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TOP MESON AND THE QUARKONIUM POTENTIAL

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We discuss the predictions for energy spectrum, radiative decays specific to $Q\bar{Q}$ bound states of quark mass $M_Q \sim 20$ GeV within the context of non-relativistic quantum mechanics and a plausible universal potential model.

Keywords: Top Meson; Quarkonium Potential; Energy Spectrum; Radiative Decays; $Q\bar{Q}$ Bound States. Non-Relativistic Quantum Mechanics.

INTRODUCTION

HADRONS are believed to be composed of quarks which are very difficult to liberate. The non-observation of free quarks suggests that the binding force is formidable, but the success of the parton model (Feynman, 1972) in describing hard scattering processes argues that quarks behave as quasi-free within hadrons. On the basis of asymptotic freedom arguments, it was anticipated that bound states of heavy quarks might be described by a non-relativistic analog of the bound e^+e^- system, a positronium (Deutsch, 1975). It will be enough to know that the Schrödinger-equation-approach does very well on the generalities of the bottomonium system and that the non-relativistic approximation should be much better for family composed of heavy top quark. The potential we have chosen in this model is the Coulomb type with a linear one incorporated (———). The latter part is neccessitated to give correct splitting of the various radial excitations of the mesons containing t quark. This approach is in line with the methods adopted by various authors (Celmaster *et al.*, 1978; Quigg *et al.*, 1979; and Gross & Martin 1979) in ψ and upsilon-meson analysis in terms of non-relativistic Schrödinger equation.

In this note, we would like to report the detailed predictions of potential model for the hypothetical $\zeta\ (t\bar{t})$ family of higher lying vector meson system in which the mass of t quark is assumed to be around 20 GeV (Krammer & Krasemann, 1979).

MODEL

We assume that the non-relativistic approximation is suitable and the same potential describes the interactions of all flavours of massive quarks. This is very much in the spirit of QCD provided all the quarks are colour triplets (Yee Jack Ng & Tye, 1978).

In the reduced radial Schrödinger equation for $u(r) = r\,R(r)$, where the Schrödinger wave function is $\psi(r) = R(r)\,Y_{lm}(\theta, \phi)$. Using the natural units $c = \hbar = 1$,

$$-\frac{u''(r)}{2\mu} + \left[V(r) - E + \frac{l(l+1)}{2\mu r^2}\right] u(r) = 0 \qquad \ldots(1)$$

Taking the potential

$$V(r) = K\alpha_s m^{-1/2} r^{\epsilon} \qquad \ldots(2)$$

where

$$-2 < \epsilon \geqslant 0 \qquad \ldots(3)$$

and define the dimensionless parameter

$$\rho = \mu^P m_0^{1-P} r \qquad \ldots(4)$$

Here m_0 is constant with dimensions of mass and power P will be discussed below. With the replacement

$$u(r) \equiv w(\rho) = w(\mu^P m_0^{1-P}\, r) \qquad \ldots(5)$$

we have

$$u''(r) = \mu^{2P} m_0^{2(1-P)} \omega''(\rho) \qquad \ldots(6)$$

so that equation (1) takes the form

$$-\tfrac{1}{2}\,\mu^{2P-1} m_0^{2(1-P)} \omega'' + \left[\frac{l(l+1)}{2\rho^2}\,\mu^{2P-1} m_0^{2(1-P)} + \frac{K\alpha_s}{\sqrt{2}}\,\mu^{-1/2-\epsilon P}\, m_0^{\epsilon(P-1)} \rho^{\epsilon} - E\right]\omega = 0 \qquad \ldots(7)$$

We now set

$$2P - 1 = \tfrac{1}{2} - \epsilon P$$

or

$$P = \frac{1}{2(2+\epsilon)} \qquad \ldots(8)$$

and divide (6) by $\mu^{2P-1} m_0^{2(1-P)}$ obtaining

$$-\tfrac{1}{2}\omega' + \left[\frac{l(l+1)}{2\rho^2} + \frac{K\alpha_s \rho^{\epsilon}}{\sqrt{2}}\,\frac{m_0^{\epsilon(P-1)}}{m_0^{2(1-P)}} - \frac{E}{\mu^{2P-1} m_0^{2(1-P)}}\right]\omega = 0 \qquad \ldots(9)$$

