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Particle Physics

MASS SPLITTING OF HADRONS IN *u, d, b, t*—QUARK SECTOR

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Broken SU(4) is exploited for deriving sum relations among hadrons in sextuplet quark model excluding strangeness and charm. It is assumed that mass operator consists of a term which transforms as a scalar in the broken sub-group $SU(3)_t$ (which operates on *u, d* and *b* quarks) and a term which transforms as a scalar in broken $SU(3)_b$ (which operates on *u, d* and *t* quarks).

Keywords : Mass Splitting; *u, d, b, t*—Quark; Hadrons; Parallelogram Laws

Introduction

The conventional approach to mass splitting is that the masses of the hadrons are given to first approximation by the masses of the constituent valence quarks marginally modified by the effects of the binding potential. These calculations would, of course, be immediately acceptable if the relatively light, weakly bound, quarks could be produced in the hadronic collisions as independent entities.

Several attempts to calculate the mass of top quark have been made (Krammer & Krasemann, 1979; and Gaillard & Maiani, 1979) often in the context of more complicated gauge theories than Weinberg-Salam, and using discrete symmetries of the Higgs system. A formula written down is

$$m_t \approx m_b \sqrt{\frac{m_u}{m_d} \cdot \frac{m_c}{m_s}}$$

This formula predicts mass between 10 to 15 GeV. No sign of the top quark has yet been seen at PETRA (Wiik, 1979). We await with impatience the next increase in e^+e^- centre of mass energy. The top meson is expected to have mass 30 GeV/c^2 (Quigg, 1979).

In this paper, certain relations among the hadrons in *u, d, b,* and *t*-quark sector are derived. We make a very weak assumption about the form of the mass splitting which separates *t* and *b* quarks but leaves the SU(2) group, based on the *u, d*-quarks, unbroken. This leads to a variety of parallelogram laws for hadrons which gives mass relations among various baryons and meson multiplets (Feldman & Mathews, 1965).

Parallelogram Laws and Mass Sum Rules

Consider a broken SU(4) multiplet of dimension N and denote the unitary sub-group based on the *u, d,* and *b* quarks as $SU(3)_t$. Suppose that N contains P

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multiplets of $SU(3)_t$ labelled by their dimension n_t. The broken SU(4) multiplet breaks further to m (SU(2) submultiplets. Each submultiplet may be specified by the $SU(3)_t$ multiplet in which it occurs, the I-spin, beauty and top quantum number. Thus

$$| n_t,, I, B, T > \qquad ...(1)$$

Individual components of the multiplet are distinguished by also specifying I_3, but will not concern us. Mixing occurs between pairs of submultiplets with the same I, B, and T but differing n_t. Suppose there are m' such pairs in N, then

$$q = m + m' \qquad ...(2)$$

which is the number of non-vanishing matrix elements is the mass operator (Eichten *et al.*, 1975) M if electromagnetic mass splitting is neglected.

The expression for M includes p terms which transform like the broken $SU(3)_t$ scalars contained in the broken SU(4) decomposition of $N \otimes \overline{N}$. These separate the original SU(4) multiplet mass into the p masses associated with the $SU(3)_t$ multiplets. The most general expression for M contains further $q - p$ terms which transform as the $I = B = T = 0$ components contained in the $SU(3)_t$ multiplets. This expression with q parameters leads to no restrictions on the SU(2) submultiplet masses.

A very weak restriction is got by including the unitary subgroup $SU(3)_b$ which operates on the u, d, and t-quarks and it is assumed that mass splitting terms in the mass operator consists of a term M_t which transforms as an $SU(3)_t$ scalar and M_b which transforms as an $SU(3)_b$ scalar (Dug, 1978)

$$M = M_0 + M_t + M_b \qquad ...(3)$$

M_0 is an SU(4) scalar: M_t makes the major contribution to the mass splitting, separating the $SU(3)_t$ multiplets of different top quantum number by separating the t-quark from u, d and b . M_b is a much smaller term which separates the beauty quark from u, d and the top quark. In the N-dimensional representation, there are p terms in the reduced matrix elements of both M_t and M_b but each contains an SU(4) singlet which can be combined with M_0. Thus, the mass operator depends on $2p - 1$ parameters. In any broken SU(4) multiplet, the number of relations among the hadrons of the mass matrix is

