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Nuclear Reaction

RESONANT REACTION RATES OF O^{16} (α, γ) Ne^{20} AFFECTING SCREENING EFFECT

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O^{16} (α, γ) Ne^{20} reaction rates have been found to be altered due to screening effect. The mean life is much shortened. It indicates that the evolutionary time scale in giant stage of stellar evolution will be reduced. The screened rates have been calculated after Mitler (1977).

Keywords: Dimensionless Parameter; Plasma; Screening Effect; Resonant Nuclear Reaction; Stellar Interiors; Core

INTRODUCTION

THE sensitive temperature for helium burning reaction is between 1.0 and 1.5 $\times 10^8$ K. C^{12} is produced by triple alpha reactions. Once C^{12} is produced, it is destroyed by (α, γ) reaction to yield O^{16}, Ne^{20}, etc.

Burbidge *et al.* (1957) considered these reactions in the giant stage of Stellar evolution and calculated the reaction rates without considering the screening effect. The recent works of Mitler (1977), Itoh *et al.* (1977) and van Horn and Salpeter (1969) have made it convenient to reinvestigate the reaction rates after taking the screening effect into consideration.

The reaction considered here falls in weak screening regime. We have known from Salpeter and van Horn (1969) and Itoh *et al.* (1977) that when the dimensionless parameter, $\Gamma \leqslant 1$ then the screening is weak.

$$\text{Here } \Gamma = \frac{z^2 e^2}{aT} \qquad \ldots(1)$$

where T is expressed in energy unit

z = atomic number

e = electronic charge

and $a = \left(\frac{3z}{4\pi n_e}\right)^{1/3}$, the mean ionic distance.

Here n_e = number density of electrons.

We get $n_e = 2\, n_\alpha$ for helium plasma.

n_α (no. density of ions) is further given by

$$n_\alpha = \frac{\rho N_A X_\alpha}{A_\alpha}\ \text{cm}^{-3} \qquad \ldots(2)$$

where ρ = density

N_A = Avogadro's number

X_α = fractional abundance of alpha particles.

A_α = 4 for helium ions

Putting $\rho X_\alpha = 10^5$ gm/cc and $T \sim 10^8$ K, one finds it to belong to weak screening regime.

Screened Correction Factor for Resonant Reactions

The reaction O^{16} (α, γ) Ne^{20} falls in weak screening regime as can be seen from the density and temperature relevant to a helium plasma. The enhancement factor due to weak screening effect is given by (Mitler, 1977)

$$f_s = e^{\Delta U/KT} \quad \ldots(3)$$

ΔU is the potential energy shift. Mitler (1977) obtained higher order correction factor for it. We are using the higher order correction factor in the present case.

In resonant reaction, correction factor is different from the non-resonant one. We denote it here by f_r. The reaction cross section in vacuum is given by

$$\sigma_{\alpha\gamma}(E) = \frac{\pi\omega}{k^2} \frac{\Gamma_\alpha(E)\,\Gamma_\gamma(E)}{(E - E_\gamma)^2 + \frac{1}{4}\Gamma^2}, \quad \ldots(4)$$

where ω is the statistical weight given by

$$\omega = \frac{(2J+1)}{(2S+1)(2I+1)} \quad \ldots(5)$$

In eqn. (5), S and I are spins of two interacting nuclei and J is the spin of compound level formed. Γ_α, Γ_γ and Γ are the widths of incoming channel, outgoing channel and total width respectively. (Γ's here are not dimensionless parameters considered earlier). E_r is the resonance energy. In a plasma, a work ΔF is done on the particle on approaching the target nucleus. This is equivalent to raising energy reference level and hence resonance energies are shifted to (Mitler, 1977)

$$E_r' = E_r - \Delta F \quad \ldots(6)$$

Mitler (1977) obtained the resonance correction factor which was given by

$$f_r = \frac{\text{Screened resonance reaction rate}}{\text{Unscreened resonance reaction rate}}$$

When $\Gamma_\alpha = \Gamma$ (i.e. $\Gamma_\alpha >> \Gamma_\gamma$), then

$$f_r = \exp\left(\frac{\triangle E}{KT}\right) \quad \ldots(7)$$

The value of $\triangle F$ is further given by

$$\triangle F = \frac{Z_1 Z_2 e^2 K_D \left(1 + \frac{X_1}{2}\right)}{\left(1 + X_1 + \frac{X_1^2}{3}\right)} \quad \ldots(8)$$

In eqn. (8)

K_D = Debye wave number.

Z_1 and Z_2 are atomic nos.

$X_1 = K_D . a.$

where a is mean ionic distance as defined earlier.

From (7), we have

$$\log_{10}(f_r) = \frac{\triangle F}{KT} \log_{10} e \qquad \ldots(9)$$

Table I shows the values of $\log_{10}(f_r)$ at various temperatures for the reaction concerned. These data are used to evaluate how logarithm of mean life decreases due to screening effect, i.e., how reaction rate increases due to screening effect. We have not calculated $\log_{10}(f_r)$ for $\Gamma_\alpha << \Gamma_\gamma$ as that does not become a necessity for the reaction $O^{16} (\alpha, \gamma)^{20}$ Ne.