We have now isolated the μ-dependence in the term $\frac{E}{\mu^{2P-1} m_0^{2(1-P)}}$. Thus the scale of the energy level spacings is given by

$$\Delta E \sim \mu^{-\frac{(1+\epsilon)}{(2+\epsilon)}} \qquad \ldots(10)$$

According to the equation (3), the quantities with dimensions of length L scale as

$$L \sim \mu^{-\frac{1}{2(2+\epsilon)}} \qquad \ldots(11)$$

The matrix elements of electric and magnetic multipole operators scale as

$$< n' \mid E_j \mid n > \sim L^j \sim \mu^{-\frac{j}{2(2+\epsilon)}} \quad \ldots(12a)$$

and

$$< n' \mid M_j \mid n > \sim \frac{L^{j-1}}{\mu} \sim \mu^{-\frac{(j+3+2\epsilon)}{2(2+\epsilon)}} \quad \ldots(12b)$$

Since the radiative decay widths are given by

$$\Gamma(E_j, M_j) \sim P_{\Upsilon}^{2j+1} \left| < n' \mid E_j \text{ or } M_j \mid n > \right|^2 \quad \ldots(13)$$

and

$$P_{\Upsilon} \sim \triangle E \sim \mu^{-\frac{2(1+\epsilon)}{(2+\epsilon)}} \quad \ldots(14)$$

We find

$$\Gamma(E_j) \sim \mu^{-\frac{(6j-2-4j\epsilon+4\epsilon)}{2(2+\epsilon)}} \quad \ldots(15)$$

and

$$\Gamma(M_j) \sim \mu^{-\frac{(6j+8+4j\epsilon+4\epsilon)}{2(2+\epsilon)}} \quad \ldots(16)$$

Probability densities $|\psi_n(r)|^2$ have dimensions of inverse volume L^{-3} so scale as

$$|\psi_n(r)|^2 \sim L^{-3} \sim \mu^{\frac{3}{2(2+\epsilon)}} \quad \ldots(17)$$

Such quantities are of interest in the decays of massive vector meson $\zeta\,(t\bar{t})$ which is $^3S_1$ bound state of a quark $\bar{t}$ and antiquark t, for which (Jackson, 1977)

$$\Gamma(V \to e^+e^-) = 16\pi\alpha_s^2 e_Q^2 \frac{|\psi_n(0)|^2}{M_\zeta^2} \quad \ldots(18)$$

where e_Q is the quark charge and $M(\zeta)$ is the vector meson mass. For $\epsilon > -1$, the scale of $M(\zeta)$ will itself be set by μ for the low-lying levels. Consequently, we find

$$\Gamma(V \to e^+e^-) \sim \mu^{-\frac{(5+4\epsilon)}{2(2+\epsilon)}}\ \epsilon \geqslant -1 \quad \ldots(19)$$

The ratios of radiative to leptonic widths are of concern for massive states

$$\frac{\Gamma(E_j)}{\Gamma(V \to e^+e^-)} \sim \mu^{\frac{(6j+4j\epsilon-2\epsilon-3)}{2(2+\epsilon)}},\ \epsilon \geqslant -1 \quad \ldots(20)$$

$$\frac{\Gamma(M_j)}{\Gamma(V \to e^+e^-)} \sim \mu^{-\frac{6j+3+4j\epsilon}{2(2+\epsilon)}},\ \epsilon \geqslant -1 \quad \ldots(21)$$

Since $j \geqslant 1$, the exponents in (20) and (21) are both negative for $\epsilon > -1$. Hence, leptonic decays will dominate over radiative transitions as the quark mass increases. Choosing the $\epsilon = 0$, as suggested by the equality of ψ and Υ mass splittings, we expect

$$\frac{\Gamma(\zeta' \xrightarrow{El} \chi_t\gamma)}{\Gamma(\zeta' \longrightarrow e^+e^-)} \Big/ \frac{\Gamma(\Upsilon' \xrightarrow{El} \chi_b\gamma)}{\Gamma(\Upsilon' \longrightarrow e^+e^-)} \propto \sqrt{\frac{m_b}{m_t}} \approx (0.5 \text{ to } 0.7) \qquad \ldots(22)$$