$$q = (2p - 1) \qquad ...(4)$$

The relations are parallelogram laws (Feldman & Mathews, 1965). We write the beauty quantum number

$$B = B' + T \qquad ...(5)$$

and the electric charges of the components of the multiplet are

$$Q = Q' + T, \qquad ...(6)$$

where B' and Q' are the usual beauty quantum numbers and the charge of the particles with $I = 1$, $B = 0$ and $T = 1$ and its components have charges $q = 2$, 1. 0.

The SU(3)$_t$ and SU(3)$_b$ multiplets are presented in the weight diagram (Fig. 1) and Table I. The projection of the weight diagram in the plane perpendicular to the I_3 axis is shown in Fig. 2. Each point on such a diagram represents an SU(2) multiplet and its position on the diagram shows its top quantum number and beauty. The SU(3)$_t$ multiplets lie on horizontal lines of constant top quantum number T and SU(3)$_b$ multiplets lie on the sloping lines of constant beauty.

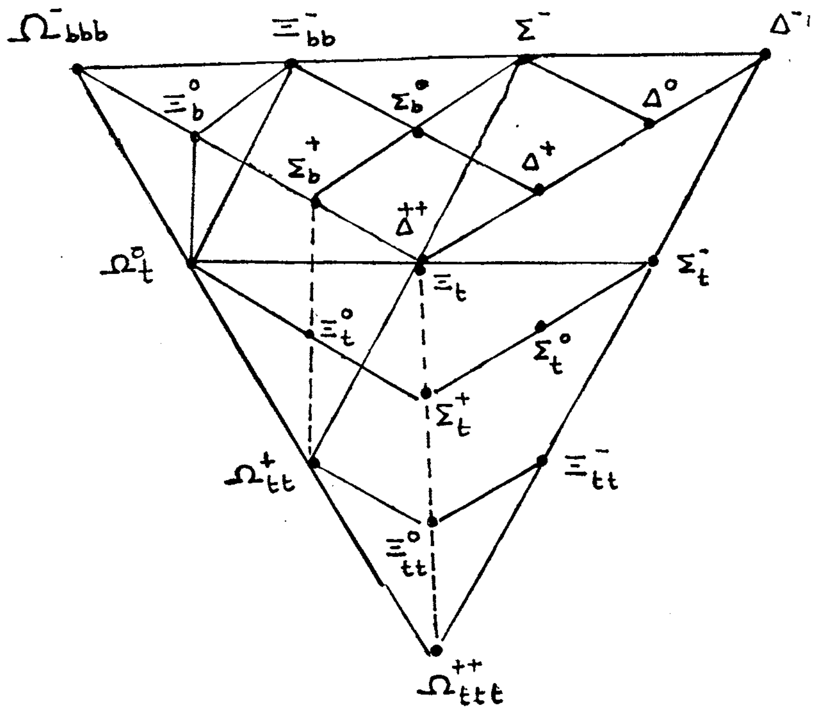

FIG. 1. The weight diagram of the symmetric baryons multiplet 20*, showing the Q' of the particles.

TABLE I

SU(3)$_t$	$n_{t'}$, n_b	SU(3)$_b$
Δ^*, Σ^*_b, Ξ^*_{bb}, Ω^*_{bbb}	10	Δ^*, Σ^*_t, Ξ^*_{bb}, Ω^*_{ttt}
Δ^*_t Ξ^*_t Ω^*_t	6	Σ^*_b Ξ^*_t Ω^*_{tt}
Ξ^*_{tt} Ω^*_{tt}	3	Ξ^*_{bb} Ω^*_t
Ω^*_{ttt}	1	Ω^*_{ttt}

