© Proc. Indian natn. Sci. Acad., 47, A, No. 4, 1981, pp. 417–427.
Printed in India.
Astrophysics

SYSTEM OF ROCHE COORDINATES FOR TIDALLY DISTORTED STELLAR MODELS

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(Received 28 November 1980)

The analysis for Kopal's system of Roche Coordinates associated with the Roche Model of a Star distorted by the tidal forces of a companion star, has been extended further to obtain the metric coefficients of the Roche coordinates when terms up to second order of smallness in tidal effects are considered.

Keywords: Roche-Coordinates; Tidal Distortions; Stellar Models; Oscillations and Stability; Binary Contact

(§1) INTRODUCTION

KOPAL (1970, 1972) introduced a system of coordinates which he called *Roche Coordinates* to study the problems of stars in close binary systems. Kopal (1970) obtained the explicit expressions for these coordinates when terms up to first order of smallness in tidal effects are considered. Mohan and Singh (1978) used these explicit expressions for the Roche coordinates to study the problems of stability and small oscillations of a tidally distorted stellar model.

In the present paper, we extend further analysis of Roche coordinates for tidally distorted stellar models to obtain explicit expressions for the metric coefficient of the Roche coordinates when terms up to second order of smallness in the tidal parameter q are considered. The analysis presented in the present paper is expected to be useful in the problems associated with stars which are tidally distorted by the gravitational effects of a companion star whose mass is too large for the effects of second order terms to be neglected. In Section 2, we introduce the system of Roche coordinates. The explicit expressions for the Roche coordinates as obtained by Kopal for the first order tidal effects are presented in Section 3. In Section 4, we extend the analysis further to obtain explicit expressions for Roche coordinates when terms up to second order of smallness in tidal effects are considered.

(§2) SYSTEM OF ROCHE COORDINATES

In this section, we present the essential features of the system of Roche coordinates as developed by Kopal. In the case of the actual stars, the greater part of their mass is concentrated very near the centre. Therefore, their structure comes much closer to the Roche model (by Roche model, we mean a model in which the whole mass of the star is supposed to be concentrated at the centre and this point mass is surrounded by an evanescent envelope in which density is assumed to vary inversely as some positive power α of distance from the centre). On the basis of some extensive numerical integrations, Chandrasekhar (1933) showed that for stars whose central density bears to the mean density a ratio of 100 or more (as is likely to be true for most of the Main Sequence stars let alone the giants), the model of a rotating configuration represents the actual form of the equipotential surfaces of a rotating star within an error of less than 1 per cent.

In the system of Roche coordinates, the equipotential surfaces of a distorted Roche model are chosen to represent the equipotential surfaces of an actual stellar model distorted by rotational and tidal forces. Choosing these equipotentials as one coordinate, the other two coordinates are chosen to form a triply orthogonal system.

Now consider the stars in a close binary system as two components of masses m and m' located at a distance R apart revolving about their common centre of gravity with angular velocity ω. Suppose a position that the above two components are referred to a rectangular system of cartesian coordinates with origin at the centre of gravity of mass m, the x-axis along the line joining the mass centres of the two components and z-axis perpendicular to the plane of the orbit of the two components (*see* Fig. 1).

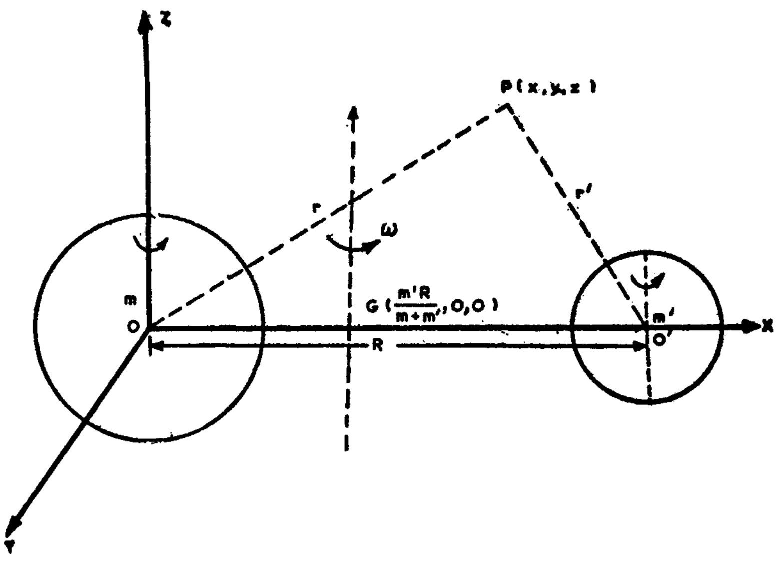

FIG. 1.

