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Plasma Physics

GENERATION OF A SECOND HARMONIC ELECTROSTATIC PULSE BY AN EM PULSE AND THIRD HARMONIC GENERATION IN A PLASMA*

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In the present paper, we have investigated the generation of a second harmonic electrostatic pulse in a collisionless hot plasma by a Gaussian (in space and time) EM pulse. On account of ($V \times B$) force (where V is the oscillation velocity of the electron in the EM pulse and B is the magnetic field of the pulse), an electrostatic pulse is generated at twice the frequency of the incident pulse. It is seen that for an EM pulse, having uniform intensity distribution along its wavefront, the pulse width of the generated Gaussian electrostatic pulse is $(1/\sqrt{2})$ times the initial EM pulse width. The electrostatic pulse generated by a Gaussian EM pulse (in space and time) displays a more complicated dependence on time. The generated electostatic pulse again interacting with the incident EM pulse generates a third harmonic EM pulse. Further, the transient self-focusing of the incident EM pulse affects the above mentioned generation phenomena significantly.

Keywords: Pulse; Ponderomotive Force; Gaussian, Harmonic, Generation.

(§1) INTRODUCTION

WHEN a high power EM wave interacts with a plasma, it can excite an electron plasma wave at twice the frequency of the pump wave (Montgomery & Tidman, 1964; and Sodha *et al.*, 1976*b*). This electron plasma wave is excited on account of the ($V \times B$) force where V is the oscillation velocity of the electron on account of the electric field of the EM wave and B is the magnetic field of the wave. Thus, if the dispersion relation of an electron plasma wave $4\omega_0^2 = \omega_p^2 + 4k_0^2 v_{th}^2$ (where ω_0 is the frequency of the incident EM wave and k_0 is its propagation vector in the plasma) is satisfied, the resonant excitation of the plasma wave occurs. As the phase velocity of the plasma wave is equal to that of the incident EM Wave, its Landau damping is negligible and it contributes significantly to the third harmonic generation by interacting with the incident EM wave (Lee *et al.*, 1974; and Sodha *et al.*, 1976*b*). Sodha and coworkers studied the generation of this plasma wave by a Gaussian EM wave and pointed out that this wave has two components, the first

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supported by the hot plasma and the second arises from the $V \times B$ force. The first component can also suffer Landau damping depending on the value of (ω_p/ω_0).

These available theories are limited for continuous EM waves. But in laser plasma interaction experiments short duration laser pulses of pulse duration, nenosecond to picosecond are commonly used. When the characteristic time of diffusion of the carriers across the beam $\tau_d(\simeq 2r_0/v_a$, r_0 being the pulse width in space and v_a is the ion speed) in a collisionless plasma and the pulse duration are comparable, the available steady state theories are no longer valid. In such situations, the nonlinear effects in the plasma depend not only on the instantaneous intensity of the pulse but also on its past history (Akhmanov *et al.*, 1968; and Sodha *et al.*, 1974, 1976 *c*, 1979).

In the present paper, we have investigated the generation of an electrostatic pulse in a collisionless hot plasma by an EM pulse at twice the frequency of the EM pulse. The initial intensity distribution of the EM pulse is taken to be Gaussian in space and time for the sake of generality. It is seen that the intensity distribution of the generated electrostatic pulse is Gaussian in time with a pulse width $= t_0/\sqrt{2}$ (here t_0 is the pulse width of the incident EM pulse in time) when the intensity distribution along the wavefront of the EM pulse is uniform. For a Gaussian pulse in space and time the ponderomotive force becomes finite and leads to the redistribution of the electrons and ions in the background (Sodha *et al.*, 1976). But this redistribution being time-dependent on account of the Gaussian nature in time of the incident pulse and hence the intensity of the generated electrostatic pulse displays a complicated dependence in time. The change in the temperature of the plasma species affects this behaviour drastically. The generated second harmonic electrostatic pulse again interacting with the incident EM pulse generates a third harmonic EM pulse. When the initial power of the incident EM pulse is more than the critical power for self-focusing, the transient self-focusing of the EM pulse occurs and the electrostatic pulse generation and the third harmonic generation gets drastically affected.

In sec. 2, we have studied the transient self-focusing of the incident EM pulse. In sec. 3 and sec 4, the generation of a second harmonic electrostatic pulse and a third harmonic EM pulse is studied. Sec. 5 presents a brief discussion of the results.

(§2) Transient Setting of Ponderomotive Nonlinearity

Consider the propagation of a Gaussian EM pulse (in space and time) along the z-axis in a collisionless plasma. The intensity distribution at $z = 0$ is given by

$$E_0 E_0^* \,|_{z=0} = E'^2_0(t) \exp[-r^2/r_0^2]$$

$$E'^2_0(t) = E_{00}^2 \exp[-t^2/t_0^2] \qquad \ldots(1)$$

where $r^2 = x^2 + y^2$ and r_0 is the initial pulse width and t_0 is the pulse duration. On account of Gaussian intensity distribution of the pulse, the ponderomotive force becomes finite and leads to the diffusion of the electrons and ions. Siegrist (1977) has numerically studied the modification in the background electron density by the ponderomotive force in the presence of a Gaussian pulse. However, one can obtain an expression for the modified background electron density in the presence of a Gaussian pulse, as follows:

In the quasihydrodynamic two-fluid approximation, one uses the following equations (Sodha *et al.*, 1979)

$$m \frac{\partial}{\partial t} V_e = -eE_s - \frac{k_B T_e}{N_e} \nabla N_e - \frac{e^2}{4m\omega_0^2} \nabla (E_0 \cdot E_0^*) \quad \text{...(2a)}$$

and

$$M \frac{\partial}{\partial t} V_i = eE_s - \frac{k_B T_i}{N_i} \nabla N_i \quad \text{...(2b)}$$

where the indices e and i of the velocity V, density N and temperature T refer to electron and ion, respectively, m and M are the masses of the electron and ion respectively, E_s is the space charge field, k_B is the Boltzmann constant, ω_0 is the angular frequency of the EM pulse and e is the electronic charge. It must be mentioned here that in writing eqn. (2*b*), the ponderomotive force on ions is not taken into account because its magnitude is less by a factor of (m/M) in comparison to that on electrons. Owing to space charge effects, electrons and ions move almost with the same velocity, i.e., $V_e \simeq V_i = V$ in an almost electrically neutral medium $N_e = N_i = N$. Adding eqns. (2*a*) and (2*b*) and neglecting electron mass m compared with the ion mass M, we obtain

$$\frac{\partial}{\partial t} V = -\frac{k_B(T_e + T_i)}{MN} \nabla N - \frac{e^2}{4mM\omega_0^2} \nabla (E_0 \cdot E_0^*) \quad \text{...(2c)}$$