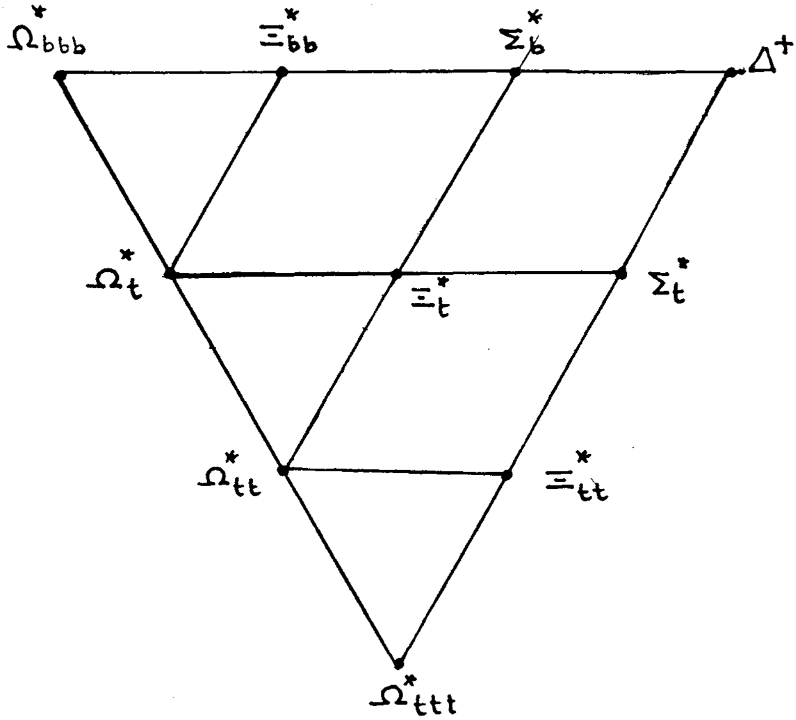

FIG. 2. The projection of the weight diagram in the plane perpendicular to the I_3 axis. The SU (3)$_t$ multiplets lie on horizontal lines and the SU(3)$_b$ multiplets on the sloping vertical lines. Parallelogram laws relate the masses of the particles which lie at the corners of the three complete parallelograms formed by these lines.

Since there is no mixing of $q = 10$, we get the following relations from eqns. 3) and (4). Thus, from Fig. 2, we have

$$\Sigma_b^* - \Delta^* + \Sigma_t^* - \Xi_t^* = 0 \qquad \ldots(7a)$$

$$\Xi_{bb}^* - \Sigma_b^* + \Xi_t^* - \Omega_t^* = 0 \qquad \ldots(7b)$$

$$\Xi_t^* - \Sigma_t^* + \Xi_{tt}^* - \Omega_{tt}^* = 0 \qquad \ldots(7c)$$

where the symbols stand for the particle masses.

The isotopic spin submultiplets of any SU(4) multiplet may also be identified through the SU(3)$_b$ multiplets to which they belong, labelled by n_b. This gives the states

$$| \, n_b, I, B, T > \qquad \ldots(8)$$

We take elements of the mass operator (3) in the mixed representation constructed from (2) and (8). Each term is diagonal in I, B, and T. Since M_b is diagonal in the $| \, n_b >$ representation, M_t in the $| \, n_t >$ representation and M_0 in both,

$$< n_t \mid M \mid n_b > = [m_0 + m(n_t) + \bar{m}(n_e)] < n_t \mid n_b > \qquad ...(9)$$

where

$$m(n_t) = < n_t \mid M_t \mid n_t > \qquad ...(10)$$

is a parameter which depends only on n_t. Also $\bar{m}(n_b)$ depends on n_b and m_0 is a constant for the whole broken SU(4) multiplet. Provided $< n_t \mid n_b >$ is non-zero, we can define

$$M(n_t, n_b) = \frac{< n_t \mid M \mid n_b >}{< n_t \mid n_b >} = m_0 + m(n_t) + \bar{m}(n_b) \qquad ...(11)$$

