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Heat Transfer

THERMAL DIFFUSION IN COMPOSITE SOLIDS

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The study of thermal diffusion in composite solids has been carried out to find out as to how the temperature distribution evolves with time and subsequently reaches a steady state. The response of the temperature distribution to the variation of thermal conductivities and diffusivities has also been studied.

Keywords : Thermal Diffusivity; Thermal Conductivity; Composite Solids.

INTRODUCTION

THERMAL diffusion studies are very important in atmospheric earth, biological and technological sciences. Structural technology relies heavily on thermal diffusion studies for material selection for reentry vehicle shields, chemical and thermal reactor components, nuclear fuel rods and combustion devices.

This investigation deals with the study of thermal diffusion in composite solids with smooth junction such as in solar cells and nuclear fuel rods. The study is useful in composite solids constituted of materials having different conductivities and the system is thin enough to permit one dimensional analysis. Theoretical and experimental studies on thermal diffusion in composite solids have been carried out by Carslaw and Jaeger (1959), Amoz (1970), Truong and Zinsmeister (1978). Carslaw and Jaeger have analysed thermal diffusion problem using Laplace transform technique but the solution is in the form of a complicated series that can provide meaningful information only for small times or large times. The accuracy of the result depends upon the number of terms used for computation. The exact analytic solution is not available so far. Here the solution of the problem is obtained through implicit finite-difference approach which transforms the energy equations in the different regions and the accompanying boundary conditions into a tri-diagonal system of linear equations. These equations are solved for temperature at various nodal points simulating the slab at different time steps, to see the response of temperature distribution to changes in thermal conductivities and diffusivities of the constituent solids. This method has the added advantage over the analytical approach that the variation of thermal properties of the material with temperature can be also accounted for without excessive complications. The size of the time interval is increased as the solution progresses in time.

FORMULATION AND SOLUTION OF THE PROBLEM

Let the composite solid extending from $x = 0$ to 1 may consist of two infinitely extending plates A and B of thickness 0.5 each that have thermal diffusivities α_A

and α_B respectively. Let there be a continuous flux across the function and the sides $x = 0$ and $x = 1$ be maintained at temperature 0 and 1 respectively with initial temperature of the composite solid as zero.

The energy equations and the accompanying initial and boundary conditions are as follows:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_A} \frac{\partial T}{\partial t} \text{ for } 0 < x < 0.5 \quad \ldots(1)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha_B} \frac{\partial T}{\partial t} \text{ for } 0.5 < x < 1.0 \quad \ldots(2)$$

$$T(x, 0) = 0 \text{ for } 0 \leqslant x \leqslant 1 \quad \ldots(3)$$

$$T(0, t) = 0 \text{ for all values of } t \quad \ldots(4)$$

$$T(1, t) = 1 \text{ for all values of } t \quad \ldots(5)$$

The flux condition at the interface may be written as

$$K_A \frac{\partial T}{\partial x} = K_B \frac{\partial T}{\partial x} \quad \ldots(6)$$

where

T is temperature at different points of the composite solid; $\alpha = \frac{K}{\rho c}$ is the thermal diffusivity (K, and C being the thermal conductivity, density and specific heat of the material of the composite solid.

Dividing the region $x = 0$ to 1 in R equal intervals and denoting the temperature at ith nodal point and mth time step by $T_{i,m}$, the second order correct implicit finite difference version (Crank & Nicholson, 1947) of (1) and (2) works to be

$$T_{i-1,m+1} + aT_{i,m+1} + T_{i+1,m+1} = - T_{i-1,m} - T_{i+1,m} + bT_{i,m} \quad \ldots(7)$$

$$\text{for } 2 \leqslant i \leqslant \frac{R}{2} - 1 \text{ and } 1 + \frac{R}{2} \leqslant i \leqslant R - 2$$

where

$$a = -2 - \frac{2(\Delta x)^2}{\alpha \Delta t}, \; b = 2 - \frac{2(\Delta x)^2}{\alpha \Delta t}$$

with $\alpha = \alpha_A$ for $1 \leqslant i \leqslant -1 + R/2$

and $\alpha = \alpha_B$ for $1 + R/2 \leqslant i \leqslant R$

For $i = R/2$, the finite difference analog of the flux condition (6) gives

$$KT_{R/2-1,m+1} - (K+1)\, T_{R/2,m+1} + T_{R/2:m+1} = - KT_{R/2-1,m} + (K+1)\, T_{R/2,m} - T_{R/2+1,m} \quad \ldots(8)$$