For further analysis, eqn. (2*c*) may be supplemented by the continuity equation

$$\frac{\partial}{\partial t} N + \nabla \cdot [N V] = 0 \quad \text{...(2d)}$$

Eqns. (2*c*) and (2*d*) can be combined to give

$$\frac{\partial^2}{\partial t^2}\left[\ln \left(\frac{N}{N_0} \right) \right] = \left[\frac{k_B(T_e + T_i)}{M} \right] \nabla^2 \left[\ln \frac{N}{N_e} \right] + \frac{e^2}{4mM\omega_0^2} \nabla^2 (E_0 \cdot E_0^*) \quad \text{...(2e)}$$

where N_0 is the electron density in the absence of the pulse. This differential equation is a wave equation with a velocity of propagation $v_a = [k_B(T_e + T_i)/M]^{1/2}$ of the order of the ion-acoustic velocity; Eqn. (2*a*) has also been derived by Siegrist (1977) from a slightly different approach. For stationary conditions, the time derivative on the left hand side of eqn. (2*e*) disappears and one obtains the usual expression of the modified density by the ponderomotive force and redistributed electron density; thus in steady state where $T_e = T_i = T_0$

$$N = N_0 \exp\left[-\frac{e^2 E_0 \cdot E_0^*}{8k_B T_0 m\omega_0^2}\right]$$

In non-stationary conditions, eqn. (2*e*) has been solved numerically for by Siegrist (1977). We have derived an analytical expression for N. Moreover, to keep the analysis more general we add a damping term phenomenologically in eqn. (2*e*). Taking the Fourier transform of the resulting equation with respect to space coordinates would give an equation of the form

$$\frac{d^2}{dt^2}\phi + \nu\frac{d\phi}{dt} + \omega^2\phi = F(t) \qquad \ldots(2f)$$

where ϕ is Fourier transform of $\ln\left[\frac{N}{N_0}\right]$ and $F(t)$ comes from the term in $E_0 \cdot E_0^*$. A Green's function solution of eqn. (2*f*) is

$$\phi = \frac{1}{\Omega}\int_{-\infty}^{t} \sin\left[\Omega(t-t')\right]\exp\left[-\frac{\nu}{2}(t-t')\right]F(t')\,dt' \qquad \ldots(2g)$$

where $\Omega = \frac{1}{2}[4\omega^2 - \nu^2]^{1/2}$. Inversion of the Fourier transform would give an equation for N. Eqn. (2*e*) gives for N as

$$N = N_0 \exp\left[-\frac{e^2 r_0^2 I}{16\,\pi m\omega_0^2 M V_{\text{th}}}\right]$$

where $V_{\text{th}} = V_a$ and ...(2*h*)

$$I = \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{0}^{\infty} dt_1 \sin\{V_{\text{th}}(k_1^2 + k_2^2)^{1/2} t_1\}(k_1^2 + k_2^2)^{1/2}$$

$$\exp\left[-(k_1^2 + k_2^2)\frac{r_0^2}{4} f_0^2(t-t_1)\right]$$

$$\exp\left[-i(k_1 x + k_2 y)\right]E'^2_0(t-t_1).$$

It must be mentioned here that in deriving eqn. (2*h*), the intensity distribution of the pulse in the plasma is consistent with eqn. (3).

The electric vector of the pulse in the plasma is governed by the wave equation.

$$\frac{\partial^2 E_0}{\partial x^2} + \frac{\partial^2 E_0}{\partial y^2} + \frac{\partial^2 E_0}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_0}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial}{\partial t}J \qquad \ldots(3a)$$

where J is the total current density in the plasma. Assuming the variation of E_0 in space and time as

$$E_0 = A_0(x, y, z, t)\exp\left[i\{\omega_0 t - k_0(z + S_0(x, y, z, t))\}\right] \qquad \ldots(3b)$$

where $k_0 = \frac{\omega_0}{c}\left\{1 - \frac{\omega_p^2}{\omega_0^2}\right\}^{1/2}$ and A_0 is a slowly varying real function of space and time and following Akhmanov *et al.*, (1968) Sodha *et al.* (1974, 1976*b*, 1979), the intensity distribution of the pulse in the plasma can be written as

$$A_0^2 = \frac{E'^2_0(\xi)}{f_0^2(\xi, z)} \exp\left[-r^2/r_0^2 f_0^2(z, \xi)\right]$$

$$S_0 = \frac{r^2}{2}\beta_0(z, \xi) + \phi_0(z, \xi)$$

$$\beta_0 = \frac{1}{f_0}\frac{df_0}{dz}$$

and f_0 is given by

$$\frac{d^2 f_0}{dz^2} = \frac{1}{R_{d_0}^2 f_0^3(z, \xi)} - \left[\frac{\omega_p^2}{\omega_0^2}\frac{f_0(z, \xi)}{\epsilon_0}\left\{\frac{e^2 r_0^2}{16\pi m\omega_0^2 M V_{\text{th}}}\right\} I_1\right] \cdot \exp\left[-\frac{e^2 r_0^2 I_2}{16\pi m\omega_0^2 M V_{\text{th}}}\right]$$

$$R_{d_0} = k_0 r_0^2,\ \xi = t - \frac{z}{v_g},\ v_g = C\,\epsilon_0^{1/2},\ \epsilon_0 = 1 - \frac{\omega_p^0}{\omega_0^2}$$

where

$$I_1 = \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_0^{\infty} dt_1 \sin\{V_{\text{th}}(k_1^2 + k_2^2)^{1/2} t_1\}\, k_1^2 (k_1^2 + k_2^2)^{1/2} E'^2_0(\xi t) \exp\left[-\frac{(k_1^2 + k_2^2)}{4}(r_0^2 f_0^2(\xi - t_1)\right] \quad \ldots(3c)$$

$$I_2 = \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_0^{\infty} dt_1 \sin\{V_{\text{th}}(k_1^2 + k_2^2)^{1/2} t_1\}\,(k_1^2 + k_2^2)^{1/2} \exp\left[-(k_1^2 + k_2^2)\frac{r_0^2}{4} f_0^2(\xi - t_1)\right] E'^2_0(\xi - t_1)$$