In this system, the coordinates of the centre of gravity G of the system as a whole are

$$\frac{m'R}{m+m'}, 0, 0;$$

and the total potential Ω of the combined forces of gravitation and rotation at an arbitrary point $P(x, y, z)$ is given by

$$\Omega = G\frac{m}{r} + G\frac{m'}{r'} + \frac{\omega^2}{2}\left[\left(x - \frac{m'R}{m+m'}\right)^2 + y^2\right], \qquad ...(1)$$

where $r^2 = x^2 + y^2 + z^2$

and $r'^2 = (R - x)^2 + y^2 + z^2$

represent the squares of the distances of P from the centres of gravity of the two components and G, as usual, denotes the constant of gravitation. The first term on right hand of (1) represents the potential arising from the mass of the component of mass m, the second term represents the disturbing potential of its companion of mass m' and the third the potential arising from the centrifugal force.

In close binary systems, the angular velocity ω is identical with Keplarian angular velocity so that

$$\omega^2 = G\frac{m+m'}{R^3} \qquad ...(2)$$

If we insert (2) in (1) and adopt m as our unit of mass, R as the unit of length and choose the unit of time such that $G = 1$, then the equation (1) may be expressed in terms of polar spherical coordinates

$$\left.\begin{aligned} x &= r\cos\varphi\sin\theta = r\lambda, \\ y &= r\sin\varphi\sin\theta = r\mu, \\ z &= r\cos\theta \qquad = r\nu, \end{aligned}\right\} \qquad ...(3)$$

as

$$\xi = \frac{1}{r} + q\left[\frac{1}{(1 - 2\lambda r + r^2)^{1/2}} - \lambda r\right] + \tfrac{1}{2}\,\omega^2 r^2(1 - \nu^2) \qquad ...(4)$$

where

$$\xi = \frac{R\Omega}{Gm} - \frac{m'^2}{2m(m+m')}$$

and $q = \dfrac{m'}{m}$

are non-dimensional parameters and ω^2 is now non-dimensional in units of $\dfrac{G.m}{R^3}$.

The surfaces generated by setting ξ = constant on left hand side of (4) are referred to as Roche equipotentials. These approximate the equipotential surfaces of a star in a binary system. The form of the Roche equipotentials depends entirely upon the values of ξ. If ξ is large the corresponding equipotentials will consist of two separate ovals closed around each of the two mass points (*see* Fig. 2). The right hand side of (4) can be large only if r or $r' = (1 - 2\lambda r + r^2)^{1/2}$ becomes small. Also if the right hand side of (4) is to be constant so must be nearly r and r'. Therefore, large values of ξ correspond to equipotentials which differ little but spheres. With decreasing values

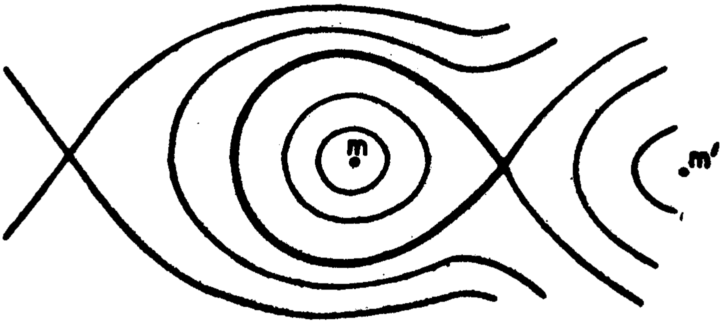

FIG. 2. Geometry (two dimensional) of the Roche surfaces (schematic). The Roche limit is marked by a heavy line.

of ξ, the ovals defined by (4) become increasingly elongated in the direction of the centre of gravity of the system until for a certain critical value of ξ (characteristic of each mass ratio) both ovals will unite in a single point on the x-axis to form a dumb-bell like configuration. These limiting values of ξ are called *Roche Limits.* Any pair of stars filling the two ovals of their Roche limit are called contact binaries. For still smaller values of ξ, the connecting part of the dumb-bell opens up and the corresponding equipotential surfaces envelope both the bodies.