If $\mid n_t >$ does not mix with any other state, the non-vanishing element

$$< n_t \mid n_b > = 1 \qquad ...(12)$$

The pair of labels (n_t, n_b) then identify a particular isotopic submultiplet and $M(n_t, n_b)$ is the mass of the multiplet. If there is mixing between $\mid n_t >$ and $\mid u'_t >$;

$$M(n_t, n_b) = < n_t \mid M \mid n_t > + < n_t \mid M \mid n'_t > \frac{< n'_t \mid n_b >}{< n_t \mid n_b >} \qquad ...(13)$$

where the transformation coefficients $<n_t \mid n_b>$ and $<n'_t \mid n_b>$ are most easily evaluated for the simpler representations by making use of the quark content of the states. The form of eqn. (11) makes it evident that the particle masses satisfy parallelogram laws which have the form

$$\sum_{n_t, n_b} i(n_t, n_b)\, M(n_t, n_b) = 0, \qquad ...(14)$$

provided

$$\sum_{n_t} i(n_t, n_b) = \sum_{b} i(n_t, n_b) = 0, \qquad ...(15)$$

where

$$i(n_t\, n_b) = \pm 1 \text{ or } 0.$$

We apply this technique to the mixed symmetry baryon multiplet $N = 20$, subgroup of sextuplet quark model with $N = 70$, which contains the usual baryon octet (spin ½). The decomposition of the mass operator is determined by

$$20 \otimes \overline{20} = 175 \oplus 84 \oplus \overline{45} \oplus 20 \oplus 15 \oplus 15 \oplus 1,$$

Hence

$$P = 4$$

The $SU(3)_t$ and $SU(3)_b$ multiplets with their dimensions n_t and n_b are represented in Table II. The weight diagram and its projection perpendicular to the I_z axis is given in Figs. 3 and 4.

TABLE II

SU(3)$_t$				$n_{t'}, n_b$	SU(3)$_b$			
N,	Λ_b,	Σ_b,	Ξ_{bb}	8′	N,	Λ_t,	Σ_t,	Ξ_{tt}
Σ_t,	Ξ_t^{6t}	Ω_t		6	Σ_b,	Ξ_t^{6b},	Ω_{tt}	
Λ_t,	Ξ_t^{3t}			$\bar{3}$	Λ_b	Ξ_t^{3b}		
Ξ_{tt},	Ω_{tt}			3	Ξ_{bb},	Ω_t		

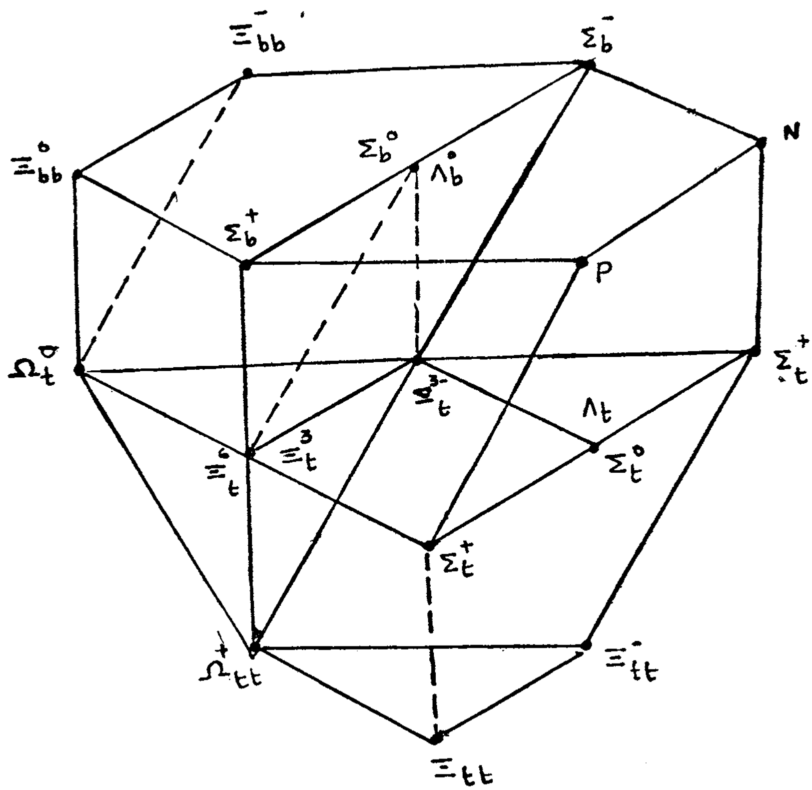

FIG. 3. The weight diagram of the mixed symmetry baryon multiplet 20, showing the Q' of the particles.