(§3) Generation of the Electrostatic Pulse

Using the fluid equations and following Sodha *et al.* (1976 *a*, *c*), the equation governing the total electron density in the plasma can be written as

$$\frac{\partial^2 N}{\partial t^2} - v_{\text{th}}^2 \nabla^2 N + 2\Gamma_e \frac{\partial N}{\partial t} - \frac{e}{m}\nabla \cdot [NE] = \nabla \cdot \left[\frac{N}{2}\nabla(V \cdot V) - V\frac{\partial N}{\partial t}\right] \quad \ldots(4)$$

where $v_{th} = [3k_B T_2/m]^{1/2}$ is the electron thermal speed, E is the sum of electric vectors of the incident EM pulse and that of the generated electrostatic pulse, V is the sum of oscillation velocities of the electron in the EM pulse and the generated electrostatic pulse and the other symbols have their usual meanings. The equation for the generated electrostatic pulse at twice the pump frequency can be obtained from eqn. (4) by substituting

$$N = N_{3e} + N_1 \exp[i\omega_0 t] + N_2 \exp[2i\omega_0 t] \text{ as}$$

$$-4\omega_0^2 N_2 - v_{th}^2 \nabla^2 N_2 + 4i\,\Gamma_e \omega_0 N_2 + \omega_p^2 \frac{N_{0e}}{N_0} N_2$$
$$+ 2(\Gamma_e + 2i\omega_0)\frac{\partial N_2}{\partial t} \simeq \nabla \cdot \left[\frac{N_{0e}}{4} \nabla V_0^2\right] - \omega_{pe}^2 \frac{N_1^2}{N_0} \quad \ldots(5')$$

Here N_1 is the density oscillation of frequency ω_0. If the pump wave frequency $\omega_0 \gg \omega_p$ that is far away from the critical density region, the density oscillation $N_1 \simeq 0$ because the pump wave is predominantly a pure EM wave and

$$\nabla \cdot E = \nabla \cdot [E_0 + E_1] \simeq \nabla \cdot E_0 \simeq -4\pi e N_1 \simeq 0.$$

On the other hand if $\omega_p \simeq \omega_0$ i.e. near the critical density region and oblique incidence the pump wave is no longer a pure EM wave and a strong electron plasma wave of frequency ω_0 is generated at the critical layer which can generate a second harmonic (Ginzburg, 1970). But in the present paper, we are considering a situation where $\omega_p \ll \omega_0$ and normal incidence, therefore, the last term on the R.H.S. of eqn. (5′) becomes unimportant in comparison to the first term. Therefore, eqn. (5′) can now be written as

$$-4\omega_0^2 N_2 - v_{th}^2 \nabla^2 N_2 + 4i\Gamma_0 N_2 + \omega_p^2 \frac{N_{0e}}{N_0} N_2$$
$$+ 2(\Gamma_e + 2i\omega_0)\frac{\partial N_2}{\partial t} \simeq \nabla \cdot \left[\frac{N_{0e}}{4} \nabla V_0^2\right] \quad \ldots(5)$$

It is obvious from the source term of eqn. (5) that one component of N_2 varies as $\exp[-2ik_0 z]$ and the second component as usual as $\exp[-ikz]$ where $k\ [\simeq (4\omega_0^2 - \omega_p^2)/v_{th}^2]^{1/2}$ is the propagation vector of the electrostatic pulse supported by the hot plasma. Hence, N_2 may be written as

$$N_2 = N'_{20}(r, z, t)\exp[-ikz] + N'_{21}(r, z, t)\exp[-2ik_0 z] \quad \ldots(6)$$

Substituting for N_2 from eqn. (6) in eqn. (5) and equating the co-efficients of $\exp[-ikz]$ and $\exp[-2ik_0 z]$, we get

$$-4\omega_0^2 N'_{20} - v_{th}^2\left[\frac{\partial^2}{\partial x^2} N'_{20} + \frac{\partial^2}{\partial y^2} N'_{20} - 2ik\frac{\partial}{\partial z} N'_{20} - k^2 N'_{20}\right]$$
$$+ 4i\Gamma_e\omega_0 N'_{20} + 2(\Gamma_e + 2i\omega_0)\frac{\partial}{\partial t} N'_{20} + \omega_p^2\left(\frac{N_{0e}}{N_0}\right) N'_{20} = 0 \ldots(7)$$

and

$$- 4\omega_0^2 N'_{21} + 4k_0^2 v_{th}^2 N'_{21} + 4i\Gamma_e\omega_0 N'_{21} + \omega_p^2 \left(\frac{N_{0e}}{N_0}\right) N'_{21}$$

$$+ \frac{e^2 N_{0e}}{4m^2\omega_0^2} \nabla^2 E_0^2 + 2(\Gamma_e + 2i\omega_0) \frac{\partial}{\partial t} N_{20} - v_{th}^2 \left[\frac{\partial^2}{\partial x^2} N'_{20} + \frac{\partial^2}{\partial y^2} N'_{20} - 2ik \frac{\partial}{\partial z} N'_{20}\right] = 0 \quad \text{...(8)}$$

Further expressing

$$N'_{20} = N_{20}(r, z, t) \exp [- iks], N'_{21} = N_{21}(r, z, t) \exp [- 2ik_0 s_0] \quad \text{...(9)}$$

and substituting for N'_{20} and N'_{21} in eqn. (7) and (8) respectively, we obtain after separating the real and imaginary parts of eqn. (7).

$$\frac{\partial}{\partial z} N_{20}^2 + \frac{1}{v'_g} \frac{\partial}{\partial t} N_{20}^2 + \frac{\partial s}{\partial x} \frac{\partial}{\partial x} N_{20}^2 + \frac{\partial s}{\partial y} \cdot \frac{\partial}{\partial y} N_{20}^2 + \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right) N_{20}^2 + \frac{4\Gamma_e\omega_0}{kv_{th}^2} N_{20}^2 = 0 \quad \text{...(10a)}$$

and

$$2\frac{\partial s}{\partial z} + \frac{2}{v'_g} \frac{\partial s}{\partial t} + \left(\frac{\partial s}{\partial x}\right)^2 + \left(\frac{\partial s}{\partial y}\right)^2 = \frac{1}{k^2} N_{10}\left[\frac{\partial^2}{\partial x^2} N_{10} + \frac{\partial^2}{\partial y^2} N_{10}\right]$$

where

$$v'_g = \frac{ikv_{th}^2}{(\Gamma_e + 2i\omega_0)} \simeq \frac{kv_{th}^2}{2\omega_0} \quad \text{when } 2\omega_0 \gg \Gamma_e \quad \text{...(10b)}$$

and from eqn. (8)

$$N_{21} \simeq - \frac{e^2 N_{0e} k_0^2 E_{00}^2 \exp(- t^2/t_0^2) \cdot \exp(- r^2/r_0^2 f_0^2)}{m^2\omega_0^2 f_0^2 \left[4\omega_0^2 - \omega_p^2 \frac{N_{0e}}{N_0} - 4k_0^2 v_{th}^2\right]} \quad \text{...(11)}$$