TABLE I

Log_{10} (f_r) for various temperatures, (1–2) 10^8 *K at density* $\rho = 10^5$ *gm* cm^{-3} *for* $\Gamma_\alpha >> \Gamma_\gamma$

Temperature T_8	K_D^{-1} in cm	X_1	$\triangle F$	$\log_{10}(f_r)$	f_r
1	0.1325×10^{-9}	1.9085	0.1318×10^{-7}	0.4203	2.6
1.2	0.1452×10^{-9}	1.7422	0.1265×10^{-7}	0.3362	2.1
1.4	0.1568×10^{-9}	1.6130	0.1220×10^{-7}	0.2778	1.9
1.6	0.1678×10^{-9}	1.5088	0.1180×10^{-7}	0.2352	1.7
1.8	0.1778×10^{-9}	1.4225	0.1145×10^{-7}	0.2029	1.6
2.0	0.1875×10^{-9}	1.3495	0.1114×10^{-7}	0.1776	1.5

RESONANT REACTION RATE CALCULATION OF $O^{16} (\alpha, \gamma)^{20}$ NE REACTION

It is known that 4.97 (2^-) MeV level of Ne^{20} cannot contribute to the resonance formation of Ne^{20} through $O^{16} (\alpha, \gamma)$-Ne^{20} reaction. Ne^{20} compound nucleus has two other levels which contribute to its resonance formation. These levels are 5.64 (3^-) and 5.80 (1^-). The rate of the resonant formation of compound nucleus when screening effect is considered comes out to be

$$\log P_r = 11.19 + \log\left(\frac{PX_\alpha}{A_\alpha}\right)\frac{\omega\Gamma_\alpha\Gamma_\gamma}{\Gamma} - 1.5 \log (AT_9) - \left(\frac{5.04}{T_9}\right) E_r + \log f_r \qquad \ldots(10)$$

where the alpha particle width Γ_α, radiation width Γ_γ, total width $\Gamma = \Gamma_\alpha + \Gamma_\gamma$ and resonance energy E_r are all expressed in MeV. A is the reduced mass in a.m.u. and $A_\alpha = 4$.

The factor $\frac{\omega\Gamma_\alpha \Gamma_\gamma}{\Gamma}$ which occurs in the equation for the reaction rates for the two levels mentioned above are 0.003 eV and < 0.15 eV, respectively. We have chosen here $\frac{\omega\Gamma_\alpha\Gamma_\gamma}{\Gamma}$ to be 0.10 eV for 5.80 MeV level after Duorah and Kushwaha (1963).

We get two different values of $\log_{10}$ (f_r) for same temperatures for 5.64 and 5.80 MeV States when $\Gamma_\alpha \ll \Gamma_\gamma$ whereas we get only one correction factor for both levels when $\Gamma_\alpha \gg \Gamma_\gamma$.

But in this reaction O^{16} (α, γ) Ne^{20}, Γ_α is not much greater in comparison to Γ_γ. So we cannot neglect Γ_γ for Γ_α. The general correction factor, therefore, for resonance reaction becomes

$$f_r = \frac{\Gamma_\alpha\left(E_r'\right)\Gamma_\gamma(E_r)}{\Gamma_\alpha(E_r)\,\Gamma_\gamma(E_r')}\exp\left(\frac{\triangle F}{KT}\right) \qquad ...(11)$$

We have not used eqn. (11), because we have $\Gamma_\alpha(E_r')$ which is not much different from $\Gamma_\alpha(E_\gamma)$ as $\triangle F$ does not come out to be a large factor (Pl. see Table I) so we write

$$\Gamma_\alpha(E_r) = \Gamma_\alpha(E_r - \triangle F) \approx \Gamma_\alpha(E_r') \qquad ...(12)$$

$$\Gamma_\gamma(E_r) = \Gamma_\gamma(E_r - \triangle F) \approx \Gamma_\gamma(E_r') \qquad ...(13)$$

Hence the general correction factor, f_r reduces to

$$f_r = \exp\left(\frac{\triangle F}{KT}\right)$$

We use the data of Table I to calculate the screened reaction rates and make the following tabulation (Table II).

TABLE II

Logarithm of mean life in years for various temperatures at $\rho X\alpha = 10^5$ g cm^{-3}

T_9	O^{16} (α, γ) Ne^{20} 5.64 MeV level		O^{16} (α, γ) Ne^{20} 5.80 MeV level	
$T_9 = \frac{T}{10^9}$	Unscreened	Screened	Unscreened	Screened
0.10	29.05	28.5297	36.09	35.5697
0.12	21.78	21.4438	27.39	26.954
0.14	16.60	16.332	21.19	20.91
0.16	14.53	14.2945	18.71	18.48
0.18	9.72	9.517	12.95	12.74
0.20	7.33	7.152	10.08	9.902

Discussion

We have used the method due to Mitler (1977) for calculation of screened resonance rates for $O^{16}(\alpha, \gamma)\,Ne^{20}$. The multiplication factor (f_r) arising out of screening correction has been calculated for this reaction (Table I). This factor decreases as temperature increases at constant density. The logarithm of mean life that we have calculated for the reaction $O^{16}(\alpha, \gamma)\,Ne^{20}$ through the two excited levels is taken into account. Therefore, the enhanced reaction rate is expected to play a crucial role in the determination of Stellar evolutionary models. The temperature that we have considered is low ($T_9 = 0.1$ to 0.2). Enhanced nuclear energy generation rate will reduce the evolutionary time scale of low/high mass stars burning helium inside the core. The reaction may be important in the abundance calculation as well. Therefore, it will be in order to do a set of new abundance calculation by taking O^{16}-Ne^{20} reaction into account.

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