Within a quarkonium family, the principal-quantum-number dependence of observables is another source of information about the nature of the potential. With the aid of an intermediate result

$$|\psi(0)|^2 = \frac{\mu}{2\pi} < \frac{dv}{dr} > \qquad \ldots(23)$$

(which is derived by direct computation from the Schrödinger equation), it is straight forward to compute (Rosner & Quigg, 1978) in WKB approximation that for power law potentials

$$|\psi(0)|^2 \sim n^{2(\epsilon-1)/2+\epsilon} \qquad \epsilon > 0$$

$$|\psi(0)|^2 \sim n^{\frac{\epsilon-2}{2+\epsilon}} \qquad -2 < \epsilon < 0 \qquad \ldots(24)$$

For $\epsilon \to 0$ both expressions imply that $|\psi_n(0)|^2$ should behave an n^{-1} for large n. [Fig. 1].

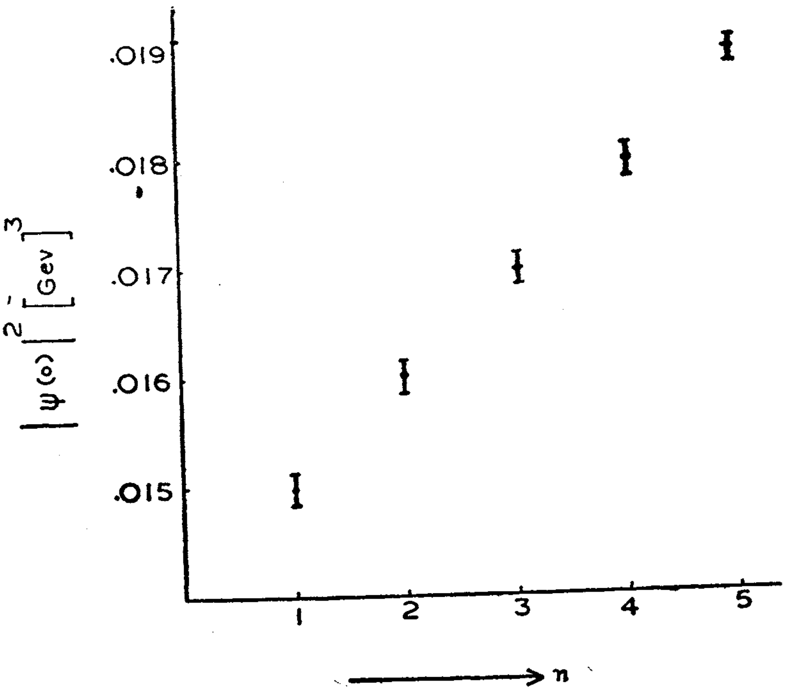

FIG. 1.

The principal quantum number dependence of $|\psi_n(0)|^2$ has been investigated by Martin (1977) who derived an equality on $|\psi_2(0)|^2/|\psi_1(0)|^2$. A theorem on the quark mass dependence of $|\psi(0)|^2$ was proved by Rosner and Quigg (1978). Quigg has assumed the $2S - 1S$ ($\zeta' - \zeta$) spacing to be 686 MeV, somewhat larger than $M_{\psi'} - M_{\psi}$ or $M_{\Upsilon'} - M_{\Upsilon}$. The $\zeta(2S) - \zeta(2P)$ spacing is 171 MeV (*vs.* 95 MeV for Υ) and the ratio

$$\frac{|\psi_{2s}(0)|^2}{|\psi_{1s}(0)|^2} \simeq 0.42$$

is larger for the ζ family than for the upsilon

$$\frac{|\psi_{2s}(0)|^2}{|\psi_{1s}(0)|^2} = 0.32.$$

Conclusion

Quarkonium families and non-relativistic quantum mechanics provide an important means for learning about the strong interactions and the colour properties qf quarks. The upsilon experience shows that quantum mechanical techniques allows us to reliably infer quark charges from bound state properties. Heavier quarkonium families may be exploited ever more fully than the ψ and Υ families. More narrow levels will exist, the non-relativist approximation is more reliable, and heavier quarks probe the potential at short distances. $\zeta(t\bar{t})$ spectroscopy of considerable richness and significance awaits the new detectors at CESR.

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