On transforming the variables (z, t) to $z = z$ and

$\xi' = \left(t - \frac{z}{v'_g}\right)$, eqn. (10a) and (10b) assume the following forms

$$\frac{\partial}{\partial z} N_{20}^2 + \frac{\partial s}{\partial x} \cdot \frac{\partial}{\partial x} N_{20}^2 + \frac{\partial s}{\partial y} \cdot \frac{\partial}{\partial y} N_{20}^2 + \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} \right) N_{20}^2 + \frac{2\Gamma_e \omega_0 N_{20}^2}{k v_{th}^2} = 0 \qquad \ldots(11a)$$

and

$$2 \frac{\partial s}{\partial z} + \left(\frac{\partial s}{\partial x} \right)^2 + \left(\frac{\partial s}{\partial y} \right)^2 = \frac{1}{k^2 N_{20}} \left[\frac{\partial^2}{\partial x^2} N_{20} + \frac{\partial^2}{\partial y^2} N_{20} \right] + \frac{\omega_p^2}{k^2 v_{th}^2} \frac{(N_{0e} - N_0)}{N_0} \qquad \ldots(11b)$$

Following Akhmanov *et al.* (1968) and Sodha *et al.* (1976*a*), the solution of eqn. (11*a*) and (11*b*) can be written as

$$N_{20}^2 = \frac{B^2}{f^2(z, \xi')} \exp\left[- r^2/a_0^2 f^2(z, \xi)\right] \exp\left[- 2k_i z\right]$$

$$S = \frac{r^2}{2} \beta(z, \xi') + \phi(z, \xi')$$

$$\beta = \frac{1}{f(z, \xi')} \frac{df}{dz}(z, \xi') \qquad \ldots(12)$$

where $k_i = 2\Gamma_e \omega_0 / k v_{th}^2$ and f is the dimensionless parameter governed by

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2 a_0^4 f^3} - \frac{\omega_p^2 f I_1 \exp\left[- r_0^2 e^2 I_2 / 16\pi M V_{th} m \omega_0^2 \right]}{k^2 v_{th}^2 r_0^2 \left[16\pi M V_{th} m \omega_0^2 / e^2 r_0^2\right]} \qquad \ldots(13)$$

The initial conditions of f are $f = 1$ and $\frac{df}{dz} = 0$ (plane wavefront) at $z = 0$, B' and a_0 are constants to be determined by the boundary condition that the amplitude of the generated electrostatic field at $z = 0$ is zero. Thus

$$B' = \frac{N_{0e}(z = 0)\, e^2 k_0^2 E_{00}^2 \exp\left[- t^2/t_0^2 \right]}{m^2 \omega_0^2 \left[4\omega_0^2 - \omega_p^2 \frac{N_{0e}}{N_0} (z = 0) - 4k_0^2 v_{th}^2 \right]} \qquad \ldots(14)$$

and

$$r_0 = \sqrt{2}\, a_0$$

By using Poisson's equation

$$\nabla \cdot E_2 = - 4\pi e N_2 \qquad \ldots(15)$$

one can obtain the electric vector (E_2) of the generated electrostatic pulse at frequency $2\omega_0$ as

$$E_2 = -iE_{00}\left[\frac{N_{0e}}{N_0}(z=0)\right]\left[\frac{k_0^2 V_{00}\omega_p^2}{\omega_0}\right]\exp[-t^2/t_0^2]$$
$$\times [G_1 \exp(-r^2/r_0^2 f^2)\exp(-ik(z+s))\exp(-k_i z)$$
$$- G_2 \exp(-r^2/r_0^2 f_0^2)\exp(-2ik_0(z+s_0))] \quad \text{...(16a)}$$

where

$$V_{00} = eE_{00}/m\omega_0,\ G_1 = \left[kf \left\{ 4\omega_0^2 - \omega_p^2 \frac{N_{0e}}{N_0}(z=0) - 4k_0^2 v_{th}^2 \right\}\right]^{-1}$$

and

$$G_2 = [N_{0e}(z)/N_{0e}(z=0)]/2f_0^2 k_0 \left\{4\omega_0^2 - \omega_p^2\frac{N_{0e}}{N_0} - 4k_0^2 v_{th}^2\right\} \quad \text{...(16b)}$$

Eqn. (16) is an expression for the electric vector of the electrostatic pulse generated at twice the frequency of the incident Gaussian EM pulse in a hot collisionless plasma. The first term in the bracket is on account of the electrostatic pulse supported by the hot plasma and the second term arises on account of the ($V \times B$) force. For an initially Gaussian pulse in time but having uniform intensity distribution ($r_0 \to \infty, f = f_0 = 1, s_0 = 0, N_{0e}(z) = N_{0e}(z=0) = N_0$) the electric vector of the generated electrostatic pulse can be written as

$$E_2 = -\frac{iE_{00}\exp(-t^2/t_0)}{[4\omega_0^2 - \omega_p^2 - 4k_0^2 v_{th}^2]}\left[\frac{k_0^2 V_{00}\omega_p^2}{\omega_0}\right]$$
$$\times\left[\frac{\exp(-ikz - k_i z)}{k} - \frac{\exp(-2ik_0 z)}{2k_0}\right] \quad \text{...(17)}$$

It is obvious from eqn. (17) that the generated electrostatic pulse is also Gaussian in time with the pulse width in time ($= t_0/\sqrt{2}$). The intensity of the generated pulse is governed by the thermal effects in the plasma. To have more insight in the characteristics of the generated electrostatic pulse, we consider its dispersion relation

$$4\omega_0^2 = \omega_p^2 + k^2 v_{th}^2$$

The phase velocity is $v_{th}\left[1 - \frac{\omega_p^2}{4\omega_0^2}\right]^{1/2}$. Hence depending on the value of ($\omega_p/2\omega_0$), the phase velocity can be comparable to the thermal velocity of the electron and the pulse gets severely Landau damped. On the other hand, the phase velocity of the electrostatic pulse arising on account of the $(V \times B)$ force (viz. second term of eqn. (16), is the same as that of the incident EM pulse and hence suffers negligible Landau damping. This electrostatic pulse can, therefore contribute, significantly to the third harmonic generation of the incident EM pulse in the subcritical density region ($\omega_p \ll \omega_0$) because in this region $k\lambda_d \gg 1$ for the electrostatic pulse supported by hot plasma, which is therefore Landau damped. Under these conditions, the intensity of the generated second harmonic electrostatic pulse is given by

$$E_2 E_2^* = \frac{E_{00}^2}{4}\left[\frac{k_0 V_{00}\omega_p^2}{\omega_0}\right]^2 \cdot \frac{\exp[-2t^2/t_0^2]}{[4\omega_0^2 - \omega_p^2 - 4k_0^2 v_{th}^2]^2} \qquad \ldots(18)$$