where $K = K_A \mid K_B$

Equations (7) and (8) constitute a set of $R-1$ linear algebraic equations and this set is augmented by the following two equations obtained with the help of (7) using (3) and (4) respectively :

$$aT_{1,m+1} + T_{2,m+1} = bT_{1,m} - T_{2,m} \quad \ldots(9)$$

$$T_{R-2,m+1} + aT_{R-1,m+1} = -T_{R-2,m} + bT_{R-1,m} - 2 \quad \ldots(10)$$

It may be observed that the right hand sides of the equations (7), (8), (9) and (10) are known at $t = 0$ through (3) and (5). This is a tri-diagonal system of linear algebraic equations and may be solved by Thomas algorithm (Brice *et al.*, 1969). Computations have been carried out for investigating the response of the transient temperature distribution to the variation of thermal conductivities and diffusivities of the constituent solids.

The finite difference grid is taken as 0.05 and the time step is chosen as 0.0025. The time step is increased by a factor of 1.1 as the solution progresses with time which is in keeping with the physical phenomenon that the variations in temperature are small as the steady state is approached.

Results and Discussion

The evolution of temperature in the composite solid through the transient state to the equilibrium state has been depicted in Fig. 1. Thermal diffusivities α_A and α_B

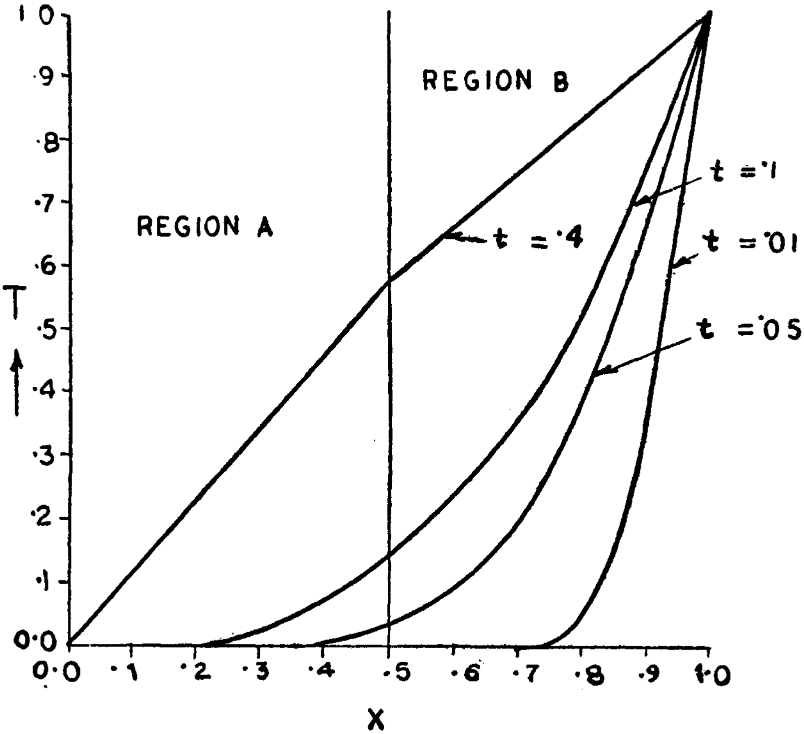

FIG. 1 Temperature history of a composite solid ($\alpha_A = 1.5$, $\alpha_B = 2.0$, $K = 0.5$)