Mixing occurs between the states

$$| 6t, \tfrac{1}{2}, -1, 1 > \quad \text{and} \quad | \bar{3}t, \tfrac{1}{2}, -1, 1 > \qquad \ldots(16)$$

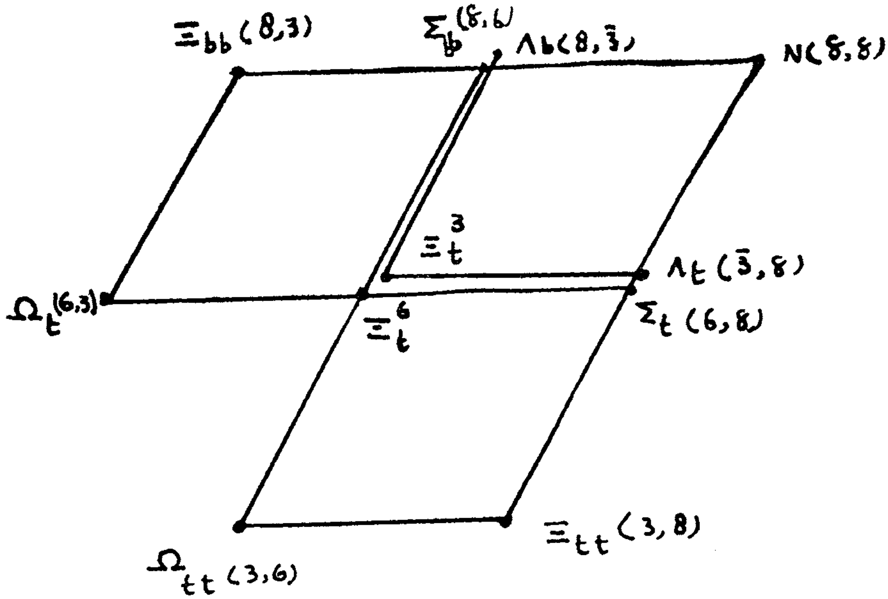

FIG. 4. The projection of the weight diagram in the plane perpendicular to the I_3 axis. Numbers in the brackets are (n^t, n_b) which label the isotopic multiplets.

and we distinguish between

$$< 6t \mid M \mid 6t = \Xi_t^{6t}, \qquad \ldots(17)$$

$$< \bar{3}t \mid M \mid \bar{3}t > = \Xi_t^{3t}, \qquad \ldots(18)$$

$$< 6t \mid M \mid \tilde{3}t > = \Xi_t^{63t}, \qquad \ldots(19)$$

with the corresponding matrix elements in the b representation denoted by suffix b.

To evaluate $< n_t \mid n_b >$ we use the quark content of the Ξ_t slates, (Lichtenberg, 1978)

$$\mid \bar{3}t > = \mid \Xi_t^{3t} = \mid utb - btu - tub + tbu + 2ubt - 2but > / \sqrt{12} \qquad \ldots(20)$$

$$\mid 6t > = - \mid \Xi_t^{6t} > = \mid utb + btu - tub - tbu > / \sqrt{4} \qquad \ldots(22)$$

The states in the b representation are obtained by interchanging top quark, and beauty quark. We then get

$$\mid 6t > = - \tfrac{1}{2} \mid 6b > + \frac{\sqrt{3}}{2} \mid \bar{3}b >, \qquad \ldots(22)$$

$$| \bar{3}t > = \frac{\sqrt{3}}{2} | 6b > + \tfrac{1}{2} | \bar{3}b >; \quad \ldots(23)$$

and hence

$$M(6, 6) = \Xi_t^{6t} - \sqrt{3}\ \Xi_t^{63t} = \Xi_t^{6b} - \sqrt{3}\ \Xi_t^{63b}, \quad \ldots(24)$$