When the incident EM pulse is having Gaussian intensity distribution in a plane transverse to the direction of propagation, the local electron density as modified by the ponderomotive force gets changed with time on account of the Gaussian nature of the pulse in time and hence the intensity variation of the generated electrostatic pulse with time is even more complicated as shown in Fig. 1.

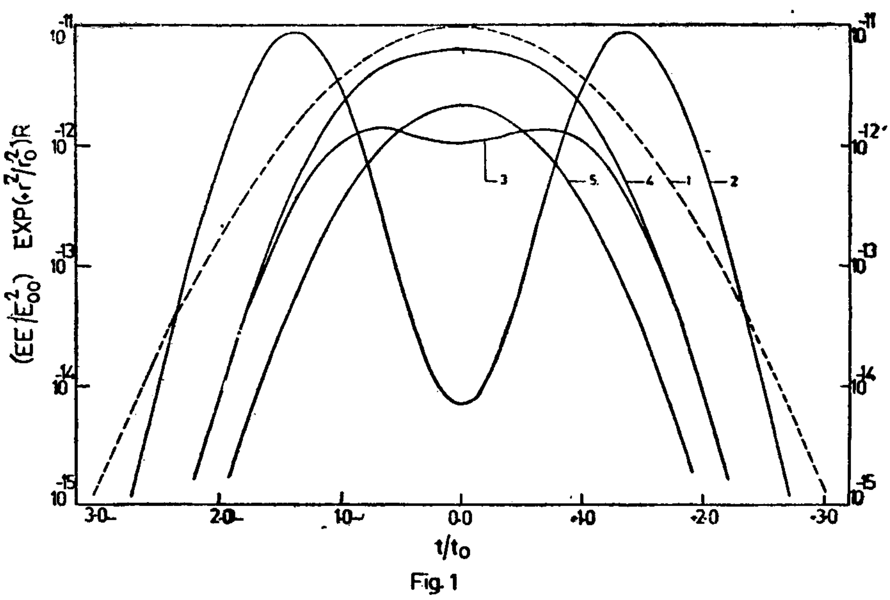

FIG. 1. Variation of the normalized electrostatic pulse intensity of the incident EM pulse) with time on the semilog graph at $z = 0$ for different v_{th}/c. The dotted curve (No. 1) is the incident EM pulse normalized by its peak intensity. The solid curves No. 2, 3, 4 and 5 are for electrostatic pulse when $\frac{v_{th}}{c}$ = .01, .02, .03 and .08 respectively. The value of the constant R is = 10^{11}, 10^{0}, 10^{2}, 10^{2} and 10^{3} for curves Nos. 1, 2, 3, 4 and 5 respectively.

(§4) Third Harmonic Generation

The wave equation governing the total electric field inside the plasma is

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial}{\partial t} J \qquad \ldots(19)$$

where J is the total current density. In writing eqn. (19), we have neglected the term $\nabla(\nabla \cdot E)$ which is justified as long as $\frac{\omega_p^2}{\omega^2}\frac{1}{\epsilon}\ln \epsilon \ll 1$, where ϵ is the effective dielectric constant of the plasma. Expressing E as

$$E = A(r, z, t) \exp[\, i\omega_0 t\,] + A_3(r, z, t) \exp[3i\omega_0 t\,] \qquad \ldots(20)$$

one can obtain the current density in the plasma at the fundamental frequency (J_0) and third harmonic $J_3(3\omega_0)$ as

$$J(\omega_0) \simeq \frac{N_{0e}e^2}{mi\omega_0}\left[A + \frac{i}{3\omega_0}\frac{\partial A}{\partial t}\right] \exp\,(i\omega_0 t) \qquad \ldots(21a)$$

$$J(3\omega_0) \simeq \frac{N_{0e}e^2}{3mi\omega_0}\left[A_3 + \frac{i}{3\omega_0}\frac{\partial A_3}{\partial t} + \frac{3}{2}\frac{N_2}{N_{0e}} A\right] \exp\,[3i\omega_0 t] \qquad \ldots(21b)$$

Substituting the expression for current density from eqn. (21a) in eqn. (19) and collecting the terms of exp ($i\omega_0 t$) and exp ($3i\omega_0 t$) on both sides, we obtain

$$\nabla^2 A + \frac{\omega_0^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega_0^2}\frac{N_{0e}}{N_0}\right] A - \frac{2i\omega_0}{c^2}\frac{\partial A}{\partial t} = 0 \qquad \ldots(22)$$

and

$$\nabla^2 A_3 + \frac{9\omega_0^2}{c^2}\left[1 - \frac{\omega_p^2}{9\omega_0^2}\frac{N_{0e}}{N_0}\right] A_3 - \frac{6\,i\omega_0}{c^2}\frac{\partial A_3}{\partial t} \simeq \frac{3}{2}\frac{\omega_p^2}{c^2}\frac{N_2}{N_0} A \qquad \ldots(23)$$