In the system of Roche coordinates (ξ, η, ζ) we take the ϵ-coordinate to be an equipotential surface of the form (4) and choose the other two coordinates η and ζ in such a way as to satisfy the conditions of mutual orthogonality with respect to ξ as well as each other. The general equations which must be satisfied by any curvilinear system of coordinates are

$$\left.\begin{aligned} \xi_x\eta_x + \xi_y\eta_y + \xi_z\eta_z &= 0, \\ \xi_x\zeta_x + \xi_y\zeta_y + \xi_z\zeta_z &= 0, \\ \eta_x\zeta_x + \eta_y\zeta_y + \eta_z\zeta_z &= 0, \end{aligned}\right\} \qquad \ldots(5)$$

where subscripts x, y, z, represent partial differentiations with respect to the respective variables. The transformation

$$(dx)^2 + (dy)^2 + (dz)^2 = h_1^2(d\xi)^2 + h_2^2(d\eta)^2 + h_3^2(d\zeta)^2 \qquad \ldots(6)$$

of the metric element is specified by the metric coefficients

$$\left.\begin{aligned} h_1^{-2} &= \xi_x^2 + \xi_y^2 + \xi_z^2, \\ h_2^{-2} &= \eta_x^2 + \eta_y^2 + \eta_z^2, \\ h_3^{-2} &= \zeta_x^2 + \zeta_y^2 + \zeta_z^2 \cdot \end{aligned}\right\} \qquad \ldots(7)$$

The direction cosines of a normal to the surface ξ = constant, η = constant and ζ = constant are given by the ratios

$$\left.\begin{aligned} l_1 &= h_1\xi_x,\ m_1 = h_1\xi_y,\ \eta_1 = h_1\xi_z,\\ l_2 &= h_2\eta_x,\ m_2 = h_2\eta_y,\ \eta_2 = h_2\eta_z,\\ l_3 &= h_3\zeta_x,\ m_3 = h_3\zeta_y,\ \eta_3 = h_3\zeta_z, \end{aligned}\right\} \qquad \ldots(8)$$

respectively. The above equations hold good for any triply orthogonal curvilinear system of coordinates (Forsyth, 1912).

Kopal (1972) and his coworkers investigated the mathematical properties of this system of Roche coordinates. Their work shows that it is not possible to obtain expressions for η and ζ in closed analytic forms. Kopal investigated two particular cases of this problem in detail. The first corresponds to the Roche coordinates of a star distorted by rotational forces alone, and the second corresponds to the Roche coordinates of a non-rotating star distorted by the tidal effects of a companion star.

(§3) Explicit Expressions for the Roche Coordinates with Terms up to First Order of Smallness in Tidal Effects

In this section, we present a particular case of Roche coordinates investigated by Kopal for a non-rotating star distorted by the tidal effects of a companion star. For a Roche model distorted by tidal forces alone, the Roche equipotentials ξ are given by (4) for $\omega^2 = 0$. These are:

$$\xi = \frac{1}{r} + q\left[\frac{1}{(1-2\lambda r+r^2)^{1/2}} - \lambda r\right] = \text{constant}, \qquad \ldots(9)$$

By expanding the radical $(1 - 2\lambda r + r^2)^{1/2}$ in terms of legendre polynomials $P_j(\lambda)$, equation (9) can be expressed as

$$(\xi - q)\, r = 1 + q \sum_{j=2}^{\infty} r^{j+1} P_j(\lambda). \qquad \ldots(10)$$

Now if we take

$$r_0 = \frac{1}{\xi - q} \qquad \ldots(11)$$

as our first approximation to the distance of the equipotential surface from the centre of the star of mass m, then Kopal has shown that up to the first order terms in q,

$$r = r_0[1 + q \sum_{j=2}^{4} r_0^{j+1} P_j(\lambda_0)], \qquad \ldots(12)$$

and

$$(1 - \lambda^2) = (1 - \lambda_0^2)\left[1 + 2q \sum_{j=2}^{4} \frac{r_0^{j+1}}{j+1} \lambda_0 P_j'(\lambda_0)\right],$$

where

$$\lambda_0 = \cos \eta .$$

In the present tidal case of the triply orthogonal system of Roche coordinates, Kopal (1972) has shown that

$$\eta = \cos^{-1}\lambda - \frac{q}{(1-\lambda^2)} \sum_{j=2}^{4} \frac{r^{j+1}}{(j+1)} P'_j(\lambda) + \ldots \qquad \ldots(13)$$

and

$$\zeta = \cos^{-1}\left(\frac{\nu}{1-\lambda^2}\right) \qquad \ldots(14)$$

respectively. Here prime denotes differentiation with respect to λ. Whereas the expression for ζ is in closed analytical form, expression for η contains only terms up to first order of smallness in q.