$$M(\bar{3}, \bar{3}) = \Xi_t^{3t} + \sqrt{3}\ \Xi_t^{63t} = \Xi_t^{3b} + \sqrt{3}\ \Xi_t^{63b} \quad \ldots(25)$$

$$M(6, \bar{3}) = \Xi_t^{6t} + \frac{1}{\sqrt{3}} \Xi_t^{63t} = \Xi_t^{3b} - \frac{1}{\sqrt{3}} = \Xi_t^{63b} \quad \ldots(26)$$

$$M(\bar{3}, 6) = \Xi_t^{3t} - \frac{1}{\sqrt{3}} \Xi_t^{63t} = \Xi_t^{6b} + \frac{1}{\sqrt{3}} \Xi_t^{63b}. \quad \ldots(27)$$

Hence according to the parallelogram, we have

$$M(6, 6) + M(\bar{3}, \bar{3}) - M(6, \bar{3}) - M(\bar{3}, 6) = 0 \quad \ldots(28)$$

Since $m' = 1$, and from Table II

$m = 11,$

we have $q = 12.$ ...(29)

Hence from the relation (4), we get the following relations

$$M(8, 3) - M(8, 8) + M(6, 8) - M(6, 3) = 0$$

or $$\Xi_{bb} - N + \Sigma_t - \Omega_t = 0; \quad \ldots(30)$$

$$M(3, 8) - M(8, 8) + M(6, 8) - M(3, 6) = 0$$

or $$\Xi_{tt} - N + \Sigma_b - \Omega_{tt} = 0 \quad \ldots(31)$$

$$M(8, 6) - M(8, 8) + M(6, 8) - M(6, 6) = 0$$

or $$\Sigma_b - N + \Sigma_t - \Xi_t^{6t} + \sqrt{3}\ \Xi_t^{63t} = 0 \quad \ldots(32)$$

$$M(8, \bar{3}) - M(8, 8) + M(\bar{3}, 8) - M(\bar{3}, \bar{3}) = 0$$

$$\Lambda_b - N + \Lambda_t - \Xi_t^{3t} - \sqrt{3}\ \Xi_t^{63t} = 0 \quad \ldots(33)$$

and

$$M(8, \bar{3}) - M(8, 6) + M(6, 6) - M(6, \bar{3}) = 0$$

$$\Lambda_b - \Sigma_b + \frac{4}{\sqrt{3}} \Xi_t^{63t} \Lambda_b - \Sigma_b + \Xi_t^{6b} - \Xi_t^{3b} - \frac{4}{\sqrt{3}} \Xi_t^{63b} \quad \ldots(34)$$

Physical states which diagonalise the mass matrix are related to group states through mixing angle θ,

$$| \Xi_t = | \Xi_t^{6t} > \cos\theta + | \Xi_t^{3t} > \sin\theta \quad \ldots(35)$$

$$| \Xi_t' > = - | \Xi_t^{6t} > \sin\theta + | \Xi_t^{3t} < \cos\theta \quad ...(36)$$

Physical masses Ξ_t and Ξ_t' are expressed in terms of the matrix elements of the mass matrix in the group representation by the following relations

$$\Xi_t^{6t} = \Xi_t \cos^2\theta + \Xi_t' \sin^2\theta, \quad ...(37)$$

$$\Xi_t^{3t} = \Xi_t \sin^2\theta + \Xi_t' \cos^2\theta, \quad ...(38)$$

$$\Xi_t^{6t} = (\Xi_t - \Xi_t') \cos\theta \sin\theta \quad ...(39)$$

The terms of the mass matrix which involve Ξ_t are determined by equations (32–34). The sum of the physical masses is given by eqns. (32) and (33).

Meson Mass Splitting

We consider the meson multiplet $N = 15$ subgroup of $N = 35$ of six quark model. We write the decomposition as

$$15 \otimes 15 = 1 \oplus 15 \oplus 15 \oplus 20 \oplus 45 \oplus 45 \oplus 84.$$

of which 1, 15, 15 and 84 contain broken SU(3) singlets.

In this case, $P = 3$.