The solution for the fundamental is given by eqn. (3) and that of the third harmonic can be obtained as follows. Expressing A_3 as

$$A_3 = A'_{30}(r, z, t) \exp[-\, ik_3 z] + A'_{31}(r, z, t) \exp[-\, 3ik_0 z] \qquad \ldots(24)$$

where A'_{30} and A_{31} are the complex functions of space and time and

$$k_3 = \frac{3\omega_0}{c}\, \epsilon_0^{1/2}(4\omega_0),\ \epsilon_0(\omega_0) = 1 - \frac{\omega_{po}^2}{9\omega_0^2} \qquad \ldots(25)$$

the equations governing A'_{30} and A'_{31} are

$$\frac{\partial^2}{\partial x^2} A'_{30} + \frac{\partial^2}{\partial y^2} A'_{30} - 2ik_3 \frac{\partial}{\partial z} A'_{30} + \frac{\omega_p^2}{c^2} \frac{[N_0 - N_{0e}]}{N_0} A'_{30} - \frac{6i\omega_0}{c^2} \frac{\partial}{\partial t} A'_{30} \simeq 0 \quad \ldots(26)$$

and

$$\frac{\partial^2}{\partial x^2} A'_{31} + \frac{\partial^2}{\partial y^2} A'_{31} - 6ik_0 \frac{\partial}{\partial z} A'_{31} - 9k_0^2 A'_{31} + 9\frac{\omega_0^2}{c^2}\left[1 - \frac{\omega_p^2}{9\omega_0^2} \frac{N_{0e}}{N_0}\right] A'_{31} - \frac{6i\omega_0}{c^2} \frac{\partial}{\partial t} A'_{31} \simeq \frac{3}{2} \frac{\omega_p^2}{c^2} \frac{N_{21}}{N_0} A_0 \exp[-2ik_0 s_0] \quad \ldots(27)$$

It must be mentioned here that in writing eqn. (23) we have taken only that component of N_2 which arises on account of $(V \times B)$ force viz., the source term of eqn. (5) because it is not Landau damped. Further writing (Akhmanov *et al.*, 1968; and Sodha *et al.*, 1974, 1976*b*, 1979).

$$A'_{30} = A_{30}(r, z, t) \exp[-ik_3 s_3(r, z, t)]$$

and

$$A'_{31} = A_{31}(r, z, t) \exp[-3ik_0 s_0(r, z, t)] \quad \ldots(28)$$

We obtain from eqn. (25) after separating the real and imaginary parts as

$$\frac{\partial}{\partial z} A_{30}^2 + \frac{1}{v'_g} \frac{\partial}{\partial t} A_{30}^2 + \frac{\partial s_3}{\partial x} \cdot \frac{\partial}{\partial x} A_{30}^2 + \frac{\partial s_3}{\partial y} \cdot \frac{\partial}{\partial y} A_{30}^2 + \left(\frac{\partial^2 s_3}{\partial x^2} + \frac{\partial^2 s_3}{\partial y^2}\right) A_{30}^2 = 0 \quad \ldots(29a)$$

and

$$2\frac{\partial s_3}{\partial z} + \frac{2}{v'_g} \frac{\partial s_3}{\partial t} + \left(\frac{\partial s_3}{\partial x}\right)^2 + \left(\frac{\partial s_3}{\partial y}\right)^2 = \frac{1}{k_3^2 A_{30}}\left[\frac{\partial^2}{\partial x^2} A_{30} + \frac{\partial^2}{\partial y^2} A_{30}\right] + \frac{\omega_p^2}{9\omega_0^2} \frac{[N_0 - N_{0e}]}{N_0 \epsilon_0(3\omega_0)} \quad \ldots(29b)$$

where

$$v''_g = c\, \epsilon_0^{1/2}(3\omega_0)$$

Eqn. (27) together with eqn. (28) gives for A_{31}

$$A_{31} \simeq \frac{3}{2} \frac{\omega_p^2}{c^2} \frac{N_{21}}{N_0} \frac{E_{00} \exp(-t^2/2\,t_0^2) \exp(-r^2/2\,r_0^2 f_0^2)}{f_0 \left[9 \frac{\omega_0^2}{c^2} \left(1 - \frac{\omega_p^2}{9\omega_0^2} \frac{N_{0e}}{N_0} \right) - 9k_0^2 \right]} \quad \ldots(30)$$

on transforming the variables (z, t) to $z = z$ and $\xi'' = t - \frac{z}{v''_g}$, eqns. (29*a*) and (29*b*) assume the following forms

$$\frac{\partial}{\partial z} A_{30}^2 + \frac{\partial s_3}{\partial x} \cdot \frac{\partial}{\partial x} A_{30}^2 + \frac{\partial s_3}{\partial y} \cdot \frac{\partial}{\partial y} A_{30}^2 + \left(\frac{\partial^2 s_3}{\partial x^2} + \frac{\partial^2 s_3}{\partial y^2} \right) A_{30}^2 = 0 \quad \ldots(31a)$$

and

$$2 \frac{\partial s_3}{\partial z} + \left(\frac{\partial s_3}{\partial x} \right)^2 + \left(\frac{\partial s_3}{\partial y} \right)^2 = \frac{1}{k_3^2} A_{30} \left[\frac{\partial^2}{\partial x^2} A_{30} + \frac{\partial^2}{\partial y^2} A_{30} \right] + \frac{\omega_p^2}{9\omega_0^2} \frac{[N_0 - N_{0e}]}{N_0 \epsilon_0(3\omega_0)} \quad \ldots(31b)$$

Following Akhmanov *et al.* (1968) and Sodha *et al.* (1976*b*) the solution of eqns. (31*a* and *b*) can be written as

$$\left.\begin{aligned} A_{30}^2 &= \frac{B''^2}{f_3^2(z, \xi'')} \exp\left[-r^2/b_0^2 f_3^2 (z, \xi'') \right] \\ s_3 &= \frac{r^2}{2} \beta_3(z, \xi'') + \phi_3(z, \xi'') \\ \beta_3 &= \frac{1}{f_3(z, \xi'')} \frac{d}{dz} f_3(z, \xi'') \end{aligned}\right\} \quad \ldots(32)$$

where $f_3(z, \xi'')$ is the dimensionless parameter governed by

$$\frac{d^2}{dz^2} f_3 = \frac{1}{k_3^2 b_0^4 f_3^3} - \frac{\omega_p^2 f_3(z, \xi'')\, I_1 \exp\left[-r_0^2\, e^2 I_2 / 16\pi M V_{\text{th}} m \omega_0^2 \right]}{9\omega_0^2 \epsilon_0(3\omega_0) \left[16\pi M V_{\text{th}}\, m\omega_0^2 / e^2 r_0^2 \right]} \quad \ldots(33)$$

