Mayer, R. F. (1974) *Nucl. Phys.*, **B71**, 226.
Moorhouse, R. G. (1966) *Phys. Rev. Lett.*, **16**, 771.
Morpurgo, G, (1965) *Physics*, **2**, 95.
Micu, L. (1969) *Nucl. Phys.*, **B10**, 521
Misra, S. P. *et al.* (1980) *Phys. Rev.*, **D21**, 1231, 2029. (Earlier references therein).
Mitra, A. N. (1962) *Nucl. Phys.*, **32**, 529.
——— (1968) *Nuovo Cim.*, **56A**, 1164.
——— (1969*a*) In : *Advances in Nuclear Physics*, Vol. 3. (Eds : M. Baranger & E. Vogt).
——— (1969*b*) *Nuovo Cim.*, **64A**, 603.
Mitra, A. N. (1973) Supermultiplet Interactions among Hadrons and Resonances. Delhi Univ. Report—*Unpublished.*
——— (1974) *Phys. Lett.*, **51B**, 149.
——— (1975) *Phys. Rev.*, **D11**, 3270.
——— (1976*a*) *Phys. Rev.*, **D14**, 855.
——— (1976*b*) *Phys. Lett.*, **62B**, 453
——— (1976*c*) *Phys Rev. Lett.*, **37**, 1647.
——— (1977) Resonances and their Quark Mechanics. *Rivista del Nuovo Cim.*, **7**, 80.
——— (1978) *Phys. Rev.*, **D17**, 729.
——— (1979*a*) *Phys. Lett.*, **89B**, 65.
——— (1979*b*) *Ann. Phys.*, (N.Y.), **120**, 316.
——— (1980*a*) Delhi University Preprint, March.
——— (1980*b*) *Z., f. Phys.* (*C*) (*in Press*).
——— (1980*c*) *V. HEP Symp.* Cochin (*to be published*).
——— *Where are the Baryonium States*? (*to be submitted for publication*).
Mitra, A. N., Chakrabarty, S., and Sood, S. K. (1977) *Phys. Lett.*, **71B**, 210.
Mitra, A. N., and Katyal, D. L. (1968) *Nucl. Phys.*, **B5**, 308.
Mitra, A. N., and Kumari, I. (1977) *Phys. Rev.*, **D15**, 261.
Mitra, A. N., and Mazumdar, R. (1966) *Phys. Rev.*, **150**, 1194.
Mitra, A. N., and Ross, M. H. (1967) *Phys. Rev.*, **158**, 1630.
Mitra, A. N., and Santhanam, I. *Z. f. Phys. C—(to be published)* (Referred to as Ms.).
Mitra, A. N., and Sen, Sutapa (1974) *Nuovo Cim. Lett.*, **10**, 685.
Mitra, A. N., and Sood, S. K. (1977*a*) *Phys. Rev.*, **D15**, 1991.
——— (1977*b*) Relativistic Hadron Couplings, A Unified Framework. *Fortschrsttle der Physik*, **25** 649–741.
Nakanishi, N. (1969) *Progr. theor. Phys., Suppl.*, **43**, 1.
Narayanaswamy, P., and Pagnamenta, A. (1968) *Nuovo Cim.*, **53A**, 635.
Novikov, V. A. *et al.* (1978) *Phys. Rep.*, **41C**, 1.
Okubo, S. (1963) *Phys. Lett.*, **5**, 165.
Politzer, H. D. (1973) *Phys. Rev. Lett.*, **30**, 1346.
——— (1974) *Phys. Rev.*, **14C**, 129.
Polyakov, A. (1975) *Phys Lett.*. **59B**, 82, 85.
Quigg, C., and Rosner, J. L. (1978) *Phys Lett.*, **71B**, 153.
——— (1979) *Phys. Rep.*, **56C** ,167.
Ravndal, F. (1971) *Phys. Rev.*, **D4**, 1466.
Reinders, L. J. (1978) *J. Phys.* (*G*), **4**, 1241.
Richardson, J. L. (1979) *Phys. Lett.*, **82B**, 272.
Rosner, J. L. (1969) *Phys. Rev. Lett.*, **22**, 689.
——— (1974) *Phys. Rep.*, **11C**.
Rosensweig, C. (1976) *Phys. Rev. Lett.*, **36**, 697.
——— (1977) *Argonne Conf. Exotic. States.*
Royen, R. Van, and Weisskopf, V. F. (1967) *Nuovo Cim.*, **50A**, 583 (Referred to as VW).
Sakurai, J. J. (1961) *Ann. Phys.* (N.Y.), **11**, 1.
Salam, A., and Strathdee, J. (1977*a*) *Phys Rev.*, **D16**, 2668.
——— (1977*b*) *Phys. Lett.*, **67B**, 429.

Salpeter, E. E., and Bethe, H. A. (1951) *Phys. Rev.*, **84**, 1232.
Salpeter, E. E. (1952) *Phys. Rev.*, **87**, 328.
Santhanam, I. *et al.* (1979) *Phys. Rev. Lett.*, **43**, 661.
Schwinger, J. (1962) *Phys. Rev.*, **128**, 2425.
——— (1967) *Phys. Rev. Lett.*, **18**, 923.
Shapiro, I. S. (1977) *Phys. Rep.*, **35C**, 129.
Simonov, Yu A. (1966) *Sov. Phys. JNP*, **3**, 461.
Smith, C. H. L. (1968) *Phys. Lett.*, **28B**, 335.
——— (1969) *Ann. Phys.* (N.Y.), **53**, 521.
Stanley, D. P., and Robson, D. (1980) *Phys. Rev. Lett.*, **45**, 235.
Sundaresan, M. K., and Watson, P. (1970) *Ann. Phys.* (N. Y.), **59**, 375.
Thornber, N. S. (1968) *Phys. Rev.*, **172**, 1395.
Veneziano, G. (1968) *Nuovo Cim.*, **57**, 190.
Weinberg, S. (1966*a*) *Phys. Rev. Lett.*, **17**, 616.
——— (1966*b*) *Phys. Rev.*, **150**, 1313.
Weisskopf, V. F. (1967) CERN Th—824.
Wick, G. C. (1954) *Phys. Rev.*, **96**, 1124.
Winger, E. (1937) *Phys. Rev.* , **51**, 106.
Wills, J. G. *et al.* (1977) *Phys. Rev.*, **D15**, 3358.
Wilson, K. G. (1974) *Phys. Rev.*, **D10**, 2445.
——— (1976) *Phys. Rep.*, **23C**, 331.
Wong, C. W., and Liu, K. F. (1980) UCLA Preprint, August.
Yukawa, H. (1953) *Phys. Rev.*, **91**, 416.
Zweig, (1964) CERN Report No. 8419/Th 412 (*Unpublished*).

Keywords : Harmonic Oscillator; Quark.

Note : This marathon work has been published within a record time of four months. Meticulous care has been taken while printing, yet as the equations amount to 300 in number, there have been difficulties at certain places in reproducing them because of technical reasons. Due to lack of time, full titles of articles have not been printed under references.

EDITOR