In the present case the metric coefficients h_1, h_2, h_3 correct up to first order terms in superficial distortion q as obtained by Kopal are:

$$h_1(\xi, \eta) = r_0^2 [1 + q \sum_{j=2}^{4} (j+2)\, r_0^{j+1} P_j(\lambda_0)],$$

$$h_2(\xi, \eta) = r_0 \left[1 + q \sum_{j=2}^{4} \frac{r_0^{j+1}}{j+1} [(j+1)(j+2) P_j(\lambda_0) - P'_{j+1}(\lambda_0)]\right]$$

and

$$h_3(\xi, \eta) = r_0(1-\lambda_0^2)^{1/2} \left[1 + q \sum_{j=2}^{4} \frac{r_0^{j+1}}{j+1} P'_{j+1}(\lambda_0)\right]. \qquad \ldots(15)$$

(§4) Explicit Expressions for the Roche Coordinates with Terms up to Second Order of Smallness in Tidal Effects

In Section 3, we presented expressions for the Roche coordinates as obtained by Kopal for the tidal case when terms up to first order of smallness in q are considered. In the present section, we extend the analysis further to obtain expressions for the Roche coordinates in the tidal case when terms up to second order of smallness in q are considered.

From (10), it follows that

$$\xi = \frac{1}{r} + q\left[1 + \sum_{j=2}^{\infty} r^j P_j(\lambda)\right]. \qquad \ldots(16)$$

Partial differentiation with respect to r, λ, μ and ν gives

$$\xi_r = -\frac{1}{r^2} + q\sum_{j=2}^{\infty} j r^{j-1} P_j(\lambda),$$

$$\xi_\lambda = q\sum_{j=2}^{\infty} r^j P_j'(\lambda), \qquad \text{...(17)}$$

$$\xi_\mu = 0$$

and

$$\xi_\nu = 0.$$

The lines that are orthogonal to the equipotentials defined by (16) are given by the differential equations

$$\frac{dr}{r\xi_r} = \frac{r d\lambda}{(1-\lambda^2)\xi_\lambda} = \frac{r d\mu}{-\lambda\mu\xi_\lambda} = \frac{r d\nu}{-\lambda\nu\xi_\lambda} \qquad \text{...(18)}$$

From (18), we obtain

$$\frac{d\lambda}{dr} = \frac{1}{r^2}\,\frac{(1-\lambda^2)\xi_\lambda}{\xi_r}, \qquad \text{...(19)}$$

or

$$\frac{d\mu}{dr} = \lambda\mu q \sum_{j=2}^{\infty} r^j P_j'(\lambda)\left[1 + q\sum_{j=2}^{\infty} j r^{j+1} P_j(\lambda) + \ldots\right] \qquad \text{...(20)}$$

and

$$\frac{d\nu}{dr} = -\frac{\lambda\nu\xi\lambda}{r^2\xi_r}$$

or

$$\frac{d\nu}{dr} = \lambda\nu q \sum_{j=2}^{\infty} r^j P_j'(\lambda)\left[1 + q\sum_{j=2}^{\infty} r^{j+1} P_j(\lambda) + \ldots\right] \qquad \text{...(21)}$$

Integration of eqns. (19) to (21) by Picard's method of successive approximation yields up to second approximation.

$$\lambda_2 = \lambda_0 - (1-\lambda_0^2)\, q\sum_{j=2}^{\infty} \frac{r^{j+1}}{(j+1)} P_j'(\lambda_0) -$$

$$2\lambda_0(1-\lambda_0^2)\, q\sum_{j=2}^{\infty} \frac{r^{j+1}}{(j+1)} P_j'(\lambda_0) -$$