The initial conditions on f_3 are $f_3 = 1$ and $\frac{df_3}{dz} = 0$ (plane wavefront) at $z = 0$; B'' and b_0 are constants to be determined by the boundary condition that the third harmonic is zero at $z = 0$. Thus

$$B'' = -\frac{3}{2} \frac{\omega_p^2}{c^2} \frac{N_{21}}{N_0} \frac{(z = 0, r = 0)\, E_{00} \exp(-t^2/2\, t_0^2}{\left[\left\{ \frac{9\omega_0^2}{c^2} \left(1 - \frac{\omega_p^2}{9\omega_0^2} \frac{N_0}{N_0} (z = 0) \right\} - 9k_0^2 \right]} \quad \ldots(34)$$

and

$$b_0 = r_0/\sqrt{3}$$

Therefore, the total electric vector of the third harmonic pulse (EM) is

$$E_3 = \frac{3}{2}\ \omega_p^2 \frac{V_{00}^2}{c^2}\ E_{00}\ \frac{N_{0e}}{N_0}\,(z = 0)\, k_0^2 \exp\left[-\frac{3}{2}\, t^2/t_0^2\right].$$

$$\left[H_1 \exp\left\{-\frac{3r^2}{2r_0^2 f_3^2}\right\} \exp\left(-ik_3(z + s_3)\right)\right.$$

$$\left. - H_2 \exp\left\{-\frac{3}{2}\frac{r^2}{r_0^2 f_0^2}\right\} \exp\left(-3ik_0(z + s_0)\right)\right]$$

where

$$H_1 = \left[f_3 \left\{ 4\omega_0^2 - \omega_p^2 \frac{N_{0e}}{N_0}\,(z = 0) - k_0^2 v_{th}^2 \right\}\right.$$

$$\left. \times \left\{ \frac{9\omega_0^2}{c^2}\left(1 - \frac{\omega_p^2}{9\omega_0^2}\,\frac{N_{0e}}{N_0}\,(z = 0)\right) - 9k_0^2 \right\}\right]^{-1}$$

$$H_2 = \frac{[N_{0e}(z)/N_{0e}(z = 0)]}{f_0^3\left[4\omega_0^2 - \omega_p^2 \frac{N_{0e}}{N_0}\,(z = 0) - k_0^2 v_{th}^2\right]\left[\frac{9\omega_0^2}{c^2}\left(1 - \frac{\omega_p^2}{9\omega_0^2}\frac{N_{0e}}{N_0}(z = 0)\right) - 9k_0^2\right]} \quad \ldots(35)$$

(§5) Numerical Results and Discussions

The following set of parameters has been used for calculations

$$\frac{\omega_p}{\omega_0} = 0.1,\ \frac{r_0\omega_p}{c} = 8.0,\ \frac{3}{4}\,\alpha\,\frac{m}{M}\,E_{60}^2 = 0.4$$

$$\omega_0 = 1.778 \times 10^{15}\ \text{rad sec}^{-1},\ \frac{v_{th}}{c} = 0.04\ \text{and}\ \frac{2r_0}{v_a t_0} \approx 0.01$$

and the results are depicted in the form of graphs in Figs. 1 — 4.

Fig. 1 illustrates the variation of the intensity of the generated electrostatic pulse at $z = 0$ against time in a collisionless hot plasma for different temperature of the plasma. The generated electrostatic pulse has not exactly a Gaussian nature in time as in the case of a pulse having uniform intensity distribution along its wavefront (eqn. 17). For an EM pulse having uniform intensity distribution along its wavefront the change in the temperature of the plasma changes the magnitude of the intensity of the generated pulse and not its Gaussian nature in time while for an EM pulse having Gaussian intensity distribution in a plane, transverse to the direction of pulse propagation the change in the temperature of the plasma species not only changes the pulse width of the generated electrostatic pulse but also the intensity shape with time. This can be understood as follows:

In the case of an EM pulse having uniform intensity distribution along its wavefront, the electric vector of the generated electrostatic pulse at $2\omega_0$ frequency is proportional to the intensity of the incident pulse on account of $(V \times B)$ force and hence the nature is Gaussian in time with pulse width $(= t_0/\sqrt{2})$ but in the case of an initially Gaussian pulse in space and time, the electric vector of the generated electrostatic pulse also depends upon the modified background electron density which also changes with time and hence the intensity of the generated electrostatic pulse exhibits a complicated dependence on time. Further the change in the temperature of the plasma species affects the background electron density, the effect on the intensity of the generated electrostatic pulse is quite significant. The symmetry in time is on account of the fact that the pulse width is much larger than the diffusion time.

Fig. 2 illustrates the variation of the dimensionless pulse width parameter of incident EM pulse and that of the generated third harmonic EM pulse with the

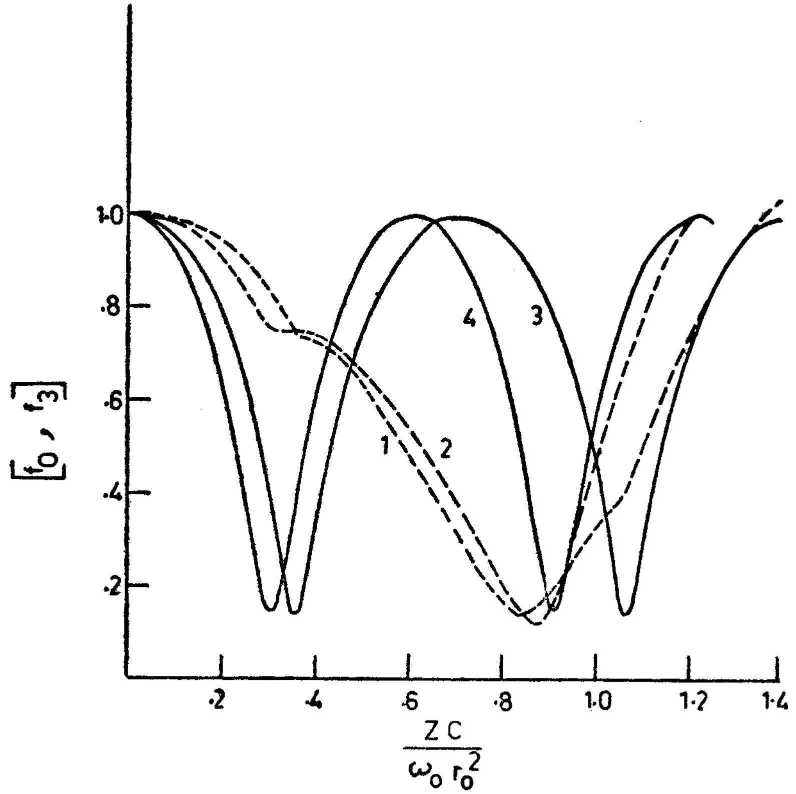

FIG. 2. Variation of dimensionless pulse width parameters f_0 (curves Nos. 1 and 3) or f_3 (curves Nos. 2 and 4) with dimensionless distance of propagation $zc/\omega_0 r_0^2$ for different times. Curves 1 and 2 occur when $\frac{t}{t_0} = .20$ and curves 3 and 4 when $\frac{t}{t_0} = .40$.

distance of propagation at different times. As time elapses, the self-focusing of the incident pulse is slower because the intensity at larger times is lesser. The focusing of the generated third harmonic pulse which depends upon the effective intensity of the incident EM pulse gets accordingly modified as shown in Fig. 2.