The multiplet structure in broken $SU(3)_t$ and $SU(3)_b$ is shown in Table III and the electromagnetic weight diagrams in Figs. 5 and 6. Mixing occurs between the states

$$| 8t, 0, 0, 0 > \quad \text{and} \quad | 1t, 0, 0, 0 >$$

and we distinguish

$$< \eta\, 8t | M | \eta, 8t > = \eta^{8t} \quad ...(40)$$

$$< \eta, 1t | M | \eta, 1t > = \eta^{1t} \quad ...(41)$$

$$< \eta, 8t | M | \eta, 1t > = \eta^{81t} \quad ...(42)$$

TABLE III

$SU(3)_t$	$n_{t'}, n_b$	$SU(3)_b$
B, π, η^{8t}	8	$\overline{T}$, π, η^{8t}
$\overline{T}_b$, $\overline{T}$	3	T_b, B
η^{1t}	1	η^{1b}

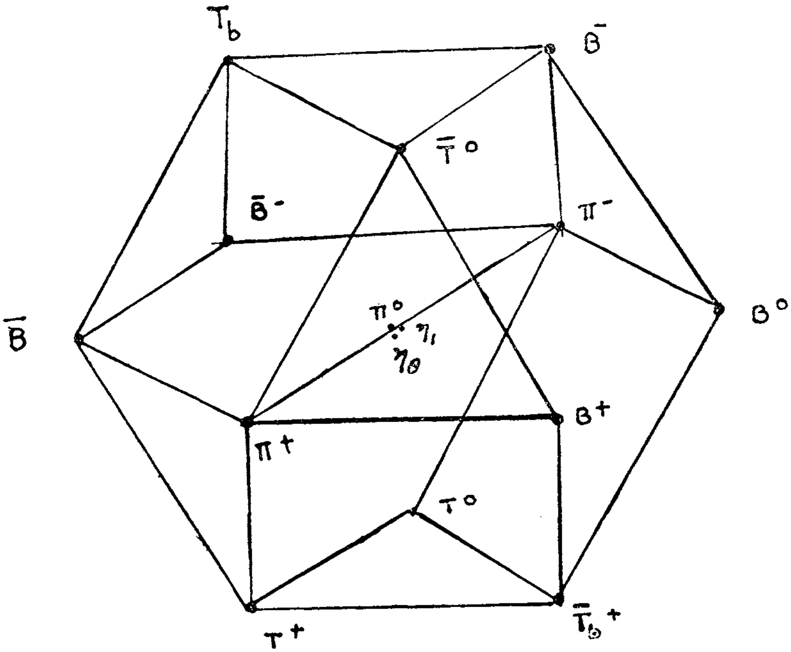

FIG. 5. The weight diagram of the meson multiplet 15.

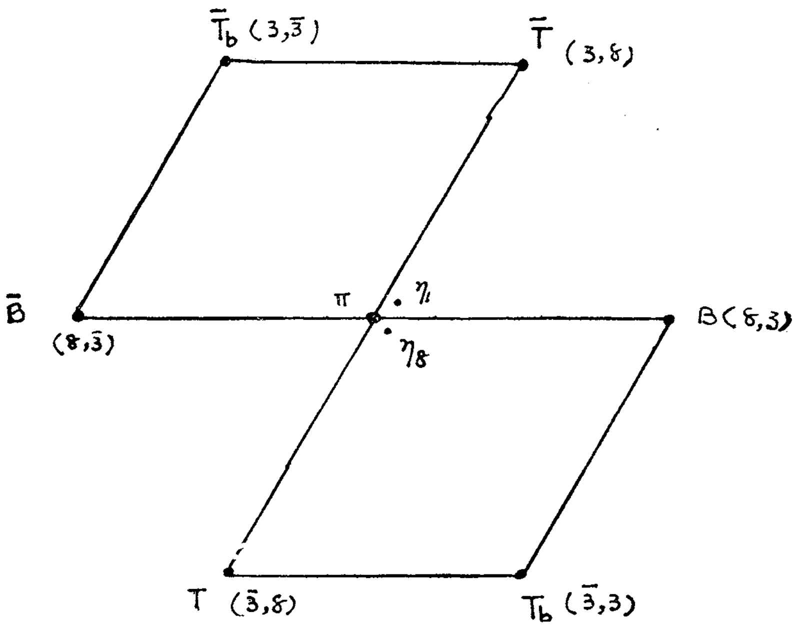

FIG. 6. The projection of the weight diagram in the plane perpendicular to the I_3 axis showing the (n_t, n_b) label of the isotopic multiplets.