(*equation continued on p.* 424)

$$2\lambda_0(1-\lambda_0^2)\, q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2} P_k'(\lambda_0) P_j'(\lambda_0)}{(j+1)(j+k+2}$$

$$-(1-\lambda_0^2)\, q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}}{(j+k+2)} P_j'(\lambda_0) P_k(\lambda_0), \qquad \ldots(22)$$

$$\mu_2 = \mu_0 + \lambda_0\mu_0 \sum_{j=2}^{\infty} \frac{r^{j+1}}{j+1} P_j'(\lambda_0) +$$

$$\mu_0 q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}(2\lambda_0^2-1)}{(j+1)(j+k+2)} P_j'(\lambda_0) P_k'(\lambda_0) +$$

$$\lambda_0\mu_0 q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{j\, r^{j+k+2}}{(j+k+2)} P_j'(\lambda_0) P_k(\lambda_0), \qquad \ldots(23)$$

and

$$\nu_2 = \nu_0 + \nu_0\lambda_0 q \sum_{j=2}^{\infty} \frac{r^{j+1}}{j+1} P_j'(\lambda_0)$$

$$+ \nu_0 q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}}{(j+1)(j+k+2)} (2\lambda_0^2-1) P_j'(\lambda_0)$$

$$\times P_k'(\lambda_0) + \lambda_0\nu_0 q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{j\, r^{j+k+2}}{(j+k+2)} P_j'(\lambda_0) P_k(\lambda_0) \ldots(24)$$

Here subscript 0 denotes the initial approximation and the subscript 2 denotes second approximation to a value, squaring and adding the set of equations (22) to (24), we find

$$\lambda_2^2 + \mu_2^2 + \nu_2^2 = \lambda_0^2 + \mu_0^2 + \nu_0^2 +$$

$$q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}}{(j+1)(k+1)} [(1-\lambda_2^2)^2 + \lambda_0^2\mu_0^2 + \lambda_0^2\nu_0^2] P_j'(\lambda_0) P_k(\lambda_0) +$$

$$2q \sum_{j-2}^{\infty} \frac{r^{j+1}}{(j+1)} P_j'(\lambda_0) [-\lambda_0(1-\lambda_0^2) + \mu_0^2\lambda_0 + \lambda_0\nu_0^2] +$$

(*equation continued on p.* 425)

$$2q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}}{(j+1)(j+k+2)} (-2\lambda_0^2(1-\lambda_0^2)+\mu_0^2(2\lambda_0^2-1)+ \nu_0^2(2\lambda_0^2-1)]\, P_j'(\lambda_0)\, P_k'(\lambda_0) +$$

$$2q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{jr^{j+k+2}}{(j+k+2)} [-\lambda_0(1-\lambda_0^2)+\lambda_0^2\mu_0^2+\lambda_0\nu_0^2] \times P_j'(\lambda_0)\, P_k'(\lambda_0)$$

On simplification, we have,

$$\lambda_2^2+\mu_2^2+\nu_2^2 = 1+2q^2 \sum_{j=2}^{\infty} \sum_{k=2}^{\infty} \frac{r^{j+k+2}}{(j+1)} \frac{(1-\lambda_0^2)(j-k)}{2(k+1)(j+k+2)} \times P_j'(\lambda_0)\, P_k'(\lambda_0) \quad \ldots(25)$$

or

$$\lambda_2^2+\mu_2^2+\nu_2^2 = 1. \quad \ldots(26)$$

Second term on the right hand side of (25) vanishes due to the fact that the terms in it occur with opposite signs alternately. The expressions (22) to (24) for λ_2, μ_2 and ν_2 which contain terms up to second order in q, therefore, satisfy the condition $\lambda^2+\mu^2+\nu^2=1$, for being the direction cosines of a normal to the equipotential surface.

The expressions for the Roche coordinates ξ and ζ are again given by (9) and (14). However, the expression for the Roche coordinate η now becomes

$$\eta = \cos^{-1}\lambda - \frac{q}{(1-\lambda^2)} \sum_{j=2}^{9} \frac{r^{j+1}}{j+1} P_j'(\lambda). \quad \ldots(27)$$