Figs. 3 and 4 illustrate the effect of the transient self-focusing of the incident EM pulse on the electrostatic power associated with the generated electrostatic pulse and the electromagnetic power of the third harmonic EM pulse respectively. Because of the self-focusing of the incident EM pulse, the electrostatic power which depends upon the intensity of the incident EM pulse and the modified background electron

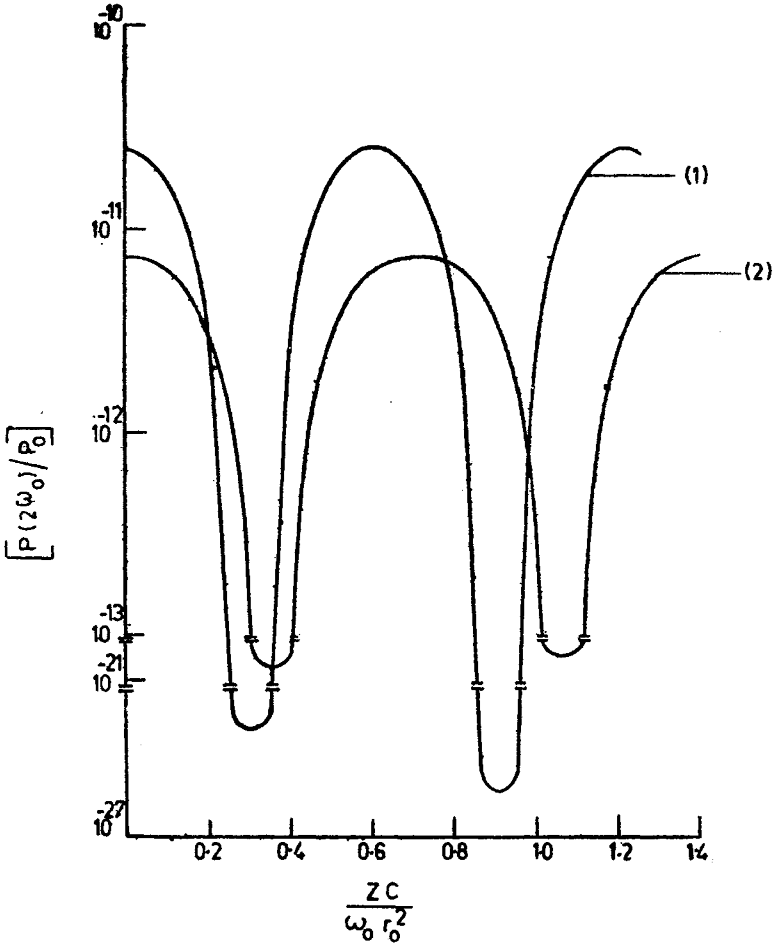

FIG. 3. Variation of the generated electrostatic pulse power normalized by the peak power of the incident EM pulse with the distance of propagation $zc/\omega_0 r_0^2$ when $\frac{t}{t_0} = .2$ (curve No. 1) and $\frac{t}{t_0} = .4$ (curve No. 2).

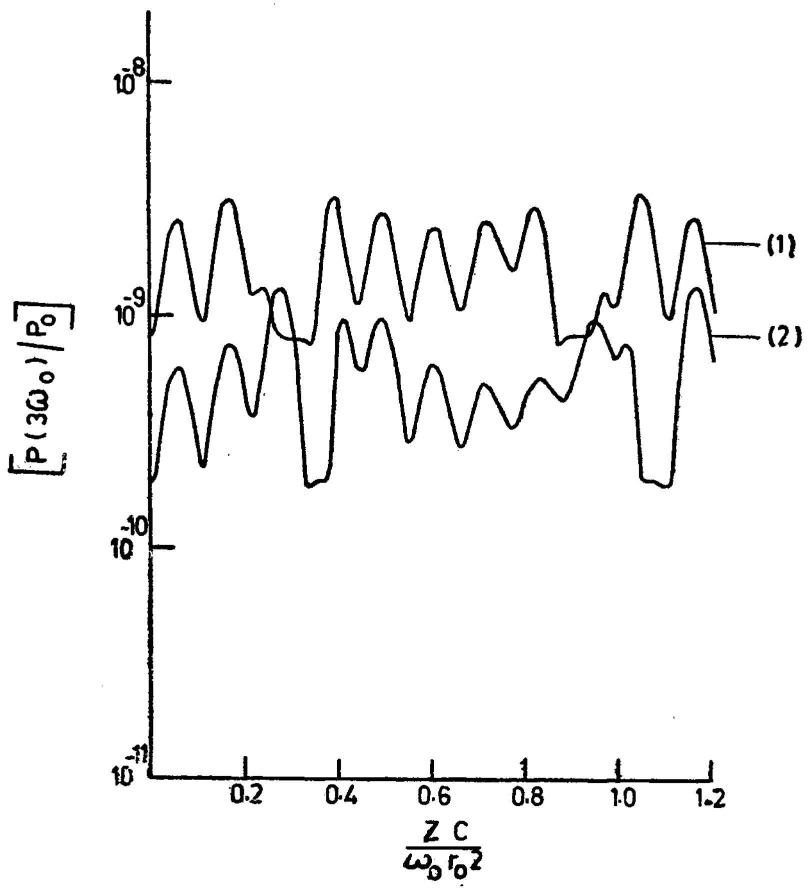

FIG. 4. Variation of the third harmonic EM pulse power normalized by the peak power of the incident EM pulse with distance of propagation $zc/\omega_0 r_0^2$ when $\frac{t}{t_0} = .2$ (curve No. 1) and $\frac{t}{t_0} = .4$ (curve No. 2).

density displays a complicated variation with the distance of propagation. This leads to a more complicated dependence with distance of propagation of the generated third harmonic power which depends upon the effective intensity of the electrostatic pulse. These effects are further modified if one abserves at different times because the intensity gets changed.

We conclude from the present analysis that the electrostatic power at twice the incident pulse frequency and the electromagnetic power at third harmonic frequency can be of significant magnitude by the interaction of an EM pulse with a plasma. The change in the temperature of the plasma affects the nature of the generated pulse drastically.

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