and the corresponding matrix elements in the b representation. Using the quark representation it follows that

$$\left| \eta, 8t > = \frac{1}{3} \right| \eta, 8b > + \frac{2\sqrt{2}}{3} \eta, 1b > \qquad ...(43)$$

$$\left| \eta, 1t > = \frac{2\sqrt{2}}{3} \right| \eta, 8b > - \frac{1}{3} \left| \eta, 1b > \right. \qquad ...(44)$$

and hence that

$$M_\eta (8, 8) = \eta^{8t} + 2\sqrt{2}\, \eta^{81t} = \eta^{8b} + 2\sqrt{2}\, \eta^{81b} \qquad ...(45)$$

We can now write the parallelogram laws, since $q = 7$.

From Fig. 5, we write

$$M(3, 3) - M(3, 8) + M_\pi(8, 8) - M(8, 3) = 0$$

or $$T_b - T + \pi - B = 0 \qquad ...(46)$$

The other is

$$M_\pi(8, 8) - M_\eta (8, 8) = 0$$

$$\pi - \eta^{8t} - 2\sqrt{2}\, \eta^{81t} = 0 \qquad ...(47)$$

CONCLUSIONS

The discovery of more flavours (c and b quarks) and of the lepton were to some extent gratifying and to some extent disquieting. The existence of the c-quark was a confirmation of theoretical predictions by Bjorken, Glashow and collaborators, but the discovery of the b-quark opened up the possibility of an unending series of flavours with increasing masses. Presently, six flavours have been proposed in order to explain the violation of CP conservation (Ellis, 1979). Hence, in this paper, we have included the top quark to get the sum rules among the hadrons in u, d, b and t quark sector excluding the strangeness and charm.

It is observed that many of the standard formulae regarding the mass differences between particles in broken SU(4) multiplets can be obtained from the very simple restriction on the mass operator (Dug, 1978) that the mass breaking terms are a combination of a scalar in $SU(3)_t$ and a scalar in $SU(3)_b$. We used the group theoretical approach enabling us to derive the parallelogram laws which connect particles lying at its corners in the electromagnetic weight diagram.

REFERENCES

Dug, D. V. (1978) Mass formula for charmed hadron-multiplets. *Il Nuovo cim.*, **43**, 365–375.

Eichten, E., Gottfried, K., Kinoshita, T., Kogut, J., Lane, K. D., and Yan, T. M. (1975) *Phys. Rev. Lett.*, **34**, 369.

Ellis, J. (1979) *Status of Gauge Theories.* Ref. Th–2701-CERN, 1979.

Feldman, G., and Mathews, P. (1967) *High Energy Physics* (*Ed.* E. H. S. Burhop). Academic Press, Vol. I.

Gaillard, M. K., and Maiani (1979) *New Quarks and Leptons,* LAPP-TH-09 Preprint. Lectures given at the Cargese Institute.

Krammer, M., and Krasemann, H. (1979) *Quarkonia.* DESY Rep., 79/20, 1979.

Lichtenberg, D. B. (1978) Mass splitting of charmed hadrons. *Nuovo. Cim.*, **28A**, 563.

PLUTO collaboration. DESY 79/56 (1979) Search for a top "threshold in hadronic e^+e^- annihilation at energies between 22 and 31.6 GeV.

Quigg, C. (1979) *Bound States of Heavy Quarks and Antiquarks.* FERMILAB-TH-79/74-THY.

Wiik, D. H. (1979) Plenary talk at neutrino-79. International Conf. on Neutrinos, Weak Interactions and Cosmology. Bergen, Norway, 1979.