The explicit expression for the metric coefficients h_1, h_2 and h_3 up to second order terms in q now become

$$h_1(\xi,\eta) = r_0^2\,[1+q\sum_{j=2}^{9}(j+2)\,r_0^{j+1}\,P_j(\lambda_0)+ q^2\sum_{j=2}^{4}\sum_{k=2}^{4} r_0^{j+k+2}\Big[(3+j(k+j+5))\,P_k(\lambda_0)\,P_j(\lambda_0) - \frac{(j+5)(1-\lambda_0^2)}{2(j+1)} P_k'(\lambda_0)\,P_j'(\lambda_0)\Big].$$

$$h_2(\xi, \eta) = r_0\Big[1 + q \sum_{j=2}^{9} \frac{r_0^{j+1}}{j+1} \left[(j+1)^2 P_j(\lambda_0) - \lambda_0 P'_j(\lambda_0)\right] +$$

$$q^2 \sum_{j=2}^{4} \sum_{k=2}^{4} \frac{r_0^{j+k+2}(j+2)}{(j+1)} \left[j(j+1) P_k(\lambda_0) P_j(\lambda_0) - \right.$$

$$\left. \lambda_0 P_k(\lambda_0) P'_j(\lambda_0)\right] +$$

$$q^2 \sum_{j=2}^{4} \sum_{k=2}^{4} r_0^{j+k+2}\left[(j+1) P_k(\lambda_0) P_j(\lambda_0) - \right.$$

$$\frac{(j+3)(1-\lambda_0^2)}{2(j+1)} P'_k(\lambda_0) P'_j(\lambda_0) -$$

$$\left.\frac{P'_k(\lambda_0) P'_j(\lambda_0)}{2(k+1)(j+1)} + \frac{jP_j(\lambda_0)}{(k+1)} (k(k+1) P_k(\lambda_0) - \lambda_0 P'_k(\lambda_0)\right]\Big].$$

$$h_3(\xi, \eta) = r_0 \sqrt{(1-\lambda_0^2)} \Big[1 + q \sum_{j=2}^{9} \frac{r_0^{j+1}}{j+1} \left[(j+1) P_j(\lambda_0) + \lambda_0 P'_j(\lambda_0)\right] +$$

$$\lambda_0 q^2 \sum_{j=2}^{4} \sum_{k=2}^{4} \frac{r_0^{j+k+2}}{j+1} \left[(j+1) P_j(\lambda_0) + \lambda_0 P'_j(\lambda_0)\right] \times$$

$$P'_k(\lambda_0) P_j(\lambda_0) + q^2 \sum_{j=2}^{4} \sum_{k=2}^{4} r_0^{j+k+2}$$

$$\left[\{j(j+1) + (k+1)(j+1)\} P'_k(\lambda_0) P_j(\lambda_0) + \right.$$

$$q^2 \sum_{j=2}^{4} \sum_{k=2}^{4} r_0^{j+k+2} \Big\{(j+1) P_k(\lambda_0) P_j(\lambda_0) -$$

$$\left(\frac{1}{2(k+1)(j+1)} + \frac{(1-\lambda_0^2)}{(j+1)}\right) \cdot P'_k(\lambda_0) P_j(\lambda_0)\Big\}\Big]. \quad \ldots(28)$$

The explicit expressions for l, m and n can also be obtained with the help of (8) which provide the respective direction cosines of a normal to the surface $\xi =$ constant, $\eta =$ constant and $\zeta =$ constant, up to second order terms of smallness in q.

Conclusions

The expressions for Roche coordinates presented in section 4 are expected to give more valuable information in case of problems associated with stars which are tidally distorted by a companion star whose mass is large enough for the terms of q^2 to be significant. This will happen in the case of stars where the mass of the companion star is more than a quarter of the mass of the primary star of the binary system such as WUMi, XTri, δLib, SX Hya for which the mass ratios are 0.48, 0.63, 0.44, 0.35 (Cf. *Close Binary Systems*, by Z. Kopal, Section VII. 4, Table 7–5, *International Astrophysics Series*, Vol. V). It may, however, be pointed out that the expression for η in (27) though different from the expression for η as presented in (13) for the first order terms in q, still contains only the first order terms in tidal parameter q. Perhaps the expression for η could also be extended to contain higher order terms in q. However, in our present study, it was observed that if we retain second order terms in q, then the system of coordinates (ξ, η, ζ) fails to satisfy the orthogonality conditions. But, of course, the effects of second order terms in tidal parameter q are incorporated through the metric coefficients h_1, h_2 and h_3 of this system of coordinates and this is indeed the outcome of our analysis.

ACKNOWLEDGEMENT

The author is thankful to Dr C. Prasad, Professor of Mathematics, Department of Mathematics, University of Roorkee, Roorkee, India, for giving valuable suggestions and constant encouragement for preparing this paper.

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