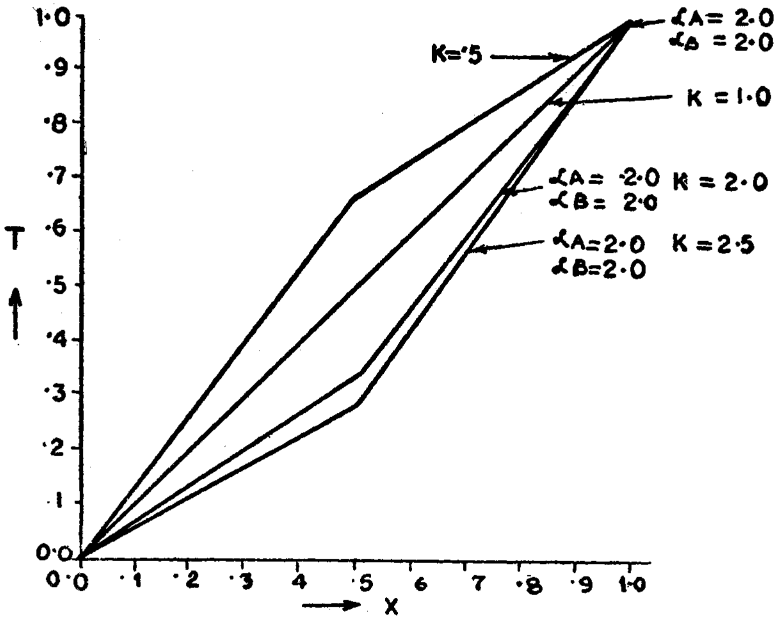

FIG. 2. Temperature response to variations in thermal conductivity ratio.

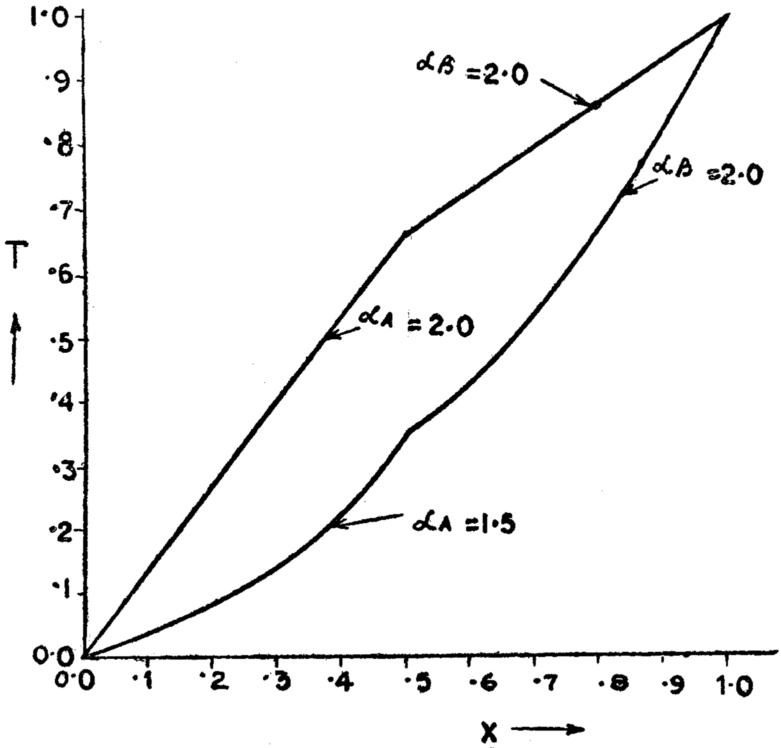

FIG. 3. Temperature response to variations in thermal diffusivity (t = 0.2, K = 0.5).

have been taken as 1.5 and 2.0 respectively and the ratio of the thermal conductivities of the solids is 0.5. Initially, the heat diffuses from $x = 1$ to the right region and the left region starts getting affected as the time progresses. The evolution of the steady state settles down to a linear profile.

Fig. 2 depicts the steady state temperature response to the variations of the conductivities of the constituent solids. When conductivity ratio becomes unity, the composite solid becomes a single slab extending from $x = 0$ to $x = 1$ and the computed result turns out to be a straight line which is as expected and this adds to our confidence in this approach.

A comparative study of the temperature distribution in a composite solid at $t = 0.2$ shows that for a given conductivity ratio the temperature distribution is still evolving for $\alpha_A = 1.5$ and $\alpha_B = 2.0$ whereas for $\alpha_A = \alpha_B = 2.0$ it has already reached an equilibrium state (Fig. 3). Thus it can interpreted that steady state is reached faster, if the diffusivity differential of the constituent solids is small.

These sample curves indicate how the temperature distribution in composite solids evolve with time from initial stages to the equilibrium state. The effects of the changes in conductivity ratio and diffusivities have also been brought out. The algorithm can, however, be used for finding the temperature response to the complete spectrum of the values of these physical